Stacks

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- Using a stack, you access only the object at the top of the stack; you cannot access, add to, or remove from any other point in the stack.
- Stacks are an abstraction, or general idea, of how the data structure functions, and they do not have specific implementations. Therefore, a stack is an Abstract Data Type (ADT) with more than one implementation.

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  - `object top()`: returns the last inserted element without removing it
  - `integer size()`: returns the number of elements stored
  - `boolean isEmpty()`: indicates whether no elements are stored

Example

Operation	Return value	Stack
`push(5)`	-	(5)
`push(3)`	-	(3, 5)
`size()`	2	(3, 5)
`pop()`	3	(5)
`isEmpty()`	`false`	(5)
`pop()`	5	()
`isEmpty()`	`true`	()
`pop()`	`null`	()
`push(7)`	-	(7)
`push(9)`	-	(9, 7)
`top()`	9	(9, 7)
`push(4)`	-	(4, 9, 7)
`size()`	3	(4, 9, 7)

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- We add elements from left to right and remove elements from right to left.
- A variable keeps track of the index of the top element. (Alternatively, a variable representing the size of the stack could also be used to determine where to add the next item.)
- The array storing the stack elements may become full. A `push()` operation may then throw an implementation-defined exception or resize the backing array. This behavior is not defined or forced by the ADT.

Array-backed Stack

- Example of an array implementation for Stack ADT where the top of the stack is index 3, the size of the stack is 4, and if you "pop" the top of the stack data 2 will come off the stack.

1	3	3	2		
0	1	2	3	4	5

Array-backed Stack

```
**procedure** PUSH($o$)
    $arr[size] \leftarrow o$
    $size \leftarrow size + 1$
**end procedure**
**procedure** POP
    $size \leftarrow size - 1$
    $item \leftarrow arr[size]$
    $arr[size] \leftarrow \texttt{NULL}$
    **return** $item$
**end procedure**
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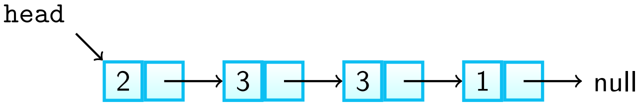
- Elements would be added and removed to the same end of the list. For a stack, this usually means that the most efficient way to do this would be to add to and remove from the front of a linked list, as this would be $O(1)$ in most cases.

Linked List-backed Stack

- Example of a linked list implementation for Stack ADT where the top of the stack is at the head, the size of the stack is 4, and if you "pop" the top of the stack data 2 will come off the stack.



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  - Linked list-backed stacks do not have either of the above two limitations.

Applications of Stacks

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- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack.
- This stack also allows for recursion to work correctly.
- This is known in general as the **activation stack**.

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