# ASTR 606: Radiative Transfer

Problem Set 2

Nikko Cleri

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**Question I.**

The power spectrum gives

- CGB: $\sim 10^{-3}$ nWm$^{-2}$ s$^{-1}$sr$^{-1}$
- CXB: $\sim 10^{-1}$ nWm$^{-2}$ s$^{-1}$sr$^{-1}$
- CUVOB: $\sim 10^{1}$ nWm$^{-2}$ s$^{-1}$sr$^{-1}$
- CIB: $\sim 10^{1}$ nWm$^{-2}$ s$^{-1}$sr$^{-1}$
- CMB: $\sim 10^{3}$ nWm$^{-2}$ s$^{-1}$sr$^{-1}$
- CRB: $\sim 10^{-4}$ nWm$^{-2}$ s$^{-1}$sr$^{-1}$

To get the number density, we take the power density from the y-axis of Figure 1 and divide by the respective energy from the x-axis. We get the following:

- CGB: $\sim 6 \times 10^{-4}$ m$^{-2}$ s$^{-1}$sr$^{-1}$
- CXB: $\sim 6 \times 10^{3}$ m$^{-2}$ s$^{-1}$sr$^{-1}$
- CUVOB: $\sim 6 \times 10^{10}$ m$^{-2}$ s$^{-1}$sr$^{-1}$
- CIB: $\sim 6 \times 10^{12}$ m$^{-2}$ s$^{-1}$sr$^{-1}$
- CMB: $\sim 6 \times 10^{15}$ m$^{-2}$ s$^{-1}$sr$^{-1}$
- CRB: $\sim 6 \times 10^{12}$ m$^{-2}$ s$^{-1}$sr$^{-1}$

So, with the exception of the CRB, the number density decreases as photon energy increases.

**Question II.**

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\begin{aligned}
\frac{1}{4\pi}\int_\Omega \frac{3}{4}(1+\cos^2\theta)\,\mathrm{d}\Omega &= \frac{3}{16\pi}\int_0^{2\pi}\int_0^{\pi}(1+\cos^2\theta)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \\
&= \frac{3}{8}\int_0^{\pi}(1+\cos^2\theta)\sin\theta\,\mathrm{d}\theta \\
&= \frac{3}{8}\int_0^{\pi}\sin\theta+\cos^2\theta\sin\theta\,\mathrm{d}\theta \\
&= \frac{3}{8}\left(2-\int_0^{\pi}\cos^2\theta\sin\theta\,\mathrm{d}\theta\right) \\
&= \frac{3}{8}\left(2-\frac{\cos^2\theta}{3}\bigg|_0^{\pi}\right) \\
&= 1
\end{aligned}
$$

So $\frac{3}{4}(1+\cos^2\theta)$ is the normalization of the Rayleigh scattering phase function.

**Question III.**

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The optical depth $\tau$ decreases towards the top of the atmosphere, so an observer in space will see emission from only the higher layers of the atmosphere with higher optical depth. The observer will also measure smaller absorption coefficients in the spectrum.

**Question IV.**

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The depth of the absorption lines from the atmosphere is determined by the temperature of the atmosphere at the depth of the source, and since the flux is proportional to the temperature we expect to see shallower absorption features from hotter parts of the atmosphere, and deeper features from cooler parts. The width of the lines is determined by the velocity distribution (so, the Doppler effect), so we expect wider lines at higher temperatures (ignoring fine/hyperfine structure effects).

**Question V.**

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In the UV regime, we expect to see more stimulated emission (things like recombination lines), due to higher energies of potentially ionizing photons.

In the IR regime, we anticipate more spontaneous emission from processes like thermal emission (reprocessed dust emission, for example), which is on the order of hundreds of Kelvin.

**Question VI.**

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Conservation of energy tells us that

$$F_{obs} = F_{in}$$

where $F_{obs}$ and $F_{in}$ are the observed and incident fluxes on the planet, respectively. The incident flux, assuming some albedo $A$, is

$$F_{inc} = (1 - A)F$$
$$F = \frac{L}{2\frac{d^2}{r_p^2}}$$

where $d$ is the distance from the planet to the star, and $r_p$ is the radius of the planet. $F$ is the flux per unit half planet surface area. We then invoke Stefan-Boltzmann to give

$$(1 - A)\frac{L}{2\frac{d^2}{r_p^2}} = \sigma T_{eq}^4$$
$$T_{eq} = \left((1 - A)\frac{L}{2\sigma\frac{d^2}{r_p^2}}\right)^{1/4}$$

For Planet K2-199b, with a host star of temperature 4648K and orbital radius of 0.038 AU, we get an equilibrium temperature of ∼2500K, which is ∼3 times the calculated temperature of the planet of 913K.

For Planet K2-199c, with the same host star of temperature 4648K and orbital radius of 0.066 AU, we get an equilibrium temperature of ∼1900K, which is ∼3 times the calculated temperature of the planet 694K.