CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2013	2
Instructors:	**Course Examiners**	
A. Boyarsky, J. Brody, L. El Hajj, S. Hachem, R. Mearns, O. Quijano Xacur, H. Proppe	A. Atoyan & H. Proppe	
Special Instructions:	Only Sharp EL 531 or Casio FX 300 MS calculators are allowed	

1 [10]. (a) Sketch a graph of the function

$$f(x) = \begin{cases} \sqrt{9-x^2} & 0 \le x \le 3 \\ |x+4| - 1 & -5 \le x < 0 \end{cases}$$

on the interval $-5 \le x \le 3$ and find the definite integral $\int_{-5}^{3} f(x)\,dx$ in terms of area (do not antidifferentiate).

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{-x^2}^{0} \cos(t^2)\,dt$,
and determine whether F is increasing or decreasing at $x = 1$.

2 [15]. Find the following indefinite integrals:

(a) $\int \frac{\sin x}{1+\cos^2(x)}\,dx$ (b) $\int \frac{x}{x^2+3x-4}dx$ (c) $\int \frac{(x^{3/2}-1)^2}{\sqrt{x}}dx$

3 [16]. Evaluate the following definite integrals (give the exact answers):

(a) $\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}}$ (b) $\int_{4}^{9} \frac{\ln x}{\sqrt{x}}dx$ (c) $\int_{0}^{\pi/4} \sec^2(\theta)\tan^2(\theta)\,d\theta$

4 [8]. Evaluate the given improper integral or show that it diverges:

(a) $\int_{0}^{\infty} xe^{-x^2}\,dx$ (b) $\int_{3}^{4} \frac{dx}{(4-x)^2}$

5. [18] (a) Sketch the curves $x = y^2 - 4y$ and $x = 2y - y^2$ and find the area enclosed by the two curves. (HINT: find first the points of intersection of the curves.)

(b) Sketch the curves defined by $y = x^2$ and $y = 2x$ and find the volume of the solid of revolution of the region bounded by these curves rotated about the axis $y = -1$.

(c) Find the average value of the function $f(x) = \ln x$ on the interval $[1, e]$.

6 [8]. Find the limit of the sequence $\{a_n\}$ as $n \to \infty$ or prove that it does not exist:

(a) $a_n = \sqrt{\dfrac{n+1}{9n+1}}$ (b) $a_n = \dfrac{(-1)^{n+1}n}{(\sqrt{n}+100)^2}$

7 [15]. Determine whether the series is divergent or convergent, and if convergent, then whether absolutely or conditionally convergent:

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}ne^{-n}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\cos\left(\frac{\pi}{n}\right)$ (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^{10}}{10^n}$

8 [5]. Find the radius and the interval of convergence of the series

$$\sum_{n=1}^{\infty}\frac{(x-4)^n}{n\,2^{n+1}}$$

9 [5]. (a) Derive the Maclaurin series of $f(x) = x^3 \sin 2x$
(HINT: start with the series for $\sin(z)$ and then let $z = 2x$).

Bonus question [5]. Suppose $\displaystyle\sum_{n-1}^{\infty} a_n$ is a convergent series. Prove that the series $\displaystyle\sum_{n-1}^{\infty} a_n^2$ also converges if $a_n \geq 0$ for all n. Also show that if the condition $a_n \geq 0$ is removed the series $\displaystyle\sum_{n-1}^{\infty} a_n^2$ may become divergent.