CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Duration**
Midterm	10 March, 2019	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks**	

1. (6+5 marks):

(a) Graph $f(x) = \begin{cases} \sqrt{1-x^2} - 1, & -1 \le x \le 0 \\ |x-1| - 1, & x > 0 \end{cases}$ on the interval $[-1, 3]$,

and use it to calculate the definite integral $\int_{-1}^{3} f(x)\,dx$ in terms of area.

(b) Use the Fundamental Theorem of Calculus to find the function $f(x)$ and the constant A such that $\int_{x}^{2} f(t)\,dt + A = x^2 + x$.

2. (6 marks): Find the antiderivative $F(x)$ of $f(x) = \dfrac{\cos^5(x) + 1}{\cos^2(x)}$ such that $F(0) = 0$.

3. (5+5 marks): Calculate the following indefinite integrals

(a) $\displaystyle\int \frac{x^2 - 2}{x^2 - x - 2}\,dx$ (b) $\displaystyle\int e^{2x}\sqrt{e^x + 1}\,dx$

4. (5 marks): Sketch the graphs of functions $x = 2y^2$ and $x = 4 + y^2$, and find the area enclosed (**do not approximate**).

5. (6+6 marks): Evaluate the following definite integrals (**do not approximate**):

(a) $\displaystyle\int_{\sqrt{2}}^{2} \sqrt{4 - x^2}\,dx$ (b) $\displaystyle\int_{0}^{\pi/4} \arctan(x)\,dx$

6. (6 marks): Sketch the region enclosed by the graphs of $y = \sqrt{x}$, $x-$axis and the line $x = 4$. Find the volume of a solid obtained by rotating this region about the axis $y = -1$.

Bonus. (3 marks): Given that $\int_{0}^{1} e^{-x}[f(x) - f'(x)]\,dx = e$ and $f(0) = 0$, find $f(1)$.