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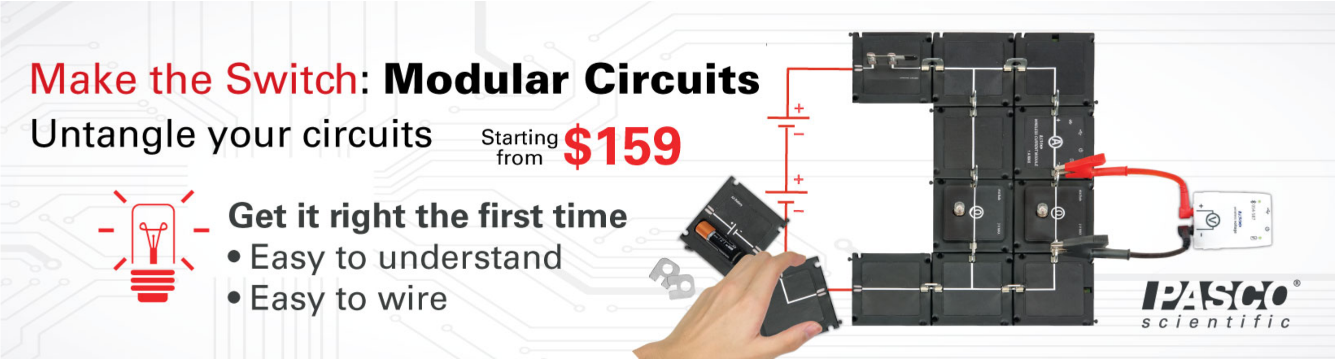

# The Magnetic Field Along the Axis of a Long Finite Solenoid

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Ampère's law is used to calculate the magnetic field inside an ideal solenoid, which has infinite length and tightly wound circular coils. The magnetic field $B$ is parallel to the central axis of the solenoid and is uniformly equal to $\mu_0 nI$,[1] where $n$ is the number of turns per unit length, $\mu_0$ is the permeability of free space, and $I$ is the current through the coils of the solenoid. However, a real solenoid in a laboratory has two ends, such that the $B$ field is not as simple as $\mu_0 nI$.

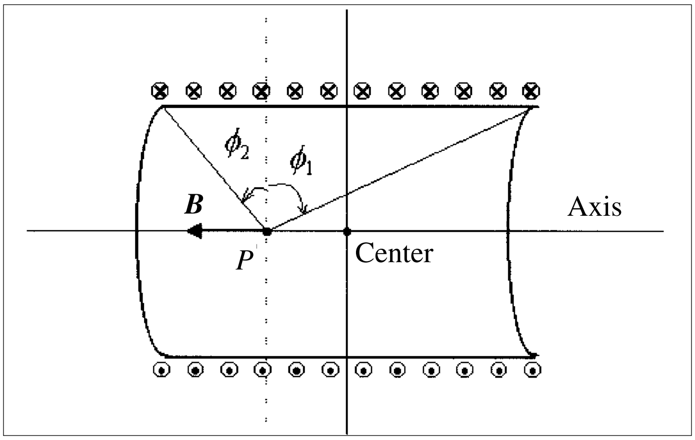


**Fig. 1. Geometry of *B* field along the axis of a tightly wound solenoid for the theoretical calculation of the field.**

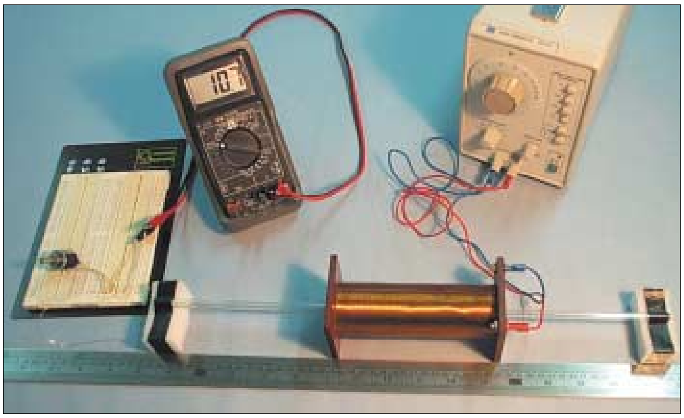

**Fig. 2. Experimental setup for measuring the magnetic field of a long finite solenoid.**

The $B$ field at the end of a sufficiently long solenoid is approximately equal to one-half of the value at the center. Both sides of a long solenoid contribute $\frac{1}{2}\mu_0 nI$ to the $B$ field at the center. If the solenoid is cut in equal halves, then the $B$ field at the cut end can be taken as $\frac{1}{2}\mu_0 nI$. A more realistic calculation, shown in textbooks,[2,3] describes the $B$ field along the axis of a tightly wound solenoid as

$$B = \frac{1}{2}(\sin\phi_2 - \sin\phi_1)\,\mu_0 nI, \qquad (1)$$

where $\phi_1$ and $\phi_2$ are the angular positions of the measuring point relative to the end points, as shown in Fig. 1. These angles are from +90° to –90°. Counterclockwise is described by a positive angle, while clockwise is described by a negative angle, when measured from the vertical axis. According to Eq. (1), the $B$ field is not uniform and depends on the measurement position. The factor, $\frac{1}{2}(\sin\phi_2 - \sin\phi_1)$, is related to the position of the measured $B$ field and to the dimensions of the solenoid. The theoretical $B$ field strength may be calculated numerically from Eq. (1); however, the $B$ field along the axis for a long finite solenoid is sought in this work.

An excellent experiment to find the spatial variation of $B$ field was demonstrated by Dietz and Keith.[4] An LRC circuit was used to probe the $B$ field. However, a digital multimeter and a handmade pick-up coil can be used for the same purpose. Figure 2 shows the experimental setup for measuring the $B$ field along the axis. An audio generator supplies an ac current, $I$ (= $I_0 \sin \omega t$), to a 15-cm long, 5.7-cm diameter solenoid with 472 turns. The solenoid is constrained to move only parallel to its central axis. Copper wire wound around a glass rod (0.8 cm in diameter) is used as the pick-up coil, which has 25 turns and a width of about 0.4 cm. This pick-up coil is fixed on the central axis of the solenoid and is connected to a 3½-digit multimeter, available from any electronic shop. When the solenoid moves along a ruler placed on the table, the multimeter can detect the variation of induced emf (in terms of ac voltage or current) in the pick-up coil at different positions along the solenoid's axis. According to Faraday's law of induction, the induced emf is created by the time variation of the magnetic flux within the cross section of the pick-up coil when an ac current is supplied to the solenoid. The magnetic field $B$ along the axis of the solenoid is similar to that described by Eq. (1) and can be written as

$$B = \frac{1}{2}(\sin\phi_2 - \sin\phi_1)\ \mu_0 n I_0 \sin\omega t. \qquad (2)$$

This time-dependent $B$ field generates the induced emf in the pick-up coil. Accordingly, the reading on the multimeter, $\varepsilon_{\mathrm{rms}}$, is proportional to the time-averaged root-mean-square of the $B$ field, which is written as

$$\begin{aligned}\varepsilon_{\mathrm{rms}} &\propto NA\left(\frac{dB}{dt}\right)_{\mathrm{rms}} = NA\omega B_{\mathrm{rms}} \\ &= NA\omega\ \frac{1}{2}(\sin\phi_2 - \sin\phi_1)\mu_0 n I_{\mathrm{rms}}, \qquad (3)\end{aligned}$$

where $N(=25)$ is the number of turns and $A$ is the cross-sectional area of the pick-up coil. According to Eq. (3), the magnetic field along the axis can be determined from the $\varepsilon_{\mathrm{rms}}$ for a specified output from the audio generator. The reading from the multimeter was more sensitive when the impedance of the pick-up coil circuit is matched to that of the solenoid circuit by adjusting the frequency of the audio generator.

Figure 3 compares the experimental results with the theoretical calculation. The solid line is the theoretical curve, based on Eq. (1), and is normalized to the $B$ field at the center. The experimental result, expressed as open squares, is the measured $\varepsilon_{\mathrm{rms}}$ after normalization to the value at the origin and is plotted against the position. The estimated experimental error is within the square shown in the figure. Experiment matches theory quite well, except outside the solenoid. This deviation is primarily due to the very weak $B$ field outside the solenoid and the finite size of the pick-up coil.

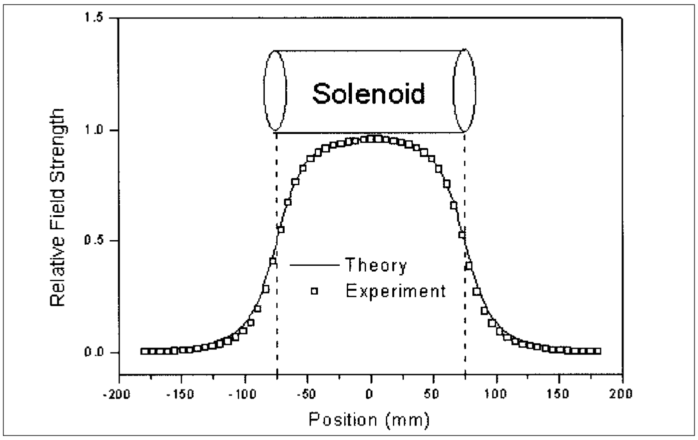


**Fig. 3. Measured $\varepsilon_{\mathrm{rms}}$ plotted against the position along the axis of solenoid. The solid line represents computer-generated theoretical data, while the square represents the experimental data after normalization to its value at the center. The two dash lines represent the edge of the solenoid.**

The multimeter can also be connected directly in series with the pick-up coil to measure the current therein. After the measured current is normalized, the results are similar. An oscilloscope can directly detect the induced emf in the solenoid. However, a simple multimeter suffices to determine the relative magnetic-field strength along the axis of a long finite solenoid, and it works well.

## References

1. See, for example, D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics*, 6th ed. (Wiley, New York, 2001), p. 698; R. Wolfson and J.M. Pasachoff, *Physics with Modern Physics for Scientists and Engineers*, 3rd ed. (Addison-Wesley, 1999), p. 785; and R.A. Serway, *Physics for Scientists and Engineers with Modern Physics*, 4th ed. (Sanders, Philadelphia, 1996), p. 875.
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3. E. Hecht, *Physics: Calculus* (Brooks/Cole, 1996), p. 799.
4. E.R. Dietz and R.W. Keith, "Inexpensive magnetic field mapping probe," *Phys. Teach.* **35**, 112 (Feb. 1997).