Lecture 9
7.2 Trigonometric Integrals
Evaluating $\int \sin^m x \cos^n x dx$

Case $n = 2k + 1$.

$$\sin^m x \cos^{2k+1} x = \sin^m x \cos^{2k} x \cos x = \sin^m x(1 - \sin^2 x)^k \cos x$$

Since $\cos x dx = d \sin x$ substitution $u = \sin x$ results in

$$\int \sin^m x \cos^{2k+1} x dx = \int \sin^m x(1 - \sin^2 x)^k \cos x dx = \int u^m (1 - u^2)^m du$$

Case $m = 2k + 1$.

$$\sin^{2k+1} x \cos^n x = \sin^{2k} x \cos^n x \sin x = (1 - \cos^2 x)^k \cos^n x \sin x$$

Since $\sin x dx = -d \cos x$ substitution $u = \cos x$ results in

$$\int \sin^{2k+1} x \cos^n x dx = \int (1 - \cos^2 x)^k \cos^n x \sin x dx = -\int u^n (1 - u^2)^k du$$

Case both $m = 2k$ and $n = 2l$.

$$\int \sin^{2k} x \cos^{2l} x dx = \frac{1}{2^{k+l}} \int (1 - \cos 2x)^k (1 + \cos 2x)^l dx$$

Evaluating $\int \tan^m x \sec^n x dx$

Case $n = 2k$.

$$\tan^m x \sec^{2k} x = \tan^m x(1 + \tan^2 x)^{k-1} \sec^2 x$$

Since $\sec^2 x dx = d \tan x$ substitution $u = \tan x$ results in

$$\int \tan^m x \sec^{2k} x dx = \int \tan^m x(1 + \tan^2 x)^{k-1} \sec^2 x dx =$$

$$= \int u^m (1 + u^2)^{k-1} du$$

Case $m = 2k + 1$.

$$\tan^{2k+1} x \sec^n x = (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x$$

Since $\sec x \tan x dx = d \sec x$ substitution $u = \sec x$ results in

$$\int \tan^{2k+1} x \sec^n x dx = \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x dx =$$

$$= \int (u^2 - 1)^k u^{n-1} du$$

Evaluating $\int \sin mx \cos nx\,dx$, $\int \sin mx \sin nx\,dx$, $\int \cos mx \cos nx\,dx$

$$\sin mx \cos nx = \frac{1}{2}(\sin{(m-n)x} + \sin{(m+n)x})$$

$$\sin mx \sin nx = \frac{1}{2}(\cos{(m-n)x} - \cos{(m+n)x})$$

$$\cos mx \cos nx = \frac{1}{2}(\cos{(m-n)x} + \cos{(m+n)x})$$