Complex Magnetic Fields Seen to Evolve From the Near Collision of Dipole Fields

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The well-known magnetic fields produced by a current-carrying solenoid or toroid are shown to gradually evolve from the near collision of magnetic dipoles, specifically current-carrying circular loops. Animations have been produced in which a number of dipoles start far enough apart to reveal their individual dipole character, and are then gradually brought closer together to approximate a solenoid or toroid. The animations have been produced by using the Law of Biot-Savart and Runge-Kutta numerical integration. These animations may give students better insight into the shape of these final field than standard derivations employing Ampere's Law and approximate symmetries.

I. INTRODUCTION

In a previous paper,[1] one of us (A.W., along with coauthors S. J. Van Hook and E. R. Weeks) described the misleading character of two-dimensional electric field line diagrams, which fail to capture essential features of inherently three-dimensional field line patterns. Such electric field line diagrams, even for simple distributions of a few point charges lying in a plane generally do not work.

The goal was to use the magnetic field produced by a circular current loop as a visual building block for the magnetic field produced by current distributions such as a solenoid or a toroid. In an introductory electromagnetism course, the magnetic fields for such current distributions are generally developed using Amperes Law, assuming one or more symmetries that, at best, are approximate. For example, the magnetic field of a solenoid much longer than its diameter is assuming translational symmetry along the solenoids axis as well as rotational symmetry. These approximate symmetry arguments are used with Amperes Law to obtain the result that, inside the solenoid, $|B| = \mu_0 I$ with lines of B parallel to the solenoids axis. Students are often skeptical of the derivation, although perhaps not the result, as the derivation requires a high ration of persuasion to actual calculation.

To supplement unpersuasive Amperes Law calculations, we decided to make animations that would show the gradual development of magnetic fields. Each animation is hosted online. The eight-ring toroid is at `http://faculty.cooper.edu/wolf/B-anims/eighttoroid_45.wmv`, the four-ring toroid is at `http://faculty.cooper.edu/wolf/B-anims/fourtoroid_45.wmv`, and the solenoid is at `http://faculty.cooper.edu/wolf/B-anims/solenoid.wmv`.

II. APPROACH

The animations are produced by recalculating the set of magnetic field lines each time the loops are brought closer together and stitching together the resulting static images. In each static image, we numerically integrate the sum of all Biot-Savart contributions at each point from all current carrying loops using a fourth order Runge-Kutta implementation. To accomplish this, each wire is modeled as a series of line segments, and the magnetic field contribution from each segment is found using the Law of Biot-Savart[2]

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_C \frac{I d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3} \tag{1}$$

where $\mathbf{B}$ is the magnetic field generated by the line segment $\mathbf{l}$ with current I. $\mathbf{r}$ starts from the midpoint of the line segment and ends where the $\mathbf{B}$-field is being calculated. To define the magnetic field lines, we followed the approach of Ref. 1 (replacing the electric field with the magnetic field)

$$\frac{d\vec{s}}{ds} = \frac{\mathbf{B}}{|B|} = \frac{B_x}{|B|}\hat{\imath} + \frac{B_y}{|B|}\hat{\jmath} + \frac{B_z}{|B|}\hat{k} \tag{2}$$

where $d\vec{s} = (dx, dy, dz)$ and represents an infinitesimal segment of the field line. The above equation was then numerically integrated with a fixed step size to arrive at a set of points that define a field line. The use of small line segments to plot magnetic field lines is justified by the complexity of an analytical solution.[3]

III. IMPLEMENTATION

The animation program is split into two main portions: the calculation of the magnetic field lines and the generation of the images. This allows animations to be viewed at different angles without recomputing the underlying data.

The calculation portion uses a list of functions as input each of which describe the wire itself and the path the wire takes through time. Each wire also specifies where field lines should start in relation to its current position. This allows magnetic field lines to be generated at evenly spaced intervals along the length of the wire. The procedure to generate each image is as follows:

1. Iterate through each wire
   (a) Split each wire into segments
   (b) Store the initial positions of each magnetic field line

2. Iterate through each initial position
   (a) Iterate through each wire segment
      i. Use Eq. (2) to calculate magnetic field contribution to the current position
   (b) Integrate contributions to determine the next position
   (c) Store position and repeat 1b with each new position unless:
      i. Current position is sufficiently close to the initial position
      ii. A specified number of iterations has completed

After the images were generated, they were combined to form a video using ffmpeg, an open source video compressor. The program itself is written in Python 2.7, and images were generated with the help of Mayavi2, a 3D scientific data visualizer with Python integration. The source code can be found at [`http://calculate.py` and `http://...generate.py`] For each image, it took ten minutes to calculate the points along the field lines and about one minute to render the image file on an Intel Core i3-2100 clocked at 3.10 Ghz, with 6 GB of RAM running Windows 7 Professional. It is important to note that calculation time depends on the length of the field lines and the number of wire segments, both of which varied widely across different animations, and even within individual animations. As a result, it took several weeks across a few computers to generate the animations.

IV. RESULTS

We primarily focused on making animations that would show the gradual development of magnetic fields from the fields of circular current loops. The initial positions of the magnetic fields for each animation were primarily chosen for aesthetic purposes. The primary motivation was to prevent the images from being cluttered, obscuring the unique shapes of each field line. We began with a small number of circular current loops of radius R. For the toroid, the center of each loop lies on a circle of radius $5R$, and but for the solenoid, they form a line of length $4R$. The current loops are gradually brought closer together to approximate the current distribution and resulting magnetic field produced by a solenoid and toroid.

Even in the initial stages, the field lines near a given loop are affected by the presence of other loops, though the basic dipole character of each loops field is evident as small circles closer to the edge of the loops. Students appreciate seeing, in even as few as four current-carrying loops (see Fig. 3), that the magnetic field begins to approach a set of concentric circles. The result is more persuasive as the number of current-carrying loops increases to eight (see Fig. 1). In the solenoid arrangement (see Fig. 2), a similar development can be seen as the magnetic field lines merge to form a line. Note that the merging of magnetic field lines from separate loops into a single field line involves passing through a moment in time where the field lines are not loops, as seen in Figs. 1(b) – 1(c) and Figs. 2(b) – 2(c). This is necessary because adjacent loops need to disconnect before they merge.

V. DISCUSSION

A. Why magnetic field lines dont work

The density of magnetic field lines may be interpreted as an indication of strength by students, similar to electric field lines.[1] Ref. 4 illustrates magnetic field tubes with magnetic field strength varying along the field line. It demonstrates a loose correlation between field strength and distance between field lines, and yet also shows how much information is missing from magnetic field lines. This is perhaps a more useful diagram for students, though it runs the risk of displaying too much information at once.

For example, in Fig. 5, four magnetic field lines are parallel inside a solenoid. At the ends of the solenoid, the lines separate, which leads to the roughly correct assumption that the magnetic field is weaker beyond the ends of the solenoid, as seen in Figs. 7. Yet Fig. 6 depicts the same solenoid, but with a denser positioning of field lines, even though both figures are based off the same magnetic field, which is shown in Fig. 8 as a vector plot. It is also not possible to simply scale or translate the field lines from one figure to arrive at the other, because the paths of the magnetic field lines are different. Therefore, magnetic field lines clearly do not reflect magnetic field strength, and their density only reflects the arbitrary placement of the lines by us.

With this issue in mind, in Fig. 9 we illustrate magnetic field lines from a finite segment of wire, and at first glance, they resemble the field lines from an infinite segment of wire. Yet in Fig. 10 we chose to use tubes to show magnetic field strength. The constant tubular thickness represents the constant field strength along each field line, while the different tubes have different strengths. The reason the field strength varies along the line segment is because points closer to the midpoint of the wire have have greater contributions from both ends of the wire. Note that the **B**-field in the center is nearly constant, so for an infinite wire, the field strength would be constant due to the lack of edge effects, as expected.

B. How magnetic field lines do work

In addition, magnetic field lines accurately represent the magnetic field created by parallel wires in simple ways. For example, in Fig. 11 the magnetic field line further from the wire demonstrates the merging of two,

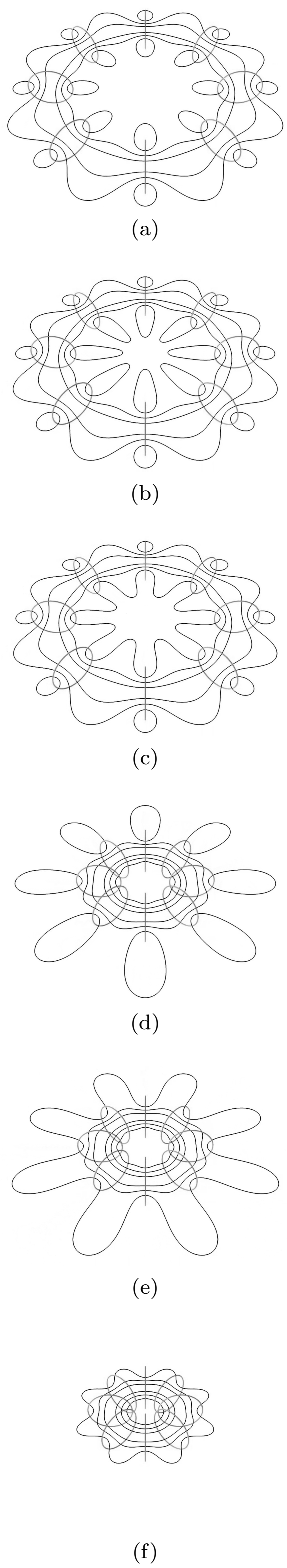

FIG. 1: Magnetic field lines from eight current-carrying loops arranged in a toroid.

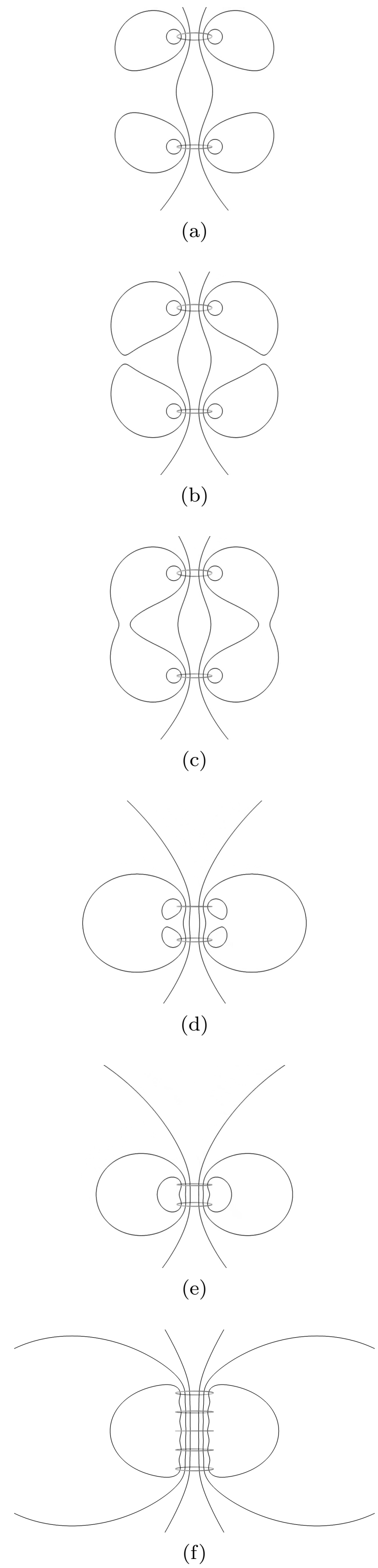

FIG. 2: Magnetic field lines from current-carrying loops along a central axis to form a solenoid.

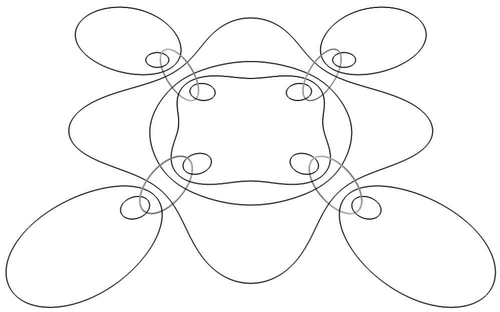

FIG. 3: Magnetic field lines from four current-carrying loops arranged in a toroid (enhanced online).

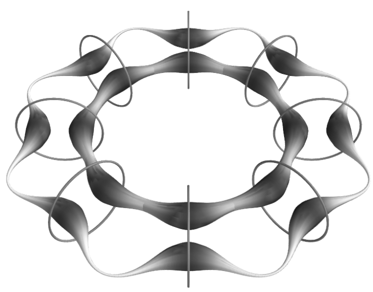

FIG. 4: Magnetic field tubes generated by a set of eight current-carrying loops arranged in a toroid. Darker and thicker portions of the tubes represent higher magnetic field strength at that position.

previously separate, field lines from each wire. This contrasts with Fig. 12, where the field lines from each wire do not merge together. Even when the currents vary in strength Fig. 13, it is clear which direction the currents are moving relative to each other, as well their relative strengths. Therefore, magnetic field lines serve as a clear

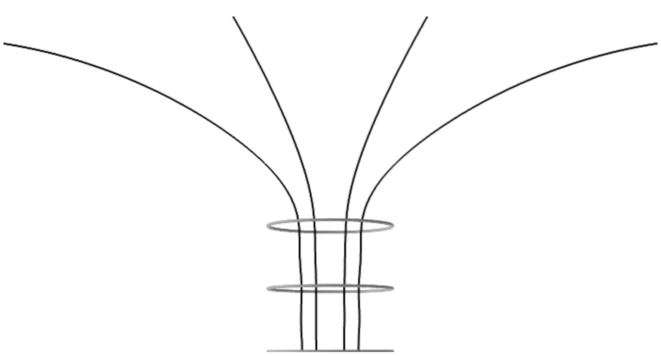

FIG. 5: Magnetic field lines from a series of current-carrying loops arranged in a line to resemble a solenoid.

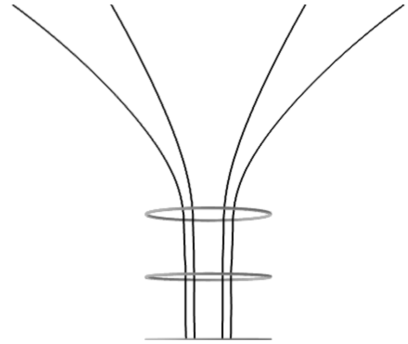

FIG. 6: Magnetic field lines from the same arrangement as Fig. 5, but with different initial positions.

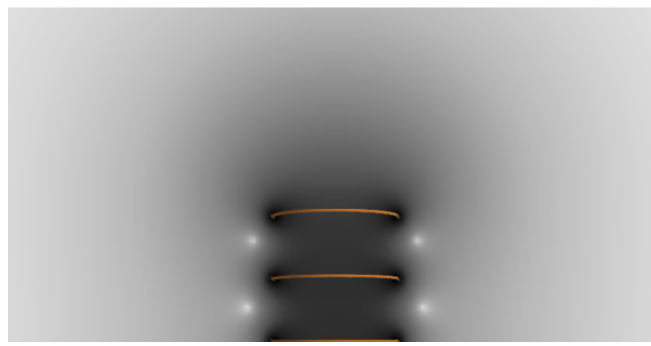

FIG. 7: Grayscale logarithmic map of magnetic field strength from the solenoid in Fig. 5. Darker regions indicate higher magnetic field strength.

and concise way of visualizing magnetic fields.

C. Unusual Observations

As these animations were created, we noticed unusual results from certain wire arrangements. In particular, non-symmetric wires did not produce simple loops, an observation that has only been explored recently.[3] We came to the same conclusion with an open circular helix

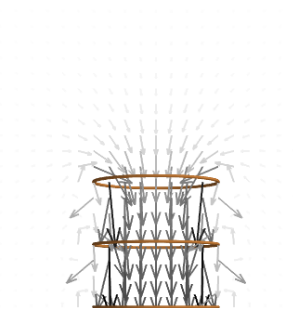

FIG. 8: Vector plot of magnetic field from the solenoid in Fig. 5.

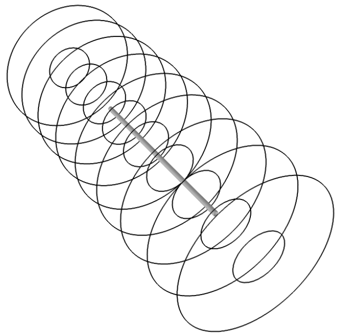

FIG. 9: Magnetic field lines generated by a single straight segment of wire of finite length.

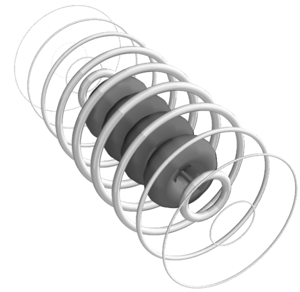

FIG. 10: Magnetic field tubes generated by the same segment of wire as Fig. 9. Darker colors and thicker tubes represent higher magnetic field strength at that location.

Fig. 15 and with two current-carrying loops Fig. 14. So even though symmetry arguments may be made to simplify the shape of the wires, resulting in simpler magnetic field lines, once those symmetries are removed, drawing conclusions from magnetic field lines becomes much more difficult. In addition, Fig. 15 shows a complex magnetic field composed of the fields of circular current loops, obscuring the dipole character of each loops field.

VI. CONCLUSION

Despite reservations about how well magnetic field lines represent the underlying field, we have shown that the closed loop nature of magnetic fields created by current-carrying wire in symmetric situations clearly visualize the behavior of particles influenced by magnetism. In addition, the circular current-carrying loop is an effective building block for demonstrating symmetric current distributions. Notably, the beauty of magnetic field lines does not necessarily break down in asymmetric situations, and instead accurately reflect how a particle would behave. Thus, magnetic field lines mostly work.

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[1] A. Wolf, S. J. Van Hook, and E. R. Weeks, Am. J. Phys. **64**, 714 (1996).
[2] D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics*, 7th ed. (John Wiley & Sons, 2004).
[3] M. Lieberherr, Am. J. Phys. **78**, 1117 (2010).

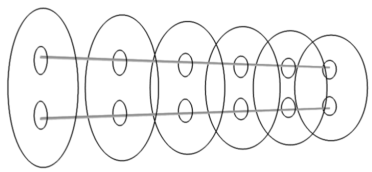

FIG. 11: Magnetic field lines from two parallel wires with currents of equal strength running in the same direction.

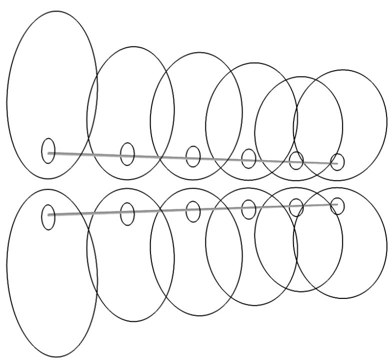

FIG. 12: Magnetic field lines from two parallel wires with currents of equal strength running in the opposite direction.

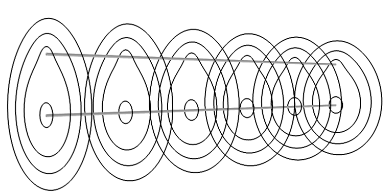

FIG. 13: Magnetic field lines from two parallel wires with currents of different strength running in the same direction.

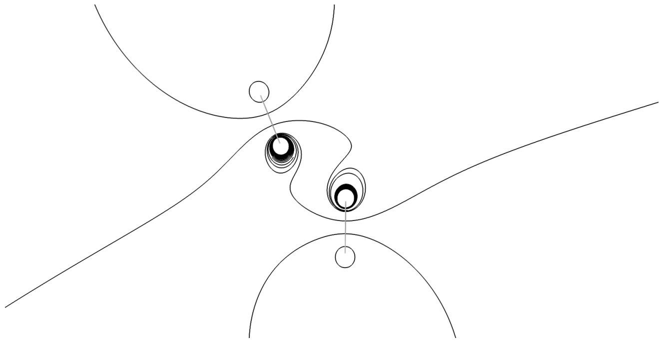

FIG. 14: Magnetic field lines from two offset loops, viewed from above.

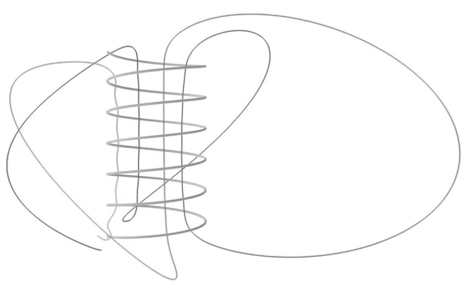

FIG. 15: A single magnetic field line from an open helical wire.