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CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2011	2
Instructors:		**Course Examiners**
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Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[10] 1. a. Sketch the graph of the function

$$f(x) = \begin{cases} 1 & x \le 1 \\ \dfrac{|1-x|}{1-x} & 1 < x \le 2 \\ x-3 & x > 2 \end{cases}$$

and find the definite integral $\int_0^3 f(x)\,dx$ in terms of area (do not antidifferentiate).

b Use the Fundamental Theorem of Calculus to calculate the derivative of

$F(x) = \int_{-x^2}^{1} e^{1-t^2}\,dt$,

and determine whether F is increasing or decreasing at $x = 1$.

[16] 2. Find the following indefinite integrals:

(a) $\displaystyle\int \frac{\cos^3(x)}{\sin^3(x)}\,dx$ (b) $\displaystyle\int x^2 \sin(2x)\,dx$ (c) $\displaystyle\int \frac{e^x}{e^{2x}-1}\,dx$

[18] 3. Evaluate the following definite integrals (give the exact answers):

(a) $\displaystyle\int_{-2}^{2} \frac{x^2+2}{x^2+4}\,dx$ (b) $\displaystyle\int_0^{\pi/4} \sqrt{4+5\tan(x)}\,\sec^2(x)\,dx$ (c) $\displaystyle\int_0^3 x\sqrt{1+x}\,dx$

[8] 4. Evaluate the given improper integral or show that it diverges:

$$\text{(a)} \int_e^{\infty} \frac{dx}{x \ln(x)} \qquad \text{(b)} \int_{-1}^{0} \frac{dx}{(1+x)^{3/4}}$$

[15] 5. a. Sketch the curves $y = x(x^2 - 3)$ and $y = x$, and find the area enclosed.

b. Find the volume of a solid obtained by rotating the region bounded by the curve $y = \sin(x)\cos(x)$ and the lines $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$ about the x-axis.

c. Find the exact average value of $f(x) = \sqrt{16 - x^2}$ on the interval $[0, 4]$.

[6] 6. Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

$$\text{(a)} \quad a_n = \frac{(3^n + 1)^2}{6^n} \qquad \text{(b)} \quad a_n = \ln(1 + 2n^2) - \ln(30 + 2n^2)$$

[12] 7. Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally :

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{1+n^3}}{n^2} \qquad \text{(b)} \sum_{n=0}^{\infty} \frac{(-3)^n}{5 + e^n} \qquad \text{(c)} \sum_{n=2}^{\infty} \frac{1}{n \ln^2(n)}$$

[6] 8. Find (a) the radius of convergence, and (b) the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)\,2^n}$.

[8] 9. (a) Use the integrability of the power series to express the function $F(x) = \int_0^x \left(\sum_{n=1}^{\infty} n\, t^{2n-1} \right) dt$ as an elementary function (i.e. sum the series for $F(x)$ within the radius of its convergence).

(b) Find the MacLaurin series for the function $x e^{-x^2}$.
(Hint: start with the series for e^z then replace z by $-x^2$)

[5] **Bonus question.** If we know that a power series about $a = -1$ is convergent at $x = 3$, can we claim convergence of the series also at $x = -4$? Explain why we can, or give a counter example if we cannot.