Test #4 will cover WebWork assignments 9, 10 and the class notes for Cramer's Rule (pages 74 to 76) and Section 6 – Vectors (pages 86 to 111), along with the problems done in class for those sections. A formula sheet will be provided.

The breakdown for Test #4 is as follows (out of 35 points):

7 pts – Vector arithmetic (3 questions)

12 pts – Applications of Vectors (3 questions)

6 pts – Proof with vectors

10 pts – Solve a system of linear equations using Cramer's rule

Good luck!

Formula sheet

$$\cos\theta = \frac{\vec{u}\cdot\vec{w}}{\|\vec{u}\|\|\vec{w}\|}$$

Area of parallelogram: $\|\vec{v}\times\vec{w}\|$

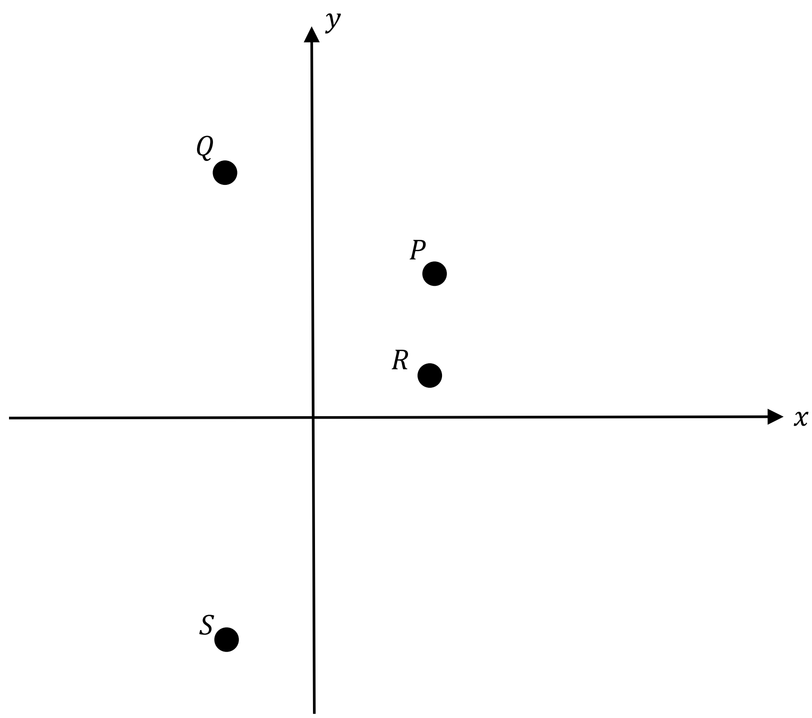
Volume of parallelepiped: $|\vec{u}\cdot(\vec{v}\times\vec{w})|$

VECTOR OPERATIONS

Operations: Addition, Subtraction, Scalar Multiplication, Norm, Dot Product, Cross Product, Scalar Triple Product

Ex 1) Given the points P, Q, R and S (shown below), construct vectors $\vec{u} = \overrightarrow{PQ}$ and $\vec{w} = \overrightarrow{RS}$ and then perform the following operations :

a. $\vec{u} + \vec{w}$
b. $\vec{u} - \vec{w}$
c. $\frac{1}{4}\vec{w}$
d. $3\vec{w} - 2\vec{u}$



Ex 2) Consider the vectors $\vec{a} = (1,0,2)$, $\vec{b} = (-1,-3,3)$ and $\vec{c} = (0,4,5)$. Find the following, if possible. If it is not possible, explain why not.

a. $3\vec{b} - 2\vec{c}$

b. $\|4\vec{a}\|$

c. $4(\vec{a} \cdot \vec{c})$

d. $\vec{c} \times \vec{a}$

e. $\vec{a} \cdot \vec{c} + 3\vec{b}$

f. $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$

g. $2\vec{a} \cdot 5\vec{c} + \|\vec{a}\|\vec{b}$

h. $(\vec{a} \cdot \vec{b}) \times \vec{c}$

i. $\vec{a} \cdot (\vec{b} \times \vec{c})$

ANGLE BETWEEN VECTORS

Ex 1) Let $\vec{u} = (1, -5, 4)$ and $\vec{w} = (-2, 3, 2)$. Find the angle between $\vec{u}$ and $\vec{w}$

Ex 2) Find the angle at B determined by the directed line segments $\overrightarrow{BA}$ and $\overrightarrow{BC}$, where $A(3,0,2)$, $B(4,3,0)$ and $C(8,1,-1)$.

APPLICATION OF VECTORS

Types: Area, Volume

Ex 1) Find the area of the triangle with vertices $P(0,-2,-2)$, $Q(5,-1,3)$ and $R(7,0,6)$.

Ex 2) Find the area of the parallelogram with vertices $P(5,-1,3)$, $Q(8,1,4)$, $R(13,2,9)$ and $S(16,4,10)$.

Ex 3) Given $\vec{u} = (5,1,-2)$, $\vec{v} = (-3,-1,-3)$ and $\vec{w} = (2,-3,-3)$

a. Find the volume of the parallelepiped whose adjacent edges are defined by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$

b. Are the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar? Explain.

Ex 4) Given the points $A(1,0,2)$, $B(3,-2,3)$, $C(4,3,4)$ and $D(2,-5,-1)$

a. Find the volume of the parallelepiped having adjacent edges defined by the points.

b. Are the points A, B, C and D coplanar? Explain.

VECTOR PROOFS

Ex 1) Let $\vec{u}$ and $\vec{w}$ be vectors in $\mathbb{R}^2$ and k be a scalar. Prove that $k(\vec{u}+\vec{w}) = k\vec{u} + k\vec{w}$

Ex 2) Let $\vec{u}$ be a vector in $\mathbb{R}^2$ and k be a scalar. Prove that $\|k\vec{u}\| = |k|\ \|\vec{u}\|$

Ex 3) Let $\vec{u}$ and $\vec{w}$ be vectors in $\mathbb{R}^2$ and k be a scalar. Prove that $(k\vec{u}) \cdot \vec{w} = k(\vec{u} \cdot \vec{w})$

Ex 4) Let $\vec{u}$ be a vector in $\mathbb{R}^2$ and c and d be scalars. Prove that $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

Ex 5) Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors in $\mathbb{R}^2$. Prove that $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

CRAMER'S RULE

Ex 1) Find the general solution of the following system of linear equations by using Cramer's rule.

$$\begin{aligned} 2x-4y+z&=1\\ -x+y-z&=2\\ x-2y&=1 \end{aligned}$$

Ex 2) Find the general solution of the following system of linear equations by using Cramer's rule.

$$\begin{aligned} x - 2y + 6z &= 0 \\ -4x + 9y - 23z &= -3 \\ 2x - 4y + 10z &= 1 \end{aligned}$$