**BAN630 – Optimization Methods for Business Analytics**

**Exam I**

**Cal State East Bay**

**I, (student’s name and NetID) fully understand the university’s policy on academic dishonesty and agree to abide by this policy.**

**Policy on Academic Dishonesty**:

By enrolling in this class the student agrees to uphold the standards of academic integrity described in the catalog at <http://www.csueastbay.edu/ecat/current/i-120grading.html#section12>.

Cheating will result in: 1) a zero score and the loss of all grading options; and/or 2) an "F" grade; and/or 3) referral to the Academic Vice President for expulsion from the University.

**Problem 1**. A farmer owns 450 acres of land. He is going to plant each acre with wheat or corn. Each acre planted with wheat yields $4000 profit, requires three workers, and requires two tons of fertilizer. Each acre planted with corn yields $2000 profit, requires two workers, and requires four tons of fertilizer. There are currently 1000 workers and 1200 tons of fertilizer available. To hedge against risk, the farmer wants to sure that the acres of wheat do not exceed the acres of corn by 150 acres, vice versa.

**W: acres of wheat; C: acres of corn**

**Max 4000W + 2000C**

**S.T. W + C <= 450; 3W + 2C <= 1000; 2W + 4C <= 1200; W – C <= 150; C – W <= 150; W, C >= 0**





Based on the Solver outputs above, please answer the following questions:

1. Due to the change in market conditions, the profit of wheat per acre increases to $5000 while the profit of corn per acre drops to $1000 per acre. What will be the new optimal profit and why?

**1000/inf + 1000/6000 < 1. The optimal solution remains the same.**

**260\*5K + 110\*1K = $1.41 million**

1. Show that the shadow price of worker/labor is $1200.

**Suppose there is one more worker. (It’s within allowable range). The two binding constraints remain to be 3W + 2C = 1001 and W – C = 150. Solving the two equations gives us new optimal solution: (260.2, 110.2). The resulting optimal profit is $1,261,200, an increase of $1,200. That is, the shadow price of worker is $1,200.**

1. The farmer has sensed more market risk so that she wants to sure that the acres of wheat do not exceed the acres of corn by 200 acres, vice versa. In addition, she expects that 50 workers will take leave because of the forthcoming holiday. What would be the impact of the above changes on the optimal profit? Explain your answer.

**50/183.33 + 50/inf + 50/550 < 1. Shadow prices remain the same.**

**400\*50 + 0\*50 – 1200\*50 = 20K – 60K = -40K.**

1. Suppose the farmer would like to grow potato as well, which generates a profit of $3000 per acre, but requires 2 workers and 4 tons of fertilizer per acre. Will it be a good idea to grow potato? Why or why not?

**Reduced cost of potato = 3000 – 1\*0 - 2\*1200 – 4\*0 = 600. Yes.**

1. The farm workers are unionized. The worker union has made a proposal to this farmer. The union can provide 250 more workers for this farmer but each worker has to be paid $800. It is a take-it-or-leave-it deal. In other words, the farmer can only accept or reject all 250 workers. Should the farmer accept the union’s proposal? Why or why not?

**The allowable increase of labor is 200. In the worst case scenario, the shadow price of labor is zero. 200\*1200 – 250\*800 = 240K – 200K = 40K.**

**The farmer can make at least $40K more in profit. So, yes.**

***Problem 2***. Candy Kane Cosmetics (CKC) produces Leslie Perfume, which requires chemicals and labor. Two production processes are available. Process 1 transforms 0.3 unit of labor and 0.4 unit of chemicals into an ounce of perfume. Process two transforms 0.2 unit of labor and 0.5 unit of chemicals into an ounce of perfume. It costs CKC $12 to purchase a unit of labor and $10 to purchase a unit of chemicals. Each year up to 5,000 units of labor and 10,000 units of chemicals can be purchased. In the absence of advertising, CKC believes it can sell 15,000 ounces of perfume. To stimulate demand for Leslie, CKC can hire the beautiful model Jenny Nelson. Jenny is paid $400 per hour for up to 150 hours. Each hour Jenny works for the company is estimated to increase the demand for Leslie Perfume by 50 ounces. Each ounce of Leslie Perfume sells for $50. Formulate an algebraic LP model that helps determine how CKC can maximize its profit by clearly defining all your variables/notations and formulating the objective function and all the constraints.

Step 1: Clearly define all the decision variables (notations)

: ounces of perfume made by process , = 1, 2.

: the hours to hire Jenny Nelson.

Step 2: Formulate the objective function

MAX

Step 3: Formulate all the constraints

s.t.:

Non-negative constraints

***Problem 3***. Based on Rothstein (1973). The Springfield City Police Department employs 30 police officers. Each officer works five days per week. The crime rate fluctuates with the day of the week, so the number of police officers required each day depends on the day of the week, as follows: Saturday, 28; Sunday, 18; Monday, 18; Tuesday, 24; Wednesday, 25; Thursday, 16; and Friday, 21. The police department wants to schedule police officers to minimize the number whose days off are not consecutive. Formulate an algebraic LP model that helps determine how to accomplish this goal by clearly defining all your variables/notations and formulating the objective function and all the constraints.

Step 1: Clearly define all the decision variables (notations)

Let , = 1(Sa), 2(Sun.), 3(Mon.), 4, (Tu.), 5(Wed.), 6(Th.), and 7(Fri.).

: the number of offices whose days off are day , of a week, < .

: There are 21 decision variables from to .

Step 2: Formulate the objective function

MIN

Step 3: Formulate all the constraints

s.t.:

Non-negative and integer constraints.

***Problem 4***. Suppose it costs $30,000 to purchase a new car. The annual operating cost and resale value of a used car are shown below. Assume that you presently have a new car. On this page, formulate an algebraic linear programming model to help determine a replacement policy that minimizes your net costs of owning and operating a car for the next six years. On the next page, solve for the optimal solution with dynamic programming. (You need one and only one car every year till the end of year 6.)

|  |  |  |
| --- | --- | --- |
| Age of car | Resale value | Operating Cost |
| 1 | $23,000 | $600 |
| 2 | $18,000 | $1,000 |
| 3 | $15,000 | $1,600 |
| 4 | $10,000 | $2,400 |
| 5 | $5,000 | $3,200 |
| 6 | $3,000 | $4,400 |

Step 1: Clearly define all the variables (parameters)

: buy a new car at the beginning of year and keep it until the beginning of year ; .

Step 2: Formulate the objective function

MIN

Step 3: Formulate all the constraints

s.t.:

: binary

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Age of car | Resale value | Operating Cost | Cumulative Operating cost | Total cost of ownership |
| 1 | $23,000 | $600 | 600 | 30000+600-23000= 7600 |
| 2 | $18,000 | $1,000 | 1600 | 30000+1600-18000=13600 |
| 3 | $15,000 | $1,600 | 3200 | 30000+3200-15000 = 18200 |
| 4 | $10,000 | $2,400 | 5600 | 30000+5600-10000 = 25600 |
| 5 | $5,000 | $3,200 | 8800 | 30000+8800-3200 = 338000 |
| 6 | $3,000 | $4,400 | 13200 | 30000+13200- 3000 = 40200 |

***Problem 5***. Solve Problem 4 for the optimal solution(s) with dynamic programming.

Let be the lowest total cost from the beginning of year to the end of year 6 (same as the beginning of year 7). .

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. (If we were at node 5, i.e., we purchased a new car at the beginning of year 5, then the best decision is to keep the new car for 2 years till the end of year 6.)

. (If we were at node 4, i.e., we purchased a new car at the beginning of year 4, then the best decision is to keep the new car for 3 years till the end of year 6.)

. (If we were at node 3, i.e., we purchased a new car at the beginning of year 3, then the best decision is to keep the new car for 4 years till the end of year 6.)

. (If we were at node 2, i.e., we purchased a new car at the beginning of year 2, then the best decision is to keep the new car for for 3 or 4 years.)

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The optimal strategy is to keep the car for 3 years and purchase another new car at the beginning of year 4 and keep it till the end of year 6. The total cost is $36400.