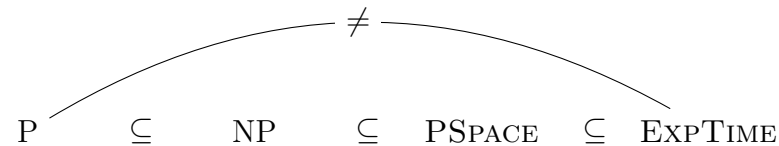


- For deterministic classes, $\mathcal{C} = \text{co-}\mathcal{C}$.
- P is considered the only *tractable* complexity class.
- **Satisfiability** and **validity** are complementary problems, hence determining that satisfiability is in \mathcal{C} immediately implies that validity is in $\text{co-}\mathcal{C}$, and vice versa.
-

sdfasdfqweasdfs

$$\begin{array}{ccccccccc} \text{P} & \subseteq & \text{NP} & \subseteq & \text{PSPACE} & \subseteq & \text{EXPTIME} & \subseteq & \text{NEXPTIME} & \subseteq \\ & & \subseteq & \text{EXPSPACE} & \subseteq & 2\text{EXPTIME} & \subseteq & \text{N2EXPTIME} & \cdots & \text{ELEM} \end{array}$$



PSPACE and EXPTIME are deterministic complexity classes, co-PSPACE is identical to PSPACE and co-EXPTIME is identical to EXPTIME

K4 is complete w.r.t. the class of all *transitive* Kripke frames, and also complete w.r.t. the class of all *irreflexive, transitive* Kripke frames. That is, $\mathbf{K4} = \text{Log } \mathbf{F}^{\text{tran}} = \text{Log } \mathbf{F}^{\text{irref,tran}}$.

P	
\sqcup	
NP	$PL_{Sat} = \mathbf{S5}_{Sat}$: NP-complete PL_{Valid}: co-NP-complete
\sqcup	
PSPACE	$\mathbf{K}_{Sat} = \mathbf{T}_{Sat} = \mathbf{S4}_{Sat} = QBF_{Sat} = QBF_{Valid}$: PSPACE-complete
\sqcup	
EXPTIME	$PDL_{Sat} = PDL_{Valid}$: EXPTIME-complete

PL : propositional logic.

QBF : the logic of quantified Boolean formulas.

PDL : propositional dynamic logic.

Table 1: The complexity of the satisfiability problem for modal logics.

NP-complete	PSPACE-complete	EXPTIME-complete	NEXPTIME-complete	undecidable
PL, S5, KD45	K_n, T_n, S4_n, $n \geq 1$	K_n^C, T_n^C, $n \geq 1$		
	S5_n, KD45_n, $n \geq 2$	S4_n^C, S5_n^C, KD45_n^C, $n \geq 2$		
NEXT(S4.3)	QBF	PDL		
		K ^{E,A}		
			S5 × S5, S5 × K	
				S5 ⁿ for $n \geq 3$,
$n \times n$ tiling (n in unary)			$n \times n$ tiling (n in binary)	$\mathbb{N} \times \mathbb{N}$ tiling

- Cf. [Halpern and Moses, 1992, Table 1, p. 350]
- In the unary numeral system, 0 is represented by the empty string ϵ , numbers $1, 2, 3, \dots$ are represented in unary as $1, 11, 111, \dots$
- C : the logic with common knowledge operator
- D : adding distributed knowledge (intersection operator) to the language does not affect the complexity
- in these cases of single-agent for $S4^C$, $S5^C$ and $KD45^C$, common knowledge reduces to knowledge.
- NEXT(S4.3): any (consistent) normal extension of S4.3.
Hemaspaandra's Theorem: every normal logic extending S4.3 is NP-complete.
- K^{E,A}: the expansion of modal logic K with the universal modality.

1 Tiling problems

A *tile* T is a 1×1 square fixed in orientation with colored edges $right(T), left(T), up(T)$ and $down(T)$ taken from some denumerable set.

A *tiling problem* takes the following form:

given a finite set \mathfrak{T} of tile types, can we cover a **certain part** of $\mathbb{Z} \times \mathbb{Z}$, using only tiles of this type, in such a way that adjacent tiles have the same color on the common edge.

There are some tiling problems:

1. $\mathbb{N} \times \mathbb{N}$ tiling.

Given a finite set \mathfrak{T} of tiles, can \mathfrak{T} tile $\mathbb{N} \times \mathbb{N}$?

The $\mathbb{N} \times \mathbb{N}$ tiling problem is undecidable, it is *re-complete*.