

Lecture 6

5.5 The Substitution Rule

Theorem 1. The Substitution Rule for Indefinite Integrals

If $u = g(x)$ is a differentiable function with $R(g) = I$ and f is continuous on I , then

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du$$

Theorem 2. The Substitution Rule for Definite Integral

If $u = g(x)$ is a differentiable function with $R(g) = I$ and f is continuous on I , then

$$\int_a^b f(g(x)) \cdot g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Integrals for Symmetric Functions

Theorem 3.

Suppose f is continuous on $[-a, a]$.

- (a) If f is even ($f(-x) = f(x)$), then $\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$
- (b) If f is odd ($f(-x) = -f(x)$), then $\int_{-a}^a f(x)dx = 0$