Question 1

Using Kernel 0

```
C:\Users\nigel\OneDrive\Desktop\Code\libsvm-3.24\windows>svm-predict.exe test.txt training.txt.model training.out
Accuracy = 84.375% (27/32) (classification)
```

Using Kernel 1

```
C:\Users\nigel\OneDrive\Desktop\Code\libsvm-3.24\windows>svm-predict.exe test.txt training.txt.model training.out
Accuracy = 81.25% (26/32) (classification)
```

Using Kernel 2

```
C:\Users\nigel\OneDrive\Desktop\Code\libsvm-3.24\windows>svm-predict.exe test.txt training.txt.model training.out
Accuracy = 90.625% (29/32) (classification)
```

Using Kernel 3

```
C:\Users\nigel\OneDrive\Desktop\Code\libsvm-3.24\windows>svm-predict.exe test.txt training.txt.model training.out
Accuracy = 43.75% (14/32) (classification)
```

Question 2

a)

Referring to the equation, we want to minimize $0.5||w||^2$, which implies we should minimize w. Using the constraint that it is subjected to, to minimize w we have:

$y(w.x + w0) = 1$

as bias $b = 0$,

$y(w.x) = 1$

Using the points given we have and setting our w as [w1 w2]:

For x1 = [1 1] transpose and y1 = 1:

$1 * [w1\ w2] . [1\ 1].T = 1$

$w1 + w2 = 1$

For x2 = [1 0] transpose and y2 = -1:

(-1) * [w1 w2] . [1 0].T = 1

w1 = -1

Therefore we get w1 = -1, w2 = 2

Thus our **w is [-1 2]**

To get γ , we use y(w.x + w0)/||w||

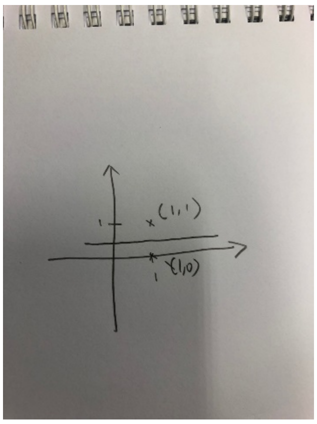
For x1 = [1 1] transpose and y1 = 1:

$1 * ([-1\ 2] . [1\ 1].T + 0)/(-1^2 + 2^2)^{0.5}$

Hence, $\boldsymbol{\gamma}$ **= $\mathbf{1/5^{0.5}}$**

b)

If bias b is allowed to be non-zero, this would allow the classifier to be optimized as it is no longer restricted and results in optimal γ and w values. Given this, since the support vectors x1 and x2 are exactly 1 unit apart as seen in the following drawing (sorry I was rushing and could not plot a better one so I drew it out PS), we can find the best fit line as shown:



The margin γ can hence be deduced to be ½

Hence, **γ=0.5**

Using the above graph, to calculate the gradient of the line we can take 2 points A(0, 0.5) and B(1, 0.5). in this case, dy/dx would be 0/1.

For every change in dx by 1, dy would change by 0. Resulting in the tangent vector being [1 0]. To get w, we can find the normal of this vector which would be [0 1], hence

w = [0 1]

Question 3

Kernel Methods

- Properties of kernel functions

1. $K(x, x') = 1$ is a kernel function.
2. Let $f : \mathcal{R}^d \to \mathcal{R}$ be any real valued function of x. Then, if $K(x, x')$ is a kernel function, then so is $\tilde{K}(x, x') = f(x)K(x, x')f(x')$
3. If $K_1(x, x')$ and $K_2(x, x')$ are kernels, then so is their sum. In other words, $K(x, x') = K_1(x, x') + K_2(x, x')$ is a kernel.
4. If $K_1(x, x')$ and $K_2(x, x')$ are kernels, then so is their product $K(x, x') = K_1(x, x')K_2(x, x')$

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The above is reference for answering the following questions.

1. $K(x, z) = K_1(x, z)K_2(x, z)$ is a Kernel

Proof:

Using the 4th property from the above and given that:

$K_1(x; z)$ and $K_2(x; z)$ are kernels over $R_n x R_n$.

since K1(x, z) and K2(x, z) are Kernels, their product K(x, z) will also be a Kernel.

2. $K(x; z) = aK_1(x; z) + bK_2(x; z)$, where $a; b > 0$ are real numbers, is also considered a Kernel.

Proof:

Using the 3rd property from above and given that:

$K_1(x; z)$ and $K_2(x; z)$ are kernels over $R_n x R_n$.

,As aK1(x, z) and bK2(x, z) are Kernels and a, b are real numbers greater than 0, the summation of these two Kernels which results in K(x, z) is a Kernel as well.

3. $K(x; z) = aK_1(x; z) - bK_2(x; z)$, where $a; b > 0$ are real numbers

In this case, K(x, z) is not a kernel as there is the potential for it to be less than 0. Given that **Kernels have the property of being non-negative**, the next part will be to prove that this is indeed not a Kernel

Proof:

As a and b are given to be real numbers that are greater than 0, we can take any non negative values. In this case lets take **a = 9** and **b = 17.**

Also given that

$K_1(x; z)$ and $K_2(x; z)$ are kernels over $R_n x R_n$.

Using the 1st property in the properties list above, K1(x, z) and K2(x, z) can be taken to be equal to 1.

Plugging these values into the given equation of:

$$K(x, z) = aK1(x, z) – bK2(x, z)$$

We get

$$K(x, z) = (9 * 1) – (17 * 1)$$

This would result in K(x, z) = -8 which is a value lesser than 0.

As this does not follow the property of non-negativity, we can prove that in this case, K is not a Kernel.

4.