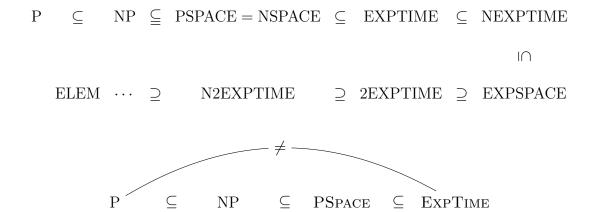
- For deterministic classes, C = co-C.
- P is considered the only tractable complexity class.
- Satisfiability and validity are complementary problems, hence determining that satisfiability is in C immediately implies that validity is in co-C, and vice versa.
- SAVITCH, W. J. (1970), Relationship between nondeterministic and deterministic tape complexities, J. Comput. System Sci. 4, 177-192.: PSPACE = NPSPACE

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• PSPACE and EXPTIME are deterministic complexity classes, co-PSPACE is identical to PSPACE and co-EXPTIME is identical to EXPTIME



 $\mathbf{K4}$ is complete w.r.t. the class of all *transitive* Kripke frames, and also complete w.r.t. the class of all *irreflexive*, *transitive* Kripke frames. That is, $\mathbf{K4} = \mathsf{Log}\ \mathsf{F}^{tran} = \mathsf{Log}\ \mathsf{F}^{irref,tran}$.

Table 1: The complexity of the *satisfiability* problem for modal logics.

NP-complete	PSPACE-complete	EXPTIME-complete	NEXPTIME-complete	undicidable
$\mathrm{PL},\mathrm{S5},\mathrm{KD45}$	$\mathbf{K}_n, \mathbf{T}_n, \mathbf{S4}_n, n \ge 1$	$\mathbf{K}_n^C, \mathbf{T}_n^C, n \ge 1$		
	$\mathbf{S5}_n, \mathbf{KD45}_n, \ n \geq 2$	$\mathbf{S4}_{n}^{C}, \mathbf{S5}_{n}^{C}, \mathbf{KD45}_{n}^{C}, n \geq 2$		
NEXT(S4.3)	QBF	PDL		
		$\mathbf{K}^{E,A}$		
			${f S5} imes {f S5}, {f S5} imes {f K}$	
				$\mathbf{S5}^n \text{ for } n \geq 3,$
	LTL			
$n \times n$ tiling (n in unary)			$n \times n$ tiling (n in binary)	$\mathbb{N} \times \mathbb{N}$ tiling

- Cf. [Halpern and Moses, 1992, Table 1, p. 350]
- $\bullet~$ PL: propositional logic.
- QBF: the logic of quantified Boolean formulas.
- PDL: propositional dynamic logic.
- In the unary numeral system, 0 is represented by the empty string ϵ , numbers $1, 2, 3, \ldots$ are represented in unary as $1, 11, 111, \ldots$
- C: the logic with common knowledge operator
- D: adding distributed knowledge (intersection operator) to the language does not affect the complexity
- in these cases of single-agent for $S4^C$, $S5^C$ and $KD45^C$, common knowledge reduces to knowledge.
- NEXT(S4.3): any (consistent) normal extension of S4.3. Hemaspaandra's Theorem: every normal logic extending S4.3 is NP-complete.
- $\mathbf{K}^{\mathsf{E},\mathsf{A}}$: the expansion of modal logic \mathbf{K} with the universal modality.
- LTL: propositional (linear) temporal logic with four temporal operators, \bigcirc (next), \Diamond (future / eventually), \square (always) and U (until).

1 Tiling problems

A tile T is a 1×1 square fixed in orientation with colored edges right(T), left(T), up(T) and down(T) taken from some denumerable set. A tiling problem takes the following form:

given a finite set \mathfrak{T} of tile types, can we cover a certain part of $\mathbb{Z} \times \mathbb{Z}$, using only tiles of this type, in such a way that adjacent tiles have the same color on the common edge.

There are some titling problems:

1. $\mathbb{N} \times \mathbb{N}$ tiling.

Given a finite set \mathfrak{T} of tiles, can \mathfrak{T} tile $\mathbb{N} \times \mathbb{N}$?

The $\mathbb{N} \times \mathbb{N}$ tiling problem is undecidable, it is *re-complete*.