

**CONCORDIA UNIVERSITY**  
**Department of Mathematics & Statistics**

Course	Number	Sections
Mathematics	205	All
Examination	Date	Duration
Midterm Test	21 October, 2017	1 h 30 min
<b>Special</b>	Only approved calculators are allowed	
<b>Instructions:</b>	<b>Show all your work for full marks</b>	

**1. (6+5 marks):**

- (a) Write the sigma notation formula for the *left* Riemann sum  $L_n$  of the function  $f(x) = (1+x)^2$  on the interval  $[-1, 2]$  using  $n$  subintervals of equal length. Use that formula to calculate the integral  $\int_{-1}^2 f(x) dx$  as the limit of  $L_n$  at  $n \rightarrow \infty$ .

(Reminder:  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ , and  $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ .)

- (b) Graph the function  $f(x) = |x-3| - 1$ , and use it to evaluate the integral  $\int_{-2}^4 f(x) dx$  in terms of area.

**2. (5 marks):** Use the Fundamental Theorem of Calculus to find the derivative  $F'(x)$  of

$$F(x) = x^2 + \int_{-2x}^{2x} [3 + \sin(t^2)] dt.$$

**3. (10 marks):** Calculate the following indefinite integrals

(a)  $\int \frac{x}{x^2 - 3x + 2} dx$       (b)  $\int \cos^4(x) dx$

**4. (6 marks):** Find the antiderivative  $F(t)$  of  $f(t) = \ln^2(x)$  such that  $F(1) = 0$ .

**5. (12 marks):** Evaluate the following definite integrals (*do not approximate*):

(a)  $\int_0^{\ln 4} \frac{e^x}{e^{2x} + 16} dx$       (b)  $\int_0^3 x^2 \sqrt{1+x} dx$

**6. (6 marks):** Sketch the graphs of the functions  $y = |x| - 2$  and  $y = 4 - x^2$ , and find the area of the region enclosed by these graphs.

**Bonus. (3 marks):** Calculate the definite integral  $\int_{-\pi}^{\pi} \frac{\sin x}{1+x^2} dx$ .