6710 Problem Set 5

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Question I.

For Polaris with mass of $5.4M_\odot$, radius $37.5R_\odot$, luminosity $1260L_\odot$:

(a) The effective surface temperature is given by

$$\begin{aligned} T_{eff} &= \left(\frac{L}{4\pi R^2\sigma}\right)^{1/4} \\ &\approx 5600 \text{ K} \end{aligned}$$

(b) For $y = 0.9$. $X = 0$, $Z = 0.1$ with half of the star available for 3α reactions, we start by finding the number of 3α reactions (which require 3 ${}^4_2$He per reaction):

$$\begin{aligned} M &= 5.4\ M_\odot \\ M_{3\alpha} &= \frac{1}{2}MY \\ &= 2.4\ M_\odot \\ N_{3\alpha} &= \frac{2.4M_\odot}{3m_{He}} \\ &= 2.393 \times 10^{56} \end{aligned}$$

We find the total energy released in this 3α process by multiplying the energy per 3α process ($Q = 7.275$ MeV, from Pols 6.4.2) by the number of 3α reactions, giving $E_{tot} = 1.74 \times 10^{57}$ MeV. We then get time as

$$\begin{aligned} t &= \frac{E_{tot}}{L} \\ &= \frac{1.74 \times 10^{57} \text{ MeV}}{1260\ L_\odot} \\ &= 5.7 \times 10^{14} \text{ s} \\ &= 1.8 \times 10^{7} \text{ yr} \end{aligned}$$

(c) The period of variability is simply given by the free fall timescale, given by

$$\begin{aligned} \tau_{ff} &= \sqrt{\frac{R^3}{GM}} \\ &\approx 1.57 \times 10^{5} \text{ s} \\ &\approx 1.8 \text{ days} \end{aligned}$$

This is perhaps off by a factor of ≈2 given that the known period of variability is around 4 days.

Question II.

For a helium flash of luminosity $L = 1 \times 10^{10} L_\odot$ in a time of 3s, the total energy released is

$$\begin{aligned} E_{tot} &= 1.0 \times 10^{10}\ L_\odot (3\text{ s}) \\ &= 1.154 \times 10^{44}\text{ ergs} \end{aligned}$$

This is equal to the change in gravitational potential energy of the core:

$$\Delta U = -\frac{3}{5} G M_c^2 \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

where M_c is the core mass (mass of a white dwarf, we use $0.7 M_\odot$), and r_i is the initial radius of the white dwarf core (we use $1.2 R_\oplus$). Solving for the final radius gives

$$r_f = \left(\frac{E_{tot}}{\frac{3}{5} G M_c^2} + \frac{1}{r_{WD}} \right)^{-1}$$

This gives the final radius to be $1.2 R_\oplus$, within 1% of the initial radius (negligible expansion, less than tens of meters).

Question III.

(a) C is produced in H/He burning, while Sn is produced in s-process neutron capture in low/mid mass stars.

(b) The runaway fusion in the deaths of massive stars form elements up to iron exothermically through heavy metal fusion, and all elements heavier than iron require endothermic neutron capture. (Assuming a typo in the problem: 'heavier than helium')

(c) Neutron star mergers lead to the most extreme neutron capture processes, meaning that the neutron capture rates are great enough to capture large numbers of neutrons before a decay. This allows for the heaviest of naturally occurring elements to form.

Question IV.

Lithium, beryllium and boron are much less abundant than other elements because they are all produced in weak reactions in the pp chain and then decay into other elements, effective 'skipping' them in stellar fusion. Stable isotopes of these elements are ${}^{6}_{3}$Li and ${}^{7}_{3}$Li, ${}^{9}_{4}$Be, and ${}^{10}_{5}$B and ${}^{11}_{5}$B.

1. Stable lithium is produced in extremely small amounts from big bang nucleosynthesis (in addition to $\frac{1}{5}$) and novae. Lithium is also produced in the pp chain by the reaction

$$^{7}_{4}\text{Be} + e^- \implies {}^{7}_{3}\text{Li} + \nu_e$$

   but this quickly reacts with a proton to form two helium nuclei.

2. ^{7}Be is produced in the pp chain in the reaction

$$^{3}_{2}\text{He} +^{3}_{2}\text{He} \implies {}^{7}_{4}\text{Be} + \gamma$$

   but this is unstable and decays into lithium in the process described in the previous point.

3. ^{8}B is produced in the third branch of the pp chain in the reaction

$$^{7}_{4}\text{Be} +^{1}_{1}\text{H} \implies {}^{8}_{5}\text{B} + \gamma$$

   but this is (yet again) unstable and decays into ^{8}Be, which we explained in the previous point is itself unstable and decays into 2 helium nuclei.

The result of all of this is that stable isotopes of these elements are not produced in most normal stellar processes, causing these anomalous abundances. The stable isotopes of these elements are often produced by the decay of larger nuclei in higher energy collisions.