- For deterministic classes, $\mathcal{C} = \text{co-}\mathcal{C}$.
- P is considered the only *tractable* complexity class.
- **Satisfiability** and **validity** are complementary problems, hence determining that satisfiability is in $\mathcal{C}$ immediately implies that validity is in co-$\mathcal{C}$, and vice versa.
- SAVITCH, W. J. (1970), Relationship between nondeterministic and deterministic tape complexities, J. Comput. System Sci. 4, 177-192.: PSPACE = NPSPACE
-
- PSPACE and EXPTIME are deterministic complexity classes, co-PSPACE is identical to PSPACE and co-EXPTIME is identical to EXPTIME

$$\begin{array}{ccccccccc}
\text{P} & \subseteq & \text{NP} & \subseteqq & \text{PSPACE} = \text{NSPACE} & \subseteq & \text{EXPTIME} & \subseteq & \text{NEXPTIME} \\
 & & & & & & & & \rotatebox{90}{$\subseteq$} \\
 & \text{ELEM} \cdots & \supseteq & & \text{N2EXPTIME} & \supseteq & \text{2EXPTIME} & \supseteq & \text{EXPSPACE}
\end{array}$$

$$\neq$$

$$\text{P} \subseteq \text{NP} \subseteq \textsc{PSpace} \subseteq \textsc{ExpTime}$$

K4 is complete w.r.t. the class of all *transitive* Kripke frames, and also complete w.r.t. the class of all *irreflexive, transitive* Kripke frames. That is, $\mathbf{K4} = \mathsf{Log}\ \mathsf{F}^{tran} = \mathsf{Log}\ \mathsf{F}^{irref,tran}$.

Table 1: The complexity of the *satisfiability* problem for modal logics.

NP-complete	PSPACE-complete	EXPTIME-complete	NEXPTIME-complete	undicidable
$\mathbf{PL}$, $\mathbf{S5}, \mathbf{KD45}$	$\mathbf{K}_n, \mathbf{T}_n, \mathbf{S4}_n$, $n \geq 1$	$\mathbf{K}_n^C, \mathbf{T}_n^C$, $n \geq 1$		
	$\mathbf{S5}_n, \mathbf{KD45}_n$, $n \geq 2$	$\mathbf{S4}_n^C, \mathbf{S5}_n^C, \mathbf{KD45}_n^C$, $n \geq 2$		
NEXT($\mathbf{S4.3}$)	$\mathbf{QBF}$	$\mathbf{PDL}$		
		$\mathbf{K}^{\mathsf{E,A}}$		
			$\mathbf{S5} \times \mathbf{S5}$, $\mathbf{S5} \times \mathbf{K}$	
				$\mathbf{S5}^n$ for $n \geq 3$,
	$\mathbf{LTL}$			
$n \times n$ tiling (n in unary)			$n \times n$ tiling (n in binary)	$\mathbb{N} \times \mathbb{N}$ tiling

- Cf. [Halpern and Moses, 1992, Table 1, p. 350]
- **PL**: propositional logic.
- **QBF**: the logic of quantified Boolean formulas.
- **PDL**: propositional dynamic logic.
- In the unary numeral system, 0 is represented by the empty string ϵ, numbers $1, 2, 3, \ldots$ are represented in unary as $1, 11, 111, \ldots$.
- C: the logic with common knowledge operator
- D: adding distributed knowledge (intersection operator) to the language does not affect the complexity
- in these cases of single-agent for $\mathbf{S4}^C$, $\mathbf{S5}^C$ and $\mathbf{KD45}^C$, common knowledge reduces to knowledge.
- NEXT($\mathbf{S4.3}$): any (consistent) normal extension of $\mathbf{S4.3}$.
  Hemaspaandra's Theorem: every normal logic extending $\mathbf{S4.3}$ is NP-complete.
- $\mathbf{K}^{\mathsf{E,A}}$: the expansion of modal logic $\mathbf{K}$ with the universal modality.
- **LTL**: propositional (linear) temporal logic with four temporal operators, $\bigcirc$ (next), $\Diamond$ (future / eventually), $\Box$ (always) and U (until).

1 Tiling problems

A *tile* T is a 1×1 square fixed in orientation with colored edges $right(T), left(T), up(T)$ and $down(T)$ taken from some denumerable set.

A *tiling problem* takes the following form:

> given a finite set $\mathfrak{T}$ of tile types, can we cover a certain part of $\mathbb{Z} \times \mathbb{Z}$, using only tiles of this type, in such a way that adjacent tiles have the same color on the common edge.

There are some titling problems:

1. $\mathbb{N} \times \mathbb{N}$ tiling.

   Given a finite set $\mathfrak{T}$ of tiles, can $\mathfrak{T}$ tile $\mathbb{N} \times \mathbb{N}$?

The $\mathbb{N} \times \mathbb{N}$ tiling problem is undecidable, it is *re-complete*.