Lecture 8
6.1 Area Between Curves

Statement 1. The area of the region bounded by the curves $y = f(x)$ and $y = g(x)$, and the lines $x = a$ and $x = b$, where $f(x)$ and $g(x)$ are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, is

$$A = \int_a^b [f(x) - g(x)]dx$$

Statement 2. The area of the region bounded by the curves $y = f(x)$ and $y = g(x)$, and the lines $x = a$ and $x = b$, where $f(x)$ and $g(x)$ are continuous for all x in $[a, b]$, is

$$A = \int_a^b |f(x) - g(x)|dx$$

Statement 3. The area of the region bounded by the curves with equations $x = f(y)$ and $x = g(y)$, and the lines $y = c$ and $y = d$, where $f(y)$ and $g(y)$ are continuous for all y in $[c, d]$, is

$$A = \int_c^d |f(y) - g(y)| dy$$

6.5 Average value of a Function

Definition 1. The average value of a function f on the interval $[a, b]$ is

$$f_{av} = \frac{1}{b-a} \int_a^b f(x)dx$$

Theorem 1. The Mean Value Theorem for Integrals
If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x)dx,$$

that is

$$\int_a^b f(x)dx = f(c)(b-a) = f_{av}(b-a)$$