Chapter: 2.1
Page: 64
Babayeu Niyazi

№21

a) ints = $[0;1]$, $[1;2,5]$, $[2,5;3,5]$

$[0;1]$ – average speed $= \frac{10-0}{1-0} = 10$ m/hr

$[1;2,5]$ – average speed $= \frac{20-10}{2,5-1} = \frac{10}{1,5} \approx 6,6$ m/hr

$[2,5;3,5]$ – average speed $= \frac{30-20}{3,5-2,5} = \frac{10}{1,5} \approx 6,6$ m/hr

b)

$t = \frac{1}{2}$, then $d \approx 8$

inst. speed $= \frac{f(1)-8}{1-\frac{1}{2}} = \frac{15-8}{0,5} = \frac{7}{0,5} = 14$ m/hr

$t = 2$, then $d = 20$

inst. speed $= \frac{f(2,5)-20}{2,5-2} = \frac{20-20}{0,5} = \frac{0}{0,5} = 0$ m/hr

$t = 3$, then $d \approx 22$

inst. speed $= \frac{f(3,5)-22}{3,5-3} = \frac{30-22}{0,5} = \frac{8}{0,5} = 4$ m/hr

c) Maximum speed occurs at the 4^{th} hour and accounts for $\frac{35}{4}$ m/hr.

UniNote

Chapter: 2.1

Page: 65

Babayev Niyazi

№22

a) ints = [0;3], [0;5], [7;10]

[0;3] – consumption $= \frac{10-15}{3-0} = \frac{-5}{3}$ g/d

[0;5] – consumption $= \frac{3{,}5-15}{5-0} = \frac{-11{,}5}{5} = -2{,}3$ g/d

[7;10] – consumption $= \frac{0-1{,}5}{10-7} = \frac{-1{,}5}{3} = -0{,}5$ g/d

b)

t = 1, then c = 14

inst. cons. $= \frac{f(2)-14}{2-1} = \frac{12-14}{1} = -2$ g/d

t = 4, then c = 6

inst. cons. $= \frac{f(3{,}5)-6}{3{,}5-4} = \frac{8-6}{-0{,}5} = \frac{2}{-0{,}5} = -4$ g/d

t = 8, then c = 1

inst. cons. $= \frac{f(9)-1}{9-8} = \frac{0{,}5-1}{1} = -0{,}5$ g/d

c) Maximum rate of gasoline consumption occurs between hours 4 and 6, since the curve is the steepest at that range.

$s = \frac{f(6)-f(4)}{6-4} = \frac{2-5}{2} = -\frac{3}{2} = -1{,}5$ g/d

Chapter: 2.2
Page: 73
Babayeu Niyazi

№2

a) $\lim_{t \to -2} f(t) = 2$ (limit exists because it is both sided)

b) $\lim_{t \to -1} f(t) = -1$ (limit exists because it has 2 sides)

c) $\lim_{t \to 0} f(t) = DNE$ (it is discontinuous. It has 2 different values as f(x) approaches to 0.

d) $\lim_{t \to -0,5} f(t) = -1$ (limit exists because it has 2 sides)

№12

$$\lim_{x \to 2} (-x^2 + 5x - 2) = -4 + 10 - 2 = 6 - 2 = 4$$

№22

$$\lim_{h \to 0} \frac{\sqrt{5h+4} - 2}{h} = \frac{(\sqrt{5h+4} - 2)(\sqrt{5h+4} + 2)}{h(\sqrt{5h+4} + 2)} =$$

$$= \frac{5h + 4 - 4}{h(\sqrt{5h+4} + 2)} = \frac{5\cancel{h}}{\cancel{h}(\sqrt{5h+4} + 2)} = \frac{5}{\sqrt{5h+4} + 2} = \frac{5}{2+2} = \frac{5}{4} = 1,25$$

№32

$$\lim_{x \to 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{x} = \frac{\frac{x+1+x-1}{x^2-1}}{x} = \frac{2\cancel{x}}{x^2-1} \cdot \frac{1}{\cancel{x}} = \frac{2}{x^2-1} = \frac{2}{-1} = -2$$

Chapter: 2.3

Page: 82

Babayeva Niyara

№2

$0<|x-2|<\delta \Rightarrow -\delta<x-2<\delta \Rightarrow -\delta+2<x<\delta+2$

$-\delta+2 \to 1$, $\quad \delta+2 \to 7$

$-\delta+2=1 \Rightarrow \delta=1$

$\delta+2=7 \Rightarrow \delta=9$

smallest: $\delta=1$

№12

$f(x)=4-x^2$

$x_0=-1$

$L=3$

$\epsilon=0{,}25$

$|x+1|<\delta \Rightarrow$

$-\delta<x+1<\delta$

$\Rightarrow -\delta-1<x<\delta-1$

$-\delta-1=-\frac{\sqrt{5}}{2} \Rightarrow \delta=\frac{\sqrt{5}}{2}-\frac{2}{2}=\frac{\sqrt{5}-2}{2}$

$\delta \cancel{-1}=\frac{\sqrt{3}}{2} \Rightarrow \delta=\frac{\sqrt{3}+2}{2}$

$\frac{\sqrt{5}-2}{2}<\frac{\sqrt{3}+2}{2}$, hence

answer is $\delta=\frac{\sqrt{5}-2}{2}$

Chapter: 2,3
Page: 82
Babayeva Niyazi

№22

$f(x)=x^2$, $L=3$, $x_0=\sqrt{3}$, $\epsilon=0{,}1$

$$|f(x)-L|<\epsilon$$

$$|x^2-3|<0{,}1$$

$$-0{,}1<x^2-3<0{,}1$$

$$2{,}9<x^2<3{,}1 \Longrightarrow \sqrt{2{,}9}<x<\sqrt{3{,}1}$$

$$|x-\sqrt{3}|<\delta \Rightarrow -\delta<x-\sqrt{3}<\delta \Rightarrow$$

$$\Rightarrow -\delta+\sqrt{3}<x<\delta+\sqrt{3}$$

$$-\delta+\sqrt{3} \swarrow \sqrt{2{,}9} \qquad \delta+\sqrt{3} \swarrow \sqrt{3{,}1}$$

$$-\delta+\sqrt{3}=\sqrt{2{,}9} \Rightarrow \delta=\sqrt{3}-\sqrt{2{,}9}$$

$$\delta+\sqrt{3}=\sqrt{3{,}1} \Rightarrow \delta=\sqrt{3{,}1}-\sqrt{3}$$

$\sqrt{3{,}1}-\sqrt{3}<\sqrt{3}-\sqrt{2{,}9}$, hence

answer is $\delta=\sqrt{3{,}1}-\sqrt{3}$

№32

$f(x)=-3x-2$, $x_0=-1$, $\epsilon=0{,}03$, $L=1$

$$\lim_{x\to -1} f(x)=-3x-2=3-2=1$$

Chapter: 2,3

Page: 82

Babayeva Niyazo

№ 32 (continuation)

$|f(x) - L| < \epsilon$

$|-3x-2-1| < 0{,}03$

$|-3x-3| < 0{,}03$

$-0{,}03 < -3x-3 < 0{,}03$

$0{,}01 > x+1 > -0{,}01$

$-0{,}99 > x > -1{,}01$

$|x+1| < \delta \Rightarrow -\delta < x+1 < \delta \Rightarrow -\delta-1 < x < \delta-1$

$-0{,}99$ $\quad$ $-1{,}01$

$-\delta-1 = -0{,}99 \Rightarrow \delta = -0{,}01$

$\delta - 1 = -1{,}01 \Rightarrow \delta = -0{,}01$

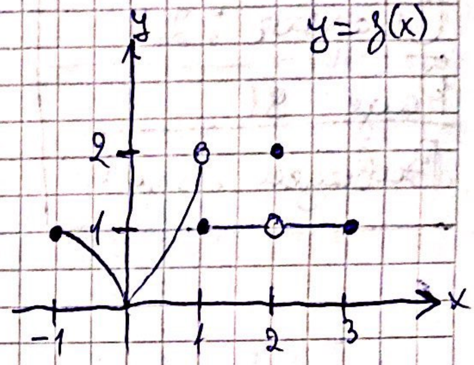
UniNote

Chapter: 2.4

Page: 90

Babayev Niyazi

№2



a) $\lim_{x \to -1^+} f(x) = 1$ → true

b) $\lim_{x \to 2} f(x) = DNE$ → false, it exists

c) $\lim_{x \to 2} f(x) = 2$ → false, $L = 1$

d) $\lim_{x \to 1^-} f(x) = 2$ → true

e) $\lim_{x \to 1^+} f(x) = 1$ → true

f) $\lim_{x \to 1} f(x) = DNE$ → true

g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ → true

h) $\lim_{x \to c} f(x)$ = exists at every c in $(-1; 1)$ → true

i) $\lim_{x \to c} f(x)$ = exists at every c in $(1; 3)$ → false

j) $\lim_{x \to -1^-} f(x) = 0$ → false

k) $\lim_{x \to 3^+} f(x) = DNE$ → true

Chapter: 2,4
Page: 91
Babayeva Niyaza

№12

$$\lim_{x\to 1^+} \sqrt{\frac{x-1}{x+2}} = \sqrt{\frac{0}{x+2}} = +\infty$$

№22

$$\lim_{t\to 0} \frac{\sin kt}{t} = \frac{k\cdot \sin kt}{kt} = k\cdot 1 = k$$

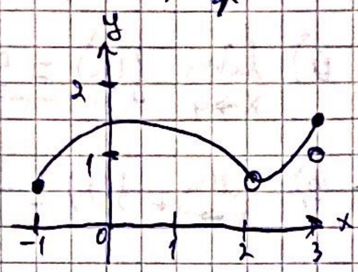
$$\frac{\sin\theta}{\theta} = 1$$

№32

$$\lim_{x\to 0} \frac{x - x\cos x}{\sin^2 3x} = \frac{0-0}{0} = 0$$

Chapter: 2.5
Page: 101
Babayeva Niyazi

№1



No, it is not continuous. It fails at $x=2$.

№13

$$y = \frac{1}{x-2} - 3x$$

$x-2 \neq 0$

$x \neq 2$

Discontinuous at $x = 2$

№22.

$$y = \tan \frac{\pi x}{2}$$

since $\frac{\pi}{2}$ is not defined, only odd numbers can fit.

continuous when x is an odd number

№32

$$\lim_{t \to 0} \sin\left(\frac{\pi}{2}\cos(\tan t)\right) = \sin\left(\frac{\pi}{2} \cdot 1\right) = 1.$$

yes, continuous.

UniNote

Scanned with CamScanner

Chapter: 2.6
Page: 114
Babayeva Noyazi

№2

a) $\lim_{x\to 4} f(x) = 2$

b) $\lim_{x\to 2^+} f(x) = -3$

c) $\lim_{x\to 2^-} f(x) = 1$

d) $\lim_{x\to 2} f(x) = \text{DNE}$

e) $\lim_{x\to -3^+} f(x) = \infty$ (asympt vert.)

f) $\lim_{x\to -3^-} f(x) = \infty$ (asympt vert)

g) $\lim_{x\to -3} f(x) = \infty$

h) $\lim_{x\to 0^+} f(x) = \infty$ (asympt vert)

i) $\lim_{x\to 0^-} f(x) = -\infty$ (asympt vert)

j) $\lim_{x\to 0} f(x) = \text{DNE}$

k) $\lim_{x\to \infty} f(x) = 0$

l) $\lim_{x\to -\infty} f(x) = 0$

№12

since $\lim_{x\to 0} \frac{\sin x}{x} = 0$,

hence $\lim_{z\to \infty} \frac{z+\sin z}{2z+7-5\sin z} = \frac{\frac{z}{z}+\frac{\sin z}{z}}{\frac{2z}{z}+\frac{7}{z}-\frac{5\sin z}{z}} = \frac{1+0}{2+0-0} = \frac{1}{2}$

Chapter: 2.6
Page: 114
Babayeva Nigar

№22

$$h(x) = \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9}$$

$$\lim_{x \to \infty} \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9} = \frac{\frac{-x^4}{x^4}}{\frac{x^4}{x^4} - \frac{7x^3}{x^4} + \frac{7x^2}{x^4} + \frac{9}{x^4}} =$$

$$= \frac{-1}{1 - 0 + 0 + 0} = -1$$

$$\lim_{x \to -\infty} \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9} = -1$$

№32

$$\lim_{x \to \infty} \frac{\sqrt[3]{x} - 5x + 3}{2x + x^{\frac{2}{3}} - 4} = \frac{x^{\frac{2}{3}} - 5x + 3}{2x + x^{\frac{2}{3}} - 4} = \frac{x^{-\frac{1}{3}} - 5 + \frac{3}{x}}{2 + x^{-\frac{1}{3}} - \frac{4}{x}} =$$

$$= \frac{0 - 5 + 0}{2 + 0 - 0} = \frac{-5}{2} = -2{,}5$$