Lecture 15

11.2 Series

Definition 1. Series is a sum of infinitely many terms

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

a_n is the n-**th term** of series.

Definition 2. The sequence $\{S_n\}_{n=1}^{\infty}$ defined by $S_n = \sum_{i=1}^{n} a_i$ is called the **sequence of partial sums** of series. The number S_n is called the n-**th partial sum**.

Definition 3. If the sequence of partial sums $\{S_n\}_{n=1}^{\infty}$ converges to a limit L, we say that the series **converges** and its sum is L, that is

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_n = L$$

Definition 4. If the sequence of partial sums $\{S_n\}_{n=1}^{\infty}$ doesn't converge, we say that the series **diverges**.

Geometric Series

Definition 5. A series of the form $\sum_{n=1}^{\infty} ar^{n-1}$ is called a **geometric series**.

The number r is called the **common ratio** of the series.

$$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} 0 & \text{if } a = 0 \\ \frac{a}{1-r} & \text{if } a \neq 0 \text{ and } |r| < 1 \\ \infty & \text{if } a < 0 \text{ and } r \geq 1 \\ -\infty & \text{if } a > 0 \text{ and } r \geq 1 \\ \text{diverges} & \text{if } a \neq 0 \text{ and } r \geq -1 \end{cases}$$

Telescopic Series

Definition 6 A series of the form $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is called a **telescopic series**.

Harmonic Series

Definition 7 A series of the form $\sum_{n=1}^{\infty} \frac{1}{n}$ is called a **harmonic series**.

Theorems About Series

Theorem 1. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

Theorem 1. Divergence Theorem

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then the series $\sum_{n=1}^{\infty} a_n$ diverges.