Lecture 4
5.3 The Fundamental Theorem of Calculus

Theorem 1. The Mean Value Theorem for Definite Integrals

If f is continuous on an interval $[a, b]$, then at some point $c \in [a, b]$

$$f(c) = \frac{1}{b-a} \int_a^b f(x)dx$$

Theorem 2. Fundamental Theorem. Part I

If f is continuous on an interval $[a, b]$, then the function

$$g(x) = \int_a^x f(t)dt$$

where $x \in (a, b)$ is continuous on $[a, b]$ and differentiable on (a, b).

Theorem 3. Fundamental Theorem. Part II

If f is continuous at every point of $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Differentiation and Integration as Inverse Processes

Theorem 3. The Fundamental Theorem of Calculus

Suppose f is continuous on an interval $[a, b]$. Then

1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.
2. $\int_a^b f(x)dx = F(b) - F(a)$, where F is any antiderivative of f, that is $F'(x) = f(x)$.