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# Managing Credit Lines and Prices for Bank One Credit Cards

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We developed a method for managing the characteristics of a bank's card holder portfolio in an optimal manner. The annual percentage rate (APR) and credit line of an account influence card use and bank profitability. Consumers find low APRs and high credit lines attractive. However, low APRs may reduce bank profitability, while indiscriminate increases in credit-lines increase the bank's exposure to credit loss. We designed the PORTICO (portfolio control and optimization) system using Markov decision processes (MDP) to select price points and credit lines for each card holder that maximize net present value (NPV) for the portfolio. PORTICO uses account-level historical information on purchases, payments, profitability, and delinquency risk to determine pricing and credit-line changes. In competitive benchmark tests over more than a year, the PORTICO model outperforms the bank's current method and may increase annual profits by over \$75 million.

*(Financial institutions: banks. Dynamic programming/optimal control: Markov, finite state.)*

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*A retentive memory may be a good thing, but the ability to forget is the true token of greatness.*

*Elbert Hubbard (19th century philosopher,  
not necessarily speaking about MDPs)*

Suppose you are considering a large purchase. You carry three credit cards with different pricing, spending limits, and terms. Which card will you use? Will the credit line or the annual percentage rate (APR) on the cards influence your decision? In industry parlance, which card will be at the "top of your wallet?" Intense competition in the banking and credit-card industry makes the answers to such questions extremely important. Credit issuers apply statistics and operations research to answer these questions. We applied modeling and optimization methods to the

problem of awarding credit lines and changing APRs to customers.

## Trends in the Credit-Card Industry

Credit cards have come a long way from their origin as charge cards, a convenient way of making payments (without the option of drawing on a revolving line of credit). The first credit-card banks arose in the early 1980s. There are now more than 7,000 US credit-card issuers and 27,000 types of credit cards (Hanft 2000). Today, consumers can use their lines of credit for various payment and personal-financing needs. On the merchant side, it is hard to find businesses that do not accept credit cards; even grocery stores and the Internal Revenue Service (IRS) accept credit

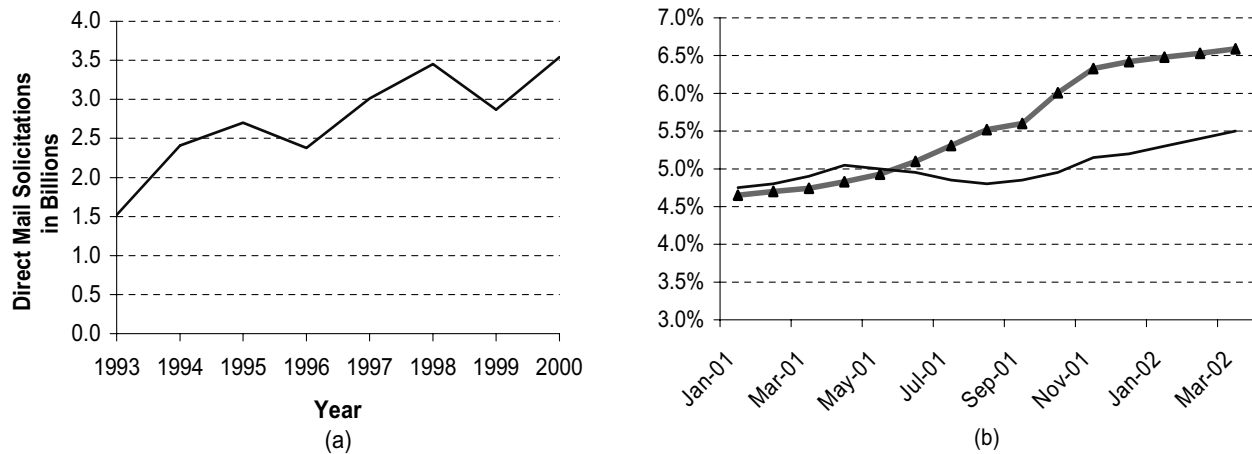


Figure 1: (a) Direct mail solicitations have shown a steady increase over the years. (b) Delinquencies and charge-offs have also increased recently (source: <http://www.cardweb.com/cardtrak/news/2002/april/29a.html>).

cards. Along the way, the industry has introduced many innovations, including chip-embedded smart cards, convenience checks, magnetic stripes for paperless transactions, fraud-detection systems, real-time purchase-transaction processing, and rewards products that grant cash or points for spending towards a variety of purchases, such as airline travel, telephone calls, and hotel stays.

With the mass marketing of credit, the average person in the US has 4.2 credit cards (Federal Reserve 2001). This level of market penetration has caused intense competition among issuers for new accounts. In 2000, firms sent out 3.54 billion direct mail solicitations (McKinley 2001) for credit cards (Figure 1). In 2001, they are estimated to have sent 4 billion. In spite of the massive amount of targeted marketing, fewer than one in 100 credit-card prospects who are good risks from a lending perspective actually respond to these offers. Pricing is highly competitive in today's environment, with zero-percent financing for periods of six months or longer common. Low-rate financing through other credit vehicles, such as home equity lines of credit, is also readily available.

At the same time, the worsening economy has adversely affected the profitability of many issuers, especially those who market to high-risk customers. Several major issuers have left the subprime market because of the high cost of customer defaults and bankruptcies. Two key measures of credit risk that

drive bank profitability are the proportion of total dollars delinquent to total debt (*delinquency rate*) and the proportion of total dollars that have been placed in default to total debt (*charge-off rate*). In March 2002, customer-delinquency rates were 5.54 percent, and default or charge-off rates stood at 6.59 percent. These rates are near the record high levels (Figure 1) experienced in the 1990–1991 recession (<http://www.cardweb.com/cardtrak/news/2002/april/29a.html>).

In recent years, banks have continued to issue new credit cards and to increase the credit lines of existing customers, and they have lowered their pricing (APRs) to remain competitive. Growth in available credit has more than kept pace with the rise in debt; consequently the *open-to-buy* (the difference between the credit line and debt) has been increasing (Consumer Federation of America 2001). During the same period, the average APR issuers charge on revolving debt has declined (Figure 2). Issuers have also increased the number of offers with variable APRs relative to fixed APRs (Federal Reserve 2001).

## Factors Influencing Profitability—Product Dynamics

Credit card profitability is driven by customers' spending and payment behavior and by the mechanics of the industry itself. When a customer makes a purchase, the issuer and the bank association (Visa or

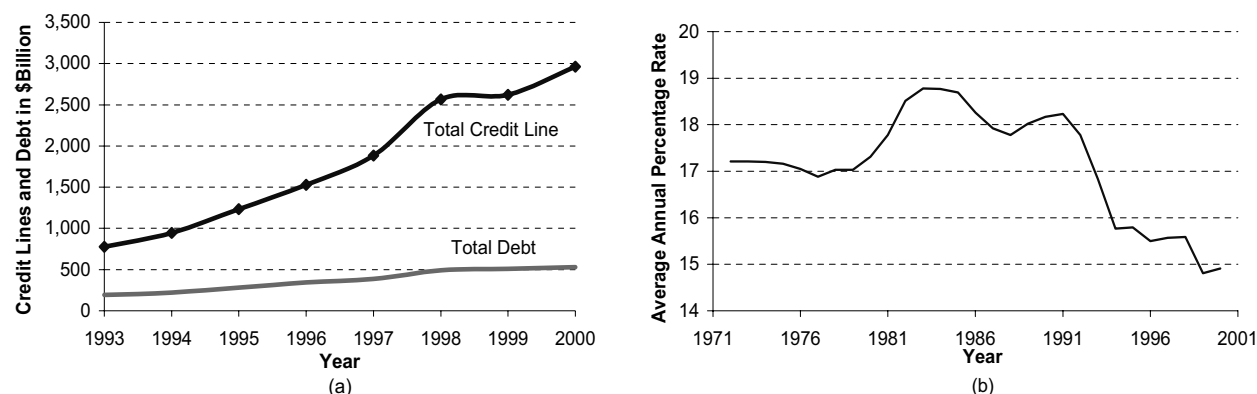


Figure 2: (a) Credit lines relative to the level of debt have increased over the years. The difference between the two curves, or the "open-to-buy," has also steadily increased (source: Consumer Federation of America 2001). (b) The average APR on cards has been reducing steadily in recent years (source: Federal Reserve 2001).

MasterCard) charge the merchant a fee. For any debt left unpaid by the due date, customers pay interest. The unpaid balance is referred to as the *revolving balance*, and the amount of interest paid is determined by the card's APR. The customer is expected to make a minimum payment on the revolving balance each month, and if this payment is missed or arrives late, the issuer assesses a late fee.

The largest component of credit-card revenue is the interest paid on card members' revolving balances. Most banks establish an APR for a credit-card account when the customer responds to an offer. For example, a customer may respond to an offer with a zero percent APR for six months and 12.9 percent APR effective at the end of the discount period. However, if a customer becomes chronically delinquent, banks will often increase his or her APR.

Merchant fees (*interchange*) are usually the second most important source of revenue for a credit-card issuer. These fees are about 30 cents per transaction plus about 2.50 percent of the amount of the transaction and are split between the issuer and a bank association (about 10 cents per transaction or sale and about 1.50 percent of the amount of transaction goes to the association). American Express charges a higher percentage, and retains all of the fees, because it is not part of a bank association.

Other main sources of profits are convenience checks, user fees, and membership fees. Banks market convenience checks to build card holders' balances

and allow them to make purchases or transfer balances from other cards at interest rates lower than their base APRs. Banks assess fees for certain customer behaviors, such as making late payments or requesting over-limit authorizations. Recently, late fees have become an important source of revenue. Since 1996, late fees have more than doubled from an average of \$13.28 to \$29.84 (<http://www.cardweb.com/cardtrak/news/2002/may/17a.html>), even as the grace period and other terms for levying fees are becoming more stringent. Finally, some card holders pay annual fees to receive such privileges as earning airline miles or credits towards future automobile or gasoline purchases.

## Factors Influencing Profitability—Customer Dynamics

Unless a bank charges a yearly fee for a credit card, it will earn no money until the customer uses the card to make purchases or payments, or to withdraw cash. A large part of issuers' portfolios consists of inactive accounts. For example, a customer may open an account just to get the 10 percent discount on the first purchase. Or a customer may *surf* (transfer) a balance into an issuer's portfolio to take advantage of a six-month introductory APR rate on balance transfers, surf out at the end of the introductory period, and become inactive thereafter. Card issuers try to motivate customers to carry revolving balances (that is,

to only partially pay off new purchases and previous balances). They do this by encouraging the customers to spend more and to carry balances on the bank's card rather than a competitor's, or by encouraging them to transfer balances to the bank from competitors' cards.

Customer attrition is a continual challenge for banks because of the intense competition for accounts. Competitors frequently tempt good customers away from their existing issuers by offering low APRs and other enticements. In addition to their direct efforts to retain customers (for example, making counteroffers), banks try to acquire new customers to replace those lost through attrition. To remain competitive, banks strive to ensure customer loyalty through pricing, proactive increases in credit lines, and such features as rewards incentives. However, overly aggressive pricing strategies can erode profit margins to unacceptable levels, and offers to induce loyalty, such as cash-back rewards, can be expensive to fulfill.

Finally, delinquencies and charge-offs can literally break the bank. The higher the credit line, the higher the balance a customer can accumulate before ceasing to make payments. When the bank finally charges off an account, it declares the customer's entire balance a loss, except for a fraction that debt collectors can recover. The bank needs the net income of many good accounts to offset the losses caused by a single default. One way banks can stimulate growth in balances and interest income is by increasing credit lines. Banks limit such policies because the increases in open-to-buy can increase losses.

Determining methods to improve profitability and manage credit loss requires sophisticated analysis and modeling. Bank One has teams constantly working on these issues to improve the products and services it offers to customers.

## **Bank One Card Services, Inc.**

Bank One Card Services, Inc., a division of Bank One Corporation, is the third largest issuer of credit cards and the largest issuer of Visa cards in the United States. The company offers credit cards for consumers and businesses under its own name and on behalf of several thousand marketing partners. These partners

include some of the leading corporations, universities, sports franchises, affinity organizations, and financial institutions in the United States. The company has 55 million card members and \$64 billion in receivables ([www.bankone.com](http://www.bankone.com)). Bank One earnings as a whole are significantly affected by the performance of Bank One Card Services, Inc.

In the Bank One Card Services organization, the decision technologies group plays an important role in developing ways to achieve management objectives. The dynamics of managing credit cards cause a tension between risk and revenue growth. For example, whereas the firm evaluates the marketing department on the number of new customers acquired and the cost of acquisition, it evaluates the risk department on how well it controls credit losses from such acquisitions programs. The optimization applications the bank implements must balance these objectives. Because it is at the center of the organization, the decision technologies group can focus on solutions that are optimal for the company as a whole.

The decision technologies group includes experts in statistics, operations research, and information systems. The group has developed and implemented several optimization models for acquiring new accounts, managing existing portfolios, and formulating test designs. Optimization staff members work with statisticians to collect data through experimental designs and to develop models that will provide input to the optimization models. They also work with information technology staff members to implement the models they develop.

The objective of every project the decision technologies group works on is to apply its models to the portfolio after properly testing them. The group conducts rigorous benchmark tests to ensure that proposed approaches are indeed better than the existing methods and other available alternatives. It routinely evaluates and compares vendor offerings to make the best choices from available solutions. The group's primary mission is to focus on analytic solutions; often, it makes discoveries and develops new and useful techniques for the bank.

The credit-card business is rich in data, and the bank is driven by data in developing its tactics and strategies. With over 1.5 billion purchase transactions



annually, it houses many terabytes of data that capture customer-payment and card-use history. It uses statistical testing extensively to develop and enhance products and techniques. In any given month, the bank creates, launches, and monitors hundreds of tests. This environment and the infrastructure the bank has created provide fertile ground for developing, testing, and validating optimization models.

## Genesis of PORTICO

The decision technologies group began the PORTICO (portfolio control and optimization) project in July 1999 when the bank asked it to evaluate approaches to improve the profitability of the bank's portfolio. Our goal was to stimulate sales and balance accumulation on Bank One card products.

There are two basic ways the bank can improve customer profitability: Take unilateral action to promote the desired behavior, or take measures that require customers to make some initial response before they adopt the desired behavior. In the first category are such actions as changes in credit lines and pricing. The second category consists of such measures as mailing convenience checks or balance-transfer checks and offering additional products; customers must respond to these offers before the bank earns financial benefits. Because of this distinction, it may be difficult to determine whether a unilateral action has produced the desired behavior. There is anecdotal evidence that increases in credit lines spur increased card use, but there is much countervailing evidence that many account holders ignore credit line increases. The effect of pricing changes is usually considered stronger, but because most price changes are increases to the APR, called *repricing*, the effect tends to be attrition or reduced card use.

When we were formulating approaches to increase customer profitability, the bank mailed notices of changes (usually increases) in credit lines along with convenience checks. Therefore, we considered systems that would help managers to make decisions about credit line changes, APR changes, and convenience-check offerings together. We developed a prototype optimization model for simultaneous actions to change credit lines and prices and to

mail convenience checks. We subsequently discovered that we needed two models, one for recurring convenience-check mailings, intended for short-term customer response, and one for the credit line and price models, intended to produce changes in customer behavior over time. We later used campaign optimization, based on projections of the likelihood of customers responding to offers, to handle checks and other response-sensitive offers. We initially focused on pricing and credit lines, which simplified the letter the bank would send to customers, because check offers must include explanations of terms and conditions. We wanted to send a letter to customers that clearly and positively explained that we were improving the pricing or credit line of their current credit card.

Our strategy was to identify the actions the bank could take on pricing and credit lines to stimulate customer use of its card products. We further wanted to improve our communication of these actions to customers. For customers who received an increase in credit line, a reduced APR, or both, we would reinforce those benefits with a letter describing the changes and emphasizing the customer's value to the bank. We also decided to focus on actions that would engender customer loyalty, card use, and ultimately, revenue growth. For this reason, we did not consider actions that would raise APRs or reduce credit lines.

## Prior Research

Some literature covers methods for granting credit initially, but much less concerns the subsequent management of credit lines and pricing. Bierman and Hausman (1970) developed statistical models using a Bayesian approach and a Markov decision model for making decisions to grant credit. Dirickx and Wakeman (1976) and Waldman (1998), among others, extended this work. Rosenberg and Gleit (1994) surveyed credit-management methods. Little research has been published that relates to adjusting the base price of card products.

Of more immediate relevance to our work is the decision to periodically change credit lines and pricing. Evidence exists that banks use increases in credit line for existing card holders as a tactical marketing

tool and routinely make such actions to encourage card use (Lunt 1992, Punch 1992). Soman and Cheema (2002) found that increasing credit lines is associated with increased spending among certain consumers. They hypothesized that these customers see the increased credit lines as a signal of their future earning potential, encouraging them to increase spending now. This line of reasoning supposes that these customers are under the impression that banks use sophisticated models to determine credit-line increases that incorporate forecasts of customers' future earnings potentials. Soman and Cheema (2002) tested this hypothesis in an experimental setting using students as subjects. However, they found that the more savvy consumers are not affected by credit-line increases.

We believe, similarly, that some customers who pay their bills in full each month may be completely insensitive to the base APR of their accounts. Pricing a card competitively, on the other hand, can lead to increased sales and use by those customers who are price sensitive.

Soman and Cheema studied the effects of credit-line changes for a customer with a single card. Because most people have multiple cards, we are more interested in seeing whether increases in credit lines shift usage to the bank's card, even if the level of a card holder's total debt does not change. In other words, new activity would be nice, but we would be equally happy with shifts in usage to our card from those of competitors. Because we see only the activity on our card, and the credit bureaus provide only aggregate data over all bankcard balances, it is difficult to differentiate between new activity and balance migration. Nonetheless, we can see the net effect on our balances, and that is really all we need to determine whether changing credit lines for certain card holders benefits the bank.

## Current Process for Credit-Line Increases and Price Reductions

As would be expected, a process was already in place for making credit-line-management decisions. This process was based on decision trees. Decision-tree analysis is commonly used in the credit-card industry for making decisions to change credit lines. In

this approach, models segment a portfolio by predicting customers' future risk, profitability, and likelihood of discontinuing card usage (that is, attrition). These models commonly rely on credit-bureau data and internal information on card use to define the segments. The variables used measure customers' payment history, purchases, bankcard revolving balances, delinquency history, and so on. The models' predicted outcomes (for example, default likelihood over the next 12 months) and variables are grouped into intervals. The inverted decision tree starts from a root variable and has as many levels as the variables being used. At each level, the tree branches into each interval of that variable, and at the bottom of the tree the leaf nodes specify the amount of credit-line increase to give. An example decision rule may be "If the current credit line is \$2,000, and balance is \$1,500, then if the risk score is 650, give a credit-line increase of \$1,000; for risk scores between 600 and 650, give a credit-line increase of \$750; and so forth." A number of commercial rule engines are available to deploy these decision criteria. Examples are JRules Version 4.0 (ILOG) and TRIAD version 7.0 (Fair Isaac and Co.). Bank One evaluates whether to change credit lines for accounts about every six months or more frequently if necessary. In addition, it may grant an ad hoc credit line increase when a card holder bumps against his or her current limit in attempting to make a large ticket purchase. The bank also evaluates credit-line increases in response to specific customer requests to inbound call centers.

## Critique on the Existing Methodology

A review of the bank's prior business practices revealed important opportunities to improve its treatment of customers. First, the bank's existing policies were designed to be competitive with the credit lines and pricing in the marketplace. Ideally, a bank would offer a credit line consistent with the customer's needs and utility. That the current policies did not stimulate sales and growth in balances was reflected in the bank's financial performance. From a risk-management perspective, the amount of incremental net credit loss the bank incurred compared

to the amount of credit line it was giving to its customers was disproportionately large. Data suggested that the bank's credit lines were more than competitive with the marketplace, but that the dollars charged off relative to outstanding balances were high.

Second, pricing changes were generally triggered by late payments or by customers asking the bank to close their accounts. The bank would increase customer's APR after a few late payments, but it made no proactive APR reductions. The bank needed a price-reduction policy; its customer-attrition rates were increasing as its competitors actively sought new accounts and offered lower prices. While the bank sought to determine how much it could reduce a customer's APR without producing negative returns, it did not consider the future revenue a customer would generate under new pricing, and it had developed its existing policies only for customers who called to request a reduction in APR. In practice, the bank focused its pricing models on pricing at the time of acquisitions or pricing convenience checks.

As a result, the bank clearly needed to evaluate a dynamic approach to adjusting credit lines and prices as customers' behavior evolved over time. This led us to consider a sequential decision methodology. The bank has many years worth of daily and monthly transactional data that we could use to model the future behavior of individual card holders. Armed with this wealth of warehoused data, we set out to model the future behavior of the bank's card holders, using a data-intensive approach to reassess credit-line management and pricing practices. The methodology we chose was the Markov decision process (MDP).

## Markov Decision Processes

MDPs are a way to model sequential decision problems. Such models typically consist of the following five features: a set of time periods, a set of states the card holders could be in during each of the time periods, a set of actions the decision maker can choose among in each of those states, a set of estimates of the immediate costs or revenues from taking each of those actions, and an understanding of which state or states taking any action would lead to in the next period.

In such models, analysts define states by providing enough information that they can accurately determine the other three features (actions available, costs or revenues, and movement dynamics). Specifying the states correctly is crucial to the whole approach and is largely an art. The rule of thumb is to use as much information (and no more) as one needs to determine these three features. As the states in the model increase, the complexity of these models and the time it takes to solve them increase exponentially. This is known as the curse of dimensionality.

If the movement between states as a result of taking actions is not deterministic, the analyst uses a *transition matrix* to model the behavior. Transition matrices are typically square matrices with as many rows and columns as the number of states in the model. The entries in the model specify the probability of moving from one state in a period to any of the states in the next period, including the originating state itself. For this reason, each row sums to one.

MDPs are well suited to modeling sequential decisions where the actions, costs or revenues, and transition probabilities depend only on the current state of the system and not on states visited or actions taken in the past, that is, when the Markovian property of being memoryless (being history independent) applies.

The analyst solves the MDP model for each state in each decision epoch to determine the optimal action that achieves an objective, such as maximizing profits or minimizing costs. For finite horizon MDPs, the solution methodology is dynamic programming.

## Choice of the MDP Approach

The MDP methodology had several attractive features from our perspective. First, we have a wealth of time-series data on our customers, and the MDP approach allows us to use it to model future customer behavior (Nair 1995, Nair and Bapna 2001). Second, the MDP model estimates the expected profitability as well as any other component of profitability (for example, balance and sales) for each period. In our case, we produced 36-month time-series projections. This feature allows us to make consistent estimates between profitability metrics for a given customer



and to account for the dynamics of customer performance under different policy scenarios. Finally, the MDP model is useful when policy actions fall on different epochs. For example, the bank may change prices less frequently than credit lines, say every six months versus every month.

While the MDP approach had advantages, it also presented some hurdles. The MDP approach requires estimation of transition matrices that can be quite large. With millions of card holders and many years of data, we believed we had enough data to overcome issues of data sparseness. In addition, it is not easy to incorporate global and local constraints in the MDP model to solve for optimal policy actions. For example, we needed to incorporate constraints on tolerance for total credit loss. Finally, we needed to handle the implications of the Markovian assumption in developing the model.

## Data Collection

We obtained 18 months of time-series data on a random sample of 3 million accounts from the bank's portfolio. We chose variables that were candidates for an MDP model and then pulled data for these variables. The variables included bureau variables, bureau scores (predicting customer risk, revenue, bankruptcy, and response to new offers), and account performance data (such as monthly purchases, cash advances, payments, balances, net cash flow (NCF), credit losses, credit line, delinquency status, APR of interest charged on the account, and response to convenience checks). In particular, NCF is an important variable for the model. It measures the total profits derived from an account in a given month. Components entering the NCF calculation include interest income, merchant interchange, operating expenses, credit losses, and other related measures.

We separated the variables into control variables and behavior variables. Control variables are factors the bank can change: APR and credit line in our model. Behavior variables are factors determined by the card holder, such as payments and purchases. We used regression-tree analysis to select a set of behavior variables that were most predictive of one-month-forward profitability as measured by NCF. To be practical, we

Credit Line Intervals		APR Intervals	
1	\$0-2,000	1	<10%
2	\$2,000-5,000	2	10-15%
3	\$5,000-7,500	3	15-17.5%
4	\$7,500-10,000	4	17.5-20%
5	\$10,000-12,000	5	>20%
6	\$12,000-14,000		
7	\$14,000-16,000		
8	\$16,000-18,000		
9	\$18,000-20,000		
10	>\$20,000		

**Figure 3:** This is an example of the levels chosen for the two control variables, credit line and APR. Each level corresponds to an interval of values.

also chose variables to span the main dimensions of customer behavior (risk, card use, revolving activity, purchases, and payments). We chose six variables to represent behavior, which we cannot identify for proprietary reasons. Our general approach, however, can incorporate other behavior variables.

We next determined the number of levels for each of the variables. For each variable, these levels were demarcated by interval break points. We chose the break points based on the results of regression-tree analysis and the frequency distributions of the variables. We established four levels for two of the variables, three levels for another two variables, and two levels for the final two variables. Based on discussions with various stakeholders, we agreed on the step size of changes to give to customers in credit lines and pricing. We defined 10 levels for the credit-line variable and five levels for the APR variable. In some cases, we bounded the interval break points for a level purposefully. For example, we grouped customers with very high credit lines together in the top-most range with the idea that we would not allow them to obtain further increases. We based the control variable groupings on business factors more than on data (Figure 3).

## Satisfying the Markovian Assumption

We spent considerable time verifying that our model would satisfy the Markovian assumption that is implicit in using an MDP approach for the problem. The Markovian assumption is that the transitions from a state in one period to another in the next

period are path independent. *Path independence* means that the probability of moving from state  $x$  to state  $y$  depends only on  $x$ , regardless of which states and actions preceded the move to state  $x$ .

If this assumption is unrealistic given the way the state is defined, one way to accommodate the assumption is to redefine the state such that it carries some (finite) history with it. One can do this, for example, by concatenating the states in two successive periods to define a new state. For example, we could redefine the state  $x$  as  $ux$  and  $vx$ , depending on whether the account got to  $x$  from  $u$  or from  $v$ . This could increase the state space considerably.

We chose not to use this concatenation method. However, to reduce the likelihood that we would violate the assumption of path independence, we identified variables that carried some history. For example, most card issuers segment their customers as *revolvers* (those who carry balances), *transactors* (those who pay off the whole amount every month), and *inactives* (those who are not using the card). Most accounts are stable in these segments over several months, and by choosing the segment as one of our behavior variables, we would increase the chance that our transitions would be path independent. We chose some other variables as three-month averages so that they would incorporate some history and in addition would reduce the volatility in the values observed.

## Defining the State and Transition Matrix

We used the set of eight variables (two controls and six behaviors) to determine the state into which we could slot an account in any month of the time-series data. For example, an account in the month of June may be in state (13; 112312), meaning that it is in the first level of credit line, the third level of APR, the first level of behavior variables 1 and 2, the second level of behavior variable 3, and so on. The semicolon separates the control variables from the behavior variables. Suppose the bank increases an account's APR from the present third level to the fourth level; then the bank has moved the account to state (14; 112312) in July. In a subsequent month, due to behavior changes, the account may move to a new state, for example, (14; 421322).

We were working with a sample of 3 million accounts and 18 months of data (that is, each account has 17 transitions); this translates to having 51 million transitions from which to create the transition matrices. Transition matrices are square and have as many rows (origins) and columns (destinations) as the number of states in the system (Figure 4). The entries in the transition matrix,  $p(j | s)$ , are conditional probabilities of the account making a transition to state  $j$  in the next period given that it is in state  $s$  in the current period. The values of all entries in a row should add up to one.

Because we have two decision variables in our model with 10 and five levels respectively, and six

Transition Record	From	To
1	(1;1)	(1;3)
2	(1;1)	(2;3)
3	(1;2)	(1;1)
4	(1;2)	(1;1)
5	(1;3)	(1;2)
6	(2;1)	(2;3)
7	(2;1)	(2;2)
8	(2;2)	(2;3)
9	(2;3)	(1;2)
10	(2;3)	(2;1)

(a)

Transition Matrix

From	To					
	(1;1)	(1;2)	(1;3)	(2;1)	(2;2)	(2;3)
(1;1)			0.5			0.5
(1;2)	1					
(1;3)		1				
(2;1)					0.5	0.5
(2;2)						1
(2;3)		0.5		0.5		

(b)

Figure 4: Using transitions data of account movement between states (Figure 4a), we can create a transition matrix (Figure 4b). In the above figures, transition record 1 shows the transition from control and behavior state (1;1) to control and behavior state (1;3) with probability 0.5. The rows in the transition matrix add to 1.

behavior variables with four, four, three, three, and two, two levels respectively, using the above procedure would result in a massive transition matrix of size  $10 \times 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 = 28,800$  rows and an equal number of columns! A data set with 51 million transitions is not sufficient to populate these transition matrices. We would clearly need to address this issue.

## Simplifying the Transition Matrix by Using Rectangular Matrices

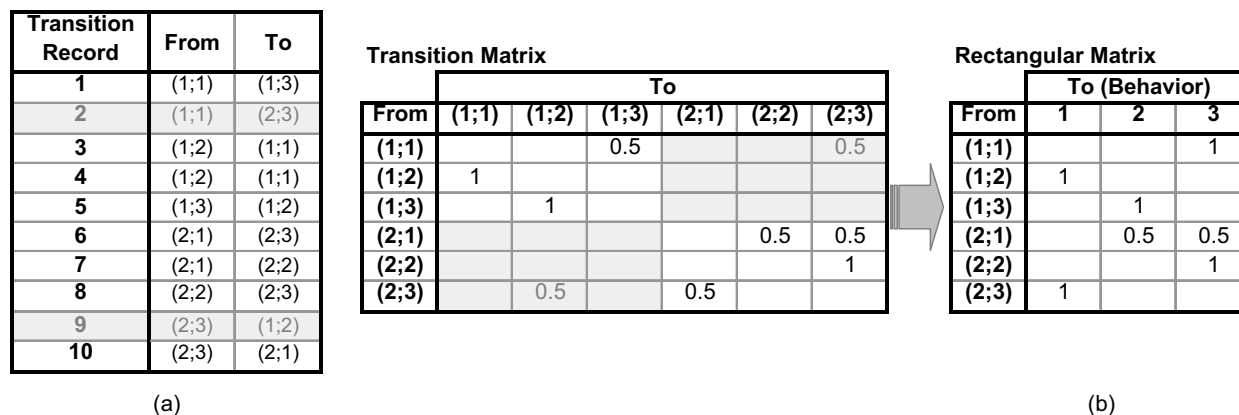
The first simplification we made was based on recognizing that changes in control variables reflect actions by the bank, not the account holder; hence they should not form part of the behavior transitions. Further, changes in control variables are much less frequent than changes in behavior variables. However, it would not be prudent to ignore control variables completely, because the behavior of a person with a credit line of \$10,000 could be quite different from that of a person with a credit line of \$2,000, all other behavior variables being at the same levels in both cases.

This realization led us to create transition matrices whose “From” states included both control and behavior variables, while its “To” states contained only behavior variables. The only accommodation we

had to make was to handle months when the bank changed the control variable a bit differently from months in which it made no changes. One possible approach is to ignore the few transitions with such changes (Figure 5). Doing this in our model would shrink the number of original cells ( $28,800 \times 28,800 = 829.44$  million) to ( $28,800 \times 576 = 16.6$  million), a 50-fold reduction.

Next, we combined low-frequency rows that had very few transitions with adjoining rows by using a simple greedy heuristic. This process is more difficult than one would think because proximity in our model had eight dimensions (one for each variable). Many rows or columns may appear close to the one that is sparse based on closeness to different variable levels—some may be close on one variable but far on others, whereas others may be the opposite. We had to decide which of these rows to choose to combine. (The details of the heuristic we used are beyond the scope of this paper and are peripheral to its contribution.) The heuristic we used reduced our rows to fewer than 14,000. A similar exercise reduced the columns to about 200, requiring us now to populate about 2 million cells.

For each of these 14,000 rows, we computed from our data the average NCF of the state defined by that



**Figure 5:** We show transitions data with the transitions between control-variable states removed and the “To” state defined only by behavior variables. We can convert this transition data to a rectangular transition matrix. For example, for states (1;1), (1;2), and (1;3) in control variable level 1, we remove the “To” states (2;1), (2;2), and (2;3) from the transition matrix in the middle. Doing this is the same as discarding the second and the ninth data points from the transition records in (a). In the resulting transition matrix, we align all remaining columns according to the behavior variables, as shown in (b).

row. Along with NCF, we also computed the various components of NCF or profitability that are important in the bank's decisions.

## Simplifying the Transition Matrix by Using Action Independent Matrices

We had to create transition matrices for each kind of action we wanted to consider in the model. This implies that in the MDP model, when the bank made a change action, the customer's behavior would be derived from that transition matrix, and when the bank was not making a change, the behavior would be derived from a no-action transition matrix. In our context, this means that the effect of the change is felt only in the period of the change (say once in six months), and in other periods, we would use the no-action transition matrix.

Earlier analysis indicated that a credit-line increase would take a few periods to change customer behavior. Thus, it would be incorrect to use a different transition matrix that we created from data that pertained only to the periods in which the bank made such changes in the past.

Instead we made the assumption that an account given a credit-line increase (say from credit-line level 1 to level 3) would behave identically to accounts that already had a higher line (in this case, credit-line level 3) and the same levels of behavior variables. For example, if an account were in state (1; 3) and the bank were to increase the control variable from 1 to 2, then from the next period on, the account would behave as if it were in state (2; 3) (the control variable changes to level 2, but the behavior variable stays in level 3).

This assumption simplified the model considerably because we no longer needed a separate transition matrix for each action. All we needed to do was to reindex to another row and proceed with making transitions as before. This approach works for changes in direction, that is, increases and decreases for both line and price. At the boundaries (the highest and lowest levels of the variables) though, no change is possible, because one cannot reindex to a level beyond the boundary levels.

Another practical problem was that in certain rare situations, reindexing would move a customer to a nonexistent row or column, because by combining them we had already eliminated many low-frequency rows and columns. In such cases, we again came up with a simple greedy heuristic to reindex instead to a proximate row or column. We omit the details of this heuristic because they are peripheral to our subject.

## PORTICO Model for Optimal Credit-Line and Price Decisions

After creating the transition matrix by using the two simplifications we discussed, we developed the MDP model fairly easily. The objective of the MDP model is to select a set of current and future actions for each state that maximizes the expected future profits (the NPV) of the portfolio subject to transitioning dynamics. NPV, in our case, is the discounted sum of future NCFs over an extended period of time. We chose a 36-month period, which was long enough to capture significant credit losses. We formulated the following MDP model, which can be solved recursively for a particular time horizon:

$$V_t(s) = \max_{a \in A_s} \left\{ r(s_a) + \beta \sum_{j \in S} p(j | s_a) V_{t+1}(j) \right\},$$

where  $V_t(s)$  is the optimal discounted NPV in state  $s$  and time  $t$ ;  $A_s$  is the set of actions available in state  $s$ ;  $r(s_a)$  is the NCF in state  $s$  when reindexed for the action  $a$  taken;  $\beta$  is the one-period discount factor;  $p(j | s_a)$  is the transition probability that specifies the likelihood of moving to state  $j$  from the (offset) state  $s_a$ ; and  $S$  is the set of states. We give a more detailed explanation of the model in the Appendix.

As for many optimization problems, we had to consider constraints in setting up the model. In our case, the constraints are business rules management established to keep track of broad bank practices. An example of these business rules is refusing to give increases in credit lines to customers who are at risk of defaulting in the future (those with risk scores below a certain threshold). Because it is difficult to incorporate such constraints in MDP models, we deal with these business rules by incorporating them into the solution

State	State Definition								Optimal Action
	Line	APR	Beh1	Beh2	Beh3	Beh4	Beh5	Beh6	
1	1	1	1	1	1	1	1	1	No action
2	1	1	1	1	1	1	1	2	No action
3	1	1	1	1	1	1	1	1	Increase line \$1,000
4	1	1	1	1	1	1	1	2	No action
5	1	1	1	1	1	1	2	1	Increase APR by 2.5%
6	1	1	1	1	1	1	2	2	No action
7	1	1	1	1	1	1	2	1	Increase line \$3,000
8	1	1	1	1	1	1	2	2	No action
9	1	1	1	1	1	2	1	1	No action
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
14000	10	5	4	4	3	3	2	2	Decrease APR by 2.5%

Figure 6: In the example of the optimal policy table, for each of the 14,000 states we specify an optimal action. In any month, once the account is slotted into one of these states, we can take the optimal action from this table.

by using a postoptimization or back-end approach. We subjected the optimal solutions produced by the MDP model in PORTICO (for the unconstrained problem) to business-rules constraints when we implemented them in a campaign. For example, we limited the percentage of accounts to be treated by an action (a change in credit line or APR) to conform to a campaign budget, and we did not treat accounts with unfavorable risk profiles.

The output of the MDP is a policy that prescribes the optimal changes in credit lines or prices (including no action) for each of the 14,000 states in our model (Figure 6). Instead of solving this model in real time, we created an easy-to-use table that prescribes an action for each state. In production, the process scores all card holders periodically (based on their credit behavior) and slots them into one of our states. Then, it uses the policy table to determine the optimal action to take. We create the transition matrix and the NCF information for the states every 12 months or so to reflect changes in the economy or bank practices (Figure 7).

## Making the Case for Testing PORTICO

The bank puts new initiatives through a vetting process to justify spending time and resources to launch, monitor, and read a live test. We compared PORTICO

with current business practices to quantify its benefits and to establish a rationale for live testing.

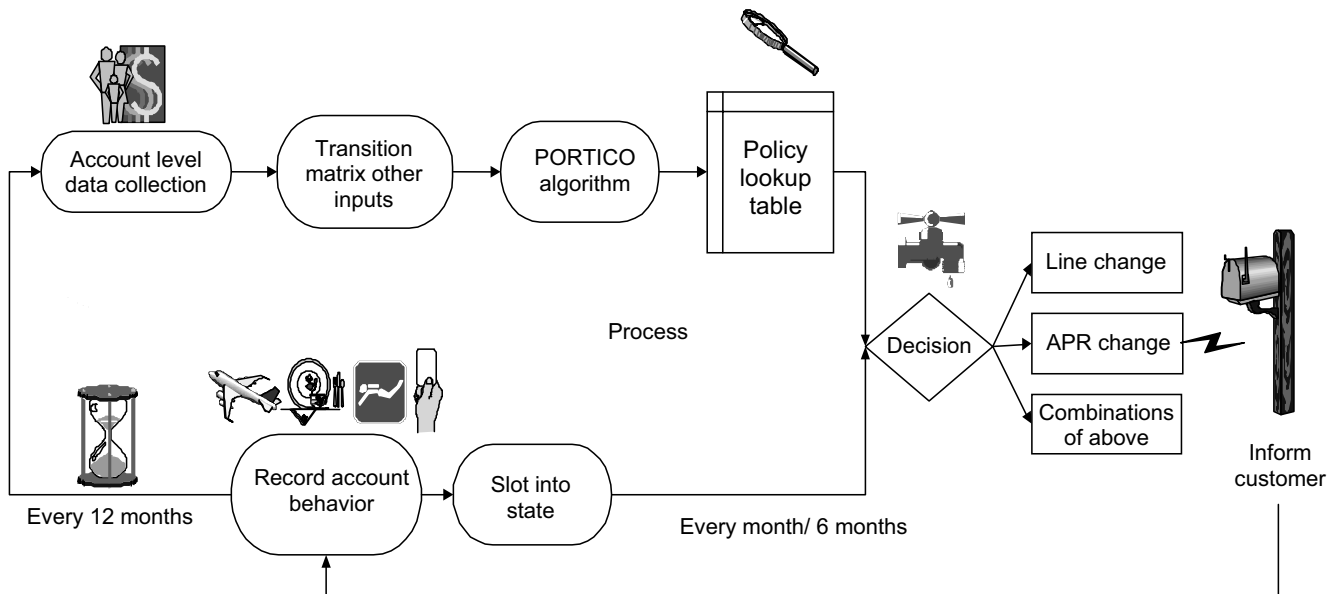
To do this, we generated two sets of numbers, one from PORTICO and the other from a time series of 3 million accounts that reflected the bank's current policies. By comparing these two estimates, we forecast a 12.5 percent improvement in NPV from applying PORTICO policies.

We also validated the transition matrices using a separate data set and compared the NCFs and delinquencies that would result from taking no action in each state of the MDP using the original transition matrix and using the time series generated by the validation data set. Through this analysis, we proved the models were robust when applied to a different random sample. We extended our analysis to confirm that the MDP model accurately projected such key profitability components as outstanding balances and net credit losses.

We next ran Monte Carlo simulations to obtain the distribution of NPV from using PORTICO. Because the MDP model provides the average NPV, it does not show the dispersion about the mean. Financial services firms are very concerned about risk, so we needed to provide information on dispersion. If the bank introduces a policy that increases overall profits but also increases volatility, the portfolio may face increased risk.

Monte Carlo simulations can easily provide this information. We do this by starting in each of the





**Figure 7:** The PORTICO model starts with collecting account-level information that is used to create the transition matrix and other inputs for the algorithm. We then use this information to run the PORTICO algorithm, which produces a table for looking up optimal policies. The table specifies the optimal action, which the bank communicates to the customer. The process also accumulates account information and periodically refreshes the transition matrix and other inputs to PORTICO.

14,000 states and letting the account randomly move from state to state using the transition matrix for a 36-month period or epoch. In review periods, we select the optimal action specified by the MDP (a credit-line change, a price change, or no action). In each period, we collect the NCF for being in that state and discount the NCFs back to the starting time period to obtain the NPV. Each start at the first month from a state provides a sample path. We had the simulation start from each state 10,000 times to obtain the distribution of the NPV for each state. We then obtained the overall NPV by conditioning on the probability of starting from each state defined in the original data set (Figure 8).

The Monte Carlo simulations also allowed us to produce other important data that did not come out of the MDP model runs. These included the distributions of outstanding balances, revenues, and charge-offs. We gathered these data in a manner similar to the one we used for NPV. We simulated the business-as-usual (BAU) and PORTICO methods to compare their effects on account balances (Figure 9). PORTICO consistently improved the average performance of

key financial drivers, including the three-year NPV and account balances. We observed no increase in the variability of these drivers, which indicated that PORTICO improved the bank's profitability without introducing undesired volatility.

We had enough information to make the case for PORTICO. We performed a swap-set analysis to determine the differences in treatments recommended by the current methodology and by PORTICO for a set of accounts that the bank was going to review for credit-line and price changes. In this analysis, we treated the accounts using both techniques and created a  $2 \times 2$  table with four boxes representing the following recommendations: change by both techniques, do nothing by both techniques, change by one but no change by the other, and vice versa.

Each of the boxes shows the type and distribution of accounts and their characteristics. This information helps managers to discern the differences in the two approaches. For example, through this analysis we determined the charge-off rates of customers treated under both approaches. BAU and PORTICO had comparable charge-off rates for all treatment groups. We

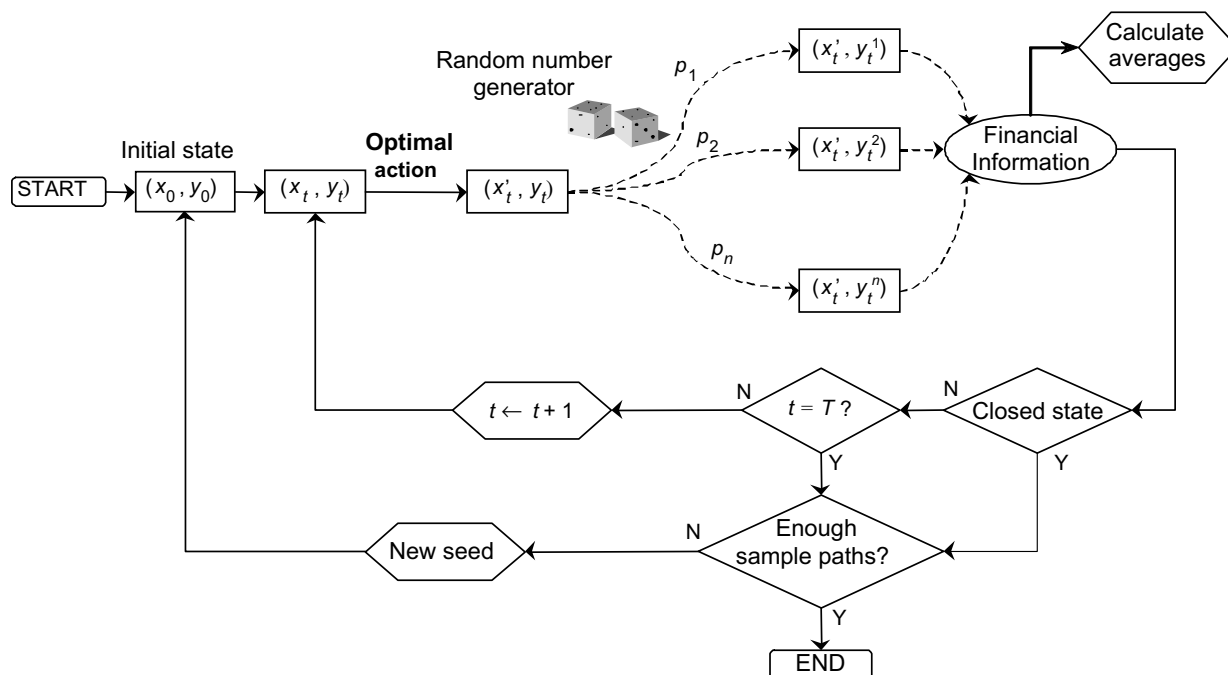


Figure 8: The Monte Carlo simulation of the optimal policy starts from each state in the model. We use the transition matrix to move the account from state to state using a random-number generator. In each period or transition, the simulation process accumulates the financial rewards. In periods when the bank needs to take action, the simulation re-indexes the account according to the optimal action recommended and makes the transitions in that transition matrix row. The simulation process repeats this sequence for 36 months to obtain one sample path. We run a total of 10,000 sample paths for each state.

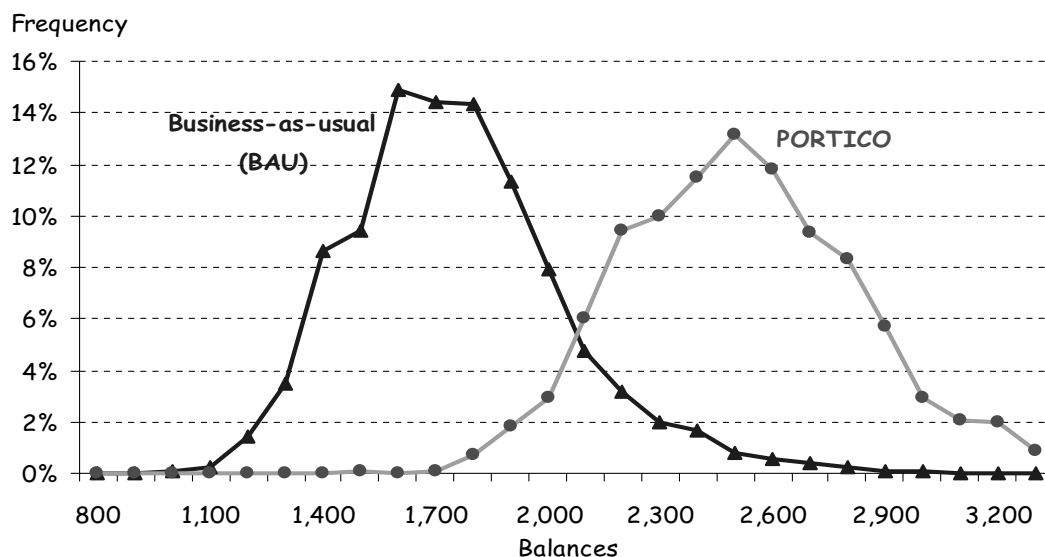


Figure 9: By simulating the distributions of account balances, we show that after implementing PORTICO, the bank would exceed its current values using the business-as-usual (BAU) method.

	BAU recommendation	
	Line increase	No increase
PORTICO recommendation		
Line increase	30%	10%
No increase	15%	45%

(a) Proportion of accounts

Accounts with line increase ...	Average line increase	
	PORTICO	BAU
In PORTICO only	\$2,000	
In BAU only		\$2,500
In PORTICO and BAU	\$3,500	\$2,750

(b) Average line increase recommended

**Figure 10:** We prepared examples of swap set analyses to help managers understand the implications of PORTICO. In (a), we show the proportion of accounts with a recommended credit-line increase policy using each of the methods. Managers focus on distinctions between the two methods. In (b), we further evaluate other metrics, such as average credit-line increases for various groups of accounts. Similar analyses can be done for such factors as projected profitability, losses, and balances.

also focused on instances in which the two methods' recommendations diverged (Figure 10). For 10 percent of the population, PORTICO recommended an average credit-line increase of \$2,000 per account and BAU recommended no increase, whereas for 15 percent of the population, BAU recommended a \$2,500 increase in credit line and PORTICO prescribed no action.

We demonstrated that PORTICO could increase portfolio profitability and that the intuition concerning customers treated under this method was within reason. The results of our analyses convinced the executives to approve a live test to complete the proof-of-concept phase of our project. This test also established the implementation path and audit procedures to be implemented with an MDP model.

## Testing and Results

In April 2000, we launched a live test of PORTICO, determining credit-line increases and APR decreases on approximately 200,000 existing accounts. The test cells consisted of accounts that the bank routed to PORTICO for decisions on credit-line increases, APR reductions, both actions, or no action. It routed a second set of accounts, intended to be statistically identical to the test set, to the existing (BAU) system for making such decisions. An unfortunate occurrence made comparisons of the test and control populations invalid, which limited what we could learn from this test. This setback delayed rollout; however, the results were sufficiently strong that we revised the model. We also established an implementation and audit process that would be easy to replicate on the next round.

In July 2001, we launched a second test of PORTICO, this time with a single control variable, credit line. At the time, the bank was particularly interested in evaluating methods for deciding on credit-line increases, and we decided not to commingle the credit-line increase and price-decrease components. We compared PORTICO and the existing policy. The existing policy focused on increasing credit lines for customers who maintained high balances and as a result typically gave credit-line increases to a smaller fraction of accounts than did PORTICO. Again, we randomly assigned accounts to the test group and the control group, and we tracked all accounts in the test for 12 months and assessed them on NPV (or discounted NCF) and loss rate. PORTICO delivered about a five percent increase in 12-month NPV in the test period over the BAU strategy (Figure 11). This translated to additional profits of over \$3 per original account. Outstanding balances increased over five percent, and the dollar loss ratio (ratio of loss dollars to average outstandings) increased only three percent. These results showed that we could increase customers' balances, increase customer profitability, and control credit losses over time.

Because the bank managed credit lines and pricing changes separately, we decided to pursue applying PORTICO for APR reductions as a stand-alone algorithm. We formulated PORTICO for recommending reductions in APR much as we did for considering credit-line increases, and it awarded price decreases to about 20 percent of accounts. After we demonstrated predicted NPV improvements, executive

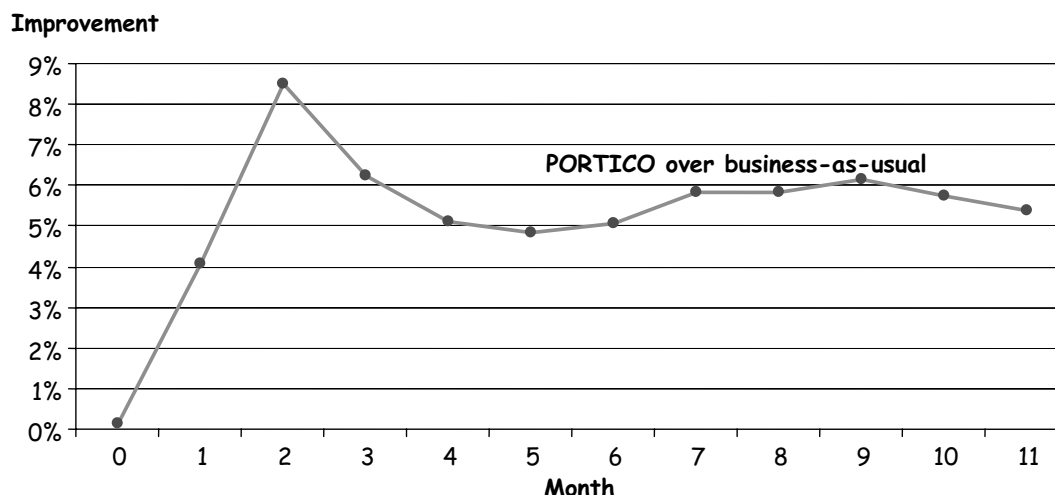


Figure 11: The test showed that over a period of 12 months PORTICO provides roughly five percent cumulative improvement in net cash flow (NCF) over the current business-as-usual (BAU) model. The improvement was most dramatic in the initial months.

leadership approved an APR reduction test in July 2002 using PORTICO. The bank plans to decide whether to deploy the APR reduction policy after it conducts a test review in July 2003.

## Credit-Line Increase Implementation and Roll-Out

In the third quarter of 2001, the bank changed the platform on which it delivered credit-line increases. In doing so, it decoupled credit-line increases from other portfolio levers, such as convenience checks and APR changes. PORTICO for credit-line changes became part of the new platform, and in November 2001, the bank began using it to treat 30 percent of its portfolio. We made some revisions in PORTICO for this implementation, the most significant being the tighter application of credit loss criteria in the postoptimization phase. As a result, the bank considered accounts eligible for credit-line increases only if their projected losses fell under a fairly stringent risk threshold. The bank also revised its existing strategy (BAU) around this time to more closely resemble PORTICO, with credit-line policies developed for segments defined by a set of behavioral variables. The bank is applying this revised BAU policy to the remaining 70 percent of accounts in the portfolio.

In August 2002, after reviewing our latest results with executive management, we decided to revise the PORTICO model for broader application. We expect to further refine it for specific behaviors of customers who have affinity-card products (for example, airline rewards). We also plan to modify the model to make pricing and credit-line decisions for customers who have multiple Bank One relationships. Although we were not interested in incorporating adverse actions (increases in prices and decreases in credit lines) in the original model, management has asked us to solve for these options. We think we can use the PORTICO framework. The decision technologies group, in conjunction with our information services department, has developed a new customer-relationship decision system that was put into production at the end 2002. The bank is setting pricing and credit-line management policies and making other offers (for example, convenience checks) using this platform. As a result, we are deploying the capability to make simultaneous decisions (credit line and price) that we estimated in the original PORTICO model. Given the tables incorporated in the model and our success in implementing the model in two other environments, we anticipate no major deployment issues.

The bank is running PORTICO and evaluating its results for much of its portfolio. Scaling up the results

obtained to date indicates the bank should profit from using it; with over 30 million customers, even a \$1 per year increase in NPV would translate to \$30 million in additional profit. Our experience to date has been that it increases NPV by about \$3 per account and we expect its impact to exceed \$75 million per year. We will put this number in perspective: Credit-card operations for Bank One brought in a net income of \$1.1 billion in 1999, a loss of \$1 million in 2000, and a profit of \$946 million in 2001 (Bank One Annual Reports). From a strategic perspective, PORTICO is clearly an important method for maximizing yield on pricing and credit-line decisions.

## Conclusion

It is a truism in the industry that financial institutions may increase revenue in the short term by acquiring subprime customers and taking on more risk. However, PORTICO opens the way for Bank One to improve its profit while balancing the dual criteria of growth and contained losses. Going forward, PORTICO holds great promise because of the flexible ways it can be applied (for example, in multi-period optimization). Ideally, PORTICO will become a linchpin in a larger strategy to optimize core actions, such as credit-line increases, targeted actions, such as convenience checks, and ad hoc actions, such as authorizations to exceed credit lines.

The MDP methodology used in PORTICO has proven very flexible in accommodating many types of actions. We have captured the dynamics of customer behavior to improve portfolio profit. In doing so, we overcame many of the practical issues associated with the MDP approach, including dimensionality reduction and state-variable definitions. We are working on using the MDP in conjunction with integer programming to incorporate business constraints more directly into PORTICO. We are also using our results to evaluate the validity of our assumption that accounts adopt the behavior of their new states once the bank takes an action.

PORTICO has given the decision technologies group considerable visibility within the bank and as a result, the bank has asked the group to develop several other optimization projects. These projects

concern acquiring customers, developing models for portfolio subgroups, managing customer relationships, and mailing convenience checks. We believe operations research and management science techniques have a bright future at Bank One, and we are pleased to have shown the business value that such approaches can yield in actual practice.

## Appendix: The PORTICO MDP Model

*States:* At each time period,  $t$ , the system (portfolio) occupies a state defined by  $(x, y)$ , where  $x$  is a set of control variables and  $y$  is a set of behavior variables. Let the highest values in  $x$  be denoted by  $x''$ , the lowest values by  $x'$ , and the set of all states by  $S$ .

*Actions:* In each state  $(x, y)$ , the set of actions  $A(x, y)$  consists of some or all of the following actions:

(1) Do nothing,  $a_i = 0$ .

(2) Increase  $x_i$  to  $\min(x_i + a_i, x_i'')$ , that is, an increase cannot take the system to a state higher than the maximum allowable.

(3) Decrease  $x_i$  to  $\max(x_i - a_i, x_i')$ , that is, a decrease cannot take the system to a state lower than the minimum allowable.

*Rewards:* In each state  $(x, y)$ , the bank receives a NCF of  $r(x, y)$ . NCFs can be positive or negative. For example, in charge-off states, the reward will be negative. This would also be true in inactive states.

*Transition Matrices:* The transitions from state to state are represented by a transition matrix  $P$ , with elements  $p(x, y; j)$ , where columns  $j$  correspond to behavior states only and  $\sum_{j \in S} p(x, y; j) = 1$  for each state  $(x, y)$ .

Because the transition matrix model transitions every month, but credit line and pricing updates may only be made periodically, the recursive functional equation for the PORTICO model is

$$\begin{aligned}
 V_t(x, y) &= \begin{cases} \max_{a \in A(x, y)} \left\{ r(x \pm a, y) + \beta \sum_{j \in S} p(x \pm a, y; j) \cdot V_{t+1}(x \pm a, j) \right\} & \text{if } t = \text{update epoch,} \\ r(x, y) + \beta \sum_{j \in S} p(x, y; j) V_{t+1}(x, j) & \text{otherwise} \end{cases} \\
 V_T(x, y) &= r(x, y).
 \end{aligned}$$



That is, in months that are not updating months, no action is taken and the system evolves into the next period according to the transition matrix. During decision epochs, actions are evaluated by reindexing the control variable part of the state definition and the transition matrix.

In the above model,  $T$  is the time horizon for which we solve the model, at which point the terminal rewards are collected as stated above, and  $\beta$  is the one-period discount factor.

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