Lecture 18
11.5 Alternating Series

Definition 1. The series

$$\sum_{n=1}^{\infty}(-1)^{n-1}a_n = a_1 - a_2 + a_3 - a_4 + \ldots$$

with $a_n > 0$ is called **alternating series**.

Theorem 1. The Alternating Series Convergence Test
If alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$, $a_n > 0$ satisfies

(i) $a_{n+1} \leq a_n$ for all n,

(ii) $\lim_{n \to \infty} a_n = 0$,

then the series is convergent.

Theorem 2. The Alternating Series Estimation

If $S = \sum_{n=1}^{\infty}(-1)^{n-1}a_n$ is the sum of alternating series that satisfies

(i) $a_{n+1} \leq a_n$ for all n,

(ii) $\lim_{n\to\infty} a_n = 0$,

then

$$|R_n| = |S - S_n| \leq a_{n+1}.$$