hw-4 (2023/10/10)　　姓名:　　　　学号:

p.19: 13-(a)　Find statement forms in **conjunctive normal form** which are logically equivalent to the following:

$$(a) \qquad (((\neg p) \vee q) \to r)$$

Your answer: (5 points)

Here we will use three methods to find some **conjunctive normal forms** (CNF) of $(\neg p \vee q \to r)$, the former two are from our textbook, while the third one is new.

In the first place, let

$$\varphi = (\neg p \vee q \to r).$$

Method-(1)

First we construct a truth table of φ's negation:

p	q	r	$\neg$	(($\neg$	p	$\vee$	q) $\to$	r)
1	1	1	0	0		1	1	
$\underline{1}$	$\underline{1}$	$\underline{0}$	1	0		1	0	
1	0	1	0	0		0	1	
1	0	0	0	0		0	1	
0	1	1	0	1		1	1	
$\underline{0}$	$\underline{1}$	$\underline{0}$	1	1		1	0	
0	0	1	0	1		1	1	
$\underline{0}$	$\underline{0}$	$\underline{0}$	1	1		1	0	

The combinations which give $\neg\varphi$ value 1 are 110, 010 and 000. Thus a **disjunctive normal form** of $\neg\varphi$ is

$$\chi = (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$$

It is clear that χ is logically equivalent to $\neg\varphi$, hence $\neg\chi$ is logically equivalent to $\neg\neg\varphi$, that is, φ.

Thus, by the **De Morgan's laws**, we have

$$\begin{aligned}
\neg\chi &= \neg[(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)] \\
&\equiv \neg(p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge \neg q \wedge \neg r) \\
&\equiv (\neg p \vee \neg q \vee \neg\neg r) \wedge (\neg\neg p \vee \neg q \vee \neg\neg r) \wedge (\neg\neg p \vee \neg\neg q \vee \neg\neg r) \\
&\equiv (\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)
\end{aligned}$$

Therefore, $(\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$ is a CNF of φ.

(**NB**: we using the symbol expression "$\alpha \equiv \beta$" to denote that formula α is logically equivalent to β)

Method-(2)

$$
\begin{aligned}
\varphi &= (\neg p \vee q \to r) \\
&\equiv \neg(\neg p \vee q) \vee r && \text{(by the meaning of } \textbf{material implication}\text{,} \\
& && \text{ cf. Example 1.4-(a), p. 7 of our textbook)} \\
&\equiv (\neg\neg p \wedge \neg q) \vee r && \text{(by the } \textbf{De Morgan's laws}\text{)} \\
&\equiv (p \wedge \neg q) \vee r \\
&\equiv (p \vee r) \wedge (\neg q \vee r) && \text{(by the } \textbf{distribution} \text{ of } (\vee\text{-}\wedge)\text{, cf. p. 10, Exercises-6-(b))}
\end{aligned}
$$

Hence $(p \vee r) \wedge (\neg q \vee r)$ is a CNF of φ.

Method-(3)

Similarly, we construct a truth table for φ (notice that, not for the negation of φ):

p	q	r	$($ $\neg$	p	$\vee$	q $)$	$\to$	r
1	1	1	0	1	1	1	1	1
$\underline{1}$	$\underline{1}$	$\underline{0}$	0	1	1	1	0	0
1	0	1	0	1	0	0	1	1
1	0	0	0	1	0	0	1	0
0	1	1	1	0	1	1	1	1
$\underline{0}$	$\underline{1}$	$\underline{0}$	1	0	1	1	0	0
0	0	1	1	0	1	0	1	1
$\underline{0}$	$\underline{0}$	$\underline{0}$	1	0	1	0	0	0

The combinations which give φ value 0 are 110, 010 and 000. Then according to these truth combinations, we can construct three **disjunctive formulas** as follows,

$$
\begin{aligned}
\varphi_1 &= (\neg p \vee \neg q \vee r) \\
\varphi_2 &= (p \vee \neg q \vee r) \\
\varphi_3 &= (p \vee q \vee r)
\end{aligned}
$$

Next, we connect above three formulas in a conjunctive form, that is,

$$\varphi_1 \wedge \varphi_2 \wedge \varphi_3 = (\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$$

It is easy to check that $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$ is a CNF of φ. And as we can see, the result in current *Method-(3)* is same as the *Method-(1)*.

(*continue on next page*)

p.26: 21 Suppose that $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_n; \therefore \mathscr{A}$ is a valid argument form. Prove that $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_{n-1}; \therefore (\mathscr{A}_n \to \mathscr{A})$ is also a valid argument form.

Your proof:

First, suppose that $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_n; \therefore \mathscr{A}$ is a valid argument form, but $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_{n-1}; \therefore (\mathscr{A}_n \to \mathscr{A})$ is not.

Then there is an assignment of truth values to the statement variables such that $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_{n-1}$ takes value T while $(\mathscr{A}_n \to \mathscr{A})$ takes value F, that is, $\mathscr{A}_n$ is T but $\mathscr{A}$ takes F. However, this contradicts to our assumption that $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_n; \therefore \mathscr{A}$ is a valid argument form. (5 points)

....作业反馈..

- 还是有同学少写题目呀，题目少写的话想给你们找分都很难了。考试的时候也差不多，尽量不要空题不做呀 ☹
- 还有很多同学写证明的时候，一句话中往往不写「定语」和「状语」，比如会出现如下情况：

  所以 φ。

  所以 φ 什么呢？φ 是重言式？φ 是矛盾式？这些都是需要额外加以说明的。