CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2014	2
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Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

[10] 1. (a) Sketch the graph of $f(x) = 2^{-x}$. Using partitioning of the interval $[-2, 2]$ into 4 subintervals of equal length, the definite integral $A = \int_{-2}^{2} f(x)\,dx$ can be approximated by either the leftpoint, or the midpoint, or the rightpoint Riemann sum. Explain which one of these three Riemann sums provides the best approximation for A, and calculate that Riemann sum.

(b) Find the derivative of the function $F(x) = 2\,e^{-x^2} - \int_{x^2}^{1} \sqrt{1+t}\,e^{-t}\,dt$, and determine whether $F(x)$ is increasing of decreasing at $x = 1$.

[11] 2. Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition:

(a) $f(x) = \dfrac{e^{-3x}}{(e^{-3x}+1)^3}$, $F(0) = 0$. (b) $f(x) = \tan^2 x$, $F\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2}$.

[15] 3. Find the following indefinite integrals:

(a) $\displaystyle\int x\ln(x+2)\,dx$ (b) $\displaystyle\int \frac{x}{x^2-4x+3}dx$ (c) $\displaystyle\int x\left(1+\frac{1}{\sqrt{x}}\right)^2 dx$.

[12] 4. Evaluate the following definite integrals (give the exact values, do **not** approximate):

(a) $\displaystyle\int_0^{\pi/4} \frac{\sec^2(x)}{4+\tan^2(x)}\,dx$ (b) $\displaystyle\int_0^{\pi/2} \cos^3(x)\,\sin^5(x)\,dx$

[8] 5. Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_{-\infty}^{0} x\,e^{-x^2}\,dx$ (b) $\displaystyle\int_{-2}^{2} \frac{dx}{(x+2)^{3/2}}$

[15] 6. (a) Plot the curve $y = \sqrt{4 - x^2}$, and the line $y = 2 - x$, and find the exact value of the area enclosed.

(b) Find the volume of a solid obtained by rotating the region bounded by the curve $y = \sqrt{3x}$ and the lines $y = 3$ and $x = 0$ about the axis $y = -1$.

(c) Find the average value of $f(x) = \sin^2(x)\cos^2(x)$ on the interval $[0, \frac{\pi}{2}]$.

[6] 7. Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

(a) $a_n = \dfrac{(-3)^{2n}}{1 + 3n^3 + 9^{n+1}}$ (b) $a_n = \dfrac{\sqrt{9 + n^2 + 4n^4}}{(n + 3n^{3/2})(4 + \sqrt{n})}$

[15] 8. Determine whether the series is divergent or convergent, and if convergent, then is it convergent absolutely or conditionally :

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{1+n}}{1+n^2}$ (b) $\displaystyle\sum_{n=0}^{\infty} e^{-n}(-3)^{n-1}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{n \ln(n)}$

[8] 9. (a) Find (a) the radius of convergence and (b) the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{n\, 3^n}$$

(b) Derive the MacLaurin series of $f(x) = x^3 \ln(1 + 2x^2)$

(HINT: start with the series for $\ln(1+z)$ where $z = 2x^2$).

[5] **Bonus Question.** It is known that for some continuous even function, $f(x) = f(-x)$, the average value of $f(x)$ on any given interval $[-a, a]$ $(a > 0)$ is equal to the length of the interval. Is this information sufficient to find $f(x)$? Find the function f if it is, otherwise explain why it is insufficient.