CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Duration**
Midterm Test	05 March, 2016	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks**	

1. (10 marks): **a.** Sketch the graph of the function

$$f(x) = \begin{cases} 1+\sqrt{4-x^2} & -2 \le x \le 2 \\ 3-x & 2 < x \end{cases}$$

on the interval $[-2, 6]$, and find the definite integral $\int_{-2}^{6} f(x)\,dx$ in terms of area (do **not** antidifferentiate).

b. Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{0}^{1-x^2} (1-t)\,e^{-t^2}\,dt$, and determine g whether F is increasing or decreasing at $x = 1$.

2. (6 marks): Find $h(x)$ if $h'(x) = \sec^2(x)\sqrt{1+\tan(x)} + \dfrac{2x}{x^2+1}$ and $h(0) = 4$.

3. (10 marks): Calculate the following indefinite integrals

$$\textbf{(a)} \quad \int \frac{x+1}{x^3+4x}\,dx \qquad \textbf{(b)} \quad \int \frac{\ln^2(x)}{x^2}\,dx$$

4. (12 marks): Evaluate the following definite integrals (*do not approximate*):

$$\textbf{(a)} \quad \int_{0}^{\frac{\ln 3}{2}} \frac{e^{2x}}{e^{4x}+9}\,dx \qquad \textbf{(b)} \quad \int_{0}^{\pi/2} \cos^5(x)\,dx$$

5. (6 marks): Find the area of the region bounded by the graphs of $f(x) = 4 + 2x - x^2$ and $g(x) = x^2 - 2x - 2$.

6. (6 marks): Find the average value of the function $f = \dfrac{x}{1+2x}$ on the interval $[0, 4]$ (do not approximate).

Bonus. (3 marks): Given that $\int_{0}^{\pi} [f(x) + f''(x)] \sin x\,dx = 2$, and $f(\pi) = 1$, find $f(0)$.