CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Duration**
Alternate Midterm	16 March, 2019	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks**	

1. (6+5 marks): **a.** Sketch the graph of the function

$$f(x) = \begin{cases} 2 + \sqrt{4 - x^2} & -2 \le x < 2 \\ |2x - 8| - 2 & 2 < x \end{cases}$$

on the interval $[-2, 5]$, and find the definite integral $\int_{-2}^{5} f(x)\,dx$ in terms of area (do **not** antidifferentiate).

b. Calculate the derivative of $F(x) = \int_{x}^{x^2} e^{-t^2}\,dt$, and determine whether F is increasing or decreasing at $x = 1$.

2. (6 marks): Find $F(x)$ if $F'(x) = \dfrac{\tan(x)}{\cos^2(x)} + \dfrac{2\,e^x}{e^x + 1}$ and $F(0) = 1$.

3. (10 marks): Calculate the following indefinite integrals

(a) $\displaystyle\int \frac{x^3 + 9}{x^2 - 9}\,dx$ **(b)** $\displaystyle\int x \ln^2(x)\,dx$

4. (12 marks): Evaluate the following definite integrals (**do not approximate**):

(a) $\displaystyle\int_0^1 x^2 \sqrt{4 - x^2}\,dx$ **(b)** $\displaystyle\int_0^{\pi/4} \sec^4(x)\,dx$

5. (6 marks): Find the average value of the function $f = \dfrac{x^5}{\sqrt{1 + x^3}}$ on the interval $[0, 2]$.

6. (5 marks): Sketch the graphs of functions $f(x) = 6 - 2x - x^2$ and $g(x) = x^2 + 2x$, and find the area of the region bounded by these graphs.

Bonus (3 marks). Given that

$$\int_0^{\pi} [f(x) + f''(x)] \sin x\,dx = 2$$

and $f(\pi) = 1$, find $f(0)$