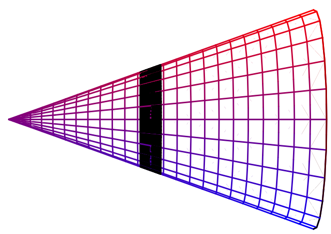
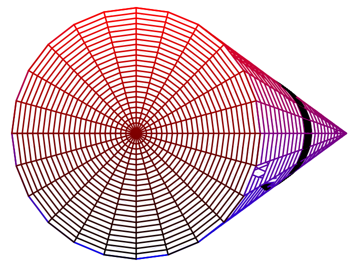
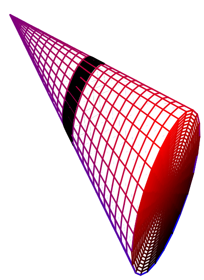
Class 7

1. Calculate the volume of the cone with circular base r and height h :

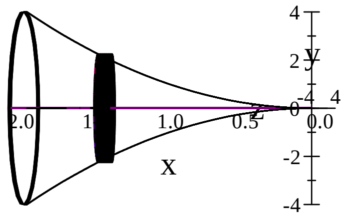
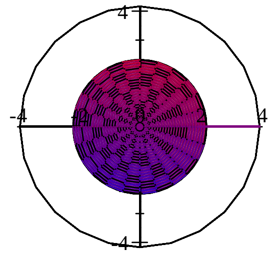
The elements are disks with volume: $V_i = \pi r_i^2 \Delta x$:$i\Delta x - (i+1)\Delta x \to r_i = \frac{ir}{n}$

$\Delta x = \frac{h}{n}$, go from $0 \to h$. $V_i = \frac{\pi r^2 h}{n^3} \sum_{i=0}^{n-1} i^2$[1] $\to \pi \sum_{i=0}^{n-1} r_i^2 \Delta x \to \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx = \frac{1}{3}\pi h r^2$



2. We can now apply the above for solids obtained by letting the curve $y = f(x)$ for $a \le x \le b$ revolve about the $x - axis$.

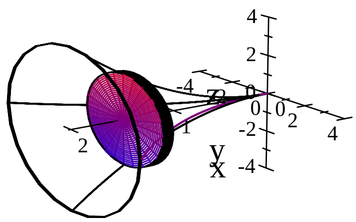
[1] This part you need only if you want to calculate the limit of the Rieman's sum.

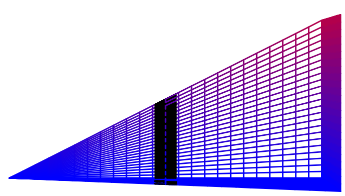
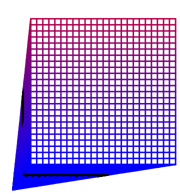
$$V = \pi \int_a^b (r(x))^2 \, dx \text{ if we rotate}$$

(a) about $x - axis$: $r(x) = f(x)$

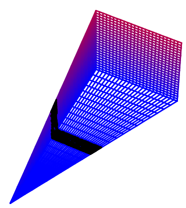
(b) about $y = a < 0 : r(x) = f(x) + a$



3. Calculate the volume of the cone with square base s and height h :

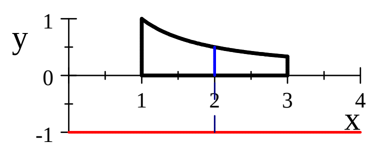
 

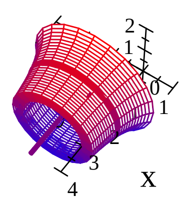
$$V = \int_0^h \left(\frac{sx}{h}\right)^2 dx = \frac{s^2}{h^2}\frac{h^3}{3} = \frac{s^2 h}{3}$$



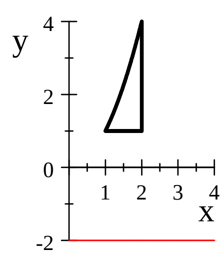
4. Calculate the volume of the solid obtained by rotating the region between the curves: $y = \dfrac{1}{x}, y = 0$ for $1 \le x \le 3$ about the line $y = -1$.

   The radius at $x : r(x) = \dfrac{1}{x} + 1 \to V = V_{1/x} - V_0 = \pi \int\limits_1^3 \left(\dfrac{1}{x} + 1\right)^2 dx -$
   $\pi \int\limits_1^3 (1)^2 \, dx = \pi \left(2 \ln 3 + \dfrac{8}{3}\right) - 2\pi = \dfrac{2}{3}\pi\,(3 \ln 3 + 1)$





   (a) $f(x) = x^2$ on $[1, 2]$ about $y = -2$ between $y = f(x)$ and $y = 1$. $V = \pi \int\limits_1^2 \left(\left(x^2 + 2\right)^2 - (1 + 2)^2\right) dx = \dfrac{158}{15}\pi$



(b) $x = f(y) = y^2$ on $[1, 2]$ about $x = -2$ between $x = f(y)$ and $x = 1$.