AN EFFICIENT PARALLEL METHOD OF THE UNDERWATER BEAM TRACING MODEL BELLHOP3D

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BELLHOP3D is a popular sound propagation calculation model based on the ray method with clear physical meanings. In this paper, the coarse-grained parallel optimization of BELLHOP3D is carried out by using MPI (Message Passing Interface). We split different azimuth and pitch angles into chunks and distribute them on separate cores. Results are merged to form the final sound field. The parallel efficiency is improved by around 95% using no more than 128 cores. Code is available at https://github.com/nj-zyq/BELLHOP3D_MPI2.git.
Keywords: underwater sound field calculation, BELLHOP3D, beam tracing method, parallel optimization

1. Introduction

Ray method[1] is a common sound field calculation method. It simplifies the wave equation through high-frequency approximation, then uses geometric acoustics to calculate sound lines, and finally adds them to the sound field. The commonly used ocean sound propagation ray models include BELLHOP[2], Eigenray[3], HWT_3D_mm[4], TRACEO[5], etc.

Since the early 1960s, the theory of ray method has been well developed because of clear physical meanings and fast calculation speed. Therefore, it still has a wide range of application scenarios, such as underwater sound field fast prediction, ocean acoustic tomography, matched field positioning and other underwater applications. Porter[2] proposed the Gaussian beam tracing theory in the 1980s, using the Gaussian function to describe the relationship between the beam width and the ray tube width and so effectively improved the calculation accuracy.

With the development of underwater application, there are higher and higher requirements in the calculation time of sound field. BELLHOP3D model is widely used and has clear structure, which is very suitable for parallel optimization. As a result, We have the possibility and necessity to use parallel optimization technology to save time in underwater sound field calculation.

In recent years, many scholars have carried out parallel optimization on the commonly used sound field calculation models. In 2011, Chen Lianrong[6] used MPI to optimize the ray model BELLHOP; In 2019, Zhang Chaojin[7] used OpenMP to optimize the ray model BELLHOP; In 2019, Ulmstedt Mattias[8] accelerated BELLHOP using GPU. However, these are researches about parallel optimization of the two-dimensional sound field calculation model. For three-dimensional(3D) models, there are also some studies in recent years. Calazan[9] carried out parallel optimization for 3D ray model trace3d; Zijie Zhu[10] and others used OpenMP and MPI to parallel optimize Tang's program[11] for the analytical solution of three-dimensional sound field on the wedge-shaped waveguide.

In this paper, we will first describe the ray method and the algorithm of BELLHOP3D, then show the results of our parallel program.

2. Serial algorithm of BELLHOP3D

2.1 Ray method theory

The three dimensional wave equation in an inhomogeneous medium without sources is

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0, \tag{1}$$

where p and c are functions of spacial positions indicating the sound pressure and sound speed, respectively. The harmonic solution of it can be written as

$$\begin{aligned} p(x,y,z,t) &= A(x,y,z)\mathrm{e}^{\mathrm{i}[\omega t - k(x,y,z)\phi_1(x,y,z)]} \\ &= A(x,y,z)\mathrm{e}^{\mathrm{i}[\omega t - k_0\phi(x,y,z)]}, \end{aligned} \tag{2}$$

where A is the amplitude, $k = \omega/c = k_0 n(x,y,z)$ is the wave number, c_0 is the reference sound speed, n is the refraction index, $k\phi_1 = k_0\phi$ is the phase and $\phi(x,y,z) = n\phi_1(x,y,z)$ is the eikonal function.

We substitute Eq. (2) into Eq. (1) to obtain intensity equation

$$\nabla^2\phi + \frac{2}{A}\nabla A \cdot \nabla\phi = 0 \tag{3}$$

and equation

$$\frac{\nabla^2 A}{A} - k_0^2\nabla\phi\cdot\nabla\phi + k^2 = 0. \tag{4}$$

When $\nabla^2 A/A \ll k^2$ and $\nabla\phi\cdot\nabla\phi = |\nabla\phi|^2$, Eq. (4) can be approximately written as

$$|\nabla\phi|^2 = n^2(x,y,z), \tag{5}$$

which is called the eikonal equation. Then, the directions of the rays are decided by the refraction index, and the sound field can be represented by these rays. We can use geometric relations to calculate how every ray propagates and sum them up to calculate the sound field.

2.2 BELLHOP3D model

BELLHOP3D[12] calculates the sound field using simple geometric relationships deprived by Eq. (5):

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}s} &= c\xi(s), \quad & \frac{\mathrm{d}\xi}{\mathrm{d}s} &= -\frac{1}{c^2}\frac{\mathrm{d}c}{\mathrm{d}x}, \\
\frac{\mathrm{d}y}{\mathrm{d}s} &= c\eta(s), \quad & \frac{\mathrm{d}\eta}{\mathrm{d}s} &= -\frac{1}{c^2}\frac{\mathrm{d}c}{\mathrm{d}y}, \\
\frac{\mathrm{d}z}{\mathrm{d}s} &= c\zeta(s), \quad & \frac{\mathrm{d}\zeta}{\mathrm{d}s} &= -\frac{1}{c^2}\frac{\mathrm{d}c}{\mathrm{d}z},
\end{aligned} \tag{6}
$$

where $c = c(x, y, z)$ is the sound speed, (x, y, z) is the ray trajectory, and (ξ, η, ζ) are angles of the ray with axes.

BELLHOP3D contains several different geometric beams. The beam width is constructed around a central ray and defined in terms of ray-centered coordinates (s, m, n). Here s is the arc length along the sound ray, and (m, n) are the normal distances from a field point to the central ray. For example, the hat-shaped beam is

$$
u_{\mathrm{hat}}(s, m, n) = \frac{1}{\sqrt{|Q(s)|}} \frac{[L_1(s) - n][L_2(s) - m]}{L_1(s)L_2(s)} \mathrm{e}^{\mathrm{i}\omega\tau(s)}, \tag{7}
$$

where L_1 and L_2 are the widths of the ray tube, $|n| < L_1$ and $|m| < L_2$, which means the beam vanishes outside that tube.

Using Gaussian beams can always obtain better accuracy, because the field at any point is the sum of contributions from a number of nearby beams, rather than just the two bracketing beams that the hat-shaped beams provide. This integration over many beams smooths out caustics and also allows leakage energy into shadow zones. The formula for the Geometric Gaussian beams is

$$
u_{\mathrm{Gaussian}}(s, m, n) = \frac{1}{\sqrt{|Q(s)|}} \mathrm{e}^{\mathrm{i}\omega\tau(s)} \mathrm{e}^{-\frac{1}{2}\frac{\left[\frac{n}{L_1(s)}\right]^2 \left[\frac{m}{L_2(s)}\right]^2}{L_1(s)L_2(s)}}. \tag{8}
$$

Due to the high accuracy of the Gaussian beam, we will use the Gaussian beam for calculation in subsequent numerical experiments.

2.3 Discrete subdivision and implementation

Users can set the ranges of the pitch angle α and azimuth angle β of the source, and corresponding numbers $Nalpha$ and $Nbeta$. Users can also set the ranges of positions of the receivers (r, θ, z), and the discrete numbers NRr, $Ntheta$ and NRz. The main function of BELLHOP3D is BELLHOPCORE. The amplitude of the sound pressure of each ray at the receiving point is calculated circularly according to the sequence of β, α, r, θ and z, then summed up.

It should be noted that BELLHOP3D is for both N $\times$ 2D and 3D computation, and all the calculation modes are contained in the loop of β (the AzimuthalAngle loop in the code). Therefore, this parallel version of BELLHOP3D achieves the parallel optimization of all these calculation modes. This paper only takes the 3D Gaussian beam as an example. The serial algorithm is in algorithm 1.

algorithm 1 Serial algorithm of BELLHOP3D

Input: Environment of the ocean and discrete information(*.env, *.bty, etc.)
Output: Sound pressure $\mathrm{P(Ntheta, NRz, NRr)}$

```
function BellhopCoreSerial
    P(Ntheta, NRz, NRr) ← 0
    for ibeta=1, Nbeta do
        for ialpha=1, Nalpha do
            function Influence3DGeoGaussianCart
                for ir=1, NRr do
                    for itheta=1, Ntheta do
                        for iz=1, NRz do
                            calculate Amp
                            P(itheta,iz,ir) ← P(itheta,iz,ir) + Amp
                        end for
                    end for
                end for
            end function
        end for
    end for
    return P
end function
```

3. Parallel algorithm

In this paper, we use MPI to achieve the parallel optimization of BELLHOP3D. The number of processes is $nprocs$. And every process read the input files at the same time, then they are assigned different azimuth angles and pitch angles.

For example, the sequence number of the $ibeta$th azimuth angle and the $ialpha$th pitch angle is $ibeta \times Nalpha + ialpha$. Then the calculation is assigned to each process in this order, and finally we use the reduce method to sum the results up to the variable $rdcP$ on process 0 and write it to the output file.

The parallel algorithm can be seen in algorithm 2.

4. Results

4.1 Problem description

We consider the case shown in Acoustics Toolbox 2020, wedge3dGaussian, about the sound field in a perfect 3D wedge using gaussian beam. The frequency of source is 10 Hz at $(0, -19.1\,\mathrm{km}, 8\,\mathrm{m})$. The depth of the receivers is 80 m. The schematic diagram of this perfect wedge waveguide is shown in Fig.1.

The top view and the perspective view of the rays[12] are shown in Fig.2. Here we have selected 10 rays in the azimuthal direction ranging from $-90°$ to $90°$. In the vertical direction we chose 21 beams from $-20°$ to $20°$. Here we can see that the energy propagating up-slope is gradually refracted back down-slope through repeated interactions with the sloping seafloor.

algorithm 2 Serial algorithm of BELLHOP3D

Input: Environment of the ocean and discrete information(*.env, *.bty, etc.)

Output: Reduces sound pressure $\mathrm{r}dcP(\mathrm{Ntheta}, \mathrm{NRz}, \mathrm{NRr})$

```
function BellhopCoreParallel
    P(Ntheta, NRz, NRr) ← 0
    rdcP(Ntheta, NRz, NRr) ← 0
    for ibeta=1, Nbeta do
        for ialpha=1, Nalpha do
            id = ibeta * Nalpha + ialpha
            if mod(id, nprocs) == myrank then
                function Influence3DGeoGaussianCart
                    for ir=1, NRr do
                        for itheta=1, Ntheta do
                            for iz=1, NRz do
                                calculate Amp
                                P(itheta,iz,ir) ← P(itheta,iz,ir) + Amp
                            end for
                        end for
                    end for
                end function
            end if
        end for
    end for
    rank0: rdcP ← MPI_REDUCE( P )
    return rdcP
end function
```

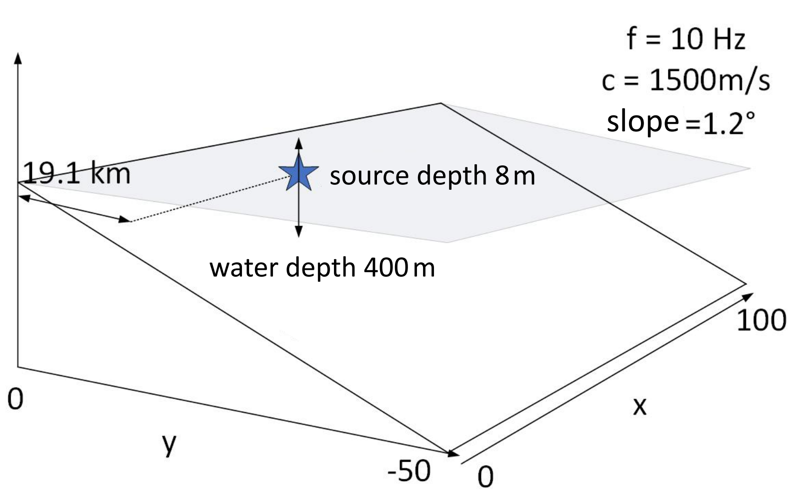
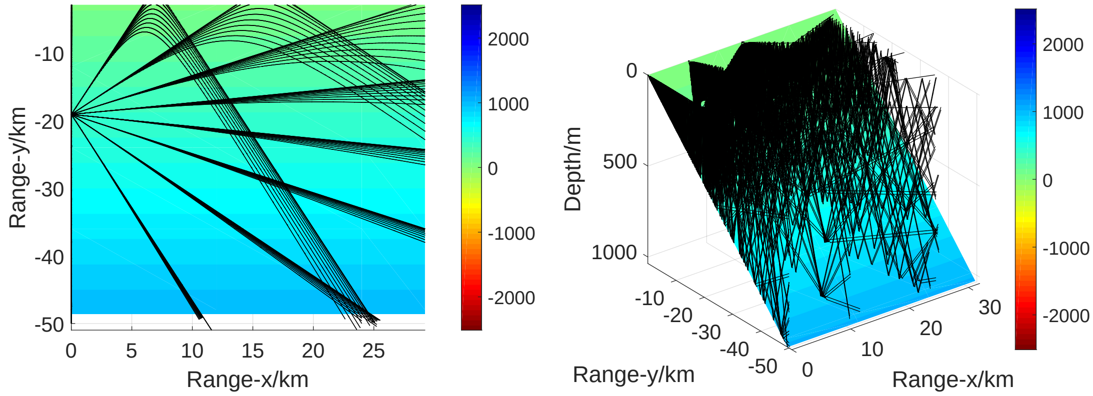


Figure 1: Schematic diagram of three dimensional environment.

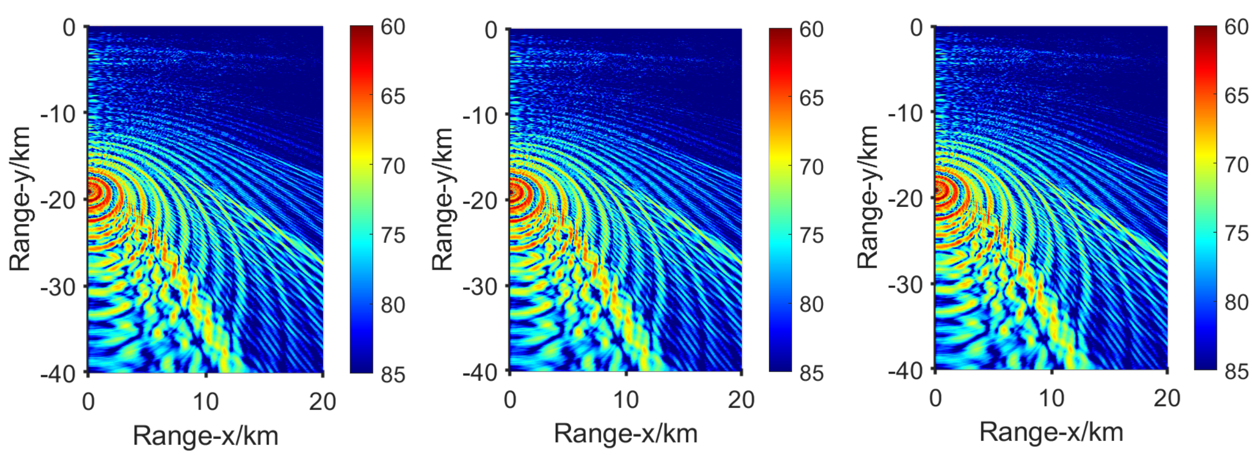


(a) The top view of the rays. (b) The perspective view of the rays.

Figure 2: Rays of the perfect wedge problem.

4.2 Error analysis

In this section, the serial BELLHOP3D and parallel BELLHOP3D are used to calculate the transmission loss of the plane with the $80\,\mathrm{m}$ receiver depth parallel to the surface. The results are in Fig.3. The numerical results show that there is no error between the serial and parallel BELLHOP3D with single process. The maximum error of sound pressure calculated by the parallel BELLHOP3D with 320 process and the serial BELLHOP3D is 5.0920e-09 and the relative error is 1.5347e-06, which is mainly caused by the reduction and sum of the last processes.



(a) Serial BELLHOP3D. (b) Parallel BELLHOP3D with 1 process. (c) parallel BELLHOP3D with 320 process.

Figure 3: Transmission loss.

4.3 Parallel performance

We use Sugon high performance computer platform to calculate the following results. The compiler is intelmpi-2020.1.217, and the compile optimization options is -O3.

The main indices of parallel performance include speedup, parallel efficiency and scalability. The speedup refers to the ratio of serial execution time to parallel execution time. Parallel efficiency refers to the ratio of speedup to the number of processes, $E_{\mathrm{p}} = T_{\mathrm{s}}/(P \times T_{\mathrm{p}})$, where P is the number of processes, T_{p} and T_{s} are the execution time of parallel and serial program, respectively. High performance computing also has two common notions of scalability. Strong scaling is defined as how the solution time varies with the number of processors for a fixed total problem size. Weak scaling is defined as how the solution time varies with the number of processors for a fixed problem size per processor.

We will then test the parallel performance of the parallel BELLHOP3D.

4.3.1 Strong scaling

We set $Nalpha$ to 160 and $Nbeta$ to 501. The result can be seem in Table.1. They illustrate that with the increasing number of processors, the parallel efficiency keeps decreasing.

Table 1: Strong Scaling

Nalpha	Nbeta	node	n-per-node	nprocs	T_{p}	T_{s}	speedup	efficiency
160	501	1	1	1	1830	1770	0.97	0.97
160	501	2	1	2	887	1770	1.99	0.99
160	501	2	2	4	452	1770	3.92	0.98
160	501	2	4	8	228	1770	7.76	0.97
160	501	2	8	16	114	1770	15.53	0.97
160	501	2	16	32	57.8	1770	30.62	0.96
160	501	2	32	64	28.4	1770	62.32	0.97
160	501	4	32	128	16.5	1770	107.27	0.84

4.3.2 Weak scaling

We set $Nalpha$ to 50 and $Nbeta$ to 50 on every processor. The result can be seem in Table.2. It can be seen that the parallel efficiency keep decreasing, but the parallel efficiency keeps around 95% with no more than 128 processors.

Table 2: Weak Scaling

Nalpha	Nbeta	node	n-per-node	nprocs	T_{p}	T_{s}	speedup	efficiency
50	50	1	1	1	62.2	60.5	0.97	0.97
50	100	1	2	2	60.4	118	1.95	0.98
50	200	1	4	4	58.3	228	3.91	0.98
50	400	1	8	8	57.1	446	7.81	0.98
50	800	1	16	16	56.1	881	15.70	0.98
100	800	2	16	32	57.4	1760	30.60	0.96
200	800	4	16	64	58.6	3510	59.90	0.94
400	800	8	16	128	58.7	6990	119.08	0.93

5. Conclusions

In this paper, MPI is used to optimize the ray model BELLHOP3D. The current version of parallel BELLHOP3D is optimized in both azimuth and pitch angles. The calculation of different azimuth angles and pitch angles is assigned to different processes to calculate the sound field respectively, and then the total sound field is calculated by sum reduction, so as to achieve the highest parallel efficiency. The numerical results show that the parallel BELLHOP3D only introduces errors in the sum reduction part, and has very high accuracy and high parallel efficiency for coarse-grained problems.

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REFERENCES

1. Jensen, F. B., Kuperman, W. A., Porter, M. B., Schmidt, H. and Tolstoy, A., *Computational ocean acoustics*, Springer (2011).

2. Porter, M. B. and Bucker, H. P. Gaussian beam tracing for computing ocean acoustic fields, *The Journal of the Acoustical Society of America*, **82** (4), 1349–1359, (1987).

3. Dushaw, B. D. and Colosi, J. A. WASHINGTON UNIV SEATTLE APPLIED PHYSICS LAB, Ray tracing for ocean acoustic tomography, (1998).

4. Zabotin, N. A., Godin, O. A., Sava, P. C. and Zabotina, L. Y. Tracing three-dimensional acoustic wavefronts in inhomogeneous, moving media, *Journal of Computational Acoustics*, **22** (02), 1450002, (2014).

5. Rodrıguez, O. C. The traceo ray tracing program, *Universidade do Algarve-Signal Processing Laboratory*, (2010).

6. Chen, L. and Peng, Z., (2011), Mpi parallel computation of sound field based on gaussian beam model. *Technical Acoustics*, vol. 30, pp. 34–36.

7. Zhang, C. and Sun, B. Parallel computation of sound field based on beam tracing model bellhop, *Journal of Applied Acoustics*, **38** (1), 1–7, (2019).

8. Ulmstedt, M. and Stålberg, J., (2019), *GPU Accelerated Ray-tracing for Simulating Sound Propagation in Water*.

9. Calazan, R. M., Rodríguez, O. C. and Nedjah, N. Parallel ray tracing for underwater acoustic predictions, *International Conference on Computational Science and Its Applications*, pp. 43–55, Springer, (2017).

10. Zhu, Z., Wang, Y., Zhu, X., Liu, W., Lan, Q., Xiao, W. and Cheng, X. Parallel optimization of three-dimensional wedge-shaped underwater acoustic propagation based on mpi+ openmp hybrid programming model, *The Journal of Supercomputing*, **77** (5), 4988–5018, (2021).

11. Tang, J., Piao, S., Yang, X., Zhang, M. and Iqbal, K. An analysis of horizontal refraction in the problem of sound propagation in wedge-shaped oceans, *OCEANS 2017-Aberdeen*, pp. 1–6, IEEE, (2017).

12. B, P. M. Bellhop3d user guide, *Technical report, Heat, Light, and Sound Research Inc*, (2016).