Entanglement Distillation Protocols in the Presence of Noise

Niyousha Najmaei (5739276)

Quantum Communication and Cryptography (CS4090)

February 28, 2023

Abstract

The need to improve the quality of entangled states in quantum computing and communication drives the investigation of entanglement distillation methods. This project compares the fidelity and success probability of two entanglement distillation methods, BBPSSW and DEJMPS, in the presence of two sources of depolarisation noise, gate noise and entanglement noise. The results demonstrat that BBPSSW outperforms DEJMPS in terms of fidelity and success probability. Additionally, it was discovered that when the gate fidelity is less than or equal to 0.8, neither of the protocols can improve the fidelity of the state.

1 Introduction

Entangled states can not be created using solely local operations and classical communications, which makes them a resource that can be used in quantum communication [1]. Since the process of entanglement generation is affected by quantum noise, creating high fidelity entangled states is a challenge. Quantum distillation is used as a method to counteract this detrimental effect by purifying entangled states and producing higher fidelity entanglement.

In this project, the performance of two entanglement distillation protocols, BBPSSW [2] and DEJMPS [3], is investigated in the presence of noise. The following subsections will provide a brief overview of these two protocols. Section 2 goes over the research questions and experiment details. The results are introduced in Section 3, and Section 4 provides a conclusion and discussion of the results.

1.1 BBPSSW protocol

BBPSSW protocol [2] is a distillation protocol in which Alice and Bob start with two pairs the Werner state with fidelity F, maximally entangled states that have been depolarised (Eq. 1), and perform the circuits depicted in Figure 1.

$$\rho = p\,|\Phi_{00}\rangle\langle\Phi_{00}| + (1-p)\frac{\mathbb{I}_4}{4} \tag{1}$$

Subsequently, using classical communication channels, they exchange their measurement outcomes, and declare success of protocol in case their measurement outcomes are the same. In case of success, Alice and Bob will share one entangled state with fidelity F_{out}. It can be shown that this protocol works if the initial fidelity is more than 0.5, i.e. $F > 0.5$. In this case, F_{out} and p_{succ} can be computed in terms of F as follows.

$$F_{out} = \frac{F^2 + (\frac{1-F}{3})^2}{p_{succ}} \tag{2}$$

$$p_{succ} = F^2 + 2F\frac{1-F}{3} + 5\left(\frac{1-F}{3}\right)^2 \tag{3}$$

Given the facts that $F(F_{out})$ is continuous, $F_{out} > F$, $F > 1/2$ and $F_{out} = 1$, Alice and Bob can obtain a state with arbitrarily high fidelity by iterating this procedure [4]. Nonetheless, in this limit, $F_{out} \to 1$, the probability of success tends to zero.

It should be noted that in case the initial state is not the Werner state, Alice and Bob can transform any state to the Werner state by deciding a priori on two unitary gates to perform on their qubits.

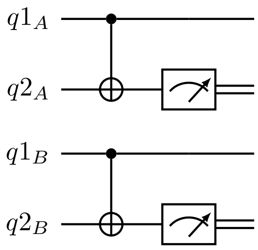

Figure 1: The circuits of BBPSSW protocol. $q1_A$ and $q2_A$ are Alice's qubits, whereas $q1_B$ and $q2_B$ are Bob's. Alice and Bob's first (second) qubits are entangled.

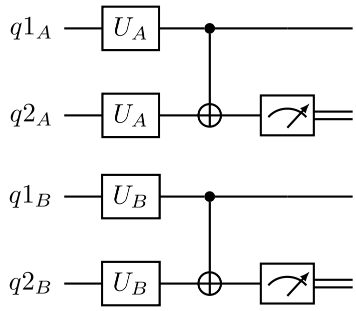

Figure 2: The circuits of DEJMPS protocol. $q1_A$ and $q2_A$ are Alice's qubits, whereas $q1_B$ and $q2_B$ are Bob's. Alice and Bob's first (second) qubits are entangled.

1.2 DEJMPS protocol

The DEJMPS protocol [3] is similar to the BBPSSW protocol, with the difference that a basis transformation is done on the two pairs of entangled state before the C-Not gate, using U_A and U_B . Alice performs U_A on her qubits which is defined as follows.

$$U_A |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle),\ U_A |1\rangle = \frac{1}{\sqrt{2}}(|1\rangle - i\,|0\rangle) \tag{4}$$

Bob, on the other hand, performs the unitary U_B on his qubits which is defined as follows.

$$U_B |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle),\ U_B |1\rangle = \frac{1}{\sqrt{2}}(|1\rangle + i\,|0\rangle) \tag{5}$$

The corresponding circuits of this protocol can be seen in Figure 2. Assuming that the initial is a mixture of bell states, $\rho = \Sigma_{ij}\lambda_{ij} |\Phi_{ij}\rangle \langle\Phi_{ij}|$, and the output state is $\rho_{out} = \Sigma_{ij}\hat{\lambda}_{ij} |\Phi_{ij}\rangle \langle\Phi_{ij}|$, the probability of success can be calculated using the following equation.

$$p_{succ} = (\lambda_{00} + \lambda_{11})^2 + (\lambda_{01} + \lambda_{10})^2 \tag{6}$$

Moreover, $\hat{\lambda}_{ij}$ can be computed as follows.

$$\hat{\lambda}_{00} = \frac{{\lambda_{00}}^2 + {\lambda_{11}}^2}{p_{succ}},\ \hat{\lambda}_{01} = \frac{{\lambda_{01}}^2 + {\lambda_{10}}^2}{p_{succ}},\ \hat{\lambda}_{10} = \frac{2\lambda_{00}\lambda_{11}}{p_{succ}},\ \hat{\lambda}_{11} = \frac{2\lambda_{01}\lambda_{10}}{p_{succ}} \tag{7}$$

2 Research Questions

In this project the performance of the BBPSSW protocol is compared to that of the DEJMPS protocol in the presence of depolarisation noise in gates and the entangled states. The research questions formulated for this purpose are as follows.

- How does the performance of the two protocols compare for different values of gate noise, ranging from perfect to fidelity=0.55, in terms of the entangled state noise?
- How does the performance of the two protocols compare for different values of entangled state noise, ranging from perfect to fidelity=0.55, in terms of the gate noise?

- What is the minimum gate fidelity for which the protocols successfully improve the fidelity of entanglement?

All results are obtained by running the protocol 100 times for each configuration of noises and protocols. The measures of performance are success probability and achieved average fidelity which is measured using Equation 8.

$$F(\rho, \sigma) = Tr\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2 \tag{8}$$

3 Results

Before delving into comparing the two protocols, a sanity check of the implemented protocols was performed by plotting the fidelity and the success probability of the BBPSSW and DEJMPS protocols when the gates are perfect and the entangled state noise varies. These results are compared to Equations 2 and 3 for BBPSSW and Equations 7 and 6 for DEJMPS, assuming the initial state is the Werner state. The plots of these comparisons can be seen in Figures 3 and 4 for BBPSSW and Figures 5 and 6 for DEJMPS. It can be seen that in case of both protocols, the output state has higher fidelity than the initial state, and the fidelity has successfully been improved.

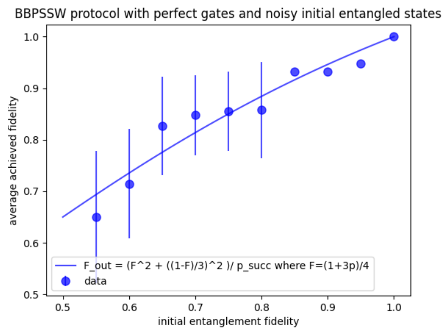

Figure 3: Average fidelity of the BBPSSW protocol with perfect gates.

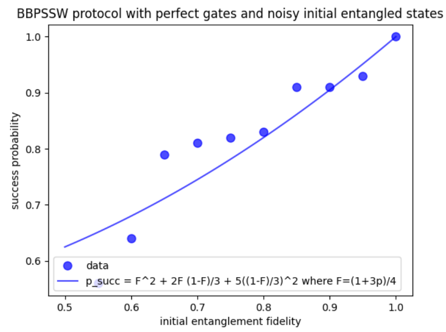

Figure 4: Success probability of the BBPSSW protocol with perfect gates.

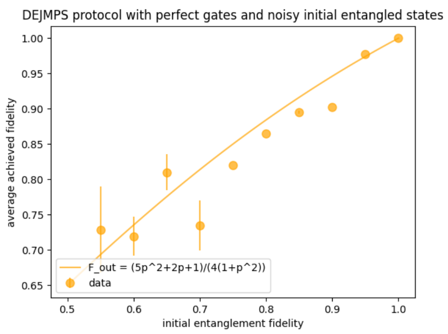

Figure 5: Average fidelity of the DEJMPS protocol with perfect gates.

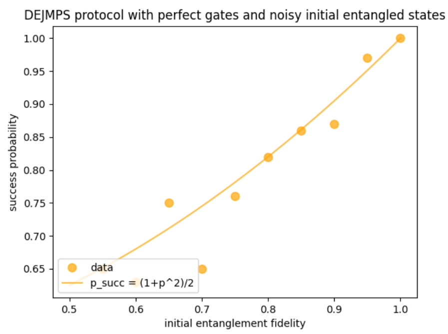

Figure 6: Success probability of the DEJMPS protocol with perfect gates.

The success probability with respect to gate fidelity and entangled state fidelity is plotted in Figures 7 and 8 for BBPSSW and DEJMPS respectively. It can be seen from the plots that the BBPSSW protocol tends to generally have a higher probability of success. A similar trend can be observed for the average fidelity of the resulting states. This result can be seen in Figures 9 and 10. On the other hand, if the fidelity is averaged on the successful runs of the protocol only, it can be observed that the DEJMPS protocol generally performs better. These results can be seen in Figures 11 and 12.

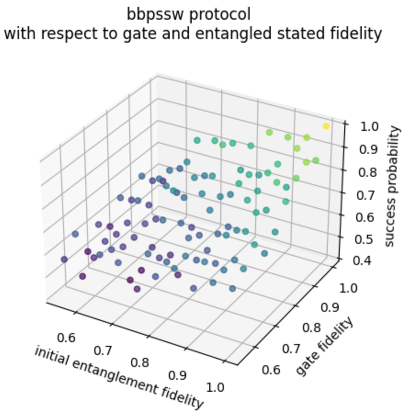

Figure 7: Success probability of the BBPSSW protocol with respect to the gate and entangled state noise.

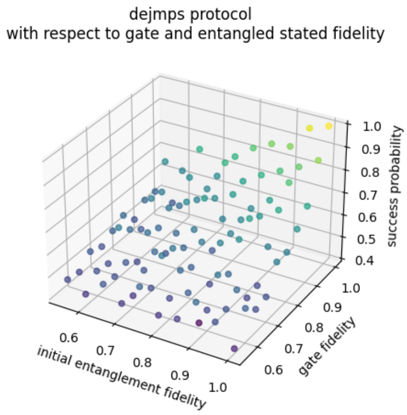

Figure 8: Success probability of the DEJMPS protocol with respect to the gate and entangled state noise.

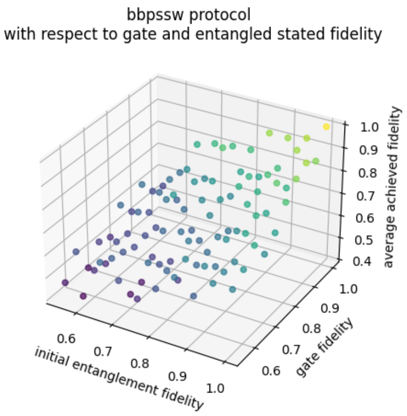

Figure 9: Average fidelity of the BBPSSW protocol with respect to the gate and entangled state noise.

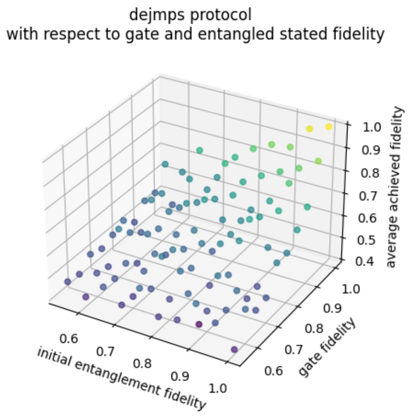

Figure 10: Average fidelity of the DEJMPS protocol with respect to the gate and entangled state noise.

A more detailed comparison of the success probability of the protocols in case of perfect gates, gates of fidelity 0.95 and 0.9 can be seen in Figures 13, 14 and 15 respectively. It is evident from the plots that the success probability of the BBPSSW protocol is higher than or equal to that of DEJMPS, in all cases except for the following combinations for gate fidelity and entangled state fidelity: (1.0, 0.55), (1.0, 0.95) and (0.9, 0.95). Moreover, as it can be seen in Figures 16 and 17, the probability of

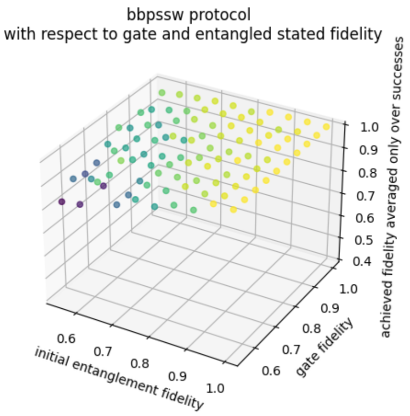

Figure 11: Average successful fidelity of the BBPSSW protocol with respect to the gate and entangled state noise.

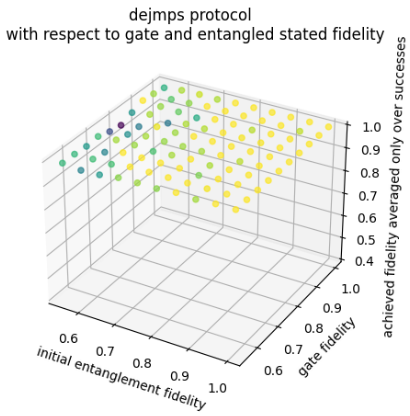

Figure 12: Average successful fidelity of the DEJMPS protocol with respect to the gate and entangled state noise.

success of the protocols deteriorates quickly for gate fidelity equal to and below 0.8. More specifically, the success probability of none of the protocols exceeds 0.75, if the gate fidelity is lower than 0.8 regardless of the initial entanglement state.

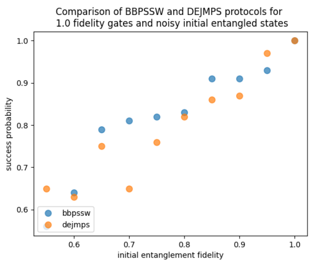

Figure 13: Success Probability of the protocols for perfect gates

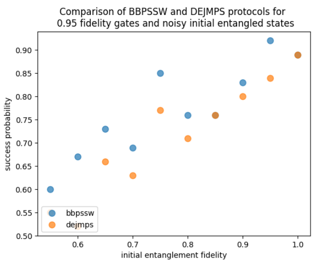

Figure 14: Success Probability of the protocols for 0.95 fidelity gates

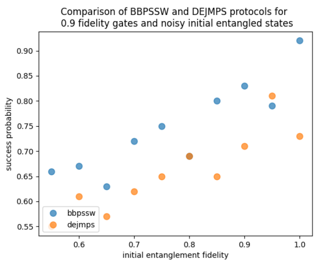

Figure 15: Success Probability of the protocols for 0.90 fidelity gates

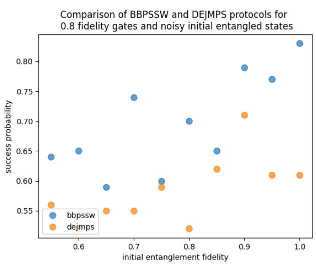

Figure 16: Success Probability of the protocols for 0.80 fidelity gates

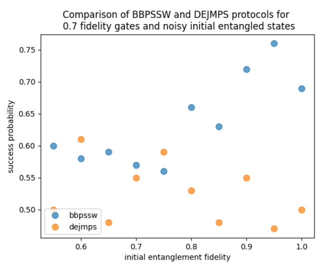

Figure 17: Success Probability of the protocols for 0.70 fidelity gates

Comparing Figures 18 and 19, it can be seen that the BBPSSW protocol performs better in terms of average fidelity in case of perfect gates and gates with 0.95 fidelity, however, the variance of fidelities obtained using the BBPSSW protocol was significantly higher than that of the DEJMPS. Similar to the case of success probability, the output fidelity of both protocols deteriorates for gate fidelity lower than 0.9, and it can be seen from Figures 20 and 21 that the protocols fail to improve the entanglement fidelity for these values of gate fidelity.

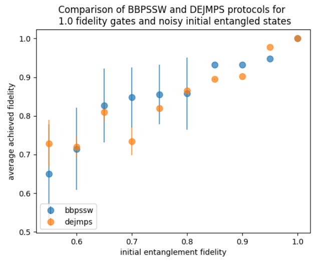

Figure 18: Average fidelity of the protocols for perfect gates

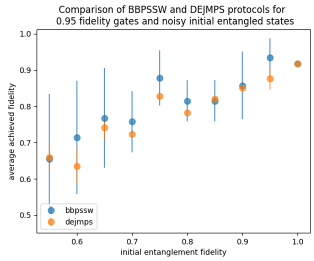

Figure 19: Average fidelity of the protocols for 0.95 fidelity gates

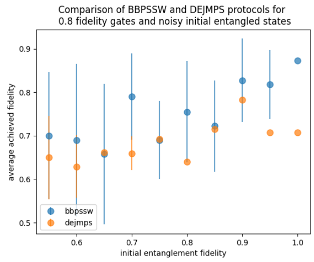

Figure 20: Average fidelity of the protocols for 0.80 fidelity gates

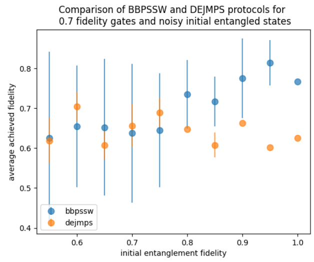

Figure 21: Average fidelity of the protocols for 0.70 fidelity gates

4 Conclusion

In this project the BBPSSW and DEJMPS protocols were compared in terms of success probability and achieved fidelity in the presence of gate and entanglement depolarisation noise. The results show that the BBPSSW protocol performs better in terms of both success probability and the average output fidelity, however, the variance of the results was higher for BBPSSW protocol. Moreover, both protocols fail to improve entanglement fidelity for gate fidelity lower than or equal to 0.8, regardless of the initial fidelity of the entangled state.

References

[1] Christopher Eltschka and Jens Siewert. “Quantifying entanglement resources”. In: *Journal of Physics A: Mathematical and Theoretical* 47.42 (Oct. 2014), p. 424005. DOI: `10.1088/1751-8113/47/42/424005`. URL: `https://doi.org/10.1088%2F1751-8113%2F47%2F42%2F424005`.

[2] Charles H. Bennett et al. “Concentrating partial entanglement by local operations”. In: *Physical Review A* 53.4 (Apr. 1996), pp. 2046–2052. DOI: `10.1103/physreva.53.2046`. URL: `https://doi.org/10.1103%2Fphysreva.53.2046`.

[3] David Deutsch et al. “Quantum Privacy Amplification and the Security of Quantum Cryptography over Noisy Channels”. In: *Physical Review Letters* 77.13 (Sept. 1996), pp. 2818–2821. DOI: 10.1103/physrevlett.77.2818. URL: https://doi.org/10.1103/physrevlett.77.2818.

[4] Paweł Horodecki and Ryszard Horodecki. “Distillation and Bound Entanglement”. In: *Quantum Info. Comput.* 1.1 (Jan. 2001), pp. 45–75. ISSN: 1533-7146.