The magnetic field of a current conducting wire

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The magnetic field of a current conducting wire

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I. MEASURING THE MAGNETIC FIELD INSIDE A CONDUCTOR

There are two possible arrangements that permit measurement of the magnetic field with a probe inside a conductor. One can either use a cylindrical tube filled with an electrolytic solution and immerse the probe into this liquid,[1] or build a conductor model consisting of a circular bundle of thin evenly spaced wires, where the space between the wires permits entry with a probe. We chose the latter model as it seemed to be more convenient for a lecture demonstration.

II. THE MODEL OF A CURRENT CONDUCTING CYLINDER

The model is made of 316 Manganin wires 0.2 mm diam with a resistance of 13.8 ohms per meter. The wires are held in place between two copper-coated epoxy plates. These plates have a circular arrangement of fine holes 5 mm apart through which the wires are passed and then soldered to the copper. The model has a radius of 50 mm and is 350 mm long.

III. THE MAGNETIC FIELD PROBE

For moderate current densities the magnetic fields to be measured will be too small for a Hall probe. One could detect them with a Förster[2] probe but this, however, would have to be a tangential probe less than 5 mm wide. It is simpler and cheaper to measure the magnetic field with a small induction coil and to use alternating current of sufficiently low frequency instead of dc. We use a flat coil 3 mm thick, inner diameter 6 mm, outer diameter 16 mm, which is embedded in a long, flat acrylic glass bar. The coil consists of 900 turns of 0.1 mm copper wire.

With a current of 500 mA at a frequency of 10 kHz delivered by a sine generator the maximum-induced voltage measured at the rim of the wire model is 30 mV. The voltage drop across the wire model is 7.5×10^{-3} V.

IV. THE INFLUENCE OF AN ALTERNATING ELECTRIC FIELD

The use of ac instead of dc is quite legitimate as one can see from Maxwell's equation:

$$c^2 \operatorname{curl} \mathbf{B}=\frac{\partial \mathbf{E}}{\partial t}+\frac{1}{\epsilon_0}\mathbf{j}.$$

For an electric field amplitude of $E_{\max}=2.1\times10^{-4}$ V/cm for our wire model which has an overall resistance of 15 $\times10^{-3}$ ohm the current density is $j=5\times10^{-2}$ A/mm^2, the first term in Maxwell's equation $\partial E/\partial t$ is about 9 orders of magnitude smaller than $1/\epsilon_0\times j$ and hence negligible. The conductivity is low enough so that the skin depth is large compared to the radius of the model. The only problem left arises from the current leads between the generator and the model.

V. SHIM CAGE

As shown in Fig. 1, the model is surrounded by a cage consisting of brass top and bottom plates 400 mm diam. The two plates are electrically connected with 120 brass bars 3 mm in diameter placed at equal distances of 10.5

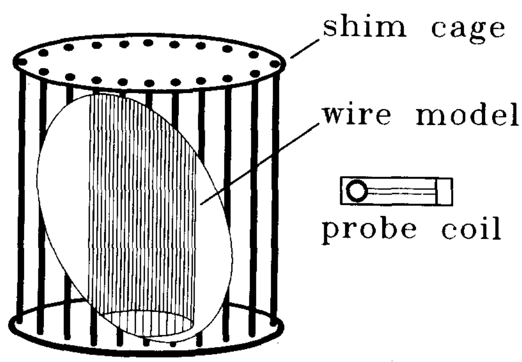

Fig. 1. Model conductor with shim cage.

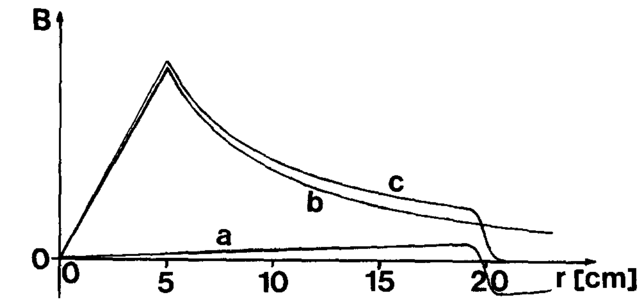

Fig. 2. The calculated field distribution of (a) The empty cage. (b) The conductor of finite length diameter 10 cm, length 35 cm. (c) Conductor and cage. B in arbitrary units.

mm around the circumference of the cage. The current from the sine generator is fed into the lower end of the model conductor through a central isolated connector through the bottom plate of the cage. It flows through the model and back to the generator via the cage's top plate and the brass bars to the center of the bottom plate.

VI. THE CALCULATED FIELD DISTRIBUTION

Figure 2 shows the result of a numerical calculation of the radial dependence of the magnetic field distribution of the empty cage and the field distribution of a cylinder of finite length. The field produced by the cage exactly compensates the missing part of the infinitely long conductor. The transition region at the radius of the cage is due to the thickness of the cage walls and its structure.

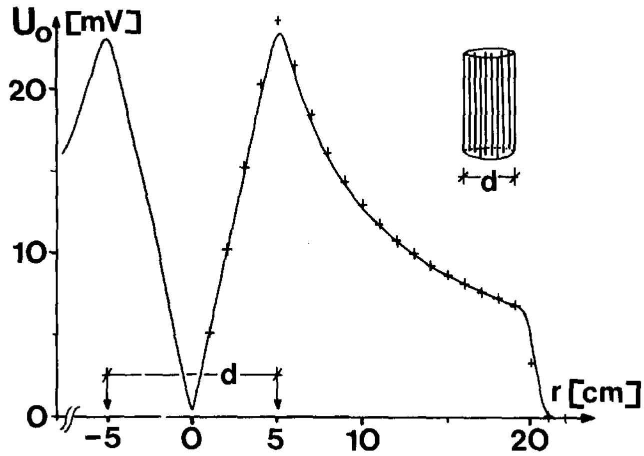

Fig. 3. Measured field distribution. The crosses represent calculated values. U_0 is the amplitude of the probe voltage, d the diameter of the conductor.

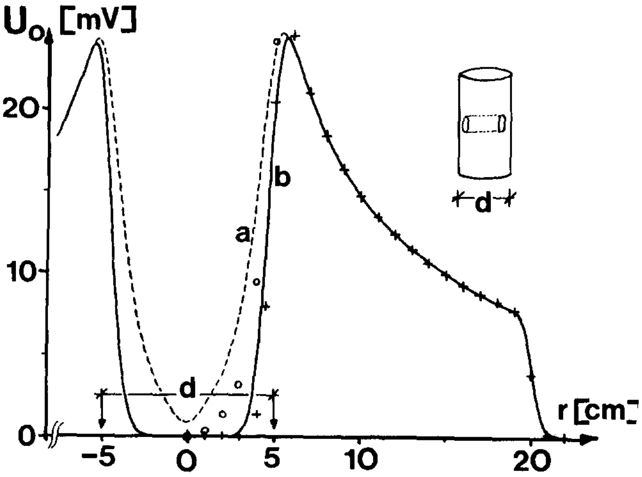

Fig. 4. Skin effect in an aluminum cylinder of diamter d for a frequency of (a) 100 Hz and (b) 1 kHz. The circles and crosses are calculated values for 100 Hz and 1 kHz, respectively. U_0 is the amplitude of the probe voltage.

VII. EXPERIMENTAL RESULTS

The calculated and the measured radial dependence of the magnetic field are shown in Fig. 3. For the calculation the magnetic field was averaged over the active area of the induction coil. For the measurement the field probe was mechanically connected to the X carriage of an XY plotter. The X axis was driven by the plotter's timebase and the Y axis by the amplified and rectified signal of the field probe.

VIII. SKIN EFFECT

In Fig. 4 the wire model has been replaced by a massive aluminum cylinder with a narrow slot for the induction probe passing radially through the center part of the cylinder. Due to the high conductivity of the solid aluminum the skin depth already becomes of the order of the cylinder's radius at frequencies below 100 Hz. The figure shows the radial field dependence inside the conductor for two different frequencies. The field outside still has the expected $1/r$ dependence. The deviation between calculated and measured field values is due to the additional surface introduced by the slot.

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[1]Phywe AG, Göttingen, Kat. Nr. 11003.10.

[2]F. Förster, "Zur Messung von magnetischen Gleichfeldern," Z. Metallkd. **46**, 358–370 (1955).