6720 Problem Set 4

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Resources: Carroll & Ostlie, Schneider
Time: ∼5 hours
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Question I.

(a) From the figures we have

$$\Delta V_{1241} \approx 350 \text{ km/s}$$
$$\Delta V_{5248} \approx 200 \text{ km/s}$$

and we know that $m_{1241} = 14$ and $m_{5248} = 12$. We can use the Tully-Fisher relation to find

$$M = -4.43 - 6.15 \log \Delta V$$
$$M_{1241} = -20.1$$
$$M_{5248} = -18.6$$

We can then use the distance modulus to find

$$d = 10^{\frac{1}{5}(m-M)+1}$$
$$d_{1241} \approx 63 \text{ Mpc}$$
$$d_{5248} = 13 \text{ Mpc}$$

(b) From Hubble's Law we have

$$v = H_0 d$$

Where v is the median of the velocity profile (dotted line), and the distances d are those found in part (a). We get Hubble constants from these measurements of NGC 1241 and NGC 5248 of $H_0 \approx 64.3$ km/s/Mpc and $H_0 \approx 88.5$ km/s/Mpc, respectively. We are in the same ballpark as the accepted Hubble parameter of $H_0 \approx 68$ km/s/Mpc.

Question II.

(a) If we consider the virial theorem we have the relationship

$$\sigma^2 \propto \frac{GM}{r}$$
$$\implies M \propto \sigma^2 r$$

We also assume the relationship between the luminosity and surface brightness is

$$L \propto Ir^2$$

And we assume a constant mass to light ratio M/L. These assumptions imply

$$\begin{aligned} M &\propto L \\ &\propto Ir^2 \\ &\propto \sigma^2 r \\ \sigma^2 &\propto Ir \end{aligned}$$

and when put in the form of the fundamental plane relation we have

$$r \propto \sigma^2 I^{-1}$$

(b) If we now assume some mass to light ratio $M/L \propto M^x$, we have

$$\begin{aligned} L &\propto M^{x-1} \\ L &\propto Ir^2 \\ M^{x-1} &\propto (\sigma^2 r)^{x-1} \\ Ir^2 &\propto (\sigma^2 r)^{x-1} \\ &\propto \sigma^{2x-2} r^{x-1} \\ r^{3-x} &\propto \sigma^{2x-2} I^{-1} \\ r &\propto \sigma^{\frac{2x-2}{3-x}} I^{\frac{-1}{3-x}} \end{aligned}$$

From the empirical fundamental plane relation given, we have

$$\begin{aligned} \frac{-1}{3-x} &= -0.85 \\ \frac{2x-2}{3-x} &= 1.4 \end{aligned}$$

which both give $x \approx 1.82$, so $M/L \propto M^{1.82}$.

Question III.

(a)

(b)

(c)

(d)

(e)

Question IV.

(a)

(b)

(c)

(d)

(e)

(f)