Class 1

1. Given is a set of n points on interval $[1, 11]$ with:

   (a) $n = 500$ points and we want to calculate their "average" of $\left\{ \frac{1}{1+\frac{n}{50}} \right\}_{n=1}^{500} =$

   $\frac{1}{50} \sum_{n=0}^{500} \frac{1}{1+\frac{n}{50}} \approx 2.408837.$

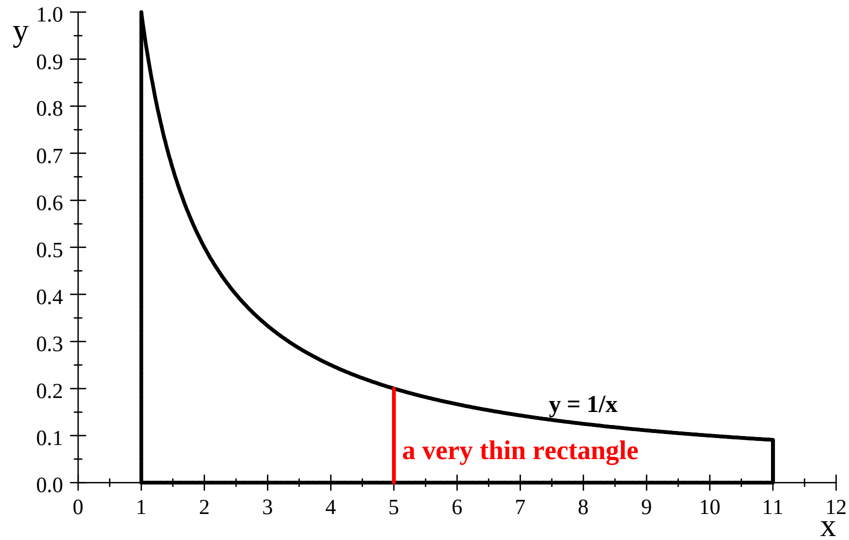
   (b) $n = 5000$ points and we want to calculate their "average" of $\left\{ \frac{1}{1+\frac{n}{500}} \right\}_{n=1}^{5000} =$

   $\frac{1}{500} \sum_{n=0}^{5000} \frac{1}{1+\frac{n}{500}} \approx 2.398987.$

   (c) $n = 50000$ points and we want to calculate their "average" of $\left\{ \frac{1}{1+\frac{n}{5000}} \right\}_{n=1}^{50000} =$

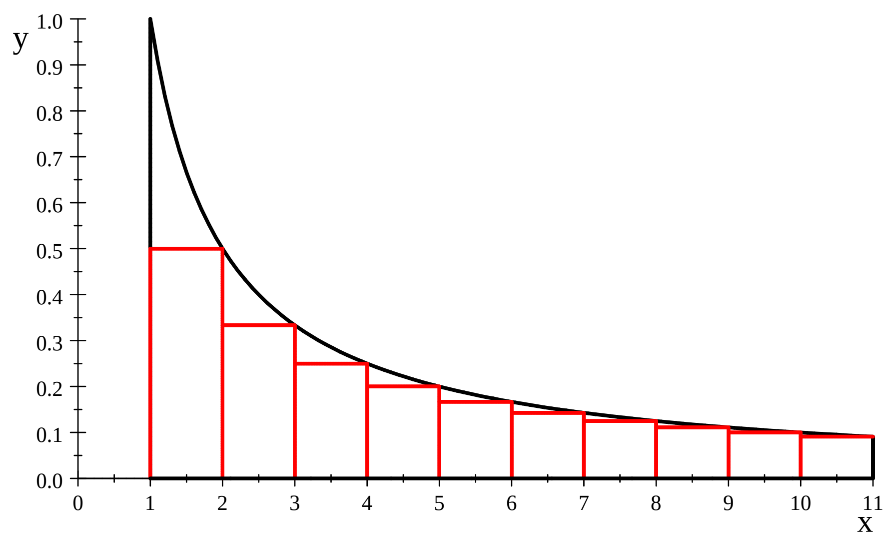
   $\frac{1}{5000} \sum_{n=1}^{50000} \frac{1}{1+\frac{n}{5000}} \approx 2.397804\,.$

   (d) The actual area is $\int_1^{11} \frac{1}{t} dt = \ln 11 \approx 2.397\,895.$

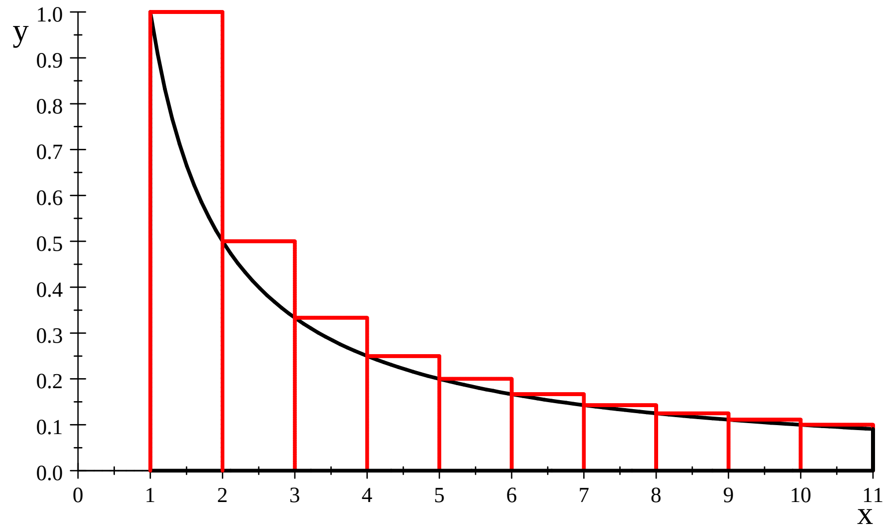


Area can by filled up with the red thin rectangles

To see a better the actual rectangles, we use essentially less points, *e.g.*, $n = 10$

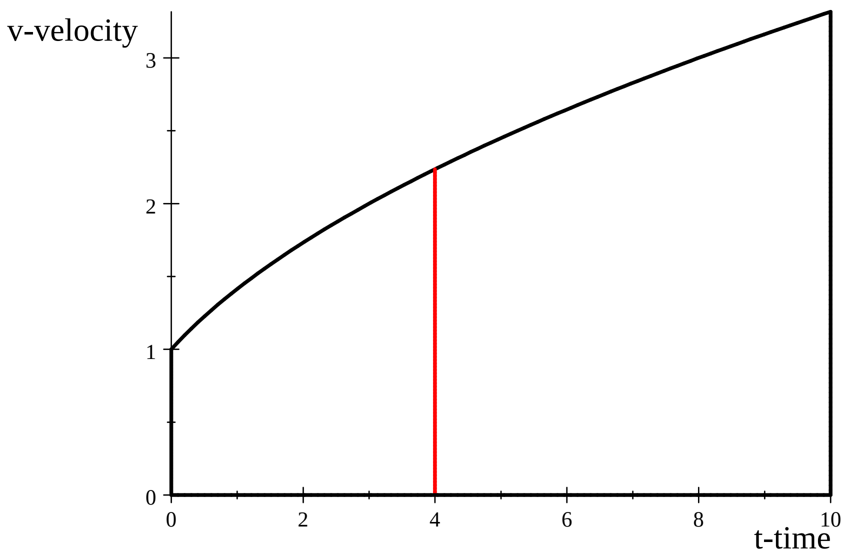


Rightpoint underestimate



Leftpoint overestimate

2. $t - time$ in hours, $s(t)$ −distance travelled in time t in $10 \times km$, $v(t) = \sqrt{1+t}$ instantaneous velocity at time t



The distance $s(t)$ is antiderivative of the velocity $v(t) = \sqrt{1+t}$ and the area is the enumeration of the distance travelled from $t = 0$ till $t = 10$ hours. The "approximation" of the area is:

$\frac{1}{500}\sum\limits_{n=0}^{5000}\sqrt{1+\dfrac{n}{500}} \approx 23.659$ we have $s\left(t\right)$ is antiderivative of $v\left(t\right) = \sqrt{1+t} \rightarrow$ with knowing that at the start $s\left(0\right) = 0$, and the antiderivative is $s\left(t\right) = \dfrac{2\left(t+1\right)\sqrt{t+1}}{3} + C \rightarrow s\left(0\right) = \dfrac{2}{3} + C = 0 \rightarrow C = -\dfrac{2}{3} \rightarrow s\left(t\right) = \dfrac{2\left(t+1\right)\sqrt{t+1}}{3} - \dfrac{2}{3} \rightarrow s = s\left(10\right) = \dfrac{22}{3}\sqrt{11} - \dfrac{2}{3} \approx 23.655km$

and the actual area is: $\int\limits_{0}^{10}\sqrt{1+t}dt = \dfrac{22}{3}\sqrt{11} - \dfrac{2}{3} \approx 23.655$.

3. Derivatives — Antiderivatives

x	$\rightarrow$	1
$\frac{x^{n+1}}{n+1}$	$\rightarrow$	x^n
$\ln x$	$\rightarrow$	$\frac{1}{x}$
e^x	$\rightarrow$	e^x
$\sin x$	$\rightarrow$	$\cos x$
$\cos x$	$\rightarrow$	$-\sin x$

$\rightarrow$

1	$\rightarrow$	$x+C$
x^n	if $n \neq -1$	$\frac{x^{n+1}}{n+1}+C$
$\frac{1}{x}$	$\rightarrow$	$\ln x + C$
e^x	$\rightarrow$	$e^x + C$
$\cos x$	$\rightarrow$	
$\sin x$	$\rightarrow$	

4. Symbolically we have:

   (a) antiderivative, called **indefinite integral** of $f(x) \rightarrow \int f(x)\,dx = F(x) + C$, e.g., $\int \sqrt{1+t}dt = \frac{2}{3}(t+1)^{\frac{3}{2}} + C$

   (b) The antiderivative with a specific (definite) value is called a **definite integral,** e.g.,

      i. Since when we start, we have travelled no distance: $s(0) = 0 = F(0) + C = \frac{2}{3}(1)^{\frac{3}{2}} + C \rightarrow C = -\frac{2}{3}$ giving the definite distance function: $S(t) = \frac{2}{3}(t+1)^{\frac{3}{2}} - \frac{2}{3}$.

      ii. The total distance travelled is then $S = \int_0^{10} \sqrt{1+t}dt = \frac{22}{3}\sqrt{11} - \frac{2}{3} \approx 23.655km$: $S(0) = \frac{2}{3}$, $S(10) = \frac{22}{3}\sqrt{11}$ therefore $S = S(10) - S(0) = F(10) - F(0)$.

   (c) interval $[a, b]$ with n equal subintervals, then $\Delta x = \frac{b-a}{n}$:

      i. **Left-point estimates**: start at $x = a$,

         A. e.g. on interval $[1, 11]$,with $n = 5000 \rightarrow \Delta x = \frac{11-1}{5000} = \frac{1}{500}$ start at $x = 1$;

         B. All the other points are then Δx apart, i.e., the function is evaluated at $x_i = a + i\Delta x$, e.g. $x_i = 1 + \frac{i}{500}$;

         C. that gives the left estimates: $\sum_{i=0}^{n-1} f(x_i)\Delta x = \sum_{i=0}^{n-1} f(a + i\Delta x)\Delta x$, e.g., for $f(x) = \sqrt{x+1}$ we get $\frac{1}{500}\sum_{i=0}^{4999}\sqrt{1+\frac{i}{500}} \approx 23.653 < 23.655 \approx \int_0^{10}\sqrt{x+t}dx$ is an underestimate, as the function $f(x) = \sqrt{x+1}$ is on interval $[0, 10]$ increasing.

ii. **Right-point estimates**: (try to think how to put this one together)