CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2012	2
Instructors: F. Balogh, A. Boyarsky, D. Challita, E. Kritchevsky, H. Proppe		**Course Examiners** A. Atoyan & H. Proppe
Special Instructions:	Only calculators approved by the Department are allowed. For full marks show your work clearly.	

MARKS

[10] 1. (a) Write in sigma notation the formula for the right Riemann sum R_n of $f(x) = 3 + 2x^2$ on the interval [0,3] partitioned into n subintervals of equal length, and calculate R_6 to approximate the area enclosed by the graph of f and $x-$ axis on that interval by the sum with $n = 6$.

NOTE: you may need the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of the function $F(x) = \int_0^{x^2} \frac{t-4}{1+\cos^2(t)}\,dt$, and find the points x of the local extrema (maximum or minimum) of F.

[8] 2. Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition:

(a) $f(x) = \sqrt{x}\,(1 - x^{-1/2})^2$, $F(1) = 1$. (b) $f(x) = \dfrac{5+\cos^2(x)}{\cos^2(x)}$, $F(0) = 5$.

[12] 3. Calculate the following indefinite integrals:

(a) $\displaystyle\int \frac{\sin(x)}{\cos^2(x)+9}\,dx$, (b) $\displaystyle\int \frac{2^x}{2^x+1}\,dx$, (c) $\displaystyle\int \frac{dx}{(x+4)(x-1)}$.

[10] 4. Evaluate the following definite integrals (give the exact answers):

(a) $\displaystyle\int_0^4 \frac{t}{\sqrt{1+2t}}\,dt$ (b) $\displaystyle\int_1^4 \sqrt{t}\,\ln(t)\,dt$

[8] 5. Evaluate the given improper integral or show that it diverges:

$$\text{(a)} \quad \int_0^4 \frac{1}{x\sqrt{x}}\,dx \qquad \text{(b)} \quad \int_0^\infty x\,e^{-2x^2}\,dx$$

[17] 6. (a) Sketch the curves $y = x^2 - 2$ and $y = |x|$ and find the area enclosed.

(b) Sketch the region enclosed by $f(x) = \cos^2(x)$ and the x-axes on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and find the volume of revolution of this region about the $x-$axis.

(c) Find the average value of the function $f(x) = \cos(x)\,\sin(2x)$ on the interval $[0, \frac{\pi}{2}]$.

[9] 7. Find the limit of the sequence $\{a_n\}$ or explain why the limit does not exist:

$$\text{(a)} \quad a_n = \frac{2^n + 5^{n+2}}{6^n} \qquad \text{(b)} \quad a_n = \frac{\sqrt{n^4 + n^3}}{n + 3n^2} \qquad \text{(c)} \quad a_n = \frac{n^2\,\cos(\pi n)}{1 + n^2}$$

[12] 8. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally, and explain why.

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n\,n}{1 + 2n} \qquad \text{(b)} \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)} \qquad \text{(c)} \quad \sum_{n=2}^{\infty} \frac{\sqrt{n+4}}{n^2 + 4}$$

[6] 9. Find the radius and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{n\,4^n}$$

[8] 10. (a) Derive the Maclaurin series of $f(x) = x^2\,e^{2x}$ (HINT: start with the series for e^z where $z = 2x$).

(b) Use differentiability of power series to find the sum $F(x) = \sum_1^\infty \frac{(x-1)^n}{n}$ within its radius of convergence.

[5] **Bonus Question.** Let f be a continuous function on the interval $[1, 4]$. Prove that $\bar{f}_{[1,4]} = \frac{1}{3}\bar{f}_{[1,2]} + \frac{2}{3}\bar{f}_{[2,4]}$, where $\bar{f}_{[1,4]}$, $\bar{f}_{[1,2]}$ and $\bar{f}_{[2,4]}$ are the average values of f on the respective intervals.