CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section**
Mathematics	205	AA
Examination	**Date**	**Pages**
Final	June 2012	2
Instructor:		**Course Examiners**
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Special Instructions:	This is a closed-book examination, no notes. Only approved calculators are allowed. Solve problems neatly for full marks.	

MARKS

[12] **1.** **(a)** Sketch the graph of a function defined piecewise as

$$f(x) = \begin{cases} |x+1| - 1 & \text{if} \quad x < 0 \\ \sqrt{4 - x^2} & \text{if} \quad 0 \le x \le 2 \end{cases}$$

(b) For $f(x)$ given in (a), calculate $\int\limits_{-2}^{2} f(x)\,\mathrm{d}x$ using the definition of definite integral as the signed area between the graph of $f(x)$ and the x axis.

(c) Use the Fundamental Theorem of Calculus to find the derivative of $F(x) = \int\limits_{\sqrt{x+3}}^{2} e^{4-t^2}\,\mathrm{d}t$, and calculate the exact value of $F'(x)$ at $x = 1$.

[15] **2.** Calculate the following indefinite integrals:

(**a**) $\displaystyle\int \frac{\sin(t)}{2 - \cos^2(t)}\,\mathrm{d}t$. (**b**) $\displaystyle\int x \ln^2(x)\,\mathrm{d}x$. (**c**) $\displaystyle\int 4\cos^4(x)\,\mathrm{d}x$.

[10] **3.** Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition:

(**a**) $f(x) = (1 + e^x)^2$, $F(0) = 2$. (**b**) $f(x) = \dfrac{x}{x^2 - 2x - 3}$, $F(1) = 0$.

[12] **4.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^{\pi/4} \frac{\sec^2(x)}{4 + \tan^2(x)}\,\mathrm{d}x$. (**b**) $\displaystyle\int_0^{3} t\sqrt{1+t}\,\mathrm{d}t$.

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_2^\infty \frac{\mathrm{d}x}{x\,\ln^2(x)}$ (**b**) $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(x-1)^{3/2}}$.

[15] **6.** **(a)** Sketch the curves $y = 1 + 2x - x^2$ and $y = |x - 1|$, and find the area enclosed.

(b) Find the volume of a solid obtained by rotating the region bounded by the curve $y = xe^{-x}$ and the lines $y = 0,\ x = 0$ and $x = 1$ about the $x-$axis.

(c) Find the average value of the function $f(x) = \sin(x)\cos^3(x)$ on the interval $[0, \pi/2]$.

[8] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(**a**) $\displaystyle a_n = \left(2 - \frac{1}{n}\right)\sqrt{\frac{3n+1}{n-1}}$ (**b**) $\displaystyle a_n = \frac{\ln(n^3 - 1)}{n+1}$

[10] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\sqrt{\frac{4n-1}{1+n^2}}$ (**b**) $\displaystyle\sum_{n=2}^{\infty} n\,e^{-n^2}$

[10] **9.** **(a)** Find the radius of convergence and the interval of the series

$$\sum_{n=0}^{\infty}\frac{(x+2)^{3n}}{8^n}$$

(b) Find the Maclaurin series for $f(x) = \dfrac{\ln(1+x^2)}{x}$.

(Hint: start with the series for $\ln(1+t)$ assuming $t = x^2$.)

[5] **Bonus Question.** Let $f(x) = \sqrt{4x - x^2}$.

(**a**) Determine the domain $[a, b]$ of f and graph this function.

(**b**) Calculate the definite integral of f over its domain: $\int_a^b f(x)\,\mathrm{d}x$.