CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Pages
Final	April 2011	2
Instructors:	F. Balogh, A. Boyarsky, J. Brody, M. Girotti, N. Hardy, H. Proppe	Course Examiners A. Atoyan & H. Proppe
Special Instructions:	Only Sharp EL 531 or Casio FX-300MS calculators are allowed	

MARKS

[8] 1. (a) Sketch the graph of the function

$$f(x) = \begin{cases} |x| - 2 & \text{if } |x| \geq 2 \\ -\sqrt{4 - x^2} & \text{if } |x| < 2 \end{cases}$$

on the interval $-3 \leq x \leq 3$ and calculate the definite integral $\int_{-3}^{3} f(x)\,dx$ as the signed area between the graph of f and the x-axis (do not antidifferentiate).

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of the function $F(x) = \int_{0}^{x^2+1} t^2 e^{-t^2}\,dt$ and find the point(s) of local maximum and/or local minimum of $F(x)$.

[10] 2. Calculate the following indefinite integrals:

(a) $\displaystyle\int \frac{(x - \sqrt{x})^2}{x^{3/2}}\,dx$ (b) $\displaystyle\int (x^2 + 1)\sin(2x)\,dx$

[10] 3. Find the antiderivative $F(t)$ of the function $f(t)$ that satisfies the given condition:

(a) $f(t) = \dfrac{t-1}{t^2 - 2t + 5}$, $F(3) = 0$. (b) $f(t) = \sin^3 t \cos^5 t$, $F\left(\dfrac{\pi}{2}\right) = 0$.

[12] 4. Evaluate the following definite integrals (give the exact answers):

(a) $\displaystyle\int_{1}^{e} \ln^2 x\,dx$ (b) $\displaystyle\int_{0}^{3} x\sqrt{9 - x^2}\,dx$

Scanned by CamScanner

[8] 5. Evaluate the given improper integral or show that it diverges:

(a) $\int_0^\infty x^2 e^{-x^3}\,dx$ (b) $\int_0^1 \frac{x}{x^2-1}\,dx$

[17] 6. (a) Sketch the curves $y=\sqrt{x+2}$ and $y=x$ and find the area enclosed.

(b) Sketch the region enclosed by $f(x)=\sin(2x)$ and the x-axes on the interval $[0,\frac{\pi}{2}]$ and find the volume of revolution of this region about the axis $y=-1$.

(c) Find the average value of the function $f(x)=\tan^2 x$ on the interval $[-\frac{\pi}{4},\frac{\pi}{4}]$.

[9] 7. Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(a) $a_n=\frac{e^n-n}{(-2)^n}$ (b) $a_n=\frac{\ln(n^2)}{n+1}$ (c) $a_n=\sqrt{n+100}-\sqrt{n}$

[8] 8. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(a) $\sum_{n=2}^\infty \frac{2\ln n}{n^2}$ (b) $\sum_{n=0}^\infty \frac{(-1)^n}{(5n+1)^{1/3}}$

[10] 9. Find the interval of convergence of the following series

(a) $\sum_{1}^\infty \frac{(3x)^n}{n!}$ (b) $\sum_{n=1}^\infty \frac{(x+1)^{3n}}{n\,8^n}$

[8] 10. (a) Find the radius of convergence of the power series $F(x)=\sum_{n=1}^\infty \frac{x^n}{n\,3^n}$.

(b) Use differentiability of power series to express the derivative $F'(x)$ of $F(x)$ as an elementary function (i.e. sum the derivative series).

[5] **Bonus Question.** Find the values of p for which the following series is convergent

$$\sum_{n=1}^\infty \frac{1}{n(1+\ln n)^p}$$

Scanned by CamScanner