CAMBRIDGE UNIVERSITY ENGINEERING TRIPOS PART IB

IB INTEGRATED COURSEWORK: EXTENDED EXERCISE REPORT

Student Name: Nirmay Jadhav Email: nj356@cam.ac.uk College: St Catharine’s

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Title: Using Multiple Tuned Mass Dampers to Reduce the Amplitude of Vibration at Resonance

Main topic area(s): *(delete as appropriate)* Vibration ~~/ Soils / Structures / Signals~~

Marker: .... Date: ....

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Marker’s Comments:

Using Multiple Tuned Mass Dampers to Reduce the Amplitude of Vibration at Resonance

Nirmay Jadhav nj356

Lab Group 87

Aims

- To explore the effects on amplitudes of vibration from using multiple tuned mass dampers in buildings, each tuned to different frequencies.
- To analyse the relationship between total mass and the amplitudes of vibration.
- To calculate the minimum total mass required to reduce the harmonic response to 10% of the response without absorbers.
- To explore the time response of the structure with multiple tuned mass dampers and compare this to the frequency response.
- To understand how to practically test these computer-generated predictions on a real structure.

Proposal Outline

Since this is a purely computational exercise, it is vital to understand how to write an algorithm that computes the response of the structure and its absorbers. To begin, the code from the A1 laboratory (see Appendix A) could be studied and used as a starting point from which to build the algorithm that can solve for n degrees of freedom as opposed to just 2. By developing the algorithm with this as a baseline, the 2 degrees of freedom can be tested for accuracy as the results should match for both. The time response can be similarly computed and compared to ensure accuracy. To explore the effects of multiple tuned mass dampers (TMDs), the degrees of freedom can be increased while adjusting the distribution of frequencies the absorbers are tuned for, which would provide the data for where the resonant peaks lie. By choosing values for the stiffness and damping rate as constant at their optimal (calculated during the lab), the distribution of the frequencies that are being tuned to can be adjusted to get the best results.

The heavy reliance on computation in this exercise meant that it was vital to ensure that the code gives stable and correct results, hence our decision to prioritise efforts into this (see Appendix B). In addition, due to the large number of iterations needed to be run to test out various distributions and values, it was necessary to automate as much as possible so as few user inputs as possible were required to generate results.

Methodology

Equations of Motion

The equations of motion for the system with n tuned mass dampers (see Figure 1) was initially calculated in matrix form to solve to give the harmonic response. The full equations of motion are here as follows in matrix form:

$$\begin{bmatrix} m_0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_n \end{bmatrix} \begin{bmatrix} \ddot{y}_0 \\ \vdots \\ \vdots \\ \ddot{y}_n \end{bmatrix} + \begin{bmatrix} \lambda_0 + \sum\lambda_n & -\lambda_1 & \dots & -\lambda_n \\ -\lambda_1 & \lambda_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ -\lambda_n & 0 & 0 & \lambda_n \end{bmatrix} \begin{bmatrix} \dot{y}_0 \\ \vdots \\ \vdots \\ \dot{y}_n \end{bmatrix} + \begin{bmatrix} k_0 + \sum k_n & -k_1 & \dots & -k_n \\ -k_1 & k_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ -k_n & 0 & 0 & k_n \end{bmatrix} \begin{bmatrix} y_0 \\ \vdots \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} f_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\boldsymbol{M\ddot{Y}} + \boldsymbol{\Lambda\dot{Y}} + \boldsymbol{KY} = \boldsymbol{F}$$

For the frequency response the input force can be thought of as $f_0 = |f_0| \cos(\omega t)$. When solving the equations of motion, a matrix inverse needs to be taken, which requires the complex version of the matrix equation:

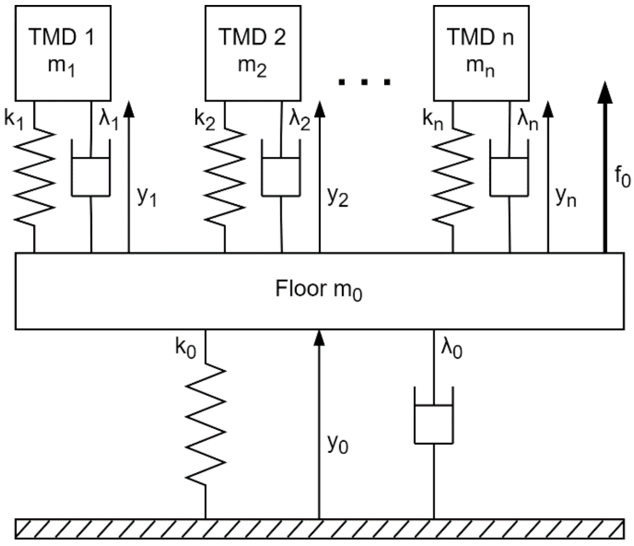
$$(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{\Lambda} + \boldsymbol{K})\boldsymbol{Y} = \boldsymbol{F}$$

$$\boldsymbol{Y} = (-\omega^2\boldsymbol{M} + i\omega\boldsymbol{\Lambda} + \boldsymbol{K})^{-1}\boldsymbol{F}$$

Values and Assumptions

Some assumptions were initially made for the properties of the building (see Table 1) so that the algorithm developed could be validated using the code available from the A1 laboratory. These included the mass of the floor as well as the stiffness of the walls and the optimal damping rate. The other advantage of keeping these consistent is that the same model building can be used in the future to practically validate the results. These values can be modified in the code, which means the model is useful for other building templates too.

Figure 1 Diagram of system with multiple tuned mass dampers



Property	Mass, m (kg)	Stiffness, k, (N/m)	Damping rate, λ (Ns/m)
Floor	3.94	2095	1.98
TMD	$\frac{\omega_t^2}{k}$	80	0.86

Table 1 Values for the properties of the structure

Each TMD needs to be tuned to a specific frequency, which can be done by the formula in Table 1 and will therefore absorb energy from the structure at this frequency. To be most effective, a single TMD would be tuned to the most problematic frequency, i.e., the natural frequency. However, adding another degree of freedom to the system causes the system to have another natural frequency, hence when using several TMDs, there are several ways to decide which frequency each one is tuned to. These frequencies were generated by the code according to a specified distribution: manual, uniform or normal with the parameters being variable. This way several options can be tested and compared easily.

Results

Responses to no TMDs and a single TMD.

The responses of the structure without any absorbers were first computed to validate the code and have a baseline to improve the performance from. The structure can be seen to have a natural frequency of 3.67Hz or 23.06 rad/s with a peak amplitude of 0.002735m from a force of 0.25N, with it taking about 15 seconds to settle to 2% of the original amplitude from a unit step force (see Appendix C). When adding a single damper, there are 2 resonances, however the maximum displacement is reduced by a factor of 5 and the time response is roughly quicker by a factor of 3 as seen in Appendix C. The A1 laboratory code and the code for using n tuned mass dampers show the same results for both cases above, hence validating the algorithm, meaning that changing the number of TMDs should provide consistent and reliable data.

Frequency Response

When changing the number of dampers, several distributions of the tuned frequencies were attempted, hence allowing for the comparison between different values of n as well as different spreads. Starting with the normal distribution, the mean was set to the natural frequency of 3.67Hz and various arbitrary standard deviations were tested. The results show that a wider standard deviation is more effective at reducing the maximum displacement (see Appendix D).

When using a uniform distribution, the mean was kept the same at 3.67Hz, however this time, the spacing between each frequency was varied. Similarly to the normal distribution, better performance was achieved with a wider distribution of tuned frequencies. Overall, the uniform distribution also performed better than the normal distributions with the maximum displacement being reduced by a larger amount with every damper added (see Figure 2). Although the curves were not as smooth, this distribution significantly spread the peak further.
A wider spread of frequencies means that each TMD can absorb energy at its tuned frequency better. This explains the reason for a wider normal distribution performing better and why the uniform distribution was even more effective.

Finally, the response with 100 dampers was also computed. The normal distribution did significantly worse with too many TMDs since all the frequencies they were tuned to are too close together to effectively absorb energy, leading to a singular peak (at a different frequency) with a large displacement. A uniform distribution performed much better, bringing the frequency down even further than before, but there are more resonant peaks than before (see Figure 3).

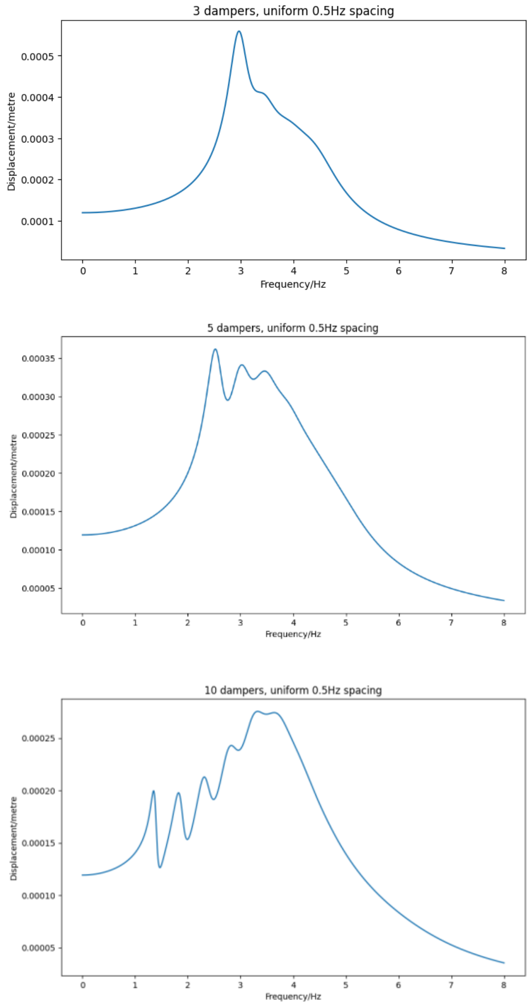


Figure 2 Frequency Response with multiple TMDs uniformly distributed

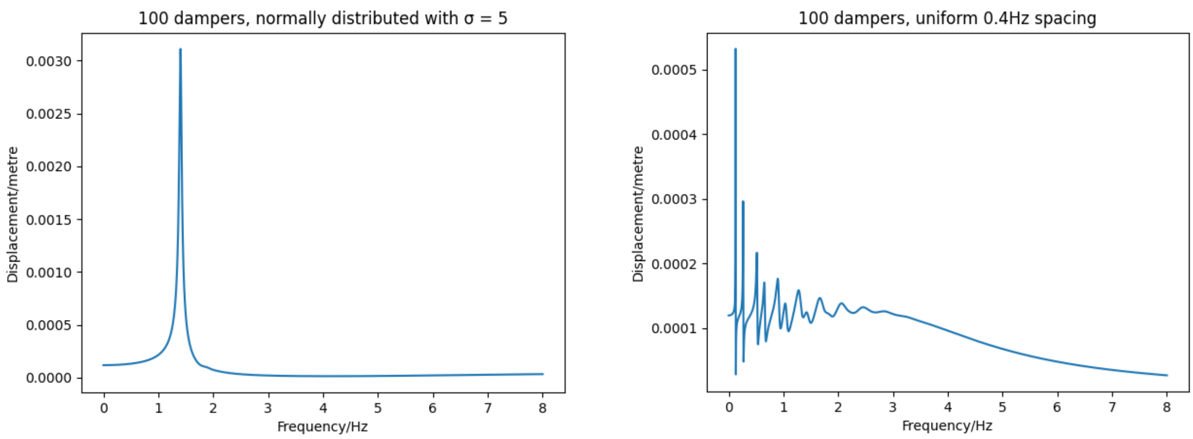


Figure 3 Frequency response of the system with 100 TMDs distributed in different ways

Time Response

The response to a unit step was also calculated using the provided code with the A1 laboratory. The overall results show that adding dampers quickly brings the oscillations down to a reasonable level. Adding too many dampers that have different tuned frequencies however causes some artefacts that have an unpredictable response. The best response out of the ones attempted was with 10 TMDs, with a uniform spacing of 0.5Hz (see Figure 4), however this response did not continuously decay

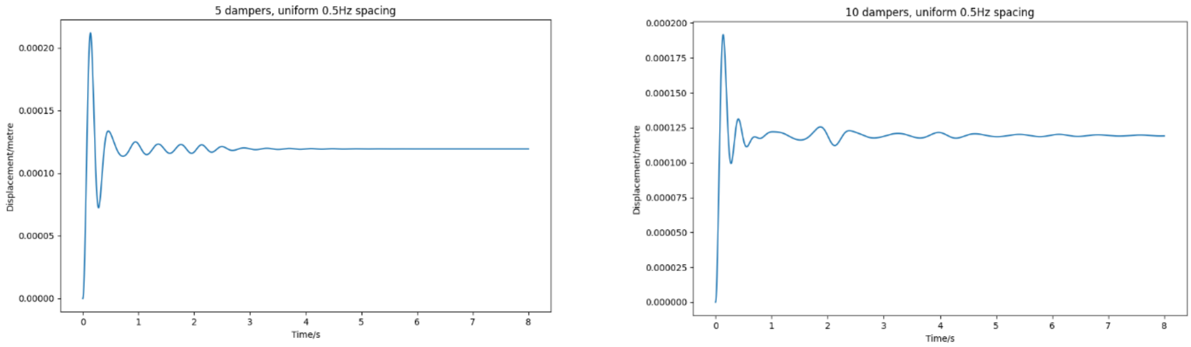


Figure 4 Time response of two well perfoming systems

down. The best response that continually decayed was one with 5 TMDs, spaced uniformly 0.5Hz apart. Unlike the frequency response, having a smaller spacing tended perform better.

Conclusions

Overall, using multiple dampers resulted in interesting responses, some of which were more predictable than other. Adding more dampers (for small values of n) improved the performance and decreased the maximum displacement of the structure. However, this investigation, despite making simplifications to reduce the computing complexity and logical complexity, is still limited by the lack of enough constraints such as practical design goals or requirements. Despite the limitations, the exploration led to the following findings:

- Using multiple tuned mass dampers reduced the maximum displacement of the structure by an order of magnitude with just 2 TMDs.
- A wider distribution of frequencies that the tuned mass dampers were tuned to yielded better performance due to each TMD being able to better absorb the energy at its frequency without the others overlapping.
- 3 dampers uniformly distributed were sufficient to reduce the harmonic response to reduce the harmonic response by a factor of about 10. It is difficult to calculate a minimum total mass since the mass and spring constant determine the frequency the TMD is tuned to.
- A narrower range of tuned frequencies had better performance for the time response and is the reverse of the harmonic response. However, more TMDs did improve the performance which matches the harmonic response.
- To practically test these computed results, it would be useful to measure the maximum displacement of the floor with the different values of tuned frequencies for each TMD. A structure with the same properties as those used in the code would be the ideal template since these exact graphs can be easily compared to. Running a sweep function to measure the response at each frequency would determine the harmonic response, which matches the method that is used in the code.

Appendix

Appendix A - A1 Vibration Absorber laboratory GitHub page
https://github.com/CambridgeEngineering/PartIB-Paper1-Vibration-Absorber-Lab

Appendix B – Multiple Tuned Mass Dampers GitHub page
https://github.com/nj356/Multiple-Tuned-Mass-Dampers

Appendix C – Frequency and Time Response of systems with 0 and 1 damper

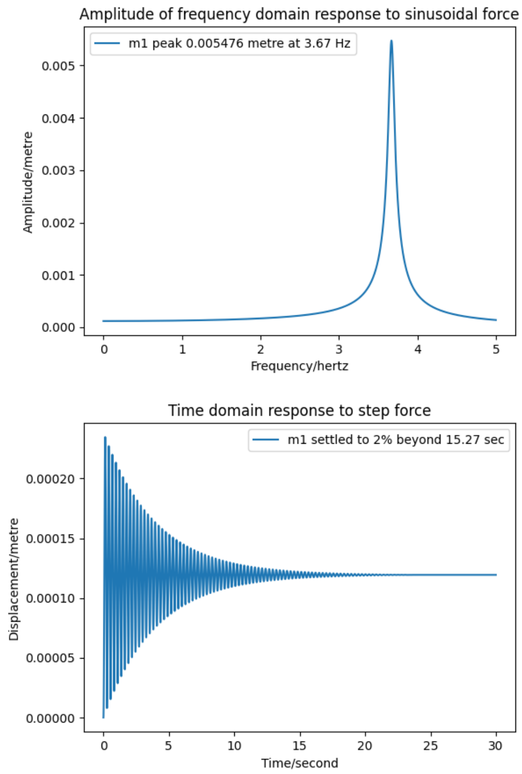


Figure 5 Frequency and Time Response with no TMDs

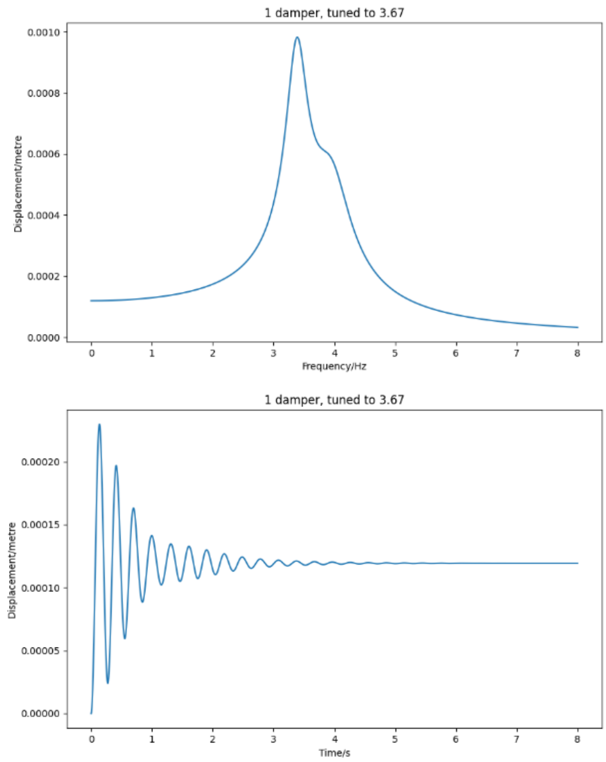


Figure 6 Frequency and Time Response with 1 TMD

Appendix D – Frequency Response of Systems with n dampers normally distributed

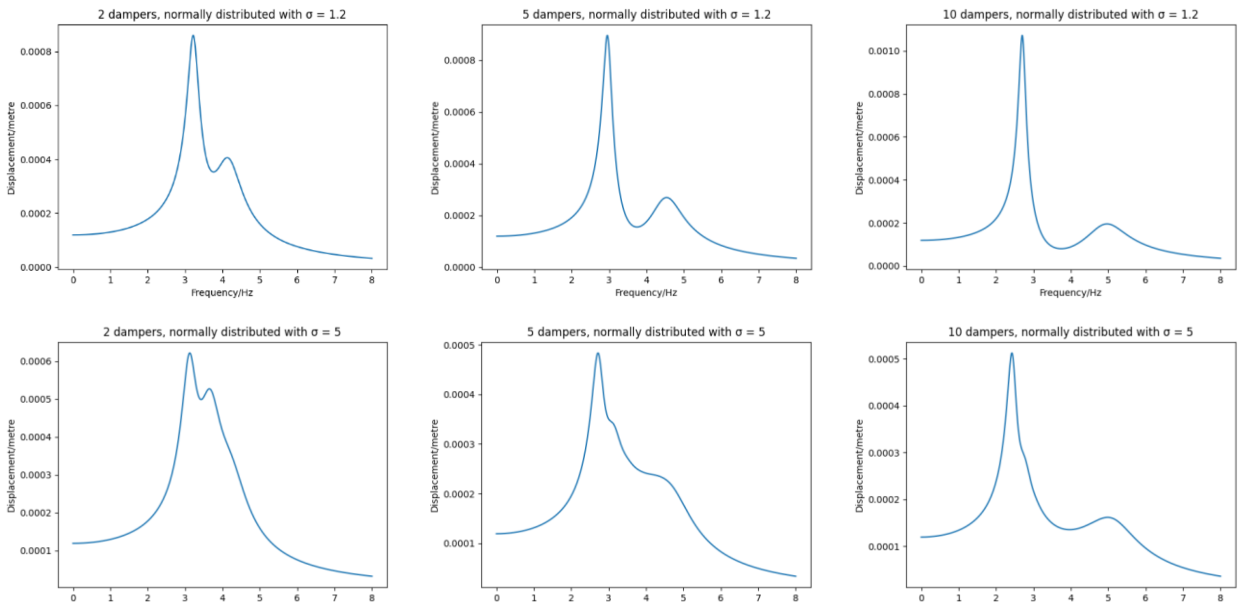


Figure 5 Frequency Response with several TMDs normally distributed about 3.67Hz