

When N is equal to the size of the input (measured by the number of coordinates)

The running time of Brute as a function of N is $N+(N)*(N-1)*(N-2)*(N-3)$ which is $\sim N^4$.

Brute Proof:

This algorithm uses four nested for loops. Starting from the outermost for loop which iterates N times, each proceeding iterates one less time than the loop above it. Therefore, I multiply the time complexities of each for loop and get $\sim N^4$.

The running time of Fast as a function of N is $N+N\log(N)+N*(N\log(N)+(N-1)*N)+(N-1)+N = N^3 + N^2(\log(N)) - N^2 + N\log(N) + 3N - 1$ which is $\sim(N^3)$.

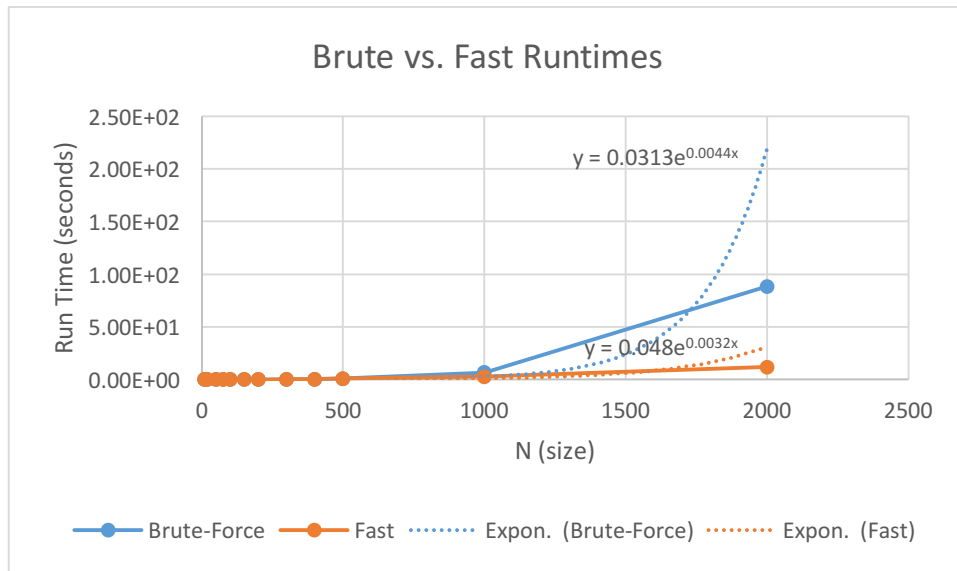
Fast Proof:

The first N represents the while loop in the constructor that loads N coordinates into an array. These coordinates are added to Arrays.sort(), which is called after we have exited out of the while loop. Arrays.sort() has a worst-case time complexity of $N*\log(N)$. After, we through a new while loop N times. Inside the while loop, we call another Arrays.sort(), which, again, will be having a worst-time complexity of $N*\log N$. Also inside the while loop, there is a for loop that gets iterated N-1 times. Inside the for loop, there is a while loop that, in the worst case scenario, will have a time complexity of N. After we exit the for loop and the while loop, we have another while loop that has a worst-case time complexity of (N-1). Lastly, we have a for loop that in the worst-case scenario, will be iterated through N times.

Plot:

N	Brute-Force Time	Fast Time
10	0.021	0.022
20	0.021	0.023
50	0.024	0.03
75	0.029	0.036
100	0.032	0.044
150	0.057	0.077
200	0.077	0.123
300	0.173	0.327
400	0.344	0.465
500	1.015	0.815
1000	6.742	2.636
2000	88.336	11.731

Graph:



Using the graph, the runtime estimates for $N=1,000,000$ are:

Brute Runtime = $0.0313e^{(0.0044(1,000,000))}$

Fast Runtime = $0.048e^{(0.0032(1,000,000))}$