CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section
Mathematics	205	CA
Examination	**Date**	**Pages**
Final	August 2011	2
Instructor:		**Course Examiners**
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Special Instructions:	Only Sharp EL 531 or Casio FX-300MS calculators are allowed	

MARKS

[8] **1.** **(a)** Sketch a graph of the function $f(x) = 4 - x^2$, write the formula in Σ-notation for its right Riemann sum $R(n)$ on the interval [0,2] with partitioning on n subintervals of equal length, and calculate the area enclosed by the graph of f and $x-$ axis on that interval as the limit of $R(n)$ at $n \to \infty$.

NOTE: you may need the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of the function $F(x) = \int_{x^2}^{1} \arctan(t)\sqrt{1+t}\,dt$ at $x = 1$

[20] **2.** Calculate the following indefinite integrals:

(a) $\int \frac{(1+\sqrt{x})^2}{x}\,dx$

(b) $\int x^2 e^{2x}\,dx$

(c) $\int \frac{x^2-1}{x^2-4}\,dx$

(d) $\int \frac{\cos^3 x}{\sin^3 x}\,dx$

[12] **3.** Evaluate the following definite integrals (give the exact answers):

(a) $\int_0^2 \frac{\arctan(\frac{x}{2})}{x^2+4}\,dx$

(b) $\int_0^3 \ln(1+x)\,dx$

[10] **4.** Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_e^\infty \frac{dx}{x\ln^2 x}$ **(b)** $\displaystyle\int_0^{\pi/2} \tan(x)\,dx$

[16] **5.** **(a)** Sketch the curves $y=\sqrt{2x}$ and $y=x$ and find the area enclosed.

(b) Sketch the region bounded by $f(x)=\sec(x)$ and the lines $y=0$, $x=0$ and $x=\frac{\pi}{4}$, and find the volume of the solid of revolution of this region about the x-axis.

(c) Find the average value of the function $f(x)=\sqrt{25-x^2}$ on the interval [-5,5].

[6] **6.** Find the limit of the sequence $\{a_n\}$ when $n\to\infty$ or prove that it does not exist:

(a) $a_n=\dfrac{(2^n+3)^2}{3^n}$ **(b)** $a_n=\dfrac{\sqrt{9+2n+n^4}-3n^2}{10+3n^2}$

[12] **7.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(a) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n}{\sqrt{1+n^2}}$ **(b)** $\displaystyle\sum_{n=0}^\infty \frac{(-2)^n}{e^n+1}$ **(c)** $\displaystyle\sum_{n=2}^\infty \frac{1}{n\ln n}$

[10] **8.** Find the radius of convergence and the interval of convergence of the series

(a) $\displaystyle\sum_{1}^\infty \frac{2^n}{n+1}(x+2)^n$ **(b)** $\displaystyle\sum_{n=1}^\infty \frac{(4x-1)^{2n}}{n^2}$

[6] **9.** **(a)** Find the radius of convergence of the power series $F(x)=\displaystyle\sum_{n=0}^\infty 9^n\,x^{2n+1}$.

(b) Express the function $F(x)$ as an elementary function (i.e. sum the series within its conversion radius).

[5] **Bonus Question.** Calculate the definite integral

$$\int_0^\pi \sin t\,\sin^9(\cos t)\,dt$$