Filtering and Control Theory beyond the Kalman Filter.

- Filtering is about estimating hidden state of a Hidden Markov Model in an online fashion. When the prior is Gaussian and transitions and observations are linear, the Kalman Filter is the Bayes-optimal and efficiently computable approach. The Kalman Filter starts to fail when dealing with nonlinear dynamics and/or multimodal distributions.

  Particle filters are perhaps the default choice in practice, but they suffer a curse of dimensionality. Gaussian sum filters are a natural multi-modal generalization of Kalman filters with a closed form density but they suffer a parameter blow-up problem.

- [Bickel et al., 2008] show that the particle filter suffers a curse of dimensionality. [Rebeschini and Van Handel, 2015] develop a *local particle filter* which escapes this curse to some extent.

- [Alspach and Sorenson, 2003] introduced the Gaussian sum filter, demonstrating the closed-form solution as well as the expoential parameter blow-up. [Psiaki et al., 2015] show how Gaussian sum filters can be viewed as a natural generalization of particle filters.

Pseudotime Inference and Principal Graphs.

- Pseodotime inference is about inferring a developmental trajectory based on a single sample of a distribution. It is very relevant in cell biology, where it is reasonable to expect that a single snapshot of a cell population would contain this information. This problem is closely related to literature on principal curves and vector quantization.

- [Hastie and Stuetzle, 1989] present principal curves as a natural generalization of principal components. [Kégl et al., 1998] present an efficient algorithm for learning principal curves which provably converges.

- [Saelens et al., 2019] does a thorough comparison of the range of methods (over 100); [Wolf et al., 2019]'s PAGA method is the clear winner in terms of efficacy and runtime. PAGA essentially involved finding a k-means partition and then using nearest-neighbor information to infer a graph on these centroids.

- [Warren et al., 2025] analyze principal curves in Wasserstein space, motivated by pseudotime inference.

Trajectory Inference and Schrödinger Bridge.

- The Schrödinger Bridge is a classical problem about inferring the most likely evolution between two distributions. It is equivalent to entropic optimal transport and can be tackled with Sinkhorn iterations. The multi-marginal case is more difficult: unless the prior has certain nice properties, it is computationally intractable to estimate Schrödinger bridges.

- [Lavenant et al., 2021] analyze a version of the Schrödinger problem where one does not have to exactly fit the given marginals but is rather optimized to fit as well as possible while respecting the prior; this is an extension to experimental work of [Schiebinger et al., 2019]. [Chizat et al., 2022] analyze a grid-free version of the estimator.

- [Hong et al., 2025] develop a smooth Schrödinger bridge using ; not obvious a priori that this can be efficiently computed. Curse of dimensionality, though.

Vignettes and Simple Demonstrations

- The parameter blow-up of the Gaussian sum filter.

- A simple analysis of the particle filter where the curse of dimensionality appears (see [Rebeschini and Van Handel, 2015]).

- Motivation and convergence of Sinkhorn algorithm in the two-marginal Schrödinger Bridge.

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