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The Magnetic Field Along the Axis of a Short, Thick Solenoid

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We commonly ask students to compare the results of their experimental measurements with the predictions of a simple physical model that is well understood. However, in practice, physicists must compare their experimental measurements with the predictions of several models, none of which may work well over the entire range of measurements. The following describes an experiment we use in the second semester of a two-semester course designed for chemistry, biochemistry, and biology majors as an example of this situation. There are three parts to the experiment. In the first part the students, working in groups of two or three, calibrate a search coil. In the second part they position the coil at various distances from the center of a thick, finite solenoid and measure the field at each position. In the third part they use three models to predict the magnetic field at each of those positions. The students must then decide if one model best predicts the results of the measurements. If no single model can do so, they must decide which model works best over which range.

In a preceding experiment the students used the Vernier Magnetic Field Sensor to map the field distribution within and just outside Helmholtz coils. They determined the region near the center over which the field variation was negligible. In the first part of this experiment the students position the search coil, mounted near the end of a 25-mm (1-in) diameter wooden dowel, at the center of the Helmholtz coils. The search coils are simply made of wire wrapped around the dowel for about 30 turns over a length of about 0.03 m. No effort is made to make any further measurements of their properties. For stability the dowels are held in place by a clamp system attached to the lab table. The students use a PASCO Model SP-9584 Low Voltage AC/DC Power Supply to generate 60-Hz currents from 0 to 3 A through the Helmholtz coils, as measured by a simple multimeter, connected as an ammeter, in series with the coils (Klinger Model KA6021) as shown in Fig. 1. The need to use the AC output from the power supply to induce a voltage in the search coil via a changing magnetic field is discussed. For each current they use a Keithley 169 multimeter to measure the voltage induced in the search coil. Any multimeter capable of measuring AC voltages with millivolt accuracy would suffice. Using Eq. (1) for the field at the center of the Helmoltz coils, the students can then associate a magnetic field with each search coil voltage.[1]

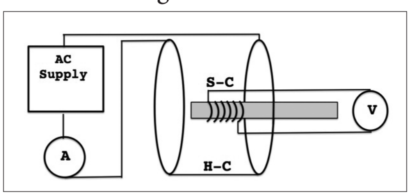

Fig. 1. Schematic diagram of the calibration circuit. The current provided by the PASCO 60-Hz supply is measured by a simple multimeter connected in series as an ammeter (A) and conveyed to the Helmholtz coils (H-C). The voltage induced in the search coil (S-C) is measured by a multimeter (V) capable of measuring millivolts accurately.

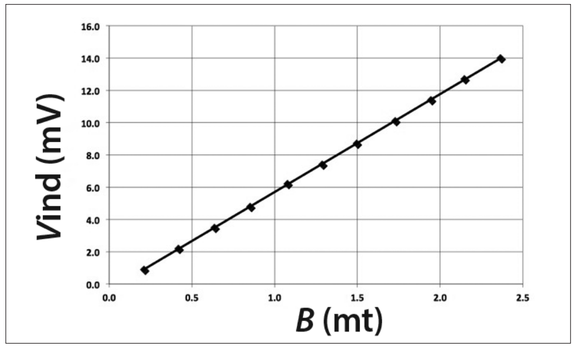

Fig. 2. Calibration curve. The voltage induced in the search coil is plotted against the magnetic field produced at the center of the Helmholtz coils. The diamonds represent the measured values and the line indicates the linear fit.

$$B = 8\mu_0 NI/R(125)^{0.5}, \qquad (1)$$

where $\mu_0 = 4\pi\times10^{-7}$ T·m/A, N is the number of turns in one of the coils, I is the current passing through the coils, and R is the radius of the coils. With the data entered into Excel they can perform a linear fit to obtain a calibration equation that relates the measured magnetic field to the induced search coil voltage. Use of the Linest function is preferable as it presents an estimate of the errors in the fitting. Figure 2 shows such a calibration plot. The slope is 6.07 $\pm$0.01 mV/mT and the y-intercept is –0.37$\pm$0.02 mV. The $\pm$ values are the standard errors. The lack of agreement of the intercept with zero indicates the presence of background AC fields. The x-intercept provides an estimate for the magnitude of the background fields and is 0.060$\pm$0.003 mT.

In the second part of the experiment the students position the center of the search coil at the center of a thick, finite solenoid. The solenoids we use in the experiment were fabricated in our shop. They are typically 0.090 m in length with inner and outer radii of R_i = 0.050 m and R_0 = 0.070 m and contain about 2500 turns. Similar finite solenoids can be purchased commercially, e.g., Leybold or PASCO. As illustrated in Fig. 3, the solenoid is connected in series to a multimeter connected as an ammeter and a PASCO SE-7351 Variable Transformer that delivers a 60-Hz current of 0.2 A to the coil. The students then measure the voltage induced in the search coil at various distances x from the center of the solenoid along its axis. Measurements are made at 0.005-m intervals inside the solenoid and 0.01 m outside the solenoid. The distances are measured

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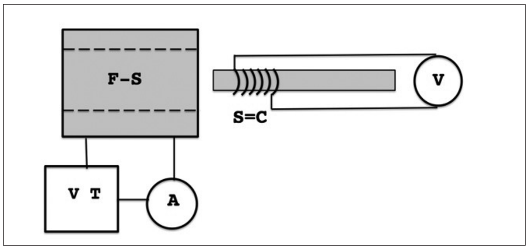

Fig. 3. Schematic diagram of the measurement circuit. The current provided by the variable transformer (V T) is measured by a simple multimeter connected in series as an ammeter (A) and conveyed to the thick, finite solenoid (F-S). The voltage induced in the search coil (S-C) is measured by a multimeter (V) capable of measuring millivolts accurately.

from the center of the solenoid to the center of the search coil. For stability the dowels are again held in place by a clamp system so that it is easier in practice to move the solenoid rather than the search coil. Using the calibration equation from the preceding part, the students then determine the measured axial magnetic field produced by the finite solenoid. The previously determined background magnetic field value, 0.060 ± 0.003 mT, is very small compared to the measured values.

In the third part the students calculate the magnetic field B at the measurement positions for three potential models: (i) an infinite solenoid, (ii) a circular loop of radius R and N turns, and (iii) a thin, finite-length solenoid of fixed radius R. In the following equations x is the distance along the axis from the solenoid center to the measurement point, I^* is the current through the solenoid, N^* is the number of solenoid turns, and L is its length. The derivation of Eq. (4) is beyond the level of high school and most introductory-level college students. However, we present the equation simply as the result of a more advanced model.

For the infinite solenoid there is no dependence on R or x. The magnetic field, $B1$, is constant.[2]

$$B1 = \mu_0 N^* I^* / L. \quad (2)$$

For the circular loop the magnetic field $B2$ is independent of L.[3]

$$B2 = \mu_0 N^* I^* R^2 / [2(x^2+R^2)^{1.5}]. \quad (3)$$

The calculation for the axial magnetic field of a finite solenoid $B3$ is more complicated.[4]

$$B3 = \mu_0 N^* I^* (\cos\beta - \cos\alpha)/2L, \quad (4)$$

where

$$\cos\beta = (x+L/2)/[(x+L/2)^2 + R^2]^{0.5} \quad (5)$$

and

$$\cos\alpha = (x - L/2)/[(x - L/2)^2 + R^2]^{0.5}. \quad (6)$$

We encourage the students to use three Excel columns to find

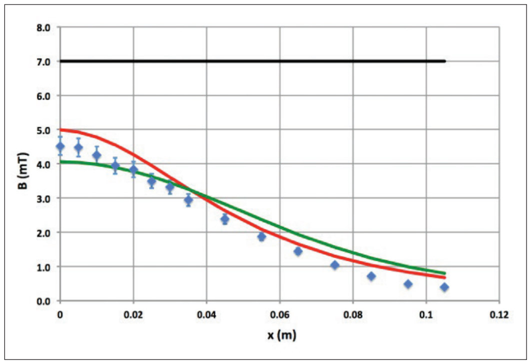

Fig. 4. Comparison of the measured magnetic field (blue diamonds) with the fields predicted by three models: infinite solenoid (black line); circular loop (red line); thin, finite solenoid (green line). The error bars indicate the estimated measurement error of 6%.

$B3$ rather than try to enter one very long equation. We suggest that they first calculate columns for $\cos\beta$ and $\cos\alpha$ and then use those values in a third column for $B3$. It is easier to find mistakes in the calculations of Eqs. (5) and (6) where cos should vary smoothly and have values between +1 and –1 than to find such errors in a single large equation.

For easy plotting, the Excel sheet should have five adjacent columns: x, $B1$, $B2$, $B3$, and $B0$, the measured values. Figure 4 is a plot of the four B values vs. x. It is clear that the infinite solenoid model does not compare well with the measured values, even inside the coil. Inclusion of this model, however, serves a practical purpose. The values should be comparable in magnitude to the measured values. If there is major disagreement, the students are alerted to check whether the calibration and measurement procedures have been done correctly.

The other two models compare reasonably well with the measured values, but at different x values. The thin, finite solenoid model appears to provide a better fit inside and close to the solenoid; the loop model, further from the solenoid. Neither model works very well over the entire range of x-values. The finite solenoid model emphasizes how the field variation depends on the "closeness" of the ends of the solenoid and should work better inside and close to the solenoid. Far from the solenoid the thickness of the windings becomes more apparent and the circular loop model with an effective R works better.

One significant error in the measurements is presumably the finite length of the search coil wiring. During the calibration procedure the search coil is positioned at the middle of the Helmholtz coils. From the previous field-mapping experiment the students would understand that the applied field was uniform over the dimensions of the search coil. However, the field produced by the finite solenoid is not uniform over the search coil. The voltmeter reading provides an average field over the coil. The error will be maximum where the field varies most nonlinearly with x—around $x = 0.02$ m. Using Eq. (4) to estimate the nonlinearity there, one finds that the average of the B values at 0.005 m and 0.035 m differs by only 3% from the value at 0.020 m. Hence the error produced by the finite size of the search coil is small. The total error due to the uncorrelated calibration intercept (0.003 mT/0.060 mT ~ 5%) and finite search coil errors is 6%. That maximum error is shown as the error bars in Fig. 4.

The students do not submit written reports. The experiments are performed by groups of two or three students. Each group presents its results for the calibration and field measurement parts orally immediately following their modeling procedure. The students are first asked which model worked best in general. Depending on their choice of R, a group might choose one model as better inside and near the coil, but the other further away. Because each group is next to their computer during their reporting, they are asked to vary the R-value they used and see how it affects their choice of the "better model."

The R-value used in the modeling shown in Fig. 4 (R = 0.063 m) is a typical value obtained by the students. One way to quickly vary R is to set up a column with the same R-value for each x. In their calculations of Eqs. (3)–(6) the students use those R-values. Changing the values in that column quickly revises the model. A better approach is to introduce the use of absolute and relative cell references in Excel if it has not been done previously. The R-value is used as an absolute address and can be quickly changed.

Although we do not ask the students to do it, use of the Solver tool[5] can shed further light on the different ranges of applicability of the two models. Briefly, one sets up a column of the squares of the differences between $B0$ and $B2$ or $B3$. Solver minimizes the sum of those squares by varying R. For the flat coil model, the best R is 0.070 m; for the finite solenoid, 0.057 m. In both cases the optimized model works best inside and near the coil and has errors far away. The other model has major errors inside and near the coil. The two models largely agree with each other far from the coil, but yield values slightly greater than the measured results.

The difficulty in obtaining good agreement for the finite solenoid model is due to Eq. 4 being applicable for a thin, finite-length solenoid. The solenoids we use are thick. Some improvement can be obtained by decomposing the thick solenoid into 10 "thin solenoids," each of which has $N^*/10$ turns. Their radii are increased in 10 steps

$$\Delta R = (R_0 - R_{\mathrm{i}})/10 \tag{7}$$

from $R_1 = R_{\mathrm{i}} + \Delta R$.

Although this extended model is readily set up in an Excel worksheet, we do not ask the students to do so. The improvement is not major and the setup would be problematic for some of the life science students. However, it might serve as an extra credit exercise.

Chia and Wang described[6] an experiment in which students measured the magnetic field along the axis of a long, finite solenoid. They found that the measured field agreed quite well with Eq. (4) except outside the solenoid. Their procedure was similar to that in the second part of this experiment. However, their probe was not calibrated so that their measurements were only relative, normalized to the field at the center of the solenoid. In this experiment the probe was calibrated so that absolute values of the field were determined. Moreover, according to the picture of their setup, their solenoid was very long compared to its thickness. In this experiment the ratio of the thickness of the solenoid to its length was 0.22. This relatively large ratio made modeling more complicated, which was the goal of the experiment.

In summary, this experiment provides the opportunity for students to compare measured results to values calculated from several approximate models, none of which provides excellent agreement over the entire range of measured values. In doing so they must discuss and decide what value to use for the effective radius of the thick coil. As a result, the students should begin to understand that complicated physical systems may not always be amenable to a single, simple model.

References

1. J. R. Reitz, F. J. Milford, and R. W. Christy, *Foundations of Electromagnetic Theory*, 4th ed. (Addison-Wesley, Reading, MA, 1993), pp. 201-203.
2. R. A. Serway and J. W. Jewett Jr., *Physics for Scientists and Engineers with Modern Physics*, 8th ed. (Brooks Cole, Belmont, CA, 2010), p. 874.
3. Ibid., p. 866.
4. M. Sadiku, *Elements of Electromagnetics*, 5th ed. (Oxford Univ. Press, New York, NY, 2010), p. 291.
5. Francis. X. Hart, "'Solvering' the physical pendulum," *Phys. Teach*. **42**, 150–153 (March 2004).
6. Chih-Ta Chia and Ya-Fan Wang, "The magnetic field along the axis of a long finite solenoid," *Phys. Teach*. **40**, 288–289 (May 2002).

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