

hw-5 (2023/10/17)

姓名:

学号:

p.36: 1-(c) Write out proofs in L for the following wfs .

$$(c) \quad (p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)$$

Your proof:

(5 points)

method-(1)

1. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow ((p_1 \rightarrow p_1) \rightarrow (p_1 \rightarrow p_2))$ (instance of $L2$)
2. $[(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow ((p_1 \rightarrow p_1) \rightarrow (p_1 \rightarrow p_2))] \rightarrow$
 $[(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)] \rightarrow ((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2))$ (instance of $L2$)
3. $((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)) \rightarrow ((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2))$ (1 + 2, MP)
4. $p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_1)$ (instance of $L1$)
5. $[p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_1)] \rightarrow [(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)]$ (instance of $L2$)
6. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)$ (4 + 5, MP)
7. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)$ (3 + 6, MP)

(the proof for (c) is not unique, of course)

以下的两个证明来自同学们，由于很多人的答案都相同，所以在此就不一一致谢了，总之感谢那些同学，谢谢你们为大家提供如此精彩的证明 ♥ ♥ ♥

method-(2)

1. $p_1 \rightarrow ((p_1 \rightarrow p_1) \rightarrow p_1)$ (instance of $L1$)
2. $(p_1 \rightarrow ((p_1 \rightarrow p_1) \rightarrow p_1)) \rightarrow ((p_1 \rightarrow (p_1 \rightarrow p_1)) \rightarrow (p_1 \rightarrow p_1))$ (instance of $L2$)
3. $(p_1 \rightarrow (p_1 \rightarrow p_1)) \rightarrow (p_1 \rightarrow p_1)$ (1 + 2, MP)
4. $p_1 \rightarrow (p_1 \rightarrow p_1)$ (instance of $L1$)
5. $(p_1 \rightarrow p_1)$ (3 + 4, MP)
6. $(p_1 \rightarrow p_1) \rightarrow ((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1))$ (instance of $L1$)
7. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)$ (5 + 6, MP)
8. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow ((p_1 \rightarrow p_1) \rightarrow (p_1 \rightarrow p_2))$ (instance of $L2$)
9. $[(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow ((p_1 \rightarrow p_1) \rightarrow (p_1 \rightarrow p_2))] \rightarrow$
 $[(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)] \rightarrow ((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2))$ (instance of $L2$)

10. $((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_1)) \rightarrow ((p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2))$ (8 + 9, *MP*)
11. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)$ (7 + 10, *MP*)

method-(3)

1. $\{(p_1 \rightarrow p_2) \rightarrow [((p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)]\} \rightarrow$
 $\{[(p_1 \rightarrow p_2) \rightarrow ((p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2))] \rightarrow [(p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)]\}$ (instance of *L2*)
2. $(p_1 \rightarrow p_2) \rightarrow [((p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)]$ (instance of *L1*)
3. $[(p_1 \rightarrow p_2) \rightarrow ((p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2))] \rightarrow [(p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)]$ (1 + 2, *MP*)
4. $(p_1 \rightarrow p_2) \rightarrow ((p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2))$ (instance of *L1*)
5. $(p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)$ (3 + 4, *MP*)
6. $[(p_1 \rightarrow p_2) \rightarrow (p_1 \rightarrow p_2)] \rightarrow [((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2)]$ (instance of *L2*)
7. $((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2)$ (5 + 6, *MP*)
8. $[((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2)] \rightarrow$
 $[p_1 \rightarrow (((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2))]$ (instance of *L1*)
9. $p_1 \rightarrow (((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2))$ (7 + 8, *MP*)
10. $[p_1 \rightarrow (((p_1 \rightarrow p_2) \rightarrow p_1) \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2))] \rightarrow$
 $[(p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_1)) \rightarrow (p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2))]$ (instance of *L2*)
11. $(p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_1)) \rightarrow (p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2))$ (9 + 10, *MP*)
12. $p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_1)$ (instance of *L1*)
13. $p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2)$ (11 + 12, *MP*)
14. $[p_1 \rightarrow ((p_1 \rightarrow p_2) \rightarrow p_2)] \rightarrow [(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)]$ (instance of *L2*)
15. $(p_1 \rightarrow (p_1 \rightarrow p_2)) \rightarrow (p_1 \rightarrow p_2)$ (13 + 14, *MP*)

(ps. 公式中的 中括号 $[]$ 和 花括号 $\{\}$ 是起辅助作用的, 为的是方便大家观看。但应注意的是, 其本身不是命题逻辑公理系统 *L* 中的符号!!!)

(continue on next page)

p.37: 5 The rule HS is an example of a legitimate additional rule of deduction for L . Is the following rule legitimate in the same sense: from the *wfs.* \mathcal{B} and $(\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C}))$, deduce $(\mathcal{A} \rightarrow \mathcal{C})$?

Your answer:

(5 points)

method-(1) (without using the **Deduction Theorem**)

1. \mathcal{B} (assumption)
2. $(\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C}))$ (assumption)
3. $(\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C})) \rightarrow ((\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\mathcal{A} \rightarrow \mathcal{C}))$ ($L2$)
4. $((\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\mathcal{A} \rightarrow \mathcal{C}))$ ($2 + 3, MP$)
5. $(\mathcal{B} \rightarrow (\mathcal{A} \rightarrow \mathcal{B}))$ ($L1$)
6. $(\mathcal{A} \rightarrow \mathcal{B})$ ($1 + 5, MP$)
7. $(\mathcal{A} \rightarrow \mathcal{C})$ ($6 + 4, MP$)

Hence this rule is a legitimate additional rule of deduction for L .

method-(2) (using the **Deduction Theorem**)

We first show that

$$\{\mathcal{B}, (\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C}))\} \cup \{\mathcal{A}\} \vdash_L \mathcal{C}.$$

We write out a deduction for above one as follows:

1. \mathcal{B} (assumption)
2. $(\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C}))$ (assumption)
3. \mathcal{A} (assumption)
4. $(\mathcal{B} \rightarrow \mathcal{C})$ ($2 + 3, MP$)
5. \mathcal{C} ($1 + 4, MP$)

Hence by the **Deduction Theorem**, we have

$$\{\mathcal{B}, (\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C}))\} \vdash_L \mathcal{A} \rightarrow \mathcal{C}.$$

as required.

.....作业反馈

- 很多同学都误解了什么是一个「 L 中的证明」, 在其中, 是不能出现“假设”、“因为-所以”这样的字眼的。因此 *p.36* 1-(c) 的证明也不能用「演绎定理」, 这个是内定理证明, 证明的序列中出现的只能是公理或者由前面的公式使用 MP 得到。还请大家特别要注意这点!