CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2018	2
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks.**	

MARKS

[10] **1.** **a.** Sketch the graph of the function

$$f(x) = \begin{cases} 3x & x \le 1 \\ x\dfrac{|x-5|}{x-5} & 1 < x < 3 \\ -3 & x \ge 3 \end{cases}$$

and find the definite integral $\int_0^5 f(x)\,\mathrm{d}x$ in terms of area (**do not antidifferentiate**).

b. Use the Fundamental Theorem of Calculus to calculate the derivative of

$F(x) = \int_0^{1-x^2} (1-t)\,e^{-t^2}\,\mathrm{d}t$,

and determine whether F is increasing or decreasing at $x = 1$.

[15] **2.** Find the following indefinite integrals:

(**a**) $\displaystyle\int \frac{\sin^3(x)}{\cos^5(x)}\,\mathrm{d}x$ (**b**) $\displaystyle\int (2x + x^2)\cos(2x)\,\mathrm{d}x$ (**c**) $\displaystyle\int \frac{x^2-8}{x^2-16}\mathrm{d}x$

[18] **3.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^{\ln 2} \frac{e^x}{e^{2x}+4}\mathrm{d}x$ (**b**) $\displaystyle\int_0^{\pi/4} \frac{\sec^2(x)}{\sqrt{1+8\tan(x)}}\,\mathrm{d}x$ (**c**) $\displaystyle\int_1^{e^2} x\ln x\,\mathrm{d}x$

[8] **4.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_e^{\infty} \frac{\mathrm{d}x}{x\,[\ln(x)]^{3/2}}$ (**b**) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{(1-x)^{5/4}}$

[16] **5.** **a.** Sketch the curves $y = x^3 - x$ and $y = 3x$, and find the area enclosed.

b. Find the volume of a solid obtained by rotating the region bounded by the curve $y = \sin(x)$ and the $x-$axis on the interval $0 \leq x \leq \pi$ about the line $y = 2$.

c. Find the exact average value of $f(x) = \sqrt{9 - x^2}$ on the interval $[-3, 3]$.

[6] **6.** Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

(**a**) $a_n = \dfrac{3^n + (-3)^n}{4^n}$ (**b**) $a_n = \ln(1 + 3n + 4n^2) - \ln(8 + 6n + 2n^2)$

[12] **7.** Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ (**b**) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2 + 1/10)^n}{(2 - 1/10)^n}$ (**c**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^3 + n}}{n^2}$

[6] **8.** Find **(a)** the radius of convergence, and **(b)** the interval convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{4^n\, n^2}$.

[9] **9.** **(a)** Use the integrability of the power series to express the function

$F(x) = \displaystyle\int_0^x \left(\sum_{n=1}^{\infty} n\, t^{n-1} \right) \mathrm{d}t$ as an elementary function

(i.e. sum the series for $F(x)$ within the radius of its convergence).

(b) Find the MacLaurin series for the function $f(x) = x^3 \sin(x^2)$.
(Hint: start with the series for $\sin z$ then replace z by x^2)

[5] **Bonus question.** If we know that $\sum_{n=1}^{\infty} a_n$ converges and each $a_n \neq 0$, can anything be said about the series $\sum_{n=1}^{\infty} 1/a_n$ - i.e. does it converge or diverge? Explain your answer.