ASTR 605 Problem Set 2

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Question I.

The flux from the star is such that

$$F_* = \frac{L_*}{\pi R_*^2}$$

and the observed luminosity from the star when eclipsed by the planet is

$$L_{*,\mathrm{transit}} = L\pi R_*^2 F_* - \pi R_p^2 F_*$$

transforming to flux space gives us the desired differential flux

$$\frac{F(t)}{F_0} = 1 - \frac{A_E(t)}{\pi R_*^2}$$

where F_0 is the flux of the star with no transit, and A_E is the obscured area by the eclipsing planet. We can now introduce the on-sky radius b, defined

$$b = \frac{a\cos i}{R_*}\left(\frac{1-e^2}{1-e\sin\omega}\right)$$

where a is the semimajor axis of the orbit and e is the eccentricity. In the circular orbit case, $e = 0$, we have

$$b = \frac{a\cos i}{R_*}$$

For the eclipse to occur, the minimum of b must be be less than the angular radius of the star. When the planet eclipses the star, we get the characteristic ingress and egress periods which are calculable with the projected area of the planet on the star. The velocity of the planet can be derived from it period from Kepler's third law $T^2 \propto a^3$.

Question II.

From the equations of motion in the restricted three body problem we have

$$\ddot{x_0} - 2\dot{y} = x_0(1+2\alpha)$$
$$\ddot{y_0} - 2\dot{x} = y_0(1+2\alpha)$$
$$\ddot{z_0} = -z_0\alpha$$

where

$$\alpha = \frac{u_1}{r_1^3} + \frac{u_2}{r_2^3}$$

by symmetry it is evident in the above equations of motion that oscillations in the z direction are stable. If we guess solutions of the form

$$x_0 = Ke^{\lambda t}$$
$$y_0 = Le^{\lambda t}$$

we get stable solutions if λ is pure imaginary. This implies that $1-\alpha$ is nonnegative. We can write this condition as

$$1-\alpha = \frac{u_1 u_2}{x_0}\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)$$

which gives stable solutions for $r_2 \geq r_1$. In the case of the triangular Lagrange points, it is known that $r_i = r_2$, so we have

$$1-\alpha = 0$$

so the triangular solutions are stable. The stability condition of $u_2 < 0.0385$ is satisfied for systems like the Sun-Jupiter system, resulting in the Trojan asteroids.

Question III.

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1. The center of mass of the system moves in a straight line with no acceleration. This is simply a consequence of the conservation of momentum of the system, where

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \sum_i F_i$$

where F_i are the external forces of the system. Since we are considering the system to be in isolation, this is satisfied trivially.

2. For the case of three masses in an equilateral triangle, we consider the forces acting on one mass m_1. We have

$$\vec{F}_{m_1} = \frac{Gm_2}{\vec{r}_{12}^2} + \frac{Gm_3}{\vec{r}_{13}^2}$$

where $|r_{12}| = |r_{13}| = r$ is the length of each leg of the triangle. In the reduced case where we consider a mass M_1 at the center of mass of the three-body system, we have

$$F_{m_1} = \frac{GM_1}{R_1^2}$$

where R_1 is the distance from mass m_1 from the center of mass of the three-body system. In this consideration,

$$M_1 = \frac{(m_2^2 + m_2 m_3 + m_3^2)^{3/2}}{(m_1+m_2+m_3)^2}$$

we then have

$$F_{m_1} = G\frac{(m_2^2 + m_2m_3 + m_3^2)^{3/2}}{(m_1 + m_2 + m_3)^2}\frac{1}{R_1^2}$$

which is the same force as the non-reduced case when considering that

$$R_1 = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

where m_i are each of the masses and r_i are there positions.