5401 Problem Set 5

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Question I.

In Exam 1, Problem 1, find the probability of finding the particle on the left half of the potential well at time t, given that at time $t = 0$, its wave function is defined as in part b.

Question II.

Shankar 7.34

Using Eqs. (7.3.23)-(7.3.25), show that

$$\langle n'|X|n\rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2} [\delta_{n,n+1}(n+1)^{1/2} + \delta_{n,n-1}m^{1/2}]$$
$$\langle n'|P|n\rangle = \left(\frac{m\hbar\omega}{2}\right)^{1/2} i[\delta_{n,n+1}(n+1)^{1/2} - \delta_{n,n-1}m^{1/2}]$$

Question III.

Shankar 7.35

Using the symmetry arguments from Exercise 7.3.3 show that $\langle n|X|n\rangle = \langle n|P|n\rangle = 0$ and thus that $\langle X^2\rangle = (\Delta X)^2$ and $\langle P^2\rangle = (\Delta P)^2$ in these states. Show that $\langle l|X^2|l\rangle = 3\hbar/2m\omega$ and $\langle l|P^2|l\rangle = \frac{3}{2}m\omega\hbar$. Show that $\psi_0(x)$ saturates the uncertainty bound $\Delta X \cdot \Delta P \geq \hbar/2$.

Question IV.

Shankar 7.36

Consider a particle in a potential

$$V(x) = \frac{1}{2}m\omega^2x^2 \quad x < 0$$
$$= \infty \quad x \leq 0$$

What are the boundary conditions on the wave functions now? Find the eigenvalues and eigenfunctions.

Question V.

Shankar 7.41

Compute the matrix elements of X and P in the $|n\rangle$ basis and compare with the result from Exercise 7.3.4.

Question VI.

Shankar 7.45

At $t = 0$ a particle starts out in $|\psi(0)\rangle - 1/2^{1/2}(|0\rangle + |1\rangle)$.

1. Find $|\psi(t)\rangle$.
2. Find $\langle X(0)\rangle = \langle\psi(0)|X|\psi(0)\rangle$, $\langle P(0)\rangle$, $\langle X(t)\rangle$, $\langle P(t)\rangle$.
3. Find $\left\langle \dot{X}(t) \right\rangle$ and $\left\langle \dot{P}(t) \right\rangle$ using Ehrenfest's theorem and solve for $\langle X(t)\rangle$ and $\langle P(t)\rangle$ and compare to part (2).