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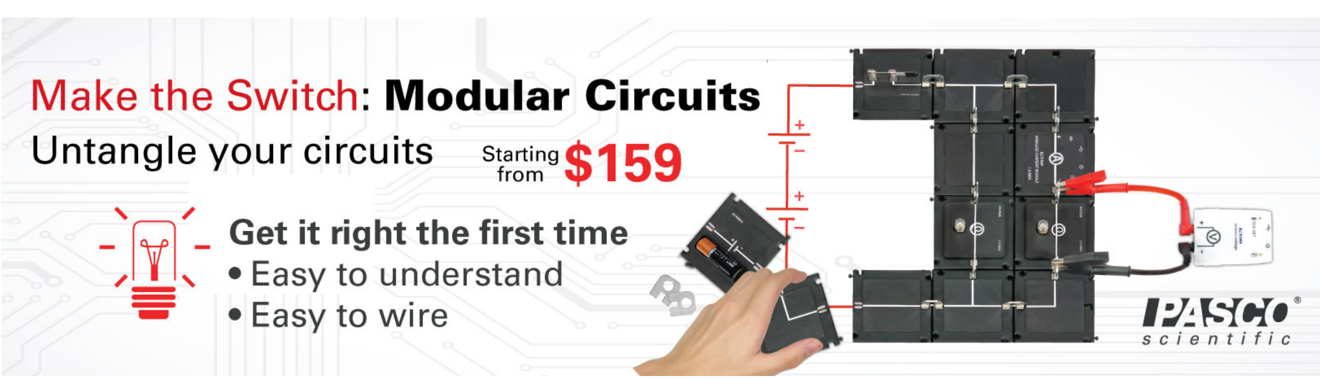

Magnetic Field of a Double-Layer Solenoid

Takao Takeuchi
Department of Mathematics and Physics, Alfred State College, Alfred, NY 14802; takeuct@alfredtech.edu

In this note we will compare the magnetic field along the axis of a single-layer solenoid and that of a double-layer solenoid with the same basic dimensions and same current flowing in the same direction. The various dimensions of a double-layer solenoid and the coordinate system are shown in Fig. 1. The length of the solenoid is L, the radius of the coil is a, the diameter of the wire is D, and the origin of the z axis is taken at the center of the solenoid. We will measure a, D, and z in units of L. Thus, in this reduced unit system, $z = 0.5$ corresponds to the right end of the solenoid and $D = 0.01$ means that the diameter of the wire is 1% of the length of the solenoid. For simplicity, we assume that the wire in the second layer is directly on the wire in the first layer. We also ignore the pitch of the windings.

What we are interested in finding is the effective number, n, of a single-layer solenoid for which the magnetic field along the axis due to a current in a single-layer solenoid is the same as that due to a double-layer solenoid ($n = n_{single} / n_{double}$). Even nonscience majors should be able to come up with the right answer.

Let us consider n at three locations: $z = 0$, 0.5, and 1, using $D = 0.01$.

a) $z = 0$

I am sure that virtually all students will correctly predict that n must be close to 2. After all, a double-layer solenoid is equivalent to two concentric solenoids. I am also sure that they can also reason that since the magnetic field at $z = 0$ due to the second layer is less than that due to the first layer (the radius is larger), it must be less than 2. Do answers depend on the radius of the solenoid a?

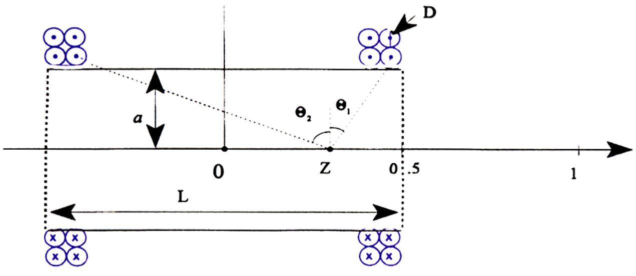

Fig. 1. Double-layer solenoid.

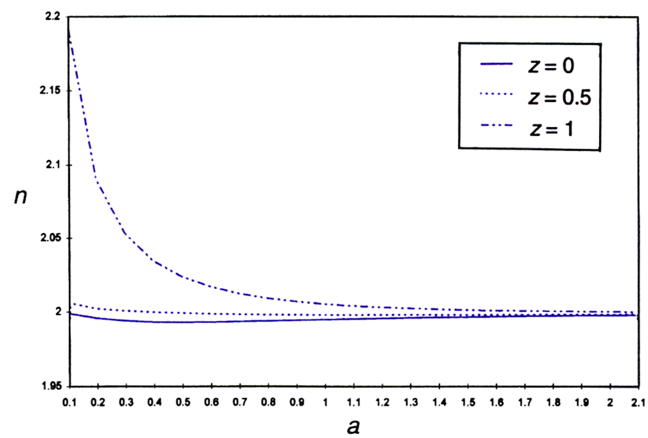

Fig. 2. Equivalent numbers of turns, *n*, as a function of solenoid radius at three locations on its axis (*D* = 0.01).

b) $z = 0.5$

It is still true that n at the edge of the solenoid is close to 2. However, it is not always less than 2. Depending on the value of a, n can be greater than 2.

c) $z = 1$

The value of n is now larger than 2. For example, it is 2.19 for $a = 0.1$ (the radius of the solenoid is 10% of the length of the solenoid). The results are summarized in Fig. 2 for $D = 0.01$ at the three locations ($z = 0$, 0.5, and 1) as a function of the solenoid radius a. The question is why it is larger than 2 at $z = 1$ (and some other locations). Many students should be able to figure out the reason easily. At $z = 1$ and some other locations, the magnetic field B_2 due to the current in the second layer is greater than the magnetic field B_1 due to the same current in the first layer.

In Fig. 3, we plotted $\Delta B = B_2 - B_1$ as a function of z for $D = 0.01$ and $a = 0.1$ to see which magnetic field is stronger, B_1 or B_2, at various z locations. The shape of the graph is quite unexpected and very interesting. For z less than about 0.5, ΔB is negative, indicating that the magnetic field B_1 due to the first layer is greater than magnetic field B_2 due to the second layer ($B_2 < B_1$). For z above about 0.5, ΔB is positive, indicating that the magnetic field due to the second layer is

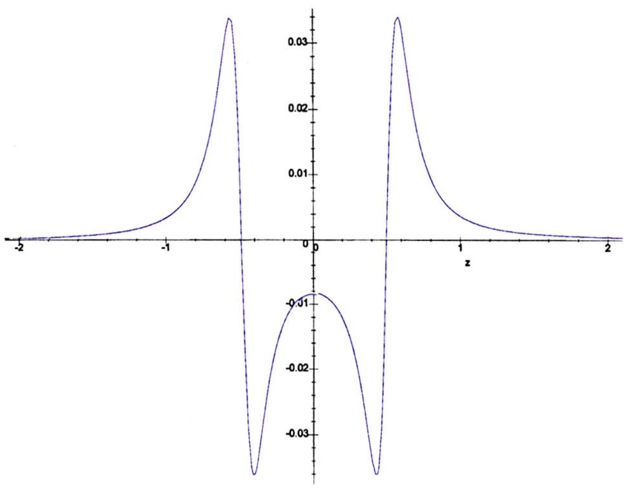

Fig. 3. Difference between magnetic fields due to the currents in two layers as a function of location along axis *z* (*D* = 0.01 and *a* = 0.1).

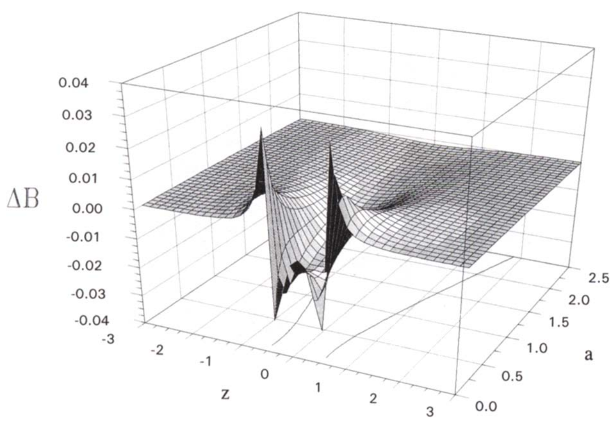

Fig. 4. Same as Fig. 3, but as a function of *z* and *a* (*D* = 0.01).

greater than that due to the first layer ($B_2 > B_1$). I like to emphasize that this graph is for a solenoid whose radius is $a = 0.1$. We assume that $\frac{1}{2}\mu_0 n I = 1$ in B_1 and B_2.

Figure 4 plots $\Delta B = B_2 - B_1$ as a function of z and a for $D = 0.01$. Notice that the foremost curve corresponds to Fig. 3 for $a = 0.1$. The two curves at the bottom of the graph indicate the location z where B_2 is the same as B_1 for various a values. It is obvious from the curves that as the radius of the solenoid increases, the location at which B_2 becomes larger than B_1 increases. To get exact values of the location z, you may want to use software like *Maple V*. The command would look like *fsolve* $(B_2 - B_1 = 0, z)$.

The expressions for B_1 and B_2 could be obtained by using the equation for a magnetic field due to a solenoid of finite length, $B = \frac{1}{2}\mu_0 n I (\sin\theta_1 + \sin\theta_2)$, where μ_0 is the permeability of free space and I is the current in the solenoid.

You may want to ask your students to explore cases where D is other than 0.01 or consider solenoids with more than two layers. No sophisticated software is required to produce graphs such as seen here; I am sure that physics majors would find it exciting to obtain graphs such as Figs. 3 and 4.

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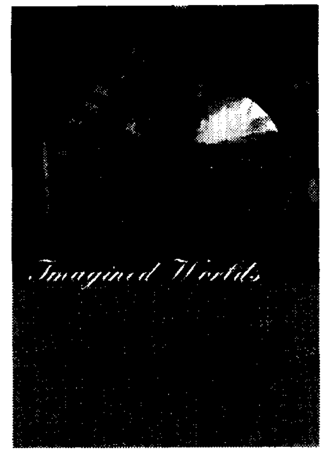

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