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1) Let the order of column matrix x be $n \times 1$

$\therefore$ Order of x^T be $1 \times n$

$\therefore$ Order of matrix $x x^T$ be $n \times n$

~~and the matrix will be like this~~

If the matrix $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$

$$x x^T = \begin{bmatrix} x_1^2 & x_1 x_2 & x_1 x_3 & \cdots & x_1 x_n \\ x_1 x_2 & x_2^2 & x_2 x_3 & \cdots & x_2 x_n \\ x_3 x_1 & x_3 x_2 & x_3^2 & \cdots & x_3 x_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n x_1 & x_n x_2 & x_n x_3 & \cdots & x_n^2 \end{bmatrix}$$

which can be rewritten as

$$xx^T = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \\ x_1 & x_2 & x_3 & \cdots & x_n \\ \vdots & \vdots & \vdots & & \vdots \\ x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$\therefore$ The rank of xx^T is 1

$\longrightarrow$ Order of $x^T x$ will be 1×1

$\therefore$ rank of $x^T x = 1$

(3)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$S_1 = |0| + |0| + |0| + |0|$$

$$= 0$$

$$S_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -1 + 0 - 1 + 0 + 0 - 1$$

$$= -3$$

$$S_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = (2)(-1)\{0\} = 0$$

$$S_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$S_4 = (-1)\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= (-1)\left[(1)[-1] + (-1)[0] \right]$$

$$= (-1)(-1)$$

$$= 1$$

$$\therefore \det(\lambda - IA) = \lambda^4 - 0 - 3\lambda^2 + 0 + 1$$

$$= \lambda^4 - 3\lambda^2 + 1$$

a).

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$(a_{11} - \lambda)(a_{22} - \lambda) \quad - a_{21} a_{12} = 0$$

$$\lambda^2 - \lambda [a_{11} + a_{22}] - a_{21} a_{12} + a_{11} a_{22} = 0$$

for $\lambda_1 = \lambda_2$

$$(a_{11} + a_{22})^2 - (4)\{(a_{22} + a_{11}) - a_{21} a_{12}\} = 0$$

$$a_{11}^2 - (2)(a_{11})(a_{22}) + a_{22}^2 + (4)(a_{12})(a_{21}) = 0$$

$$(a_{11} - a_{22})^2 + \cancel{4 a b = 0}\ 4 (a_{12})(a_{21}) = 0$$

$$(4)(a_{12})(a_{21}) = -(a_{11} - a_{22})^2$$

$$(a_{12})(a_{21}) \leq 0$$

$$|A| = a_{11}a_{22} - a_{21}a_{12} \geq 0.$$

(2). For real and distant eigen value, the Condition is

~~then the~~ $D > 0$

$$(a_{11} - a_{22})^2 + (4)(a_{21})(a_{12}) > 0 \quad \text{—— (1)}$$

So if a_{12}, a_{21} is of Same Sign the (1) will be always true, So ~~theorem~~ therefore

→ If a_{12}, a_{21} are of Same Sign then A has real and Distinct eigen Values

→ But converse is not true, Consider both a_{21} and a_{12} are not of Same Sign but

$|4a_{21}a_{12}| < (a_{11} - a_{22})^2$ then the condition

$$(a_{11} - a_{22})^2 + 4(a_{21})(a_{12}) > 0$$

holds true.

(c).

$$A = \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{bmatrix}$$

⇒ charactoristic equation

$$\lambda^2 - \lambda[1+1] - \cancel{*}\varepsilon^2 + 1 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4(1)(-\varepsilon^2+1)}}{(2)}$$

$$= \frac{2 \pm \sqrt{4 + 4\varepsilon^2 - 4}}{2}$$

$$= 1 \pm \sqrt{1 + \varepsilon^2 - 1}$$

$$= 1 \pm \sqrt{\varepsilon^2}$$

$$= 1 \pm \varepsilon$$

$\lambda_{max} = 1 + \varepsilon$, $\lambda_{min} = 1 - \varepsilon$ | $\lim_{\varepsilon \to 0} \frac{1+\varepsilon}{1-\varepsilon} = 1$

5).

$$A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$$

→ characteristic equation of A

$$\lambda^2 - \lambda[2a] + [a^2 - bc] = 0$$

$$D = \sqrt{(2a)^2 - 4(a^2 - bc)}$$

for λ → Unreal Number

$$D < 0$$

$$4a^2 - 4(a^2 - bc) < 0.$$

$$4bc < 0$$

$$bc < 0.$$

∴ Either

$S \in b \in (-\infty, -0]$ & $c \in [0, \infty)$ GT

or

$S \in b \in [0, \infty)$ & $c \in (-\infty, -0)$ GT

→ Either way

~~A~~ A $S \cap T = \phi$

6) $A = \begin{bmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1+x^2-\lambda & 7 & 11 \\ 3x & 2x-\lambda & 4 \\ 8x & 17 & 13-\lambda \end{bmatrix}$$

$$|A-\lambda I| = (1+x^2-\lambda)\left[(2x-\lambda)(13-\lambda) - (4)(17)\right]$$
$$- 8x \left[(7)(13-\lambda) - (11)(17)\right]$$
$$+ 8x \left[(7)(4) - (11)(2x-\lambda)\right]$$

6).

$$A = \begin{vmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{vmatrix}$$

$$|A| = \{1+x^2\}\{26x - 68\} - 3x\{91 - 187\}$$

$$+ 8x\{28 - 22x\}$$

$$= 26x - 68 + 26x^3 - 68x^2 - 273x + 561x$$

$$+ 224x - 176x^2$$

$$f(x) = 26x^3 - 244x^2 - 33x + 538x - 68$$

$f(-1) < 0$

$f(1) > 0$, $\therefore$ there exist $f(x)$ such that

$$f(a) = 0$$

Since $|A| = 0$ for $x = a$

$\therefore$ there must be an eigen value to be

zero since determinant is product of eigen values

2)

$$A|B = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & -5 & 5 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 4 & -3 & 0 \\ -1 & -5 & 5 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 4 & -3 & 0 \\ 0 & -3 & 8 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 4 & -3 & 0 \\ 0 & 0 & \frac{23}{4} & 0 \end{array}\right]$$

Since $R(A|B) = 3$

$\rightarrow$ There must be only one solution to this equation

8)

Let the vector Basis be

$$A = \cancel{\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}} \quad A = \begin{bmatrix} 1 & 3 & -2 \\ 1 & 4 & 2 \\ 2 & 8 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -3 \end{bmatrix}$$

Since three vectors linearly Independent they donot pass through origin.

Sol 1Q vector basis is

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 1 & 4 & 2 \\ 19 & 74 & 30 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 17 & 68 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

Since they are linearly Independent they do pass through origin.

10)

$$\begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0-0+3 \\ -1+0+1 \\ a+0-b \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 0 \\ a-b \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

$$\boxed{a-b=3} \quad \text{— (1).}$$

$$\begin{vmatrix} -\lambda & -2 & -3 \\ -1 & 1-\lambda & -1 \\ a & 2 & b-\lambda \end{vmatrix} = 0$$

$$0 = -\lambda\left[(1-\lambda)(6-\lambda) + 2\right]$$

$$+1\left[(-2)(6-\lambda) + 6\right]$$

$$+a\left[2 + 3 - 3\lambda\right]$$

$$= -\lambda\left[\lambda^2 - \lambda\{1+6\} + 2\right)$$

$$+1\left[6 - 12 + 2\lambda\right]$$

$$+a\left[5 - 3\lambda\right]$$

$$= -\lambda^3 + \lambda^2[1+b] - \cancel{2\lambda} + 6 - 2b \cancel{+2\lambda} + 5a - 3a\lambda$$

$$0 = -\lambda^3 + \lambda^2[1+b] - 3a\lambda + 6 - 2b + 5a \quad — (2)$$

For $\lambda = 1$, $2 \to 1$

$$-1 + 1\{1+b\} - 3a + 6 - 2b + 5a = 0$$

$$\cancel{-1} \cancel{+1} + b - 3a + 6 - 2b + 5a = 0$$

$$2a - b + 6 = 0$$

From (1) $\quad a - b = 3$

$$2a \cancel{-b} + 6 - a \cancel{+b} + 3 = 0$$

$$a + 9 = 0$$

$$a = -9$$

$$b = -12$$

∴ characteristic equation is

$$-\lambda^3 + \lambda^2\{1-12\} - (3)(-9)\lambda + 6 - (2)(-12) + (5)(-9) = 0$$

$$-\lambda^3 + \lambda^2[-11] + 27\lambda + 6 + 24 - 45 = 0$$

$$-\lambda^3 - 11\lambda^2 + 27\lambda - 15 = 0$$

By Camely-hamilton theorem

$$P^3 = -11P^2 + 27P - 15I$$

$$P^4 = -11P^3 + 27P^2 - 15P$$

$$= [-11]\left[-11P^2 + 27P - 15I\right] + 27P^2 - 15P$$

$$= [-11]\left[-11P^2 + 27P - 15I\right] + 27P^2 - 15P$$

$$= 121P^2 - 297P + 165I + 27P^2 - 15P$$

$$= 148P^2 - 312P + 165I \quad - (3)$$

$$P^2 = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ -9 & 2 & -12 \end{bmatrix} \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ -9 & 2 & -12 \end{bmatrix}$$

$$P^9 = \begin{bmatrix} 29 & -8 & 38 \\ 8 & 1 & 14 \\ 106 & -4 & 169 \end{bmatrix}$$

$$P^9 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 37 \\ 7 \\ 110 \end{bmatrix}$$

ii)

11 $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 4 & -24 \\ 0 & -1-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix}$.

$$= (1-\lambda)\left[(-1-\lambda)(1-\lambda) - 0\right]$$

$$= (1-\lambda)\left[\lambda^2 - \lambda[-1+1] - 1\right]$$

$$= (1-\lambda)(\lambda^2 - 1)$$

$$= (1-\lambda)^2(1+\lambda)$$

$\lambda = -1, \lambda = 1, \lambda = 1$

For $\lambda = 1$

$$\begin{bmatrix} 0 & h & -2h \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0x + hx_1 - 2hx_3 \\ -2x_2 + 4x_3 \\ 0 \end{bmatrix} = 0.$$

$hx_1 - 2hx_{3}$

$2x_2$

$$\begin{bmatrix} hx_1 - 2hx_1 \\ -2x_2 + 4x_2 \\ 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} -hx_1 \\ 2x_2 \\ 0 \end{bmatrix} = 0.$$