CONCORDIA UNIVERSITY

Math-205, Midterm Examination

23 May, 2012

Instructions: This is a closed-book test, notes are not allowed.

Answers without showing any intermediate work will not be accepted. The marks for each question are indicated, with 100% = 40 marks.

[10] **1.** (a) Write the sigma notation formula for the right Riemann sum R_n of $f(x) = 3x^2$ on the interval [1,2] with partitioning on n subintervals of equal length, and calculate the definite integral $\int_1^2 f(x) dx$ as the limit of R_n at $n \to \infty$.

(Reminder: $\sum_{k=1}^{n} k = n(n+1)/2$, $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$)

- (b) Calculate the derivative of $F(x) = \int_{0}^{x^2} \frac{t-4}{1+\cos^2(t)} dt$, and find the points x where the local extrema (maximum or minimum) of F(x) are reached.
- [5] **2.** Find the function F(x) such that F(0) = 1 and $\frac{dF}{dx} = \frac{\sec^2(x)}{1 + \tan(x)} + xe^{-x^2}$.
- [12] 3. Calculate the following indefinite integrals

(a) $\int x (x^{-1} + \sqrt{x})^2 dx$ (b) $\int \frac{t^2 + 4t}{t^2 + 4} dt$ (c) $\int \frac{e^x}{e^{2x} - 9} dx$

[8] 4. Evaluate the following definite integrals (give the exact values, do not approximate):

(a) $\int_{0}^{4} \frac{x}{\sqrt{2x+1}} dx$ (b) $\int_{0}^{1} x^{2} \cos(\pi x) dx$

- [5] **5.** Find the mean value of the function $f(x) = \sin^3(x)\cos^2(x)$ on the interval $[0, \pi/2]$.
- [2] **Bonus.** Assuming f(x) is a continuous function of x, find F'(x) for

 $F(x) = \int_{1}^{x} [x + f(t)]dt$

NOTE: the answer is **not** F'(x) = x + f(x).