Lecture 21

11.9 Representation of Functions as Power Series

Definition 1. A function f is said to be **analytical function at the number** a if it can be represented by a power series about a with a positive radius of convergence.

Differentiating and Integrating of Power Series

Theorem 1. Term by term differentiation and integration of power series

(1) If the series $\sum_{n=0}^{\infty} c_n(x-a)^n$ converges to the sum $f(x)$ on an interval $(a-R, a+R)$, that is $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ for $a-R < x < a+R$, then $f(x)$ is differentiable on $(a-R, a+R)$ and

$$f'(x) = \sum_{n=1}^{\infty} c_n n(x-a)^{n-1}$$

(1) If the series $\sum_{n=0}^{\infty} c_n(x-a)^n$ converges to the sum $f(x)$ on an interval $(a-R, a+R)$, then

$$\int f(x)dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1}$$