

# Perspectives on Aerodynamic Design Optimization

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**CFD-based aircraft design optimization has matured significantly in the last few years thanks to improvements in CFD solvers, mesh deformation, sensitivity computation, and optimization tools.** We review our recent developments for each of these components, and present open-source tools made available for aerodynamic shape optimization. A variety of applications is presented, including the optimization of a supercritical airfoil starting from a circle, a web application that optimizes airfoils within a few seconds, aircraft aerodynamic and aerostructural optimization, and aeropropulsive optimization. We also review our experience with solving the Aerodynamic Design Optimization Discussion Group (ADODG) benchmarks and other problems in aerodynamic shape optimization. Among the ADODG benchmarks, we focus on the RANS-based problems and discuss some of the issues encountered, including the comparison between Euler and RANS results, and design space multimodality. The availability of these benchmarks and the open-source tools is expected to enable further studies and benchmarks in CFD-based aerodynamic design optimization and MDO.

## I. Introduction

While CFD-based aircraft design optimization was introduced decades ago, several challenges have prevented its widespread use in industry, academia, and government laboratories. These challenges include: (A) CFD solver robustness, (B) scalability with number of design variables, (C) efficient and accurate gradient computation, (D) robust mesh deformation, (E) availability of specialized software, (F) practical industrial constraints, and (G) consideration of aircraft design disciplines other than aerodynamics. This paper reviews the efforts at the University of Michigan MDO Lab to address these challenges. Challenges A through F pertain more specifically to aerodynamic shape optimization, while Challenge G broadens the scope to aircraft design optimization. The list of challenges above is not meant to be comprehensive, but just the main challenges that motivated our developments in the last few years.

The overall result of addressing these challenges was MACH-Aero, an open-source framework for aerodynamic shape optimization.\* By releasing it open source, we addressed challenge E. For each challenge, we provide a short description of the developed solution and how it is integrated in MACH-Aero, with citations to the articles that describe the developments in much more detail.

The Aerodynamic Design Optimization Discussion Group (ADODG) benchmarks played a crucial role in our developments. By developing optimization problems that everyone could run and compare, it motivated us to do a specially thorough job of getting the best possible optimized shapes and addressing computational bottlenecks. It also motivated us to make all the results publicly available and to generate benchmarks within and beyond the scope of the ADODG. Finally, solving and discussing the ADODG benchmarks helped us understand our results and their limitations, motivating further developments.

This paper is structured as follows. In Sec. II.A, we provide an overview of the developments that address the challenges listed above. In Sec. III, we summarize the ADODG benchmarks and cite the efforts towards solving them. In Sec. IV, we highlight applications that we developed and investigated beyond the ADODG benchmarks. We end with concluding remarks in Sec.V.

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\*<https://github.com/mdolab/MACH-Aero>

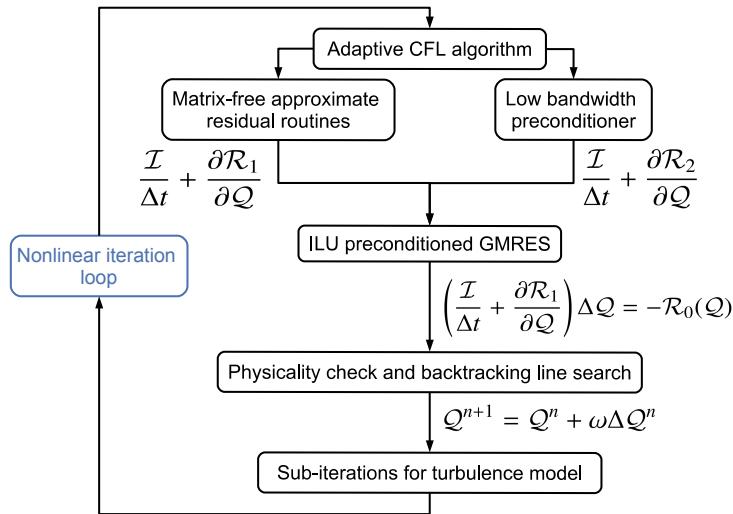
## II. Review of Developments

### A. CFD solver robustness

Integrating CFD in a numerical optimization cycle demands additional requirements on the robustness of the CFD solver. The reason for this is that a CFD solver is more likely to fail during optimization because the optimizer does not share the intuition of a designer, and will provide bad design shapes to the CFD solver. Therefore, it is crucial that the CFD solver be able to solve for designs that might not make much sense.

If the CFD solver fails to converge during an optimization iteration, it interrupts the optimization process, which must then be restarted. Some optimizers allow the user-provided functions to return a “fail” flag, which the optimizer takes into account by taking a less ambitious step for the next function evaluation. While this might prevent the optimization from exiting outright, the optimization might still fail nevertheless because the information from failed function evaluations is not as rich as a normal evaluation. Inaccurate information is still better than a “fail”, as long as it is sufficiently accurate to provide the correct trends—having the right sign in each gradient component is usually sufficient to guide the optimizer away from a bad design. Therefore, it is usually worthwhile to force the CFD solver to converge to a solution that we know does not reflect the real physics.

To this end, we developed a Jacobian-free approximate Newton–Krylov strategy for robustly solving the RANS equations for a wide range of geometries. Yildirim et al. [1] describes this strategy in detail so we only include a brief summary here. Figure 1 shows an overview of the method. The solver updates the state at each nonlinear iteration using the backward Euler time stepping formula. To alleviate the difficult startup phase, we use an adaptive CFL algorithm based on pseudo-transient continuation [2]. This starts with a small time step for robustness and then increases the step size rapidly as it approaches the solution, which has the favorable stability properties of the backwards Euler method during the initial stage, while approaching a Newton-type algorithm as the time step approaches infinity. At each nonlinear iteration, the update vector is obtained by inexactly solving a large linear system. We use two levels of approximation of the exact flow Jacobian residuals ( $R_0$ ):  $R_1$  and  $R_2$ .  $R_1$  is an approximation that leaves out several terms, but it is still closer to the exact Jacobian than  $R_2$  (which is a first order Jacobian approximation).  $R_1$  is used for the GMRES matrix vector products and  $R_2$  is used for the preconditioner. Since this is inexact, we make sure that the thermodynamic quantities do not vary by more than 20%, and we use a line search with backtracking to insure a decrease in the unsteady residual norm.



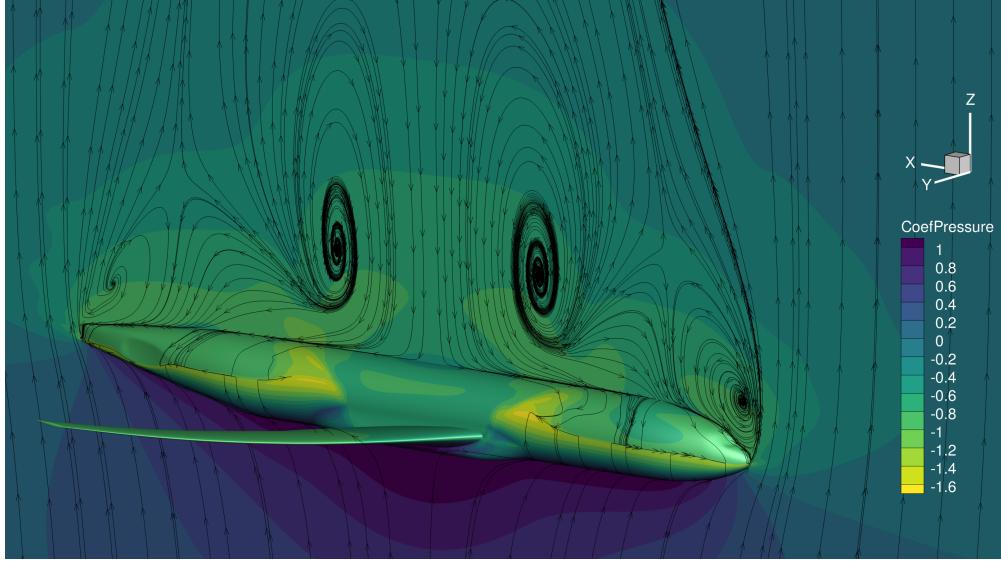
**Fig. 1 Overview of the Jacobian-free approximate Newton–Krylov strategy (courtesy of Anil Yildirim)**

We implemented the ANK approach in the ADflow open-source CFD solver.<sup>†</sup> ADflow can converge steady state

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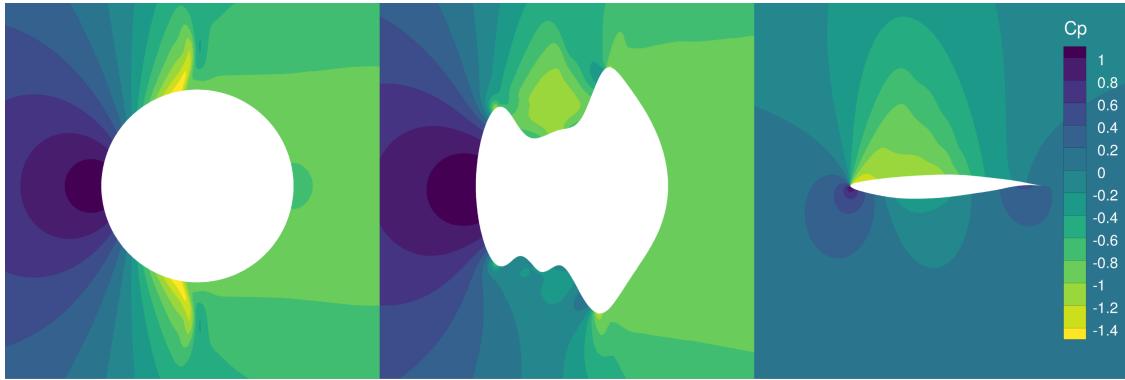
<sup>†</sup><https://github.com/mdolab/adflow>

solutions to the RANS equations even if the flow field is inherently unsteady. This is due to the backwards Euler algorithm; the solver can stabilize the physically unstable modes. An example of such a flow solution is shown in Fig. 2, where NASA’s Common Research Model (CRM) is solved for 90 deg angle of attack at  $M = 0.85$ . The solution in this case is not physical, but the important thing is that the flow solver is able to converge. This case was inspired by the 90 deg angle-of-attack solution by Burgess and Glasby [3].



**Fig. 2** Converged RANS solution for the CRM configuration at 90 deg angle of attack and  $M = 0.85$  demonstrated the robustness of the ADflow solver (courtesy of Anil Yildirim).

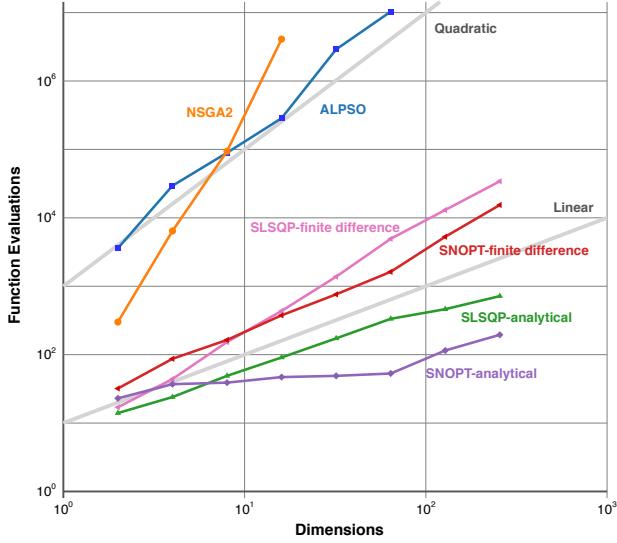
An example of such an optimization is the one performed by He et al. [4] using MACH-Aero, which starts from a circle and converges to a supercritical airfoil shape in a single fully automated optimization, as shown in Fig. 3. Again, the flow solution for the circle and some of the intermediate shapes is not physical, but the derivatives with respect to shape have the right trend for drag reduction, and as the shape approaches the optimum, the RANS solution becomes valid. As long as the optimizer can eventually converge to a case where the solution is valid, inaccurate intermediate results are irrelevant.



**Fig. 3** Lift- and moment-constrained drag minimization starting from a circle;  $M = 0.734$ ,  $C_l = 0.824$ ,  $C_m \geq -0.092$  [4].

## B. Scaling with number of design variables

Aerodynamic shape optimization requires a large number of shape design variables to achieve the best possible performance. For transonic wing design optimization, Lyu et al. [5] showed that at least 200 shape variables are required to take full advantage of aerodynamic shape optimization and that beyond this number of variables, there are diminishing returns. Only gradient-based optimization algorithms can handle this number of variables efficiently. Figure 4 shows the scalability of two gradient-free optimization algorithms (NSGA2 and ALPSO) compared to two gradient-based optimization algorithms (SLSQP and SNOPT) for a multidimensional Rosenbrock function. The results in this figure demonstrate that gradient-based algorithms are the only viable option for more than 100 design variables. Furthermore, for a given gradient-based optimizer, there is a large difference between using finite differences (the default way gradient-based optimizers compute the gradients) and an analytic method (which in this case is based on the symbolic differentiation of the function and is thus fast). Therefore, there is a big motivation for developing methods for computing gradients efficiently. We discuss such methods in the next section.



**Fig. 4 Gradient-based optimization is the only hope for handling the large numbers of design variables required for aerodynamic shape optimization [6].**

To compare gradient-based and gradient-free optimization algorithms for a more practical problem, Lyu et al. [6] benchmarked various optimizers for a CFD-based wing twist optimization problem. The problem was limited to nine twist design variables and a rather coarse mesh so that the optimization with the gradient-free algorithms was achievable. The optimization problem was to minimize the drag coefficient subject to a lift coefficient constraint. Most gradient-based algorithms achieved optimality within 14 to 230 function evaluations, while the gradient-free algorithms required over 8,000 evaluations.

For the vast majority of our work, we have used SNOPT, a gradient-based algorithm that implements sequential quadratic programming and can handle nonlinear constraints [7]. To facilitate its use in our framework, we have wrapped SNOPT (a Fortran library) with Python through the pyOptSparse wrapper [8]. <sup>‡</sup> In addition to facilitating the integration with the other modules of the framework, pyOptSparse provides a common interface to various optimization algorithms; to try a different algorithm, only a flag needs to be changed in the main run script. Using this feature, we were able to run all the results shown in Fig. 5 with minimal setup effort.

<sup>‡</sup><https://github.com/mdolab/pyoptsparse>

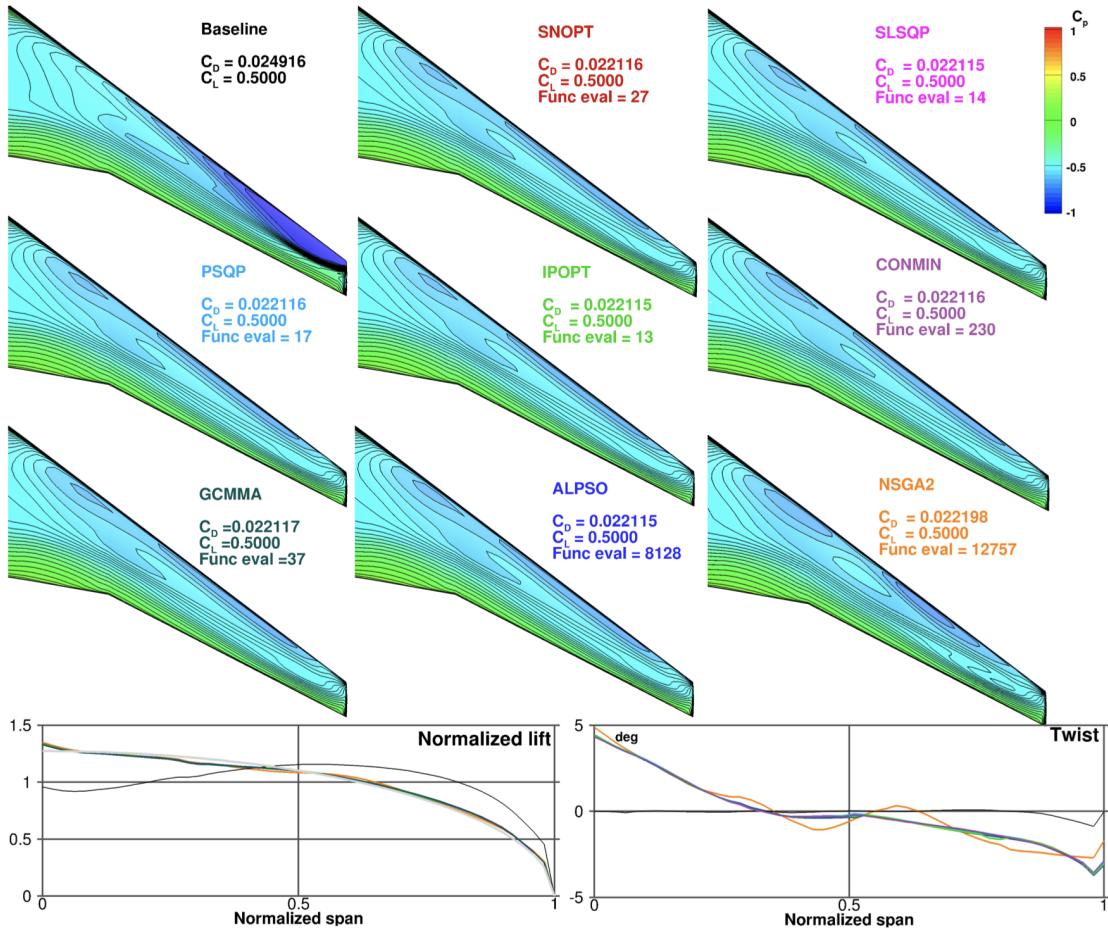
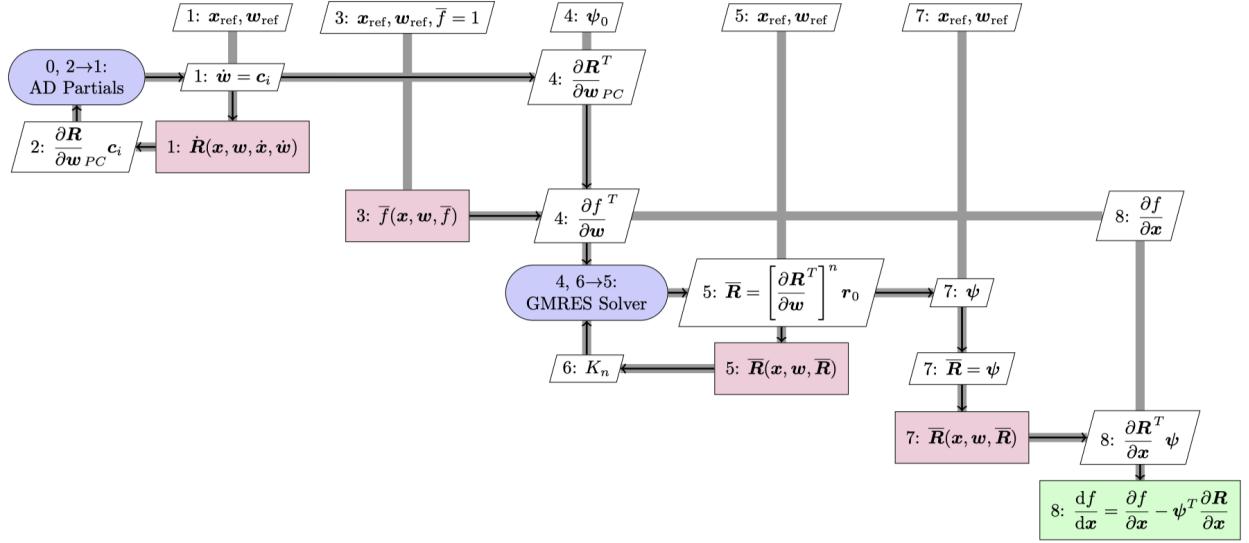


Fig. 5 Optimizer comparison for a wing design problem with nine twist variables shows that gradient-free methods cost 2–3 orders of magnitude more to optimize this simple problem [6].

### C. Effective computation of derivatives

As illustrated in Fig. 4, a good gradient-based algorithm is not sufficient; it is also necessary for the gradients to be computed accurately and efficiently. Adjoint methods compute derivatives with respect to large numbers of design variables efficiently [9], but require a high implementation effort. To address this issue, we have developed a general recipe for effective adjoint method implementation [10], which we have applied to both ADflow and OpenFOAM [11, 12]. By *effective*, we mean that the approach for adjoint development provides a good balance between implementation effort, accuracy, and efficiency. The approach described by Kenway et al. [10] is a hybrid approach where the code for computing the partial derivatives in the adjoint method is derived by using automatic differentiation (AD) tools. For the solution of the adjoint equations, we adopt a Jacobian-free GMRES strategy, which we found to be the most scalable [10]. The overall approach is illustrated in Fig. 6 using an extended design structure matrix (XDSM) representation [13].



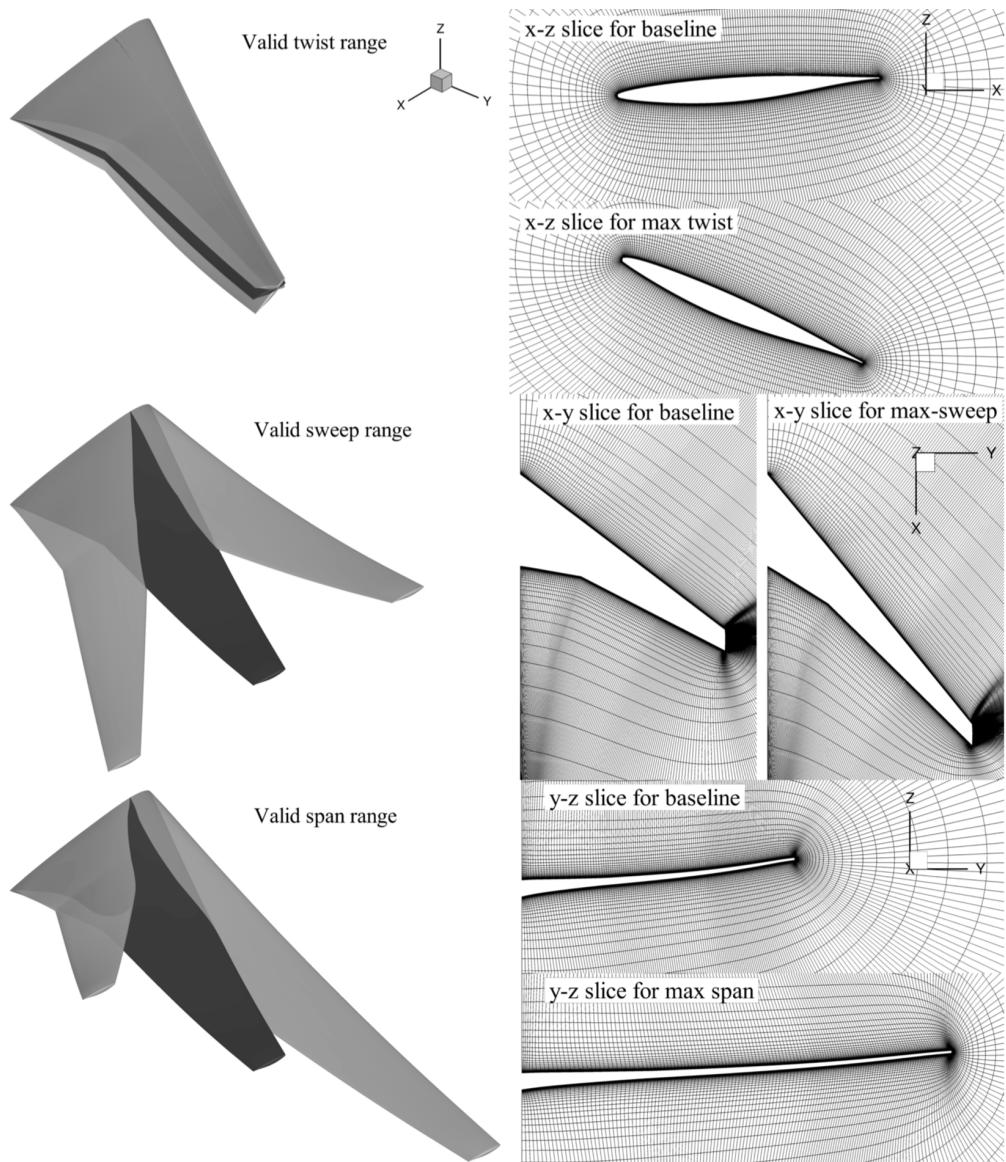
**Fig. 6 XDSM of the hybrid-adjoint approach [10].** The transpose of the Jacobian ( $[\partial R / \partial w]_{PC}^T$ ) is computed using forward mode AD and coloring in the 0, 2 $\rightarrow$ 1 loop. The adjoint equation right-hand side vector ( $[\partial f / \partial w]^T$ ) is computed using reverse-mode AD in step 3. Then, the adjoint equation is solved using GMRES and the resulting adjoint vector is used in step 8 to compute the desired total derivatives.

### D. Mesh deformation

A shape optimization cycle requires a geometry engine to translate new shape design variables to new shapes and automatically obtain a new mesh for the new design surface. The new mesh is not re-generated, but rather, it is a deformation of the baseline mesh. The robustness of the mesh deformation is essential for the same reasons cited for flow solver robustness in Sec. II.A: If it fails, it jeopardizes the optimization process. To address this need, we implemented IDWarp, an efficient analytic method for volume mesh deformation <sup>§</sup>, which was also crucial in the airfoil optimization starting from a circle described in Sec. II.A [4].

The implementation is based on the inverse distance weighting proposed by Luke et al. [14], with some improvements. The most significant of these improvements is the efficient computation of derivatives via reverse-mode AD. More specifically, IDWarp computes the derivatives of all mesh point coordinates with respect to changes in the surface mesh point coordinates. This is one of the partial derivatives in the derivative chain between the aerodynamic force coefficients and the shape variables. Figure 7 shows examples of mesh deformations performed with IDWarp for a wing mesh.

<sup>§</sup><https://github.com/mdolab/idwarp>

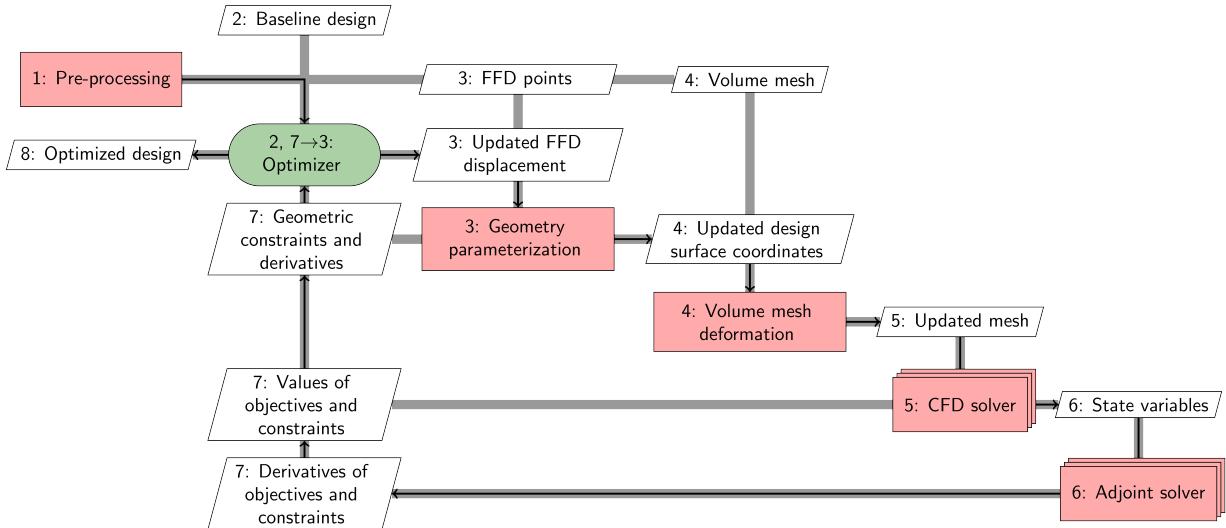


**Fig. 7** Mesh deformations for wing planform optimization showing twist, sweep, and span variables (courtesy of Ping He).

## E. Software availability

One of the reasons why aerodynamic design optimization has not been more widely used is that it requires a complex set of tools that have not been readily available in the aerodynamic design community at large. Commercial CFD solvers have not implemented adjoint solvers until recently, and they have not been developed with optimization in mind. Other CFD-based optimization capability has been restricted to a few research groups in academia and government laboratories. One exception is the open-source CFD solver SU2, which includes an adjoint solver and provides aerodynamic shape optimization capability [15].

The capability described in the present paper is integrated in the MACH-Aero framework and is available under an open source license.<sup>¶</sup> An XDSM showing the aerodynamic shape optimization in MACH-Aero is shown in Fig. 8. All the components shown in the diagonal are wrapped in Python for modularity and ease of use. The optimizers are provided through the pyOptSparse interface introduced in Sec. II.B. The geometry parametrization can use either our free-form deformation (FFD) implementation, pyGeo [16], or OpenVSP [17]. The volume mesh deformation is provided by IDWarp, which we introduced in Sec. II.D. Either ADflow or OpenFOAM are available to solve the flow using a common Python interface. We envision that other CFD solvers could be easily integrated by wrapping them using the same Python interface. Finally, as previously mentioned, both CFD solvers include an adjoint solver to efficiently compute the gradients for design optimization.



**Fig. 8 XDSM of the open-source aerodynamic shape optimization framework, MACH-Aero.**

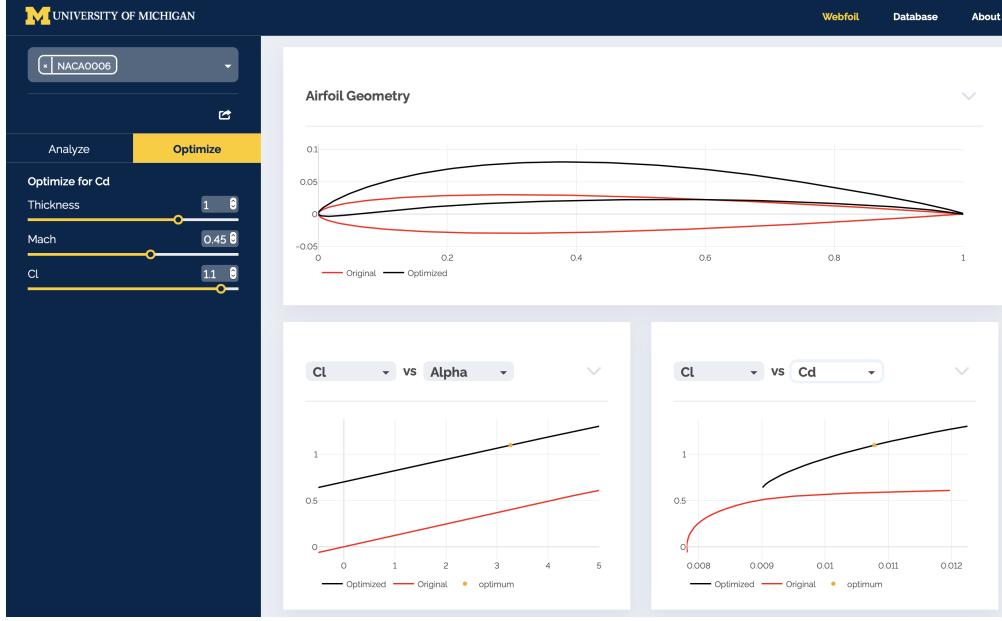
Even though gradient-based optimization with the adjoint method enables efficient aerodynamic shape optimization, it still requires hours in a parallel computer to fully converge an optimization. To make aerodynamic shape optimization more accessible, we have developed a web-based data-driven approach to airfoil design that takes just a few seconds for optimization [18].<sup>¶</sup> Figure 9 shows a screen shot of this online tool, which includes a database of over 1,500 airfoils.

## F. Practical industrial design constraints

Based on our collaborations with industry, we identified several constraints that had to be enforced, which required new developments. Geometric constraints, such as variable fuel volume, wing thickness, leading edge radius, and trailing edge angle constraints are linear and were relatively easy to implement [5]. To consider geometric constraints implicitly, it is also possible to use a data-driven approach [19].

<sup>¶</sup><https://github.com/mdolab/MACH-Aero>

<sup>¶</sup><http://webfoil.engin.umich.edu>



**Fig. 9** Webfoil is an online database and airfoil optimization tool.

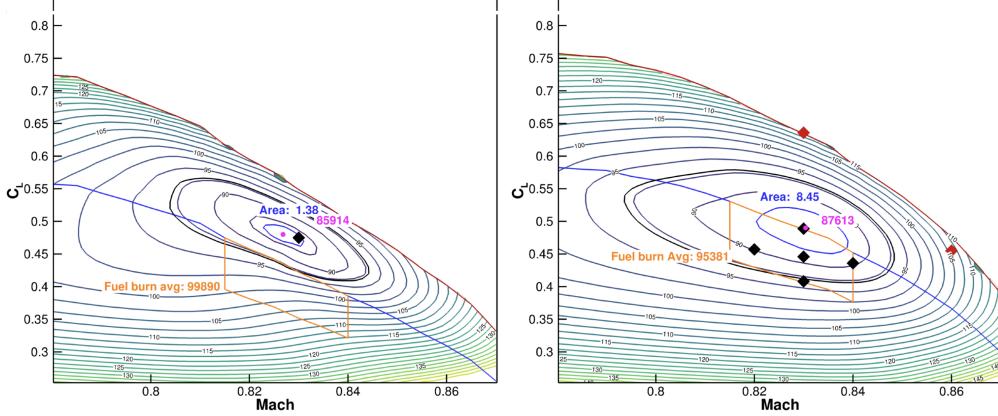
Other constraints, such as buffet and flutter, are highly nonlinear and require much more development effort. Buffet is a phenomenon that is caused by shock-induced separation and is undesirable because it causes vibration. We developed a constraint formulation for buffet based on a separation sensor function [20], which was shown to be effective in various applications [21–26]. We have found that constraining buffet is crucial in transonic wing design. In Fig. 10 we compare the fuel burn contours for a single-point optimization without buffet constraints to a multipoint optimization with buffet constraints. The unconstrained single-point optimization achieves a high performance outside the buffet margin and thus this high performance is not usable in practice. The buffet-constrained optimization moves the area of high performance and the 30% margin boundary so that the highest performance region is usable.

We developed an approach that is mathematically identical to constrain cavitation in hydrofoil design optimization [27, 28]. This approach could also be potentially used for low-speed and high-lift performance, where we would formulate a minimum lift coefficient constraint to be achieved without major separation.

We also made progress towards constraining flutter using two different approaches [29, 30]. Jonsson et al. [31] review existing methods for constraining flutter in optimization, including the approaches cited above. Like flutter, many other practical constraints require the consideration of disciplines beyond aerodynamics, which is the subject of the next section.

## G. CFD-based multidisciplinary design optimization

Aerodynamics is not enough to achieve a feasible aircraft with high performance. One of the most important other disciplines is structures, which couples with aerodynamics to determine the wing performance. The coupling between the disciplines manifests itself both in analysis and design. The analysis requires solving a coupled system with both disciplines because the aerodynamic model provides the forces to the structural model, while the structural model determines the displacements and hence the aerodynamic shape. From the design point of view, there is a trade between structural weight and aerodynamic drag that depends on the chosen objective function. Objective functions such as fuel burn and take-off weight depend on both structural weight and drag, but in different proportions. Fuel burn places more emphasis on reducing drag, while take-off weight places more emphasis on reducing structural weight. More broadly, multidisciplinary design optimization (MDO) provides a way to model a coupled system and find optimal multidisciplinary design trades [32].



**Fig. 10** Fuel burn contours and buffet boundary over the space of flight conditions (lift coefficient and Mach number) for a single-point optimization without buffet constraints (left) and multipoint optimization with buffet constraints (right). The red line represents the buffet boundary and the blue line represents the 30% margin to the boundary, which must not be exceeded in cruise flight conditions [20].

Given the motivation for efficient and accurate gradient computation, we developed a coupled-adjoint approach that computes gradients accounting for the aerostructural coupling [33]. To achieve this, we coupled ADflow to the open-source structural solver TACS, which has an adjoint solver [34]. The result was the MDO for aircraft configurations with high fidelity (MACH) framework. Using this approach, we can compute derivatives of aerodynamic forces and structural stresses with respect to both structural sizing and aerodynamic shape variables. This enabled us to perform the simultaneous design optimization of aerodynamic shape including wing planform variables and structural sizing for various aircraft design applications, including planform optimization starting from the CRM baseline [24], multimission fuel burn minimization [35], tow-steered composite optimization [26], and morphing wing optimization [36, 37].

The coupled-adjoint approach has also been generalized for an arbitrary number of disciplines [9], which lead to the modular analysis and unified derivatives (MAUD) architecture for MDO [38]. The MAUD architecture has been integrated into and improved upon in the OpenMDAO framework [39].

OpenMDAO has facilitated the coupling of CFD with other disciplines and the corresponding coupled adjoint derivative computations. Other MDO work beyond aerostructural optimization includes the simultaneous optimization of aerodynamic shape and propulsion system [40], and optimization of wing, mission, and allocation [41, 42].

### III. ADODG benchmarks

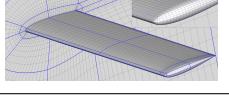
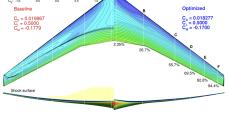
The development of the ADODG benchmarks was motivated by the fact that no such benchmarks were in place when the ADODG was formed. Each researcher had solved different problems using different approaches, making it difficult to compare the various approaches. The ADODG benchmarks enable researchers to solve the same problem so that the results are more comparable. A series of aerodynamic shape optimization problems of increasing complexity in the geometry and physics was developed.

Before the ADODG benchmark cases were established, the MDO Lab had been focusing on CFD-based wing aerostructural design optimization, which we introduced in Sec. II.G. However, our early efforts solved the Euler equations for the aerodynamics and it was difficult to interpret the optimal design trends because they included both aerodynamic and structural effects. The ADODG benchmark cases (and Case 4 in particular), forced us to do more detailed studies considering aerodynamics alone. Understanding the aerodynamic shape optimization results then helped us understand the aerostructural results a lot better.

## A. Summary of the ADODG benchmark cases

The ADODG benchmark problems are listed in Table 1, together with the references that tackled the respective problems. The objective for all problems is to minimize the drag coefficient (or a sum of drag coefficients in the multipoint cases). Most cases are subject to a lift coefficient constraint as well as thickness constraints. Some cases include a moment constraint as well.

**Table 1 ADODG benchmarks**

Case	Description	References
	1 Drag minimization of the NACA 0012 in transonic inviscid flow.	[4, 43–58]
	2 Drag minimization of the RAE 2822 in transonic viscous flow	[4, 43, 44, 46, 49–53, 56, 59]
	3 Lift-constrained drag minimization of a rectangular wing in inviscid subsonic flow	[44, 46, 49]
	4 Lift-constrained drag minimization of the CRM wing in viscous flow	[5, 6, 44, 46, 49, 57, 60, 61]
	5 Lift-constrained drag minimization of the CRM wing-body-tail configuration at flight Reynolds number	[60, 62]
	6 Multimodal subsonic inviscid lift-constrained drag minimization	[63, 64]

The two first cases are airfoil shape optimization problems: Case 1 starts from a NACA 0012 airfoil baseline and Case 2 starts from the RAE 2822 airfoil. These two cases use different aerodynamic models: Case 1 solves the Euler equations, while Case 2 is based on the RANS equations. Another difference between these two cases is that Case 1 does not enforce a lift constraint, while Case 2 does. Based on the number of references, these two cases have been the most solved by far. This is in large part due to the lower cost of solving two-dimensional cases.

Cases 3 and 4 optimize wing shapes; Case 3 is a subsonic case that solves the Euler equations, and Case 4 is a transonic case using RANS. Case 3 optimizes only twist, while Case 4 includes both twist and airfoil shapes. All wing cases include a volume constraint, where the volume of the optimized wing cannot decrease relative to the baseline wing.

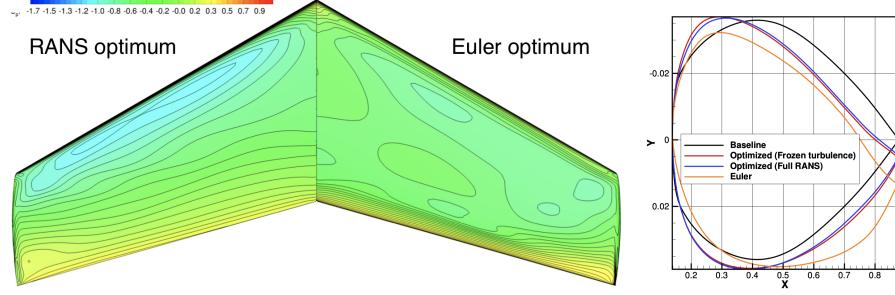
Case 5 is an extension of Case 4 that adds the fuselage and tail. The baseline shape for Case 4 is actually the wing from the CRM full aircraft configuration clipped at the fuselage intersection. Case 5 restores the full CRM configuration and provides a benchmark for a more realistic and industrially relevant design problem. It uses the flight Reynolds number (as opposed to the wind tunnel one) and the thickness constraints prevent the thickness from decreasing relative to the baseline CRM wing. It also includes a tail rotation angle design variable to trim the aircraft at various flight conditions. Case 5 adds two off-design flight conditions to prevent buffet [20].

Finally, Case 6 is a wing shape optimization problem similar to Case 3 with the addition of airfoil shape variables and planform variables (chord variation, sweep, span, and dihedral). The idea is to provide as much freedom in the design space exploration as possible and to determine the characteristics of the wing design problem in terms of multimodality.

## B. Euler- versus RANS-based optimization

Although Cases 1 and 3 do not represent the real physics, they challenge the numerical methods of the CFD, the parametrization approach, and the optimizer. Case 1 in particular has received much attention even though the resulting shape is of no practical value. Such cases do have some value and Case 1 in particular has inspired and demonstrated

new techniques. However, beyond a certain point, one might fall into the trap of developing solutions to issues that do not exist in practice. While RANS is more costly, many of the issues encountered with Euler-based shape optimization, such as non-unique solutions, disappear when using RANS. In addition, we have found that for transonic wing shape optimization, Euler models resulted in completely different shapes, as shown in Fig. 11 [65].



**Fig. 11** Comparison between Euler- and RANS-based aerodynamic shape optimizations starting from the ONERA M6 wing. The RANS-optimized pressure distribution is better behaved and the optimal airfoils show large differences in shape [65].

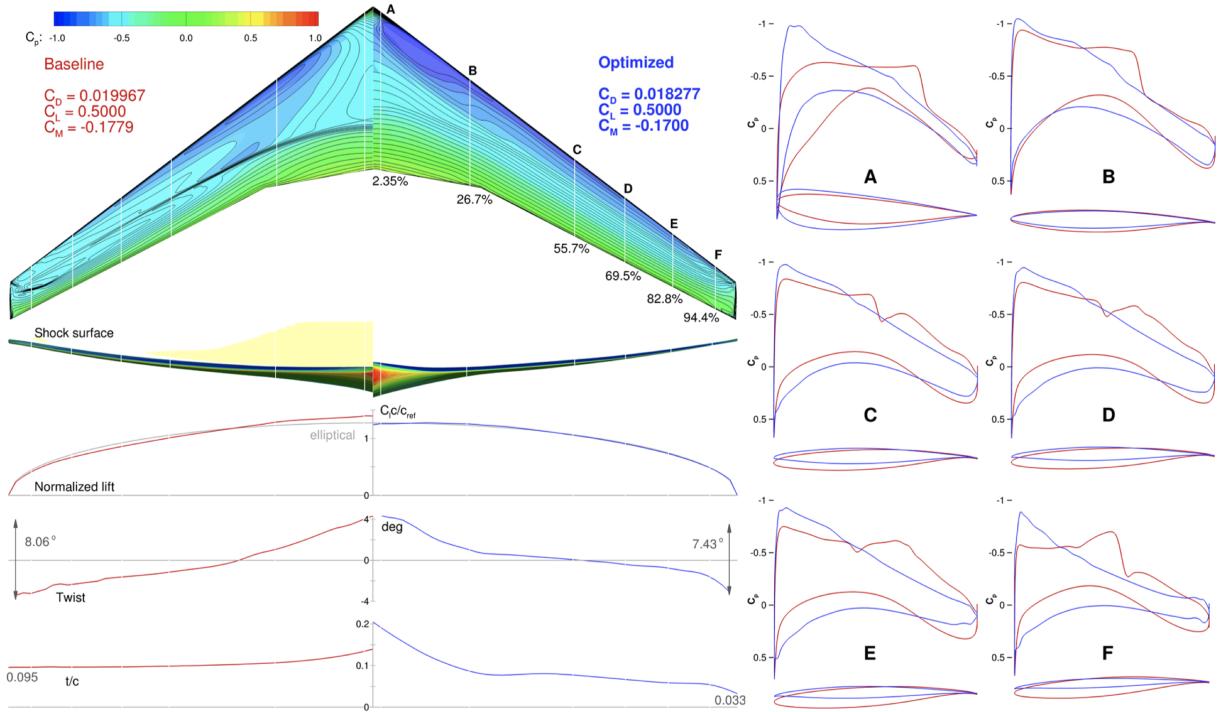
### C. RANS-based wing design optimization

As soon as the ADODG benchmark cases were established, we focused our first efforts on Case 4. This was because this was the most similar to the wing aerostructural design optimization that we had already been performing (see Sec. II.G) [66].

Lyu et al. [5] reported our first results on Case 4.1, which was a single point optimization. The resulting wing, shown in Fig. 12, exhibits much thinner wing sections than the baseline CRM in the outboard sections, which also have sharper leading edges. This is because the case specifies that the airfoil thickness is allowed to be as low as 25% of the original CRM wing. The optimizer exploited this generous allowance to reduce the viscous pressure drag on the outboard of the wing, while thickening the inboard of the wing to satisfy the volume constraint. The thickening of the inboard increases the viscous pressure drag, but the decrease in the outboard more than compensates for this increase. Since the chord in the inboard is much larger, it is more efficient to increase the volume in this region to satisfy the volume constraint. Méheut et al. [60] verified this explanation by analyzing our optimal geometry with a drag decomposition tool. Not everyone achieved the same result. In some cases the problem was not setup exactly as stated, but we suspect that in other cases, the gradients were not accurate enough. We found that the trade between outboard and inboard viscous pressure drag is subtle and requires a large number of iterations with accurate gradients for the optimizer to find it.

Lyu et al. [5] also included the solution for a five-point multipoint problem before the ADODG added a series of multipoint problems to Case 4. The optimized shape for the multipoint case has less sharp leading edges. While the multipoint optimum does not exhibit a shock-free solution like its single-point counterpart, it represents an optimal compromise between the different flight conditions with a much more robust solution over the flight condition space.

Various multipoint cases (Cases 4.2–4.7) were then added to the Case 4 ADODG benchmark, ranging from three to nine points. Kenway and Martins [67] solved and discussed all these cases, including a post-optimality study of the performance of the optimal wings over a range of flight conditions (defined by the lift coefficient and Mach number). The corresponding contour plots are shown in Fig. 13, where the contours were obtained by evaluating  $ML/D$  in a grid of RANS solutions for each optimal wing. In these plots, we can see that the larger the number of points considered, the more robust the design is to variations in the flight conditions. This robustness is achieved with a small penalty in the maximum performance. One particularly interesting result is that of Case 4.5, which results in two maximum performance points that have a region of lower performance between them.



**Fig. 12** Single-point optimization result for Case 4 [5].

#### D. Multimodality in aerodynamic shape optimization

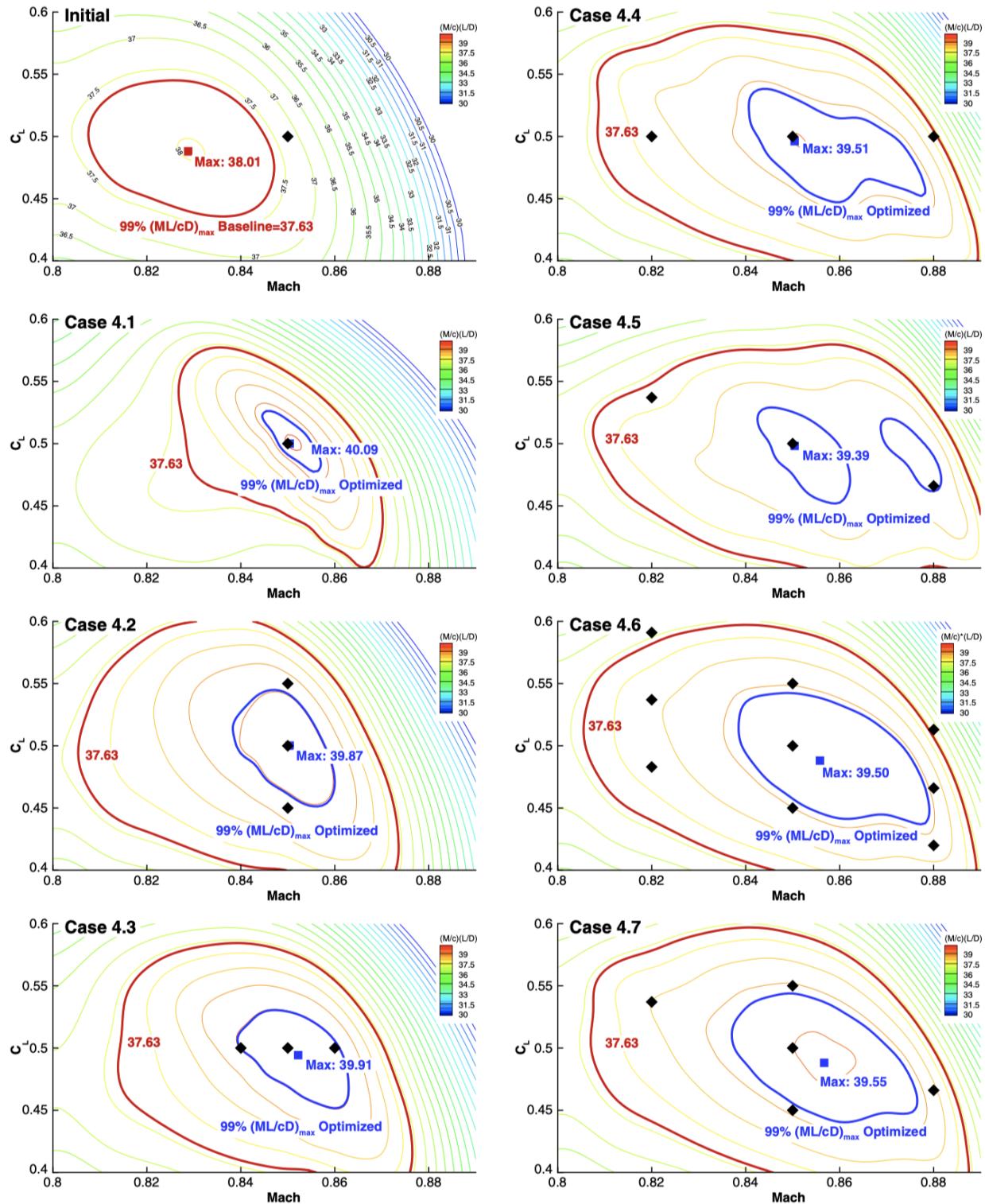
A popular belief in the research community is that these aerodynamic shape optimization problems *have to* be multimodal. However, our studies have found that this concern is largely unwarranted. It is impossible to prove that we have found the global optimum, but to show that the problem is multimodal, we only need to find a second local minimum. Therefore:

*“An optimum should be assumed to be the global one until proven otherwise.”*

Having said that, a reasonable effort must be made to look for multiple optima by starting from different designs.

In our first study of Case 4, we tried to find multiple local minima by starting from random perturbations of the original shape [5]. All optimizations converged to essentially the same shape. While we saw some difference in the shape design variables, the differences in drag between the various designs was only 0.1 drag counts. Follow-up work by Yu et al. [61] found that refining the computational mesh brought these optimized shapes even closer to each other. That work included a starting design that consisted of a CRM planform with NACA 0012 airfoils and no twist, which also converged to the same Case 4 optimum. He et al. [4] did a similar multi-start study for Case 2 and found that the optimization always converged to the same optimum airfoil. Therefore, our hypothesis is that the design space for twist and airfoil shape optimization is unimodal from the physical point of view. Around the global optimum, however, numerical noise causes gradient-based optimizers to converge to slightly different designs.

We believe that researchers often identify spurious multiple local minima because of numerical issues with either the computed gradients or the optimization algorithm. A careful verification of the computed gradients against an equally accurate method is recommended. In ADflow, the adjoint gradients have been verified against the complex-step method [68]. We also recommend that practitioners pay close attention to the optimality and feasibility tolerances, as well as convergence histories. When using gradient-free algorithms, claimed optima are even more suspect because unlike the mathematical optimality criteria of gradient-based algorithms, the optimality criteria for most gradient-free algorithms are based on heuristics.



**Fig. 13** Aerodynamic performance in the flight condition space for all optimal wings for Case 4 [67].

### E. Full configuration aerodynamic optimization and MDO

Case 4 considers only the wing from the Common Research Model (CRM) configuration and compensates by the lack of horizontal tail by imposing a moment coefficient constraint. While this design freedom provided an excellent test for the overall optimization procedure by requiring robustness to changes in the shape and accurate gradients, the optimal wings were too thin to be practical.

Case 5, builds upon Case 4, by considering the full CRM configuration (wing, fuselage, and horizontal tail), also known as the Drag Prediction Workshop (DPW) 4 geometry [69]. By adding back the fuselage and horizontal tail, we can enforce trim by changing the horizontal tail rotation, and capture the true effect of the wing shape on trim drag. Chen et al. [62] investigated this effect and created a surrogate model for the trim drag to use when the horizontal tail is not available in the CFD model.

As mentioned in Sec. II.G, aerodynamics is not enough for aircraft design. Inspired by the success of the ADODG benchmarks, we developed an aerostructural wing design benchmark based on Case 5. This required creating a wingbox structure (loosely based on the Boeing 777 structural arrangement) and a new jig outer mold line [24]. The structure and shape were designed such that they achieve the shape of the original CRM when analyzed using static aeroelastic analysis at the CRM nominal flight condition ( $C_L = 0.5$ ,  $M = 0.85$ ). The benchmark is called the undeflected CRM (uCRM) and has been used in various studies [26, 36, 37, 70].

## IV. Other applications

The CFD-based optimization efforts in the MDO Lab are summarized in Table 2. They include the applications that were already mentioned as well as two other applications that go beyond aircraft applications: the aerodynamic shape optimization of wind turbine blades and the hydrodynamic and hydrostructural optimization of hydrofoils.

While we are not involved in experimental work, we have had two optimization results that have been the subject of experimental measurements. One was the hydrofoil design optimization of Garg et al. [27], who was built and tested in a water tunnel. The experimental results matched the numerical predictions [28]. The other result that was tested was the tow-steered aerostructural optimization of Brooks et al. [26]. Aurora Flight Sciences built a one-third-scale model of the optimized wingbox [71] that underwent structural testing at NASA Armstrong Research Center. \*\*

## V. Conclusions

This paper provides an outline of the methods developed in the MDO Lab to enable CFD-based aircraft design optimization. The cited references provide much more detail on the methods themselves and the applications. The common denominator in all of this work is accurate and efficient computation of derivatives via adjoint methods, which together with gradient-based optimization enables the solution of large-scale CFD-based aerodynamic shape and MDO for aircraft configurations and other applications. Several challenges were tackled over the last several years to make the optimization more scalable, more robust, and more practical. Much of the developed software, including all the components required for aerodynamic shape optimization are available under an open-source license in the MACH-Aero framework. ††

The ADODG benchmark cases motivated many of the reported developments. Even cases that are not realistic from the practical point tested the limits of the methods, which ultimately motivated improvements in accuracy and robustness. One exception to this are the Euler-based cases, which include issues that we do not think are worth tackling because they disappear when using RANS. Working closely with industry has been invaluable in identifying the challenges that needed to be solved for practical applications.

Given the contributions above and the fact that much of the code that we developed is available under an open-source license, we expect that the use CFD-based optimization will continue to increase and become more widespread than ever. The recent developments in the OpenMDAO framework, including coupled-adjoint derivative computation, have facilitated the implementation of CFD-based MDO, and we expect further developments in this area.

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\*\*The results from this testing have not been published yet.

††<https://github.com/mdlolab/MACH-Aero>

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**Table 2 List of optimization problems solved with MACH.**

Application	References
Aerodynamic shape optimization	<p>2-D transonic aerodynamic shape optimization [4, 72]</p> <p>3-D transonic aerodynamic shape optimization [5, 62, 67, 73]</p> <p>Optimization of novel configurations [74–76]</p> <p>Formulation of buffet constraint for wing design optimization [20]</p> <p>2-D and 3-D supersonic aerodynamic shape optimization [77]</p> <p>Optimization with spatial integration constraints [78, 79]</p> <p>Simultaneous design optimization of shape, trajectory, and aircraft allocation [41]</p>
Aerostructural design optimization	<p>Optimization of a transport configuration [24, 35, 66]</p> <p>Optimization with tow-steered composite structures [26, 70]</p> <p>Optimization of morphing trailing edge device [36, 37]</p> <p>Optimization with flutter constraints [29, 80, 81]</p>
Aeropropulsive design optimization	<p>Boundary layer ingestion modeling [82]</p> <p>Design optimization of a boundary layer ingestion system [40, 83–85]</p>
Design optimization of wind turbines	<p>Aerodynamic shape optimization of wind turbine blades [86, 87]</p>
Optimization of hydrofoils	<p>Hydrodynamic hydrofoil shape optimization [88]</p> <p>Hydrostructural optimization of metallic and composite hydrofoils [27, 28, 89]</p>