6710 Problem Set 2

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Question I.

In addition to eclipsing binaries, the radius of stars can be directly measured using interferometry. The CHARA array, run by Georgia State University, is currently the most powerful optical interferometer in the world, achieving a 1σ angular resolution of 200 μas (2×10^{-4}"). Consider the following types of stars:

Table 1: Radii and space densities of stars.

Stellar Type	Example	Radius	Density (stars/pc^3)
Main Sequence	Sun	$\sim 1 R_\odot$	4×10^{-3}
K Giant	Arcturus	$\sim 25 R_\odot$	4×10^{-5}
M Supergiant	Betelgeuse	$\sim 1000 R_\odot$	5×10^{-8}
White Dwarf	Sirius B	$\sim 1 R_\oplus$	5×10^{-3}

(a) What kind of star is most often measured with interferometry?

Larger stars are more often measured with interferometry per star (M supergiants like Betelgeuse are the best candidates since they give the best SNR, see Pols 1.1).

(b) About how many stars can have radii measured with 10% precision (S/N = 10) by CHARA?

If we assume a stellar density of supergiants from Table , we can use the angular resolution to find a maximum distance for SNR of 10. If d is the distance to a star and we assume a stellar radius of Betelgeuse 1000 $R_\odot$, we can find the resolution limited distance:

$$\begin{aligned}
\tan\theta &= \frac{R}{d} \\
d &= \frac{R}{\theta} \\
&= 2300 \text{ pc} \\
V &= 5.3 \times 10^{10} \text{ pc}^3
\end{aligned}$$

This gives us $\sim$ 2600 supergiants. This population dominates the number of stars visible to CHARA, and is slightly supplemented by K giants like Arcturus.

(c) Beyond the angular resolution uncertainty, what else might contribute uncertainty to the physical radius measurement?

Optical affects of the atmosphere, stellar variability, and dust attenuation may all play a role in the uncertainty.

Question II.

The density profile of the sun can be modeled with an empirical best fit formula:

$$\rho(r) = [519(r/R)^4 - 1630(r/R)^3 + 1844(r/R)^2 - 889(r/R) + 155] \text{ g cm}^{-3} \tag{1}$$

(This fit comes mostly from astroseismology data, which we will discuss later in the course.) Here $R = 6.96 \times 10^{10}$cm, the radius at the surface of the Sun. Plot the enclosed mass profile $m(r)$ of the Sun. Label the axes, mark the average density $\bar{\rho}$ and radius where $\rho(r) = \bar{\rho}$.

We know

$$\begin{aligned} m(r) &= \int_0^r 4\pi r'^2 \rho \, \mathrm{d}r' \\ &= 4\pi \left[\frac{519}{7R^4} r^7 - \frac{1630}{6R^3} r^6 + \frac{1844}{5R^2} r^5 - \frac{889}{4R} r^4 + \frac{155}{3} r^3 \right] \end{aligned}$$

We can solve for the average density by solving $m(r = R)$ and dividing by the volume, gives $\bar{\rho} = 2.1$ g cm^{-3}, which is within reason of the known value $\bar{\rho} = 1.4$ g cm^{-3}. We can solve for the radii at which the Sun, given this density profile, has this density $\bar{\rho}$, shown in Figure 1, which has four solutions as a rank 4 polynomial, and indicates some weird behavior of this density profile . We also see that the endpoint behavior of this is density profile is incorrect as it yields a negative density at the surface of the Sun.

Question III.

Assume a star made solely of ionized hydrogen.

(a) Create (by sketch or computer) a plot of log density $\log(\rho)$ versus log temperature $\log(T)$ (where log is the base 10 logarithm). Mark the boundaries between regions dominated by ideal gas pressure, radiation pressure, and nonrelativistic electron degeneracy pressure (i.e, where $P_{gas} = P_{rad}$ and $P_{gas} = P_e$).

We use the following for the $P_{rad} = P_{gas}$:

$$\frac{T}{\rho^{1/3}} = 3.2 \times 10^7 \mu^{-\frac{1}{3}}$$

And for $P_{rad} = P_e$:

$$\frac{T}{\rho^{2/3}} = 1.21 \times 10^5 \mu \mu_e^{-\frac{5}{3}}$$

These give us lines with slopes 1/3 and 2/3 respectively (see Pols 3.3.7) See solid lines in Figure 2

(b) Now consider the Sun's density profile given in Problem 2. At what radii do you expect ideal gas pressure, radiation pressure, and nonrelativistic electron degeneracy pressure to dominate?

From this density profile we take the solar limits (or close to the limits) to show the Sun's temperature vs. density behavior. we expect to see a transition from electron degeneracy pressure dominance to ideal gas dominance at a radius of .3 $R_\odot$. We do not see a transition to radiation dominance which further indicates that this density profile is incorrect.

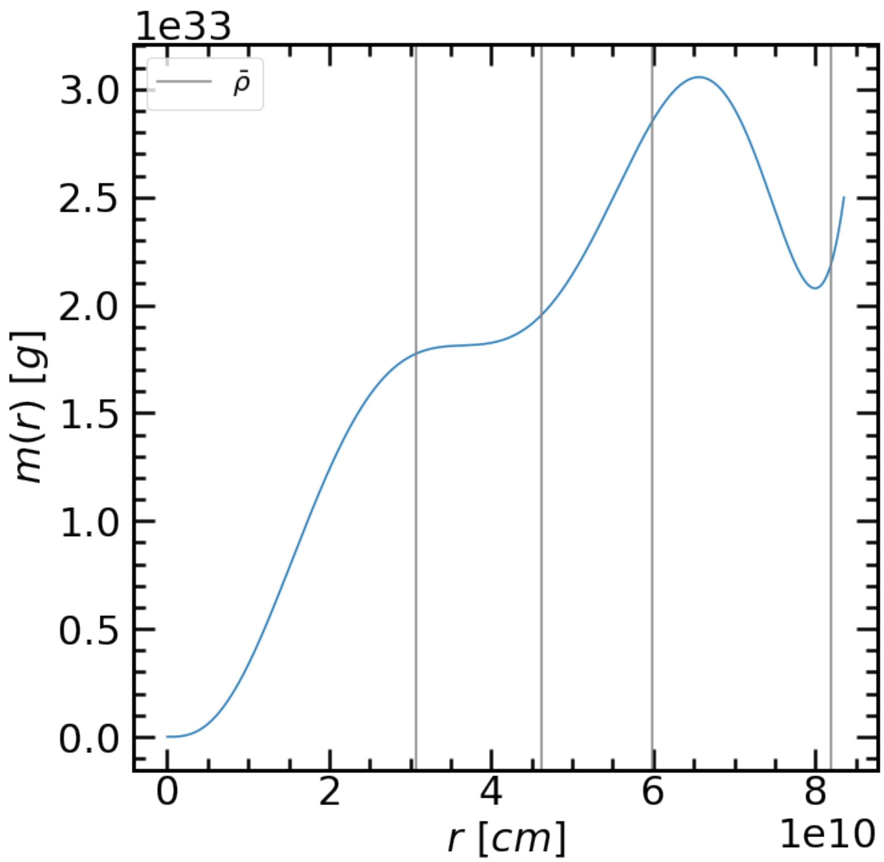

Figure 1: Mass-radius relation for problem 2. Gray lines represent radii which solve $\bar{\rho} = \rho(r) = 1.4$ g cm^{-3}. The four real solutions show that this density profile has some incorrect behavior.

Question IV.

Plot a second set of boundary lines in the $\log(\rho) - \log(T)$ plot for a star made of purely ionized metals (i.e., elements heavier than helium).

See dotted lines in Figure 2.

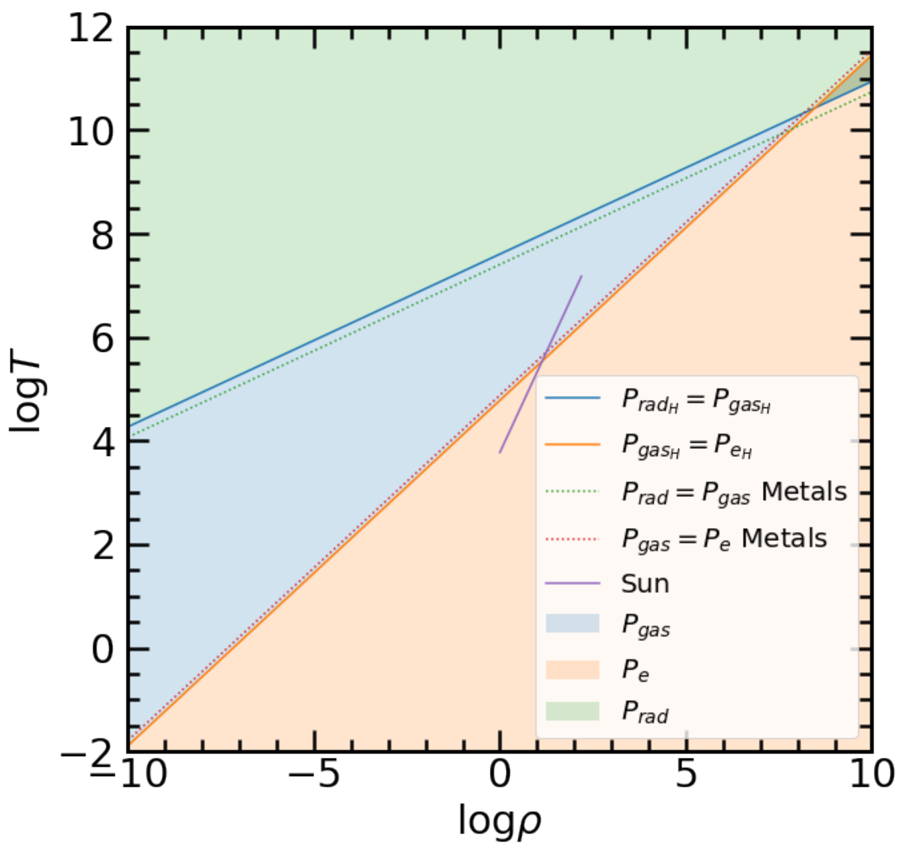

Figure 2: Temperature-density relation for problems 3 and 4.