Lecture 1
Ch 5. Integrals
5.1 Areas and Distances

Area Problem. Find the area of the region S that lies under the curve $y = f(x)$ from a to b.

Definition 1. The area A of the region S that lies under the graph of the continuous function f from a to b is the limit of the sum of areas of approximating rectangles

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} (f(x_1^*)\triangle x + f(x_2^*)\triangle x + \cdots + f(x_n^*)\triangle x)$$

where $\triangle x = \frac{b-a}{n}$ and $x_{i-1} \le x_i^* \le x_i$ for $i = \overline{1, n}$.

Sigma Notation

$$\sum_{i=1}^{n} a_1 = a_1 + a_2 + \cdots + a_n$$

Algebraic Rules for Finite Sums

1. Sum Rule: $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
2. Difference Rule: $\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$
3. Constant Multiple Rule: $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$
4. Constant Value Rule: $\sum_{i=1}^{n} c = c\sum_{i=1}^{n} 1 = cn$

Area in Sigma Notation

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\triangle x$$

Some Useful Formulas for Finite Sums

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Distance Problem. Find the distance traveled by an object during a certain time period if the velocity is known at all the time.

Geometrically, distance = area under the graph of velocity.

Example. A rainstorm hit Portland, Maine, in October 1996, resulting in record rainfall. The rainfall rate $R(t)$ on October 21 is recorded in cm per hour, in the following table, where t is the number of hours since midnight. Compute the total rainfall during this 24 hour period and indicate on the graph how this quantity can be interpreted as an area.

$t(h)$	0-2	2-4	4-9	9-12	12-20	20-24
$R(t)(cm/h)$	0.5	0.3	1.0	2.5	1.5	0.6