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Adjoint-Based Nonlinear Output Space Mapping for Accelerated Aerodynamic Shape Optimization

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A nonlinear parametric response correction technique for aerodynamic shape optimization by physics-based surrogates is proposed. In particular, the nonlinear physics-based surrogates are realized using a quadratic extension of the linear output space mapping where the model coefficients are determined through zero- and first-order conditions with the gradients calculated using adjoint methods. The effectiveness of the proposed surrogate modeling approach is investigated using the responses of transonic inviscid fluid flow simulations. The results indicate that the proposed nonlinear response correction, in the context of optimization, may provide a significantly better high- and low-fidelity model alignment throughout the design space than the linear method.

I. Introduction

PDE-constrained aerodynamic design can be accelerated using surrogate-based optimization (SBO) techniques[1]. Physics-based SBO techniques have been shown to be more efficient than approximation-based SBO[2]. The physics-based SBO techniques can be broadly categorized as parametric and nonparametric[3]. The parametric techniques utilize an explicit formulation, typically a linear one, where the relationship between the high-fidelity model and the surrogate is quantified by a number of parameters that need to be extracted, usually realized by solving a separate nonlinear minimization problem, in order to identify the model. In nonparametric techniques, the relationship between the low- and high-fidelity models is directly extracted from the model responses. The formulation of nonparametric techniques is generally more complex than the parametric ones, and involves more restrictive assumptions regarding their applicability. However, nonparametric methods are normally characterized by a better generalization capability than parametric techniques.

This work focuses on extending parametric techniques to improve their generalization capabilities. In particular, the work investigates whether a nonlinear parametric response correction technique provides a better generalization capability than a linear mapping, in a local optimization context, when given a set of models with variable physics. More specifically, a nonlinear output space mapping technique using adjoint sensitivities is proposed. The effectiveness of the proposed approach is investigated for surrogate modeling of the responses of transonic inviscid fluid flow simulation past airfoils at constant lift.

II. Optimization Methodology

A. Surrogate-based Optimization Algorithm

Nonlinear PDE-constrained minimization problem of the following form is considered[3]

$$\mathbf{x}^* = \arg\min_{\mathbf{l}\le\mathbf{x}\le\mathbf{u}} H(\mathbf{f}(\mathbf{x})) \;\text{ s.t. }\; \mathbf{g}(\mathbf{x}) \le 0,\; \mathbf{h}(\mathbf{x}) = 0, \tag{1}$$

where $\mathbf{x}$ is the design variable vector, $\mathbf{x}^*$ is the optimized design, H is a scalar valued objective function, $\mathbf{f}(\mathbf{x})$ is a vector with the figures of merit, $\mathbf{g}(\mathbf{x})$ is a vector with the inequality constraints, $\mathbf{h}(\mathbf{x})$ is a vector with the equality constraints,

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and $\mathbf{l}$ and $\mathbf{u}$ are the design variable lower and upper bounds, respectively. The solution of problem (1) is accelerated using surrogate-based modeling and optimization within a trust region framework[3]

$$\mathbf{x}^{(i+1)} = \underset{\mathbf{l}\le\mathbf{x}\le\mathbf{u},\,\|\mathbf{x}-\mathbf{x}^{(i)}\|\le\delta^{(i)}}{\arg\min}\; H(\mathbf{s}^{(i)}(\mathbf{x})), \tag{2}$$

where $\mathbf{x}^{(i)}$, $i = 0, 1, \ldots$, is a sequence of approximate solutions to (1), $\mathbf{s}^{(i)}(\mathbf{x})$ is a surrogate model of $\mathbf{f}(\mathbf{x})$ at iteration i, and $\delta^{(i)}$ is the trust region search radius[4] at iteration i. The pattern search algorithm[5] drives the solution of (2).

B. Response Correction by Nonlinear Output Space Mapping with Adjoints

A generalized form of the linear output space mapping[6] (LOSM) is proposed to allow for nonlinear correction. The nonlinear output space mapping (NOSM) surrogate model form is assumed to be

$$\mathbf{s}(\mathbf{x}) = \mathbf{H}(\mathbf{x})\circ\mathbf{c}(\mathbf{x}) + \mathbf{G}(\mathbf{x}), \tag{3}$$

where both $\mathbf{H}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are the second-order polynomials defined as

$$\mathbf{H}(\mathbf{x}) = \mathbf{H}_0 + \mathbf{G}_h^T\mathbf{x} + \mathbf{x}^T\mathbf{H}_h\mathbf{x}, \tag{4}$$

$$\mathbf{G}(\mathbf{x}) = \mathbf{G}_0 + \mathbf{G}_g^T\mathbf{x} + \mathbf{x}^T\mathbf{H}_g\mathbf{x}, \tag{5}$$

with $\mathbf{H}_0$ and $\mathbf{G}_0$ being vectors, $\mathbf{G}_h$ and $\mathbf{G}_g$ being rectangular matrices and $\mathbf{H}_h$ and $\mathbf{H}_g$ being symmetric (three-dimensional) matrices. Let us further assume that both $\mathbf{f}(\mathbf{x}^{(k)})$ and $\nabla\mathbf{f}(\mathbf{x}^{(k)})$ are available (with the gradients obtained using adjoint methods) for all $k = 1, ..., p$, where p is the number of the training points. We want to establish the surrogate model by imposing the following conditions:

$$\mathbf{s}(\mathbf{x}^{(k)}) = \mathbf{f}(\mathbf{x}^{(k)}), \tag{6}$$

$$\nabla\mathbf{s}(\mathbf{x}^{(k)}) = \nabla\mathbf{f}(\mathbf{x}^{(k)}), \tag{7}$$

for $k = 1, \ldots, p$. Because we have

$$\nabla\mathbf{h}(\mathbf{x}) = \mathbf{G}_h + 2\mathbf{H}_h\mathbf{x}, \tag{8}$$

$$\nabla\mathbf{g}(\mathbf{x}) = \mathbf{G}_g + 2\mathbf{H}_g\mathbf{x}, \tag{9}$$

the conditions (6) and (7) can be rewritten as

$$[\mathbf{H}_0 + \mathbf{G}_h^T\mathbf{x}^{(k)} + (\mathbf{x}^{(k)})^T\mathbf{H}_h\mathbf{x}^{(k)}]\circ\mathbf{c}(\mathbf{x}^{(k)}) + \mathbf{G}_0 + \mathbf{G}_g^T\mathbf{x}^{(k)} + (\mathbf{x}^{(k)})^T\mathbf{H}_g\mathbf{x}^{(k)} = \mathbf{f}(\mathbf{x}^{(k)}), \tag{10}$$

$$[\mathbf{G}_h + 2\mathbf{H}_h\mathbf{x}^{(k)}]\circ\mathbf{c}(\mathbf{x}^{(k)}) + [\mathbf{H}_0 + \mathbf{G}_h^T\mathbf{x}^{(k)} + (\mathbf{x}^{(k)})^T\mathbf{H}_h\mathbf{x}^{(k)}]\circ\nabla\mathbf{c}(\mathbf{x}^{(k)}) + \mathbf{G}_g + 2\mathbf{H}_g\mathbf{x}^{(k)} = \nabla\mathbf{f}(\mathbf{x}^{(k)}), \tag{11}$$

for $k = 1, \ldots, p$. Note that both (10) and (11) are linear with respect to model coefficients. Furthermore, equations (10) and (11) form a $p(n+1) \times (n+1)(n+2)$ system. The necessary condition for the system to have a unique least-square solution is that $p \ge n + 2$.

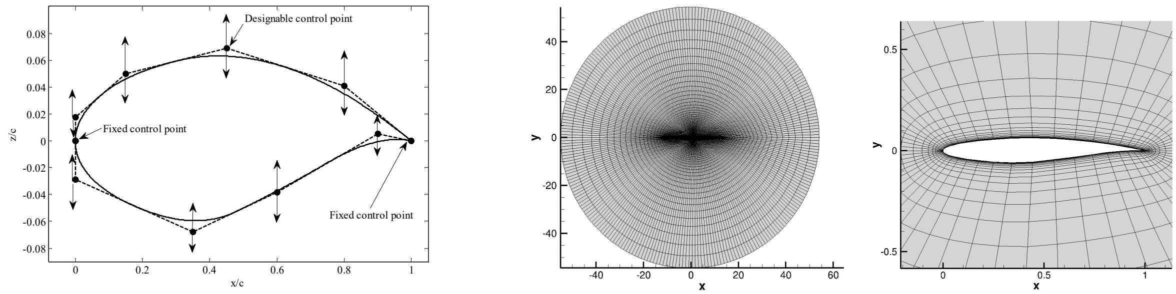
III. Numerical Investigation

A. Description

The numerical investigation considers a design case involving airfoil shape optimization in transonic inviscid flow. The investigation is performed as follows. For the given problem formulation (described below), the proposed NOSM response correction (Sect. II.B) is utilized to construct surrogates for several preselected designs within the search space. In particular, three versions of the surrogate $\mathbf{s}(\mathbf{x})$ are constructed for the chosen designs by modeling the polynomials $\mathbf{H}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ as follows: (1) $\mathbf{H}(\mathbf{x})$ constant, $\mathbf{G}(\mathbf{x})$ linear, (2) $\mathbf{H}(\mathbf{x})$ linear, $\mathbf{G}(\mathbf{x})$ linear, and (3) $\mathbf{H}(\mathbf{x})$ constant, $\mathbf{G}(\mathbf{x})$ quadratic. For each setup type, the NOSM response correction is evaluated for the figures of merit and the constraints. The error of the surrogate and low-fidelity model predictions, relative to the high-fidelity model, are calculated and compared.

The design problem involves a lift-constrained drag minimization of the airfoil shapes in transonic inviscid flow. The objective is to minimize the drag coefficient $C_d(\mathbf{x})$. The Mach number is $M_\infty = 0.734$, and the lift coefficient is set constant (by varying the angle of attack) at $C_l(\mathbf{x}) = 82.4$ l.c. (1 l.c. = 1 lift count = 1.0E−2). The cross-sectional area is constrained to be larger or equal to the value of the baseline. The pitching moment coefficient at the quarter-chord is constrained as $C_{m,c/4} \ge -0.092$. The design variables are the vertical coordinates of eight control points in a B-spline airfoil shape parameterization constrained to be within upper and lower bounds of 0.07 and −0.07, respectively (Fig. 1a).

The flow is simulated using computational fluid dynamics (CFD) where the high-fidelity model utilizes a fine mesh. The low-fidelity model is the same as the high-fidelity one, but with a coarser mesh and relaxed flow solver convergence criteria. More specifically, the CFD models solve the compressible Euler equations, and the adjoint equations, on an O-type mesh (Fig. 1b) using the Stanford University Unstructured code[6]. The high-fidelity model ($\mathbf{f}(\mathbf{x})$) uses a 256 × 256 mesh, whereas the low-fidelity model ($\mathbf{c}(\mathbf{x})$) uses a 64 × 64 mesh and a maximum number of solver iterations set to 300.



(a) **(b)**

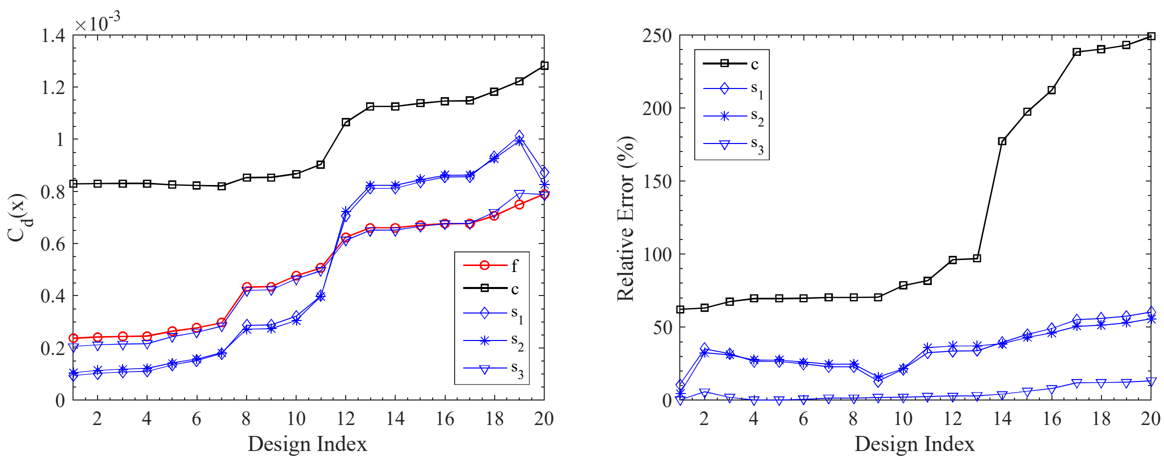
Figure 1. Airfoil description and computational mesh setup: (a) B-spline control points, (b) O-type mesh.

B. Results

The quality of the surrogate models constructed using the three aforementioned correction schemes (1) through (3), here, denotes as s_1 through s_3, is compared with the coarse model in Fig. 2 using 20 samples selected randomly from the design space. It can be observed that NOSM provides considerable accuracy improvement with respect to the coarse model. Furthermore, the quadratic correction performs better than the linear ones for the considered test case. Moreover, it should be emphasized that using NOSM permits low-fidelity model correction using a very small number of samples, which is a considerable advantage over LOSM.

IV. Conclusion

Simulation-based aerodynamic shape optimization using multi-fidelity models is considered. A nonlinear parametric response correction technique utilizing adjoint sensitivities is proposed. In particular, the proposed technique extends the linear output space mapping to allow for nonlinear mappings. Consequently, the physics-based surrogate model is better equipped to handle nonlinear discrepancies between the multi-fidelity models throughout the design space. Thus, the generalization capability of the surrogate is improved. Furthermore, because of using adjoint sensitivities, the computational cost of low-fidelity model correction is greatly reduced. The next step in this work is to investigate how well the proposed nonlinear surrogates perform in local search.



(a) **(b)**

Figure 2. Results of surrogate modeling with the proposed approach for inviscid transonic flow past airfoils at M_∞ = 0.734, and C_l(x) = 82.4 l.c.: (a) evaluation of the drag coefficient using the high- and low-fidelity models and the proposed surrogate models, (b) relative error with respect to the high-fidelity model response in percent.

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