hw-3 (2023/09/26) 姓名: 学号:

p15: 11-(a) Show, using Proposition 1.14 and 1.17, that the statement form $((\neg(p \vee (\neg q))) \to (q \to r))$ is logically equivalent to each of the following.

(a) $((\neg(q \to p)) \to ((\neg q) \vee r))$

.... *Recall that*: ....

Proposition 1.14: If $\mathscr{B}_1$ is a statement form arising from the statement form $\mathscr{A}$ by substituting the statement form $\mathscr{B}$ for one or more occurrences of the statement form $\mathscr{A}$ in $\mathscr{A}_1$, and if $\mathscr{B}$ is logically equivalent to $\mathscr{A}$, then $\mathscr{B}_1$ is logically equivalent to $\mathscr{A}_1$.

Proposition 1.17 (De Morgan's Laws): Let $\mathscr{A}_1, \mathscr{A}_2, \cdots \mathscr{A}_n$ be any statement forms. Then:

1. $(\bigvee_{i=1}^{n}(\neg\mathscr{A}_i))$ is logically equivalent to $(\neg(\bigwedge_{i=1}^{n}\mathscr{A}_i))$.
2. $(\bigwedge_{i=1}^{n}(\neg\mathscr{A}_i))$ is logically equivalent to $(\neg(\bigvee_{i=1}^{n}\mathscr{A}_i))$.

....

Your answer:

Let

$$\varphi = ((\neg(p \vee (\neg q))) \to (q \to r)) \qquad \text{and} \qquad \chi = ((\neg(q \to p)) \to ((\neg q) \vee r)).$$

It suffices to show that if $\neg(p \vee (\neg q))$ is logically equivalent to $(\neg(q \to p))$, and $(q \to r)$ is logically equivalent to $(\neg q) \vee r)$, then φ is logically equivalent to χ according to **Prop. 1.14**.

But it is easy to check, say, using truth table, that

$$\begin{aligned} \neg(p \vee (\neg q)) &\leftrightarrow (\neg(q \to p)) \qquad \text{and} \\ (q \to r) &\leftrightarrow (\neg q) \vee r) \end{aligned}$$

are tautologies, which means that $(\neg(p \vee (\neg q)))$ and $(\neg(q \to p))$, $(q \to r)$ and $(\neg q) \vee r)$ are logically equivalent, respectively.