Lecture 22

11.10 Taylor and Maclaurin Series

Theorem 1. Taylor Series Expansion
If f has a power series representation (expansion) at a, that is if

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n, \qquad |x-a| < R,$$

then its coefficients are given by the formula

$$c_n = \frac{f^{(n)}(a)}{n!}.$$

This series is called the **Taylor series** of the function of the function f at a.

Definition 1. If $a = 0$, the Taylor series is called **Maclaurin series**.

Definition 2. Partial sum of the Taylor series

$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{n!}(x-a)^i$$

is called the n-**th degree Taylor polynomials** of f at a.

Definition 3. $R_n(x) = f(x) - T_n(x)$ is called the **reminder** of the Taylor series.

Theorem 2. If $f(x) = T_n(x) + R_n(x)$, where T_n is the n-th degree Taylor polynomial of f at a and

$$\lim_{n\to\infty} R_n = 0$$

for $|x-a| < R$, then f is equal to the sum of its Taylor series on the interval $|x-a| < R$.