Concordia University
Comp 232 Sample Review Questions

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1. State truth value of: If $1+1=2$ or $1+1=3$ then $2+2=3$ and $2+2=4$.

☐ **True** ☐ **False** ☐ **Justify**

2. Are the following propositions logically equivalent ? $p \to (\neg q \wedge r)$, $\neg p \vee \neg(r \to q)$.

☐ **Yes** ☐ **No** ☐ **Justify**

3. Determine whether the following proposition is a tautology: $((p \to \neg q) \wedge q) \to \neg p$

☐ **Tautology** ☐ **Not Tautology** ☐ **Justify**

4. $P(x)$ represents $x + 2y = xy$. What is the truth value of each of the following ?

a) $\exists y \forall x P(x, y)$

☐ **True** ☐ **False** ☐ **Justify**

b) $\neg \forall x \exists y \neg P(x, y)$

☐ **True** ☐ **False** ☐ **Justify**

5. $P(m, n)$ means $m \leq n$, where the domain of discourse for m and n is the set of non negative integers. What is the truth value of the following statements ?

a) $\exists n \forall m P(m, n)$

☐ **True** ☐ **False** ☐ **Justify**

b) $\forall m \exists n P(m, n)$

☐ **True** ☐ **False** ☐ **Justify**

6. Are the following statements valid ?

a) $\forall x \forall y [P(x) \wedge \neg Q(y)] \equiv \forall x P(x) \wedge \neg \exists y Q(y)$

☐ **Valid** ☐ **Not Valid** ☐ **Justify**

b) $\forall x \forall y [P(x) \vee Q(y)] \equiv \forall x P(x) \vee \forall y Q(y)$

☐ **Valid** ☐ **Not Valid** ☐ **Justify**

7. Variable x represents students, y represents courses. $M(y)$ rep. y is a math course, $F(x)$ rep. x is a freshman, $B(x)$ rep. x is a full-time student, $T(x, y)$ rep. x is taking y. Write the statement in good English without using variables in your answers.

a) $\forall x \exists y T(x, y)$

b) $\exists x \forall y T(x, y)$

c) $\forall x \exists y [(B(x) \wedge F(x)) \to (M(y) \wedge T(x, y))]$

8. Suppose the variables x and y represent real numbers, and $L(x, y) : x < y$, $Q(x, y) : x = y$, $E(x) : x$ is even, $I(x) : x$ is an integer. Write the statement using these predicates and any needed quantifiers.

a) Every integer is even.

b) If $x < y$, then x is not equal to y.

c) There is no largest real number.

9. Determine whether the following argument is valid or not valid:

She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

☐ Valid ☐ Not Valid ☐ Justify

10. Place the correct symbol from the list $\subseteq$, $=$, $\supseteq$ between each pair of sets below

a) $A \cup B$, $A \cup (B - A)$
b) $A \cup (B \cap C)$, $(A \cup B) \cap C$
c) $(A - B) \cup (A - C)$, $A - (B \cap C)$
d) $(A - C) - (B - C)$, $A - B$

11. Suppose $f : \quad R \to Z$ where $f(x) = \lceil 2x - 1 \rceil$.

a) Is f one to one ?

☐ Yes ☐ No ☐ Justify

b) Is f onto Z ?

☐ Yes ☐ No ☐ Justify

12. Suppose $g : \quad R \to R$ where $g(x) = \lfloor \frac{x-1}{2} \rfloor$. List the answer for each.

a) If $S = \{x | 1 \leq x \leq 6\}$, find $g(S)$

b) If $T = \{2\}$, find $g^{-1}(T)$

13. For each of the following statements below state whether it is True or False:

a) For all integers a, b, c, if $a|c$ and $b|c$, then $(a + b)|c$.

☐ True ☐ False ☐ Justify

b) For all integers a, b, c, d, if $a|b$ and $c|d$ then $(ac)|(b + d)$.

☐ True ☐ False ☐ Justify

c) If a and b are rational numbers (not equal to zero), then a^b is rational.

☐ True ☐ False ☐ Justify

d) If $f(n) = n^2 - n + 17$, then $f(n)$ is prime for all positive integers n.

☐ True ☐ False ☐ Justify

e) If $a \equiv b (mod\ m^2)$ then $a \equiv b (mod\ m)$.

☐ True ☐ False ☐ Justify

14. List the answer(s) for each.

a) **Find the smallest integer** $a > 1$ **such that** $(a + 1) \equiv 2a \ mod \ 11$.

b) **Find integers** a **and** b **such that** $(a + b) \equiv (a - b) \ mod \ 5$.

c) **Solve for** a **if** $a = (5^4 \ mod \ 7)^3 \ mod \ 13$.

15. List a complete proof for each proposition showing all steps with references.

a) **Consider the statement: If 7n+4 is an even Integer then n is an even Integer. Prove this statement two ways: by Contraposition and by Direct methods.**

b) **Prove that given a non negative Integer** n **, there is a unique non negative Integer** m **such that:** $m^2 \leq n < (m + 1)^2$

c) **Use the Principle of Mathematical Induction to prove that** $5|(7^n - 2^n)$ **for all** $n \geq 0$.

d) **Let** $a_1 = 2, a_2 = 9$ **and** $a_n = 2a_{n-1} + 3a_{n-2}$ **for** $n \geq 3$. **Prove that** $a_n \leq 3^n$ **for all positive integers** n. **Use Strong Induction.**

16. **If Relation** R **is on set** $\{a, b, c, d\}$ **represented by** $M_R = \begin{matrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{matrix}$ **determine if R is:**

a) **Reflexive**

☐ **True** ☐ **False** ☐ **Justify**

b) **Symmetric**

☐ **True** ☐ **False** ☐ **Justify**

c) **Antisymmetric**

☐ **True** ☐ **False** ☐ **Justify**

d) **Transitive**

☐ **True** ☐ **False** ☐ **Justify**

17. **If Relation** R **is on the set of all integers and** xRy **iff** $x \equiv y \ mod \ 7$ **determine if R is:**

a) **Reflexive**

☐ **True** ☐ **False** ☐ **Justify**

b) **Symmetric**

☐ **True** ☐ **False** ☐ **Justify**

c) **Antisymmetric**

☐ **True** ☐ **False** ☐ **Justify**

d) **Transitive**

☐ **True** ☐ **False** ☐ **Justify**

18. If Relation R is on the set of all integers and $(x, y) \in R$ iff $x \geq y^2$ determine if R is:

a) Reflexive

☐ True ☐ False ☐ Justify

b) Symmetric

☐ True ☐ False ☐ Justify

c) Antisymmetric

☐ True ☐ False ☐ Justify

d) Transitive

☐ True ☐ False ☐ Justify

19. Consider R and S are relations on $\{a, b, c, d\}$, where $R = \{(a, b), (a, d), (b, c), (c, c), (d, a)\}$ and $S = \{(a, c), (b, d), (d, a)\}$ Find value of each:

a) R^2

b) R^3

c) $S \circ R$

d) The transitive closure of R

20. Suppose A is the set composed of all ordered pairs of positive integers. Let R be the relation defined on A where $(a, b)R(c, d)$ means that $a + d = b + c$. Is R an equivalence relation ?

☐ True ☐ False ☐ Justify

21. Find the value of each:

a) The smallest equivalence relation on $\{1, 2, 3\}$ that contains $(1, 2)$ and $(2, 3)$.

b) The smallest partial order relation on $\{1, 2, 3\}$ that contains $(1, 1), (3, 2), (1, 3)$

22. Let R be the relation defined on the set of integers defined by $(a, b) \in R$ if and only if $a \geq b$. Is R a partial order ?

☐ True ☐ False ☐ Justify

Answers

1. False 2.Yes 3. Tautology 4 a) False 4 b) False 5 a) False 5 b) True 6 a) Valid 6 b) Valid 7 a) Every student is taking a course 7b) Some student is taking every course 7 c) Every full-time freshman is taking a math course 8 a) $\forall x(I(x) \to E(x))$ **8 b)** $\forall x \forall y(L(x,y) \to \neg Q(x,y))$ **8 c)** $\forall x \exists y L(x,y)$ **9. Valid 10 a)** $=$ **10 b)** $\supseteq$ **10 c)** $=$ **10 d)** $\subseteq$ **11 a) No 11 b) Yes 12 a)** $\{0,1,2\}$ **12 b)** $5 \le x < 7$ **13 a) False:** $a = b = c = 1$ **13 b) False:** $a = b = 2, c = d = 1$ **13 c) False** $(\frac{1}{2})^{\frac{1}{2}} = \frac{\sqrt{2}}{2}$ **which is not a Rational 13 d) False ,** $f(17)$ **is divisible by** 17 **13 e) True 14 a) 12 14 b)** $b = 0, \pm 5, \pm 10, \pm 15, \ldots$ **;** a **any integer 14 c) 8 15 a) 15 b) proofs see below 16 a) True 16 b) False 16 c) False 16 d) False 17 a) True 17 b) True 17 c) False 17 d) True 18 a) False 18 b) False 18 c) True 18 d) True 19 a)** $\{(a,a),(a,c),(b,c),(c,c),(d,b),(d,d)\}$ **19 b)** $\{(a,b),(a,c),(a,d),(b,c),(c,c),(d,a),(d,c)\}$ **19 c)** $\{(a,a),(a,d),(d,c)\}$ **19 d)** $\{(a,a),(a,b),(a,c),(a,d),(b,c),(c,c),(d,a),(d,b),(d,c),(d,d)\}$ **20 Yes : Reflexive:** $a+b=b+a$**; Symmetric: if** $a+d=b+c$**, then** $c+b=d+a$**; Transitive: if** $a+d=b+c$ **and** $c+f=d+e$**, then** $a+d-(d+e)=(b+c)-(c+f)$**, therefore** $a-e=b-f$**, or** $a+f=b+e$**. 21 a)** $\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ **21 b)** $\{(1,1),(2,2),(3,3),(3,2),(1,3),(1,2)\}$ **22** T

15a) Hint: For the Direct proof use Backward reasoning to decide on the initial form of $7n+4$

15b) There are two parts to the proof. Hint: Use Backward reasoning to decide on the relationship between m **and** n **in the Existence part of the proof.**
Use a proof by Contradiction in the Uniqueness part of the proof

15c) Prove $P(n):\ 5 \mid (7^n - 2^n) \forall n \ge 0$

Step1 (Base case) Prove $P(1):\ 5 \mid (7^1 - 2^1)$

Proof $5 \mid 5 \to 5 \mid (7-2) \to 5 \mid (7^1 - 2^1) \to P(1)$

Step2 (Inductive hypothesis) Assume $P(k):\ 5 \mid (7^k - 2^k)$

Step3 (What must be proved in the inductive Step4)

Prove $P(k) \to P(k+1):\ 5 \mid (7^k - 2^k) \to\ 5 \mid (7^{k+1} - 2^{k+1})$

Step4 (Proof of the inductive step) Prove $P(k+1):\ 5 \mid (7^{k+1} - 2^{k+1})$

Proof

$P(k) \to 5 \mid (7^k - 2^k) \to 5 \mid 7(7^k - 2^k)$	**Assumption, then Def. of Division**
$P(1) \to 5 \mid (7-2) \to 5 \mid 2^k(7-2)$	$P(1)$**, then Def. of Division**
$\to 5 \mid [7(7^k - 2^k) + 2^k(7-2)]$	**Addition, Def. of Division**
$\to 5 \mid [7^{k+1} - 7 \times 2^k + 7 \times 2^k - 2^{k+1}]$	**Multiplication**
$\to 5 \mid [7^{k+1} - 2^{k+1}]$	**Cancel**
$\to P(k+1)$	
$\to P(n):\ 5 \mid (7^n - 2^n)\ \forall n \ge 0$	**By Mathematical Induction**

15 c) Prove $P(n): a_n \le 3^n\ \forall n \ \epsilon\ Z^+$ **Using Strong Induction**

Step1 (Base cases) Prove $P(1): a_1 \le 3^1$ **and** $P(2): a_2 \le 3^2$

Proof LHS $= a_1 = 2$**, RHS** $= 3^1 \to a_1 \le 3^1 \to P(1)$

LHS $= a_2 = 9$**, RHS** $= 3^2 \to a_2 \le 3^2 \to P(2)$

Step2 (Inductive hypothesis) Assume $P(k): a_k \le 3^k$ **for** $1 \le k < n$ **where** $n \ge 3, n \ \epsilon\ Z^+$

Step3 (What must be proved in the inductive Step 4)

Prove $P(k) \to P(k+1):\ a_k \le 3^k$ **for** $1 \le k < n \to a_{k+1} \le 3^{k+1}$

Step4 (Proof of the inductive step) Prove $P(k+1): a_{k+1} \le 3^{k+1}$

Proof

a_{k+1}	$= 2a_k + 3a_{k-1}$	**Since** $k+1 \ge 3$ **use recursive definition of** a_n
	$\le 2 \times 3^k + 3 \times 3^{k-1}$	**By Assumption replace** a_k **and** a_{k-1}
	$= 2 \times 3^k + 3^k$	**Multiplication**
	$= 3 \times 3^k$	**Addition**
	$= 3^{k+1}$	**Multiplication**
$\to a_{k+1} \le 3^{k+1}$		
$\to P(k+1)$		
$\to P(n): a_n \le 3^n\ \forall n \ \epsilon\ Z^+$		**Using Strong Induction**