Exam Review

Math for Computer Science – 201-H01-HR

1. Convert each of the following numbers to the requested base.

   (a) Convert 534_8 to base 10.

   (b) Convert $A3.F5_{16}$ to base 8.

   (c) Convert 12.3 to binary.

2. Perform each of the following arithmetic operations in the required base.

   (a) $E3.B_{16} - 4C.2_{16}$

   (b) $275_8 \times 33_8$

   (c) $10110_2 / 101_2$

3. Convert -55 and 76 to the requested form. Then compute the sum $-55 + 76$ using the same form.

   (a) One's Complement Form

   (b) Two's Complement Form

4. Convert the number 83.125 to the requested form.

   (a) Single Precision Form

   (b) Double Precision Form

5. Give an example of a calculation that would give an overflow error. Give an example of a conversion error.

6. List all of the subsets of $\{\Delta, \Gamma, \Phi\}$.

7. Consider the following sets.

$$S = \{x \in \mathbb{Z} \mid x \text{ is prime and } x < 17\} \text{ and } T = \{3, 5, 7, 9, 11, 13\}$$

(a) List the elements of S.

(b) Write T using set-builder notation.

(c) Are S and T equal? Are S and T equivalent? Give reasons why or why not.

(d) Find $S \cup T$ and $S \cap T$.

8. Use a Venn Diagram to show that for sets A and B which are subsets of some universal set,
$$\overline{(A \cap \overline{B})} = \overline{A} \cup B.$$

9. Let p be the statement "everyone is eating grapefruit" and let q be the statement "rates of scurvy are increasing". Write each of the following as an English sentence.

   (a) $p \wedge q$

   (b) $q \rightarrow (\sim p)$

10. Show that the two logic statements $(\sim p) \vee ((\sim q) \wedge r)$ and $(p \overline{\wedge} q) \wedge (p \rightarrow r)$ are equivalent by

    (a) using a truth table and

    (b) simplifying the statements using properties of logic statements.

11. Determine whether the following logic argument is valid using a truth table.
    Hypothesis: $p \wedge (\sim q)$, $(\sim p) \vee q$
    Conclusion: q

12. Let A and B be Boolean variables. Simplify the following Boolean expression and then draw a diagram of the corresponding logic gates.

$$(A \cdot (\overline{B})) + ((1 + A) \cdot B)$$

13. Draw a black-box diagram for the function

$$f(x) = 3x^2 - 2x.$$

14. Consider the relation

$$R = \{(x, y) \in \mathbb{Z}^2 \mid xy \text{ is odd } \}.$$

(a) What are the domain and range of this relation?

(b) Show that the relation is symmetric and transitive but not reflexive.

15. Write out the first five terms of the recursive sequence $a_1 = -3$, $a_2 = 2$, and $a_n = 4a_{n-1} + a_{n-2}$ for $n \geq 3$.

16. Write a formula for the nth term of each of the following sequences. Indicate if the sequence is geometric or arithmetic.

    (a) $144, 96, 64, \frac{128}{3}, \frac{256}{9}, \ldots$

    (b) $-\frac{1}{6}, -\frac{1}{8}, -\frac{1}{12}, -\frac{1}{24}, 0, \ldots$

17. Consider the recursive sequence $a_1 = 3, a_n = 10a_{n-1} + 8$ for $n \geq 2$. Use mathematical induction to prove that $a_n = \frac{35(10^{n-1})-8}{9}$ for all $n \geq 1$.

18. You deposit \$3800 in a bank account on May 1, 2010 that pays an annual interest rate of 3.6%, compounded monthly. Determine the value of the investment on September 13, 2011.

19. You take out a mortgage for \$350,000 at an annual interest rate of 4.8%, compounded monthly. If you make equal payments at the end of each month for 30 years to pay off the mortgage, what should the amount of the monthly payments be?

20. Consider the system of linear equations

$$\begin{aligned} 2x - 3y + z &= 4 \\ x + 5y &= 3. \end{aligned}$$

(a) Use trial-and-error to find a particular solution to this system of linear equations.

(b) Is $x = 4$, $y = 3$, $z = 0$ a solution to this system of linear equations? Explain.

(c) Without using any elementary row operations, determine how many solutions the system of linear equations has.

21. How many solutions does the following system of linear equations have? Compute the rank of the coefficient and augmented matrices. Do not find the solution(s), if they exist.

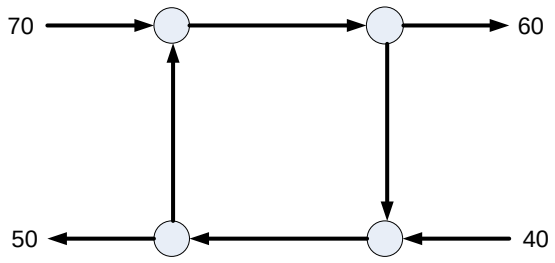
$$\begin{aligned} 4x - 3y + z &= 7 \\ x + 4y - 5z &= 1 \\ 2x - 11y + 11z &= 4 \end{aligned}$$

22. Find the general solution to the following system of linear equations.

$$\begin{aligned} 3x + 7y - 2z &= 9 \\ 5x - 3y + 4z &= 37 \\ x - 5y + 3z &= 14 \end{aligned}$$

23. You are allocating weekly hours for workers at your ski factory. There are 3 types of workers. The first type is paid $8 per hour and can create 2 skis per hour. The second type of worker is paid $10 per hour and can create 2.5 skis per hour. The third type of worker is paid $12 per hour and can create 2.5 skis per hour. You have a total weekly payroll of $9400, you must assign a total of 1000 hours of work per week to your employees and you must produce 2250 skis per week. How many working hours should you allocate to each type of employee in order to meet these requirements?

24. Determine the general solution for the traffic flows shown in the following diagram.



25. Let

$$A = \begin{bmatrix} -1 & 5 & 6 \\ 2 & 0 & -4 \end{bmatrix}, B = \begin{bmatrix} 8 & 3 \\ -4 & 5 \\ 0 & 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} -2 & 4 & 3 \\ -1 & -2 & 6 \\ 4 & 3 & 0 \end{bmatrix}.$$

For each of the following matrix operations, determine whether or not the computation is possible. If so, perform the computation. If not, give the reason why not using a size argument.

(a) $A + B$

(b) AC

(c) BC

26. Classify each of the following matrices as a row matrix, column matrix, zero matrix, identity matrix, or square matrix.

(a) $\begin{bmatrix} -4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

27. Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, determine which of the following is A^{-1} without making any effort to compute A^{-1}. Explain why your choice is the correct inverse.

$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 1 & 1/2 \\ 1/3 & 1/4 \end{bmatrix}$ $\quad$ $\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

28. Consider the following system of linear equations.

$$\begin{aligned} x + y + z &= 0 \\ x + 5y + 7z &= 6 \\ 6x + y - 2z &= -11 \end{aligned}$$

(a) Write the system of linear equations in matrix form.

(b) Use the Gauss-Jordan method to find the inverse of the coefficient matrix.

(c) Find the unique solution to the system of linear equations.

29. Bob wants to send the message, "DANGER", to Alice. Bob's secret key is $A = \begin{bmatrix} 3 & -4 \\ 5 & 7 \end{bmatrix}$.

    (a) Find the ciphertext matrix that Bob sends to Alice.

    (b) What is Alice's secret key?

30. You are turning an old office building into condominiums. A one-bedroom apartment will cost \$90,000 to build, take up 800 square feet of space, and will generate \$10,000 in profit. A two-bedroom apartment will cost \$120,000 to create, take up 1000 square feet of space, and will generate \$13,000 in profit. The total amount of space available in the building is 50,000 square feet and your total budget is \$5,850,000. How many of each type of apartment unit should you build in order to maximize your profits? Solve this method first using the Simplex Method and then verify your results with the graphical method.