Question 1

Following Bayes' Ball Algorithm, nodes are dependent on each other only if the 'ball' can reach from one node to the other through open gates.

For the case where actual values are unknown:

X2 and X6 are independent of each other. This is so as not all the gates from X2 to X6 are open. Gates are open from X2 to X9, however, that is where the gate from X9 to X6 is closed, hence X2 and X6 are independent of each other.

For the case where X7 and X11 are known:

In this situation, X7 becomes a gate that is not passable. However, X7 can be bypassed by going through X8 instead. Once again, going from X8 to X9, X6 will still be inaccessible. However, as we know the value for X11, after the 'ball' is passed through X11, it can go backwards and, in this fashion, go through node X9 to reach node X6. Hence, this shows that X2 and X6 are dependent of each other.

Question 2

Given that each node can take up to 2 values, we can calculate the free parameters for each parameter.

For X1, we get 1.

For X2, X3, X4, X5, we get 2 for each. Total 8.

For X6, we get 1.

For X7 and X8, we get 2 for each. Total 4.

For X9 we get 8.

For X10 and X11, we get 2 each. Total 4.

Summing it all up, the number of free parameters is 26.

In the case where X3 and X9 take up 3 different values while the rest take up 5 different values, we get a different calculation for the number of free parameters.

For X1, we get 4.

For X2, we get 20.

For X3, we get 10.

For X4, we get 12.

For X5, we get 20.

For X6, we get 4.

For X7 and X8, we get 20 for each. Total 40.

For X9, we get 250.

For X10, we get 12.

For X11, we get 20.

Summing it all up, the number of free parameters is 392.

Question 3

Using Bayes' rule formula, we have P(A|B) = P(B|A) * P(A) / P(B). In the table, it is shown that X3 is independent of X2, and X10 is independent of X9.

a) P(X3=1|X4=1)
 = P(X4=1|X3=1) * P(X3=1) / P(X4=1)
 = 0.1 * 0.3 / (0.1 * 0.3 + 0.5 * 0.7)
 = 0.078947

b) P(X5=2|X3=2,X11=2,X1=2)
 = P(X3=2,X11=2,X1=2|X5=2) * P(X5=2) / P(X3=2,X11=2,X1=2)
 = P(X3=2|X5=2) * P(X11=2|X5=2) * P(X1=2|X5=2) * P(X5=2) /
 P(X3=2) * P(X11=2) * P(X1=2)

 As X11 and X1 are not dependent of X5,

 = P(X3=2|X5=2) * ~~P(X11=2)~~ * ~~P(X1=2)~~ * P(X5=2) / P(X3=2) * ~~P(X11=2)~~ * ~~P(X1=2)~~
 = P(X3=2|X5=2) * P(X5=2) / P(X3=2)

 Applying Bayes' rule, we get
 = P(X5=2|X3=2)
 = (0.5 * 0.5) + (0.4 * 0.5)
 = 0.45