Test 3 Review

Math for Computer Science – 201-H01-HR

1. Consider the relation

$$R = \{(3, 1), (-2, 4), (0, 3), (2, 1)\}.$$

   (a) State the domain and range of this relation.

   (b) Graph this relation.

   (c) Is this relation a function?

2. Consider the relation

$$R = \{(x, y) \in \mathbb{Z}^2 \mid y \text{ is an integer multiple of } x\}.$$

Show that R is reflexive and transitive, but not symmetric.

3. Draw a black-box diagram for the function

$$f(x) = 8x^2 - 5x$$

4. A database currently contains 4.6 Gb of data and it takes 0.6 seconds to search the database for a specific bit of data. For each extra 1 Gb of data added to the database, searches take 0.1 seconds longer. Likewise for each reduction in the size of the database by 1 Gb, searches become 0.1 seconds quicker. Assume that the relationship between these two quantities is linear.

   (a) Clearly define the variables for the two quantities.

   (b) Construct the linear function that models this scenario.

   (c) Draw the graph of the linear model.

5. Solve each of the following quadratic equations. Factor, if possible.

    (a) $3x^2 - 9x + 6 = 0$

    (b) $5x^2 - 7x - 13 = 0$

6. Write out the first five terms of each of the following sequences.

    (a) $a_n = n^2 - 3n + 5$ for $n \geq 1$

    (b) $a_1 = 3$, $a_2 = -4$, $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$

7. Give a formula that will generate the terms of each of the following sequences. If the sequence is arithmetic or geometric, please indicate which.

   (a) $-\frac{1}{2}$, $-\frac{1}{4}$, 0, $\frac{1}{4}$, $\frac{1}{2}$, ...

   (b) -1, 5, 17, 41, 89, ...

   (c) 8, 12, 18, 27, $\frac{81}{2}$, $\frac{243}{4}$, ...

8. You can obtain a loan at an annual interest rate of 4.2%. You can afford to pay back $2800 in two years' time. Compute the amount of money you would receive today if you get a simple discount loan. Then compute the amount of money you would receive today if you get a simple interest loan.

9. Consider the recursive sequence $a_1 = 4$, $a_n = 5a_{n-1} + 2$ for $n \geq 2$. Use mathematical induction to show that

$$a_n = \frac{9}{2}\left(5^{n-1}\right) - \frac{1}{2}.$$

10. You deposit $1700 in a bank account on January 1, 2010 at an annual interest rate of 4.2%, compounded monthly. You withdraw the full balance on April 19, 2010. How much money will you receive?

11. Which is better, an annual interest rate of 5.3%, compounded quarterly, or an annual interest rate of 5.2%, compounded weekly?

12. Compute the mortgage payments that must be made at the end of every month if the current value of the mortgage is $370,000, the annual interest rate is 4.8%, compounded monthly, and the mortgage will be paid over a 25 year period.

13. Which is worth more, $1,000,000 paid today or payments of $5,000 at the end of each month for the next 25 years? Use an annual interest rate of 3.6%, compounded monthly, in your calculations.

14. Consider the following system of linear equations.

$$3x - 2y + 5z = 0$$
$$-4x + y = 0$$

(a) Verify that $x = 1$, $y = 4$, $z = 1$ is a solution to this system of linear equations.

(b) Find another solution to the system of linear equations.

(c) How many solutions does the system of linear equations have?

15. Four cities have a water sharing agreement. Consider the water pipelines shown in the following diagram. City A produces a net surplus of 7 billion litres of water, city B has a net deficit of 3 billion litres of water, city C has a net surplus of 1 billion litres of water, and city D has a net deficit of 5 billion litres of water. Create a system of linear equations that describes all the possible water flows in between cities that would satisfy each city's water requirements.