

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Duration
Midterm Test	03 March, 2018	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	Show all your work for full marks	

1. (10 marks): a. Sketch the graph of the function

$$f(x) = \begin{cases} \sqrt{9-x^2} - 1 & -3 \leq x \leq 0 \\ |2x-3| - 1 & 0 < x \leq 2 \end{cases}$$

on the interval $[-3, 2]$, and find the definite integral $\int_{-3}^2 f(x) dx$ in terms of area (do **not** antidifferentiate).

- b. Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{-x^2}^0 (t-1) \cos^4(t+1) dt$, and determine whether F is increasing or decreasing at $x = 1/2$.

2. (6 marks): Find $G(x)$ if $G'(x) = \sec^3(x) \tan^3(x)$ and $G(0) = 1$.

3. (10 marks): Find the following indefinite integrals

$$(a) \int \ln(\sqrt{x}) dx \quad (b) \int \frac{x+1}{x^3+x} dx$$

4. (12 marks): Find the exact values of the following definite integrals (*do not approximate*):

$$(a) \int_0^a x\sqrt{a-x} dx, \text{ where parameter } a > 0 \quad (b) \int_0^{\pi/2} (2 - \sin x)^2 dx$$

5. (6 marks): Sketch the region bounded by the graph of $f(x) = 4x - x^2$ and the x-axis, and find the volume of the solid obtained by revolution of that region about the axis $y = -2$.

6. (6 marks): Find the average value of $f(x) = (x-3)^2$ on the interval $[2, 5]$. Sketch the graph of f , and draw a rectangle whose base is the interval $[2, 5]$ and whose height is the average value of f on that interval.

- Bonus. (3 marks):** Evaluate the limit by recognizing it as a Riemann sum for a function $f(x)$ on the interval $[0, 1]$ and then using integration.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right)$$