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WeBWorK assignment due : 03/03/2019 at 11:59pm EST.

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1. (1 point) Use the Midpoint Rule to approximate

$$\int_{-2.5}^{4.5} x^3 dx$$

with $n = 7$.

Correct Answers:

- `91`

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2. (1 point) Consider the function $f(x) = \frac{x^2}{3} - 2$.

In this problem you will calculate $\int_0^4 \left(\frac{x^2}{3} - 2\right) dx$ by using the definition

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left[\sum_{i=1}^{n} f(x_i)\Delta x\right]$$

The summation inside the brackets is R_n which is the Riemann sum where the sample points are chosen to be the right-hand endpoints of each sub-interval.

Calculate R_n for $f(x) = \frac{x^2}{3} - 2$ on the interval $[0,4]$ and write your answer as a function of n without any summation signs. You will need the summation formulas on page 383 of your textbook (page 364 in older texts).

Hint: $x_i = \frac{4i}{n}$ and $\Delta x = \frac{4}{n}$.

$R_n =$ ______

$\lim_{n\to\infty} R_n =$ ______

Correct Answers:

- `-2*4 + 4**3*(n+1)*(2*n+1)/(6*(3)*n**2)`
- `-0.888888888888889`

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3. (1 point) Let $\int_{-4}^{5} f(x)dx = 2$, $\int_{-4}^{-1} f(x)dx = 2$, $\int_{2}^{5} f(x)dx = 4$.

Find $\int_{-1}^{2} f(x)dx =$ ______

and $\int_{2}^{-1} (2f(x) - 2)dx =$ ______

Correct Answers:

- `-4`
- `14`

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4. (1 point) If $f(x) = \int_3^{x^2} t^5 dt$

then

$f'(x) =$ ______

$f'(5) =$ ______

Solution: One way to solve this problem is to compute the definite integral and then take the derivative of the result. This is called the BITD (beat it to death) method. Luckly, there is a shortcut in this case. First, note that with $y = x^2$ we get the following:

$$f(y) = \int_3^y t^5\, dt$$

Then, applying the Fundamental Theorem of Calculus, remembering that since we substituted y for x^2 we will need to apply the chain rule, we get:

$$f'(y) = (y^5)dy = (x^2)^5(2x^1) = 2x^{11}$$

Then plugging 5 into the formula, we get the answer to question two as shown.

$$f'(5) = 2(5)^{11} = 97656250$$

Correct Answers:

- `2*x^11`
- `97656250`

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5. (1 point) Given

$$f(x) = \int_0^x \frac{t^2 - 36}{1 + \cos^2(t)} dt$$

At what value of x does the local max of $f(x)$ occur?

$x =$ ______

Solution:

Solution:

First, note that we don't need to do any computation to compute the first derivative, which we will use to check for local maxima and minima. By applying the Fundamental Theorem of Calculus, we see that:

$$f'(x) = \frac{x^2 - 36}{1 + \cos^2(x)}$$

Now, we can use this derivative to find the critical points of the function. We set this to zero and solve for x to get:

$$\frac{x^2 - 36}{1 + \cos^2(x)} = 0$$

$$x^2 - 36 = 0$$

$$(x + 6)(x - 6) = 0$$

$$x = 6 \text{ or } x = -6$$

Checking on either side of these two points shows that -6 is the local maximum for which we are looking.

Correct Answers:

- `-6`

6. (1 point) Evaluate the definite integral.
$\int_0^1 x^2 \sqrt[7]{e^x}\, dx =$ ____________________

Correct Answers:

- `e^(1/7)[7-2(7)^2+2(7)^3] -2(7)^3`

7. (1 point) Evaluate the definite integral.
$\int_1^7 \sqrt{t}\ln(t)\, dt =$ ____________________

Correct Answers:

- `2/3*7^(3/2)*ln(7)-4/9*[7^(3/2)-1]`

8. (1 point) Use the Fundamental Theorem of Calculus to carry out the following differentiation:
$\frac{\mathrm{d}}{\mathrm{d}x}\int_1^{\sqrt{x}} t^t \mathrm{d}t =$ ____________.

Correct Answers:

- `sqrt(x)**sqrt(x)/2/sqrt(x)`

9. (1 point) **Integration by Parts:** This is the most important integration technique we've discussed in this class. It has a wide range of applications beyond increasing our list of integration rules.

$\int z^3 \ln z \mathrm{d}z =$ ____________________.

$\int e^t \cos t \mathrm{d}t =$ ____________________.

$\int_0^{2\pi} \sin(x)\sin(x+1)\mathrm{d}x =$ ____________________.

Correct Answers:

- `z^4*(4*log(z)-1)/16`
- `1/2 exp(t) (cos(t) + sin(t))`
- `1.69740975483297`

10. (1 point) Use integration by parts to evaluate the integral.

$$\int xe^{4x} dx$$

Answer: ____________________ $+C$

Correct Answers:

- `0.25 * (x * e^(4 * x) - 0.25 * e^(4 * x))`

11. (1 point) Use integration by parts to evaluate the definite integral.

$$\int_1^6 \sqrt{t}\ln t dt$$

Answer: ____________________

Correct Answers:

- `2/3 * 6^(3/2) * ln(6) - 4/9 * (6^(3/2) - 1)`

12. (1 point) Evaluate the definite integral.

$$\int_1^8 \ln x^{30} dx$$

Answer: ____________________

Correct Answers:

- `30*( 8*ln(8)-8 -(1*ln(1)-1) )`

13. (1 point) Evaluate the indefinite integral.

$$\int \ln(x^2+18x+77)dx$$

____

Correct Answers:

- `(x + 7)*ln(x + 7) + (x + 11)*ln(x + 11) - 2*x`

14. (1 point)
Definition: The AREA A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x]$$

Consider the function $f(x) = \frac{\ln(x)}{x}, \quad 3 \le x \le 10$. Using the above definition, determine which of the following expressions represents the area under the graph of f as a limit.

- A. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{10}{n}\frac{\ln(3+\frac{10i}{n})}{3+\frac{10i}{n}}$
- B. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{7}{n}\frac{\ln(3+\frac{7i}{n})}{3+\frac{7i}{n}}$
- C. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{7}{n}\frac{\ln(\frac{7i}{n})}{\frac{7i}{n}}$
- D. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{10}{n}\frac{\ln(\frac{10i}{n})}{\frac{10i}{n}}$
- E. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\ln(3+\frac{7i}{n})}{3+\frac{7i}{n}}$

Correct Answers:

- `B`

15. (1 point)

Definition: The AREA A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x]$$

(a) Use the above Definition to determine which of the following expressions represents the area under the graph of $f(x) = x^3$ from $x = 0$ to $x = 1$.

- A. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right) \frac{1}{n}$
- B. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{5}{n}$
- C. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right) \frac{5}{n}$
- D. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$

(b) Evaluate the limit that is the correct answer to part (a). You may find the following formula for the sum of cubes helpful:

$$1^3 + 2^3 + 3^3 + ... + n^3 = \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Value of limit = ____________

Correct Answers:

- `D`
- `1/4`

16. (1 point)

Find the derivative of $f(x) = x\sin(x) + \cos(x) + C$ to complete the following integration formula:

$\int$ ____________ $dx = x\sin(x) + \cos(x) + C$

Correct Answers:

- `x*cos(x)`

17. (1 point)

Using an upper-case "C" for any arbitrary constants, find the general indefinite integral

$$\int 8x^{-3/4} dx$$

Integral = ____________

Correct Answers:

- `4*8*x^(1/4)+C`

18. (1 point)

Using an upper-case "C" for any arbitrary constants, find the general indefinite integral

$$\int (-10x^3 - 1x - 10)\, dx$$

Integral = ____________

Correct Answers:

- `1/4*-10*x^4+1/2*-1*x^2+-10*x+C`

19. (1 point)

Using an upper-case "C" for any arbitrary constants, find the general indefinite integral

$$\int (2 - \sqrt{x})^2\, dx$$

Integral = ____________

Correct Answers:

- `1/2*x^2-4/3*2*x^(3/2)+(2)^2*x+C`

20. (1 point)

Evaluate the integral

$$\int_1^2 \frac{5y + 1y^7}{y^3}\, dy$$

Integral = ____________

Correct Answers:

- `(31/5*1+1/2*5)`

21. (1 point)

Evaluate the integral

$$\int_1^9 \frac{5x - 10}{\sqrt{x}}\, dx$$

Integral = ____________

Correct Answers:

- `52/3*5+4*-10`

22. (1 point)

Evaluate the integral

$$\int_4^9 \left(-1\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 dx$$

Integral = ____________

Correct Answers:

- `2*(5)^2*ln(3)+10*-1*5+65/2*(-1)^2-2*(5)^2*ln(2)`

23. (1 point)

Evaluate the following integral by making the given substitution:

$$\int x^2\sqrt{x^3+8}\, dx, \quad u = x^3 + 8$$

Note: Any arbitrary constants used must be an upper-case "C".

____

Correct Answers:

- `2/9*(x^3+8)^(3/2)+C+c`

24. (1 point)
Evaluate the following integral by making the given substitution:

$$\int \frac{-9\sin(\sqrt{x})}{\sqrt{x}}\,dx, \quad u=\sqrt{x}$$

Note: Any arbitrary constants used must be an upper-case "C".

Correct Answers:

- `-2*-9*cos(sqrt(x))+C+c`

25. (1 point)
Evaluate the indefinite integral

$$\int -2(2-x)^6\,dx$$

Note: Any arbitrary constants used must be an upper-case "C".

Correct Answers:

- `-1*-2/7*(2-x)^7+C+c`

26. (1 point)
Evaluate the indefinite integral

$$\int \frac{-3(\ln(x))^2}{x}\,dx$$

Note: Any arbitrary constants used must be an upper-case "C".

Correct Answers:

- `-3/3*(ln(x))^3+C+c`

27. (1 point)
Evaluate the indefinite integral

$$\int -7\,\sqrt[3]{x^3+1}\,x^5\,dx$$

Note: Any arbitrary constants used must be an upper-case "C".

Correct Answers:

- `-7/7*(x^3+1)^(7/3)--7/4*(x^3+1)^(4/3)+C+c`

28. (1 point)
Evaluate the definite integral (if it exists)

$$\int_0^4 \frac{-5x}{\sqrt{1+2x}}\,dx$$

If the integral does not exist, type "DNE".

Correct Answers:

- `10*-5/3`

29. (1 point)
Find the average value of the function $g(x)=-4x^2\sqrt{1+x^3}$ on the interval [0, 2].

$g_{ave}=$ ______

Correct Answers:

- `26/9*-4`

30. (1 point)
Find the average value of the function $f(t)=-2\sec(t)\tan(t)$ on the interval $[0,\pi/4]$.

$f_{ave}=$ ______

Correct Answers:

- `-2*4/pi*(sqrt(2)-1)`

31. (1 point)
Evaluate the integral

$$\int_{\pi/2}^{3\pi/4} -4\sin^5(x)\cos^3(x)\,dx$$

Correct Answers:

- `-4*(-11/384)`

32. (1 point)
Evaluate the integral

$$\int_0^{\pi/2} -10\cos^2(x)\,dx$$

Correct Answers:

- `-10*(pi/4)`

33. (1 point)
Evaluate the integral

$$\int_0^{\pi/2} 2\sec^4(t/2)\,dt$$

Correct Answers:

- `8*2/3`

34. (1 point)
Evaluate the integral

$$\int -2\tan^4(x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

Correct Answers:

- `-2/3*tan(x)^3-(-2)*tan(x)+-2*x+C+c`

35. (1 point)
Evaluate the integral

$$\int -3\sin(3x)\cos(x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

Correct Answers:

- `-1*(-3)/8*cos(4*x)-(-3)/4*cos(2*x)+C+c`

36. (1 point)
Evaluate the integral

$$\int_0^{2\sqrt{3}} \frac{5x^3}{\sqrt{16-x^2}}\,dx$$

______

Correct Answers:

- `5*40/3`

37. (1 point)
Evaluate the integral

$$\int \frac{7}{x^2\sqrt{25-x^2}}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

______

Correct Answers:

- `-1*7/25*sqrt(25-x^2)/x+C+c`

38. (1 point)
Evaluate the integral

$$\int \frac{-6}{x^2\sqrt{16x^2-9}}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

______

Correct Answers:

- `-6/9*sqrt(16*x^2-9)/x+C+c`

39. (1 point)
Evaluate the integral

$$\int -10\sqrt{5+4x-x^2}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

______

Correct Answers:

- `-10*(-1/4*(-2*x+4)*(5+4*x-x^2)^(1/2)+9/2*asin(-2/3+1/3*x))+C+c`

40. (1 point)
Which of the following is the correct form of the partial fraction decomposition of $\frac{x^4}{x^4-1}$?

- A. $1+\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+1}$
- B. $-1+\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{x^2+1}$
- C. $-1+\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+1}$
- D. $1+\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{x^2+1}$

Correct Answers:

- `A`

41. (1 point)
Evaluate the integral

$$\int_0^1 \frac{1x-1}{x^2+3x+2}\,dx$$

______

Correct Answers:

- `-5*1*ln(2)+3*1*ln(3)`

42. (1 point)
Evaluate the integral

$$\int \frac{10x^2}{(x+1)^3}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

______

Correct Answers:

- `10*(ln(abs(x+1))+2/(x+1)-1/(2*(x+1)^2))+C+c`

43. (1 point) Calculate the following antiderivatives:

(a) $\int 13t-2t^5-9\,dt =$ ______ $+C$.

(b) $\int \frac{1}{u^{5/4}}+5.5\sqrt{u}\,du =$ ______ $+C$.

(c) $\int \frac{1}{5x^6}\,dx =$ ______ $+C$.

Correct Answers:

- `-2/6*t^6+13/2*t^2+(-9)*t`
- `4/(-1)*u^(-1/4)+5.5*2/3*u^(3/2)`
- `1/5*1/(-5)*x^(-5)`

44. (1 point) Evaluate the indefinite integral.

$\int 3\sec^2 x-2e^x\,dx =$ ______ $+C$.

Correct Answers:

- `3*tan(x)-2*e^x`

45. (1 point) Evaluate the integral by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.

$$\int_0^9 |9x-2|\,dx$$

______

Correct Answers:

- `346.944444444444`

46. (1 point) In a certain city the temperature (in degrees Fahrenheit) t hours after 9am was approximated by the function

$$T(t) = 80+13\sin\left(\frac{\pi t}{12}\right)$$

Determine the temperature at 9 am. ______
Determine the temperature at 3 pm. ______

Find the average temperature during the period from 9 am to 9 pm. ______

Correct Answers:

- `80`
- `93`
- `88.2760570407786`

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47. (1 point)

Find the average value of the function $h(r) = -27/(1+r)^2$ on the interval [1, 6].

$h_{ave} =$ ______________

Correct Answers:

- `-9*3/14`

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48. (1 point)

Find the average value of the function $f(t) = -4te^{-t^2}$ on the interval [0, 5].

$f_{ave} =$ ______________

Correct Answers:

- `-4/10*(1-exp(-25))`