Lecture 3
4.9 Antiderivatives

Definition 1. A function F is an **antiderivative** of f on an interval I if

$$F'(x) = f(x) \quad \text{for all} \quad x \in I$$

Table of antiderivatives

$$
\begin{array}{ll}
f(x) = x^n & F(x) = \frac{1}{n+1}x^n + C \\
f(x) = sin(kx) & F(x) = -\frac{1}{k}cos(kx) + C \\
f(x) = e^{kx} & F(x) = \frac{1}{k}e^{kx} + C \\
f(x) = a^{kx} & F(x) = \frac{1}{k \ln a}a^{kx} + C, \quad a > 0, a \neq 1 \\
f(x) = \frac{1}{x} & F(x) = \ln|x| + C
\end{array}
$$

$$
\begin{array}{ll}
f(x) = \sin kx & F(x) = -\frac{1}{k}\cos kx + C \\
f(x) = \cos(kx) & F(x) = \frac{1}{k}\sin kx + C \\
f(x) = \sec^2(kx) & F(x) = \frac{1}{k}\tan kx + C \\
f(x) = \csc^2(kx) & F(x) = -\frac{1}{k}\cot kx + C \\
f(x) = \frac{1}{\sqrt{1-k^2x^2}} & F(x) = \frac{1}{k}\sin^{-1} kx + C \\
f(x) = \frac{1}{1+k^2x^2} & F(x) = \frac{1}{k}\tan^{-1} kx + C
\end{array}
$$

Antiderivative Linearity Rules

1. Constant Multiple Rule

   If $g(x) = kf(x)$ then $G(x) = kF(x)$

2. Negative Rule

   If $g(x) = -f(x)$ then $G(x) = -F(x)$

3. Sum Rule

   If $h(x) = f(x) + g(x)$ then $H(x) = F(x) + G(x)$

4. Difference Rule

   If $h(x) = f(x) - g(x)$ then $H(x) = F(x) - G(x)$