6710 Problem Set 1

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Question I.

Parallax is critically important for understanding stellar properties. But despite its simplicity (just geometry!), most stars in the sky have been inaccessible to reliable parallax measurements - until very recently.

(a) Can you reliably measure parallax to any stars with your eye?

If the pupil of the human eye is an aperture with diameter of the order $\sim$ 1cm and we consider the best case, which is observing at the violet end of the visible wavelengths ($\lambda = 400$nm), we have the Rayleigh criterion as follows:

$$\begin{aligned}\theta &= \frac{1.22\lambda}{D} \\ &= \frac{1.22(400 \text{ nm})}{1 \text{ cm}} \\ &= 4.88 \times 10^{-5} \text{ radians} \\ &= 10.07 \text{ arcsec}\end{aligned}$$

Since we know that the closest stars (other than the sun) require better than 1 arcsec angular resolution to resolve parallax at $S/N = 1$, we cannot reliably measure parallax to any stars.

(b) Galileo's telescope had a 3 cm aperture. How many stars could he resolve parallax for?

Using the Rayleigh criterion again, and using the same wavelength:

$$\begin{aligned}\theta &= \frac{1.22\lambda}{D} \\ &= \frac{1.22(400 \text{ nm})}{3 \text{ cm}} \\ &= 1.63 \times 10^{-5} \text{ radians} \\ &= 3.36 \text{ arcsec}\end{aligned}$$

Due to the limits of the Rayleigh criterion, Galileo's telescope also does not have the resolution to reliably measure parallax to any stars.

(c) The *Hipparcos* satellite was a dedicated astrometry and parallax experiment that operated from 1989-1993. It had an angular resolution of 1 mas (0.0012). How many stars could *Hipparcos* measure parallax for with at least 10% precision? (*Clearly state your assumption about the density and distribution of stars around us.*)

10% precision means that we can measure objects of angular size 0.012 arcsec. If we assume a uniform stellar density of 0.14 stars/pc^{-3}, we can resolve stars with parallax of to this precision at distances of

$$\begin{aligned} d &= \frac{1}{p} \\ &= \frac{1}{.012} \\ &= 83.3 \text{ pc} \end{aligned}$$

where p is the parallax angle. From this we get a volume:

$$\begin{aligned} V &= \frac{4}{3}\pi d^3 \\ &= 2.4 \times 10^6 \text{ pc}^3 \end{aligned}$$

With the assumed stellar density, this yields $\sim 3 \times 10^5$ stars in this volume visible to *Hipparcos.*

(d) *Gaia* launched in 2013 and achieves an astrometric precision of 7μas (7×10^{-6}"). It is still operating, and had a major data release in April 2018. For how many stars can Gaia measure parallax with at least 10% precision? (*Again, clearly state your assumptions.*)

With the same assumptions as before (uniform stellar density of 0.14 stars/pc^{-3}), our new distance limit is given by

$$\begin{aligned} d &= \frac{1}{p} \\ &= \frac{1}{7 \times 10^{-5}} \\ &= 14 \text{ kpc} \end{aligned}$$

So we get a volume of

$$\begin{aligned} V &= \frac{4}{3}\pi d^3 \\ &= 4.3 \times 10^{12} \text{ pc}^3 \end{aligned}$$

Gives us $\sim 6 \times 10^{11}$ stars visible to this resolution by *Gaia.* These assumptions limit the accuracy of this calculations since *Gaia* is capable of observing parallaxes outside of the Milky Way.

Question II.

Let's use an H-R diagram to age-date two star clusters. Download the files `cluster1.dat` and `cluster2.dat` from the course HuskyCT page. These are the $(B - V)$ colors and apparent V magnitudes for stars in two open clusters in the Milky Way, compiled from the WEBDA database (`http://webda.physics.muni.cz`).

(a) Plot both in an H-R diagram, labeling your axes and orienting the y-axis to be brightest at the top and faintest at the bottom. You can make two separate H-R diagrams, or color-code each cluster and plot both in the same diagram.

See Figure 1.

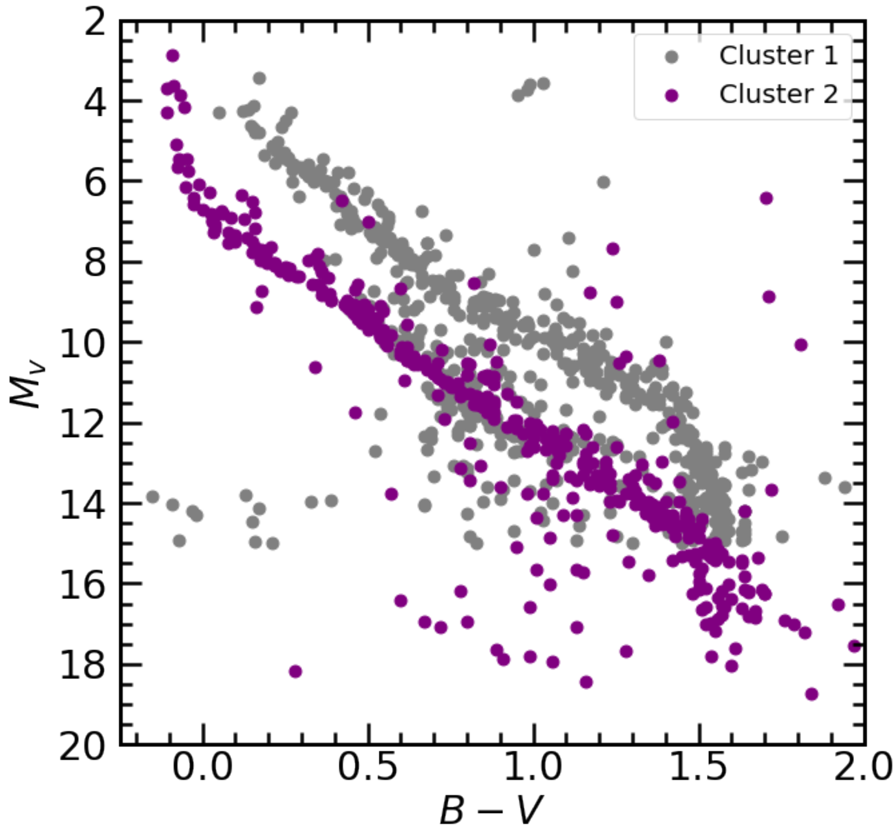

Figure 1: H-R diagram for question 2

(b) What is the approximate age of each cluster?

The oldest stars on the cluster 1 main sequence are $B - V \sim 0.1$ which corresponds to A stars, so cluster 1 is $\lesssim 10^8$yr. The oldest in the cluster 2 main sequence are $B - V < 0$ which corresponds to a young population of O and B stars $\lesssim 10^7$yr.

(c) The two clusters are offset on the y-axis. Why?

This means that cluster 2 is likely being attenuated by dust since its V-band magnitude is offset (dimmer).

(d) One of the clusters has a rather broad main sequence in the H-R diagram, almost like parallel tracks. What's going on here?

This indicates that part of the cluster is obscured by dust, which can be caused by a few factors. There could be partial obscuration in the line of sight, which is most likely, or there can be an uneven attenuation due to gas and dust in the cluster. The most likely explanation for the latter would be remnants of a supernova.

Question III.

An eclipsing binary star system has a period of 5 days. Two absorption-line components are observed with maximum radial velocities of 50 km/s and 100 km/s.

(a) What is the mass of each star?

Since the binaries are eclipsing we can assume the inclination $i = 0$. We do the following to determine the ratio of masses where $v_1 = 50$km/s , $v_2 = 100$km/s:

$$\begin{aligned} \frac{M_1}{M_2} &= \frac{v_2}{v_1} \\ &= 2 \end{aligned}$$

Since we know the period $P = 5$days:

$$\begin{aligned} a &= \frac{P}{2\pi}(|v_1| + |v_2|) \\ &= 1.03 \times 10^{10} \text{ m} \end{aligned}$$

Where $a = a_1 + a_2$ is the sum of the semimajor axes. We now find the sum of the masses as

$$\begin{aligned} M_1 + M_2 &= \frac{4\pi^2}{G}\left(\frac{a^3}{\sin^3 i}\right)\frac{1}{P^2} \\ &= 3.46 \times 10^{30} \text{ kg} \\ &= 1.74\ M_\odot \end{aligned}$$

We now solve the system for the individual masses as

$$\begin{aligned} \frac{M_1}{M_2} &= 2 \\ M_1 &= 2M_2 \\ 2M_2 + M_2 &= 1.74\ M_\odot \\ M_2 &= .58\ M_\odot \\ M_1 &= 1.16\ M_\odot \end{aligned}$$

(b) The fast star is smaller, and takes 1 hr to go from first contact to minimum in the lightcurve. Totality of the eclipse lasts for 10 hr. What is the size of each star? Which one is on the main sequence?

We know

$$\begin{aligned} R_2 &= \frac{v}{2}(t_2 - t_1) \\ R_1 &= \frac{v}{2}(t_3 - t_1) \end{aligned}$$

Where $v = |v_1| + |v_2| = 150$ km/s, $(t_2 - t_1) = 1$ hour, and $(t_3 - t_1) = 11$ hours since totality lasts for 10 hours + the time to totality of 1 hour. This yields

$$
\begin{aligned}
R_2 &= 2.7 \times 10^8 \text{ m} \\
&= 0.39 \ R_\odot \\
R_1 &= 2.9 \times 10^9 \text{ m} \\
&= 4.3 \ R_\odot
\end{aligned}
$$

If we consider the mass-radius relation, we see that the neither star is significantly far from the main sequence, but the smaller star is further off. If either of these are not on the main sequence, the smaller star is closer to becoming a white dwarf in the end of its lifetime.

Question IV.

Make an H-R diagram of the 314 brightest stars in the sky (limited by apparent magnitude $m_v <$ 3.55), drawn from this link: `http://www.astro.utoronto.ca/ garrison/oh.html` (You want the columns `B-V` and M_v.) Is this set representative of where most stars live on the H-R diagram? In other words, are any kinds of stars over- or under-represented in a sample limited by apparent brightness?

In Figure 2 we show the H-R diagram for the 314 brightest stars, and Figure 3 shows the H-R diagram for these 314 stars along with the two clusters from question 2 for reference. We see that the behavior of the brightest stars is distinct from the behavior of the clusters, and we see overrepresentation of giants and supergiants, along with extremely bright O and B stars.

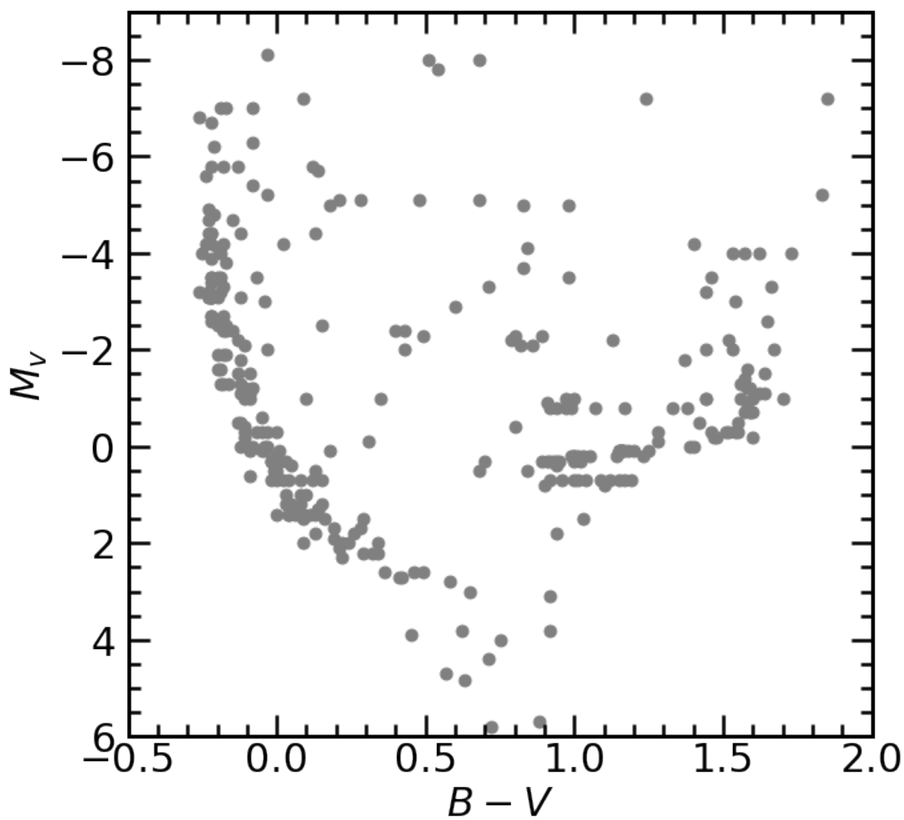

Figure 2: H-R diagram for question 4 with just the brightest 314 stars shown.

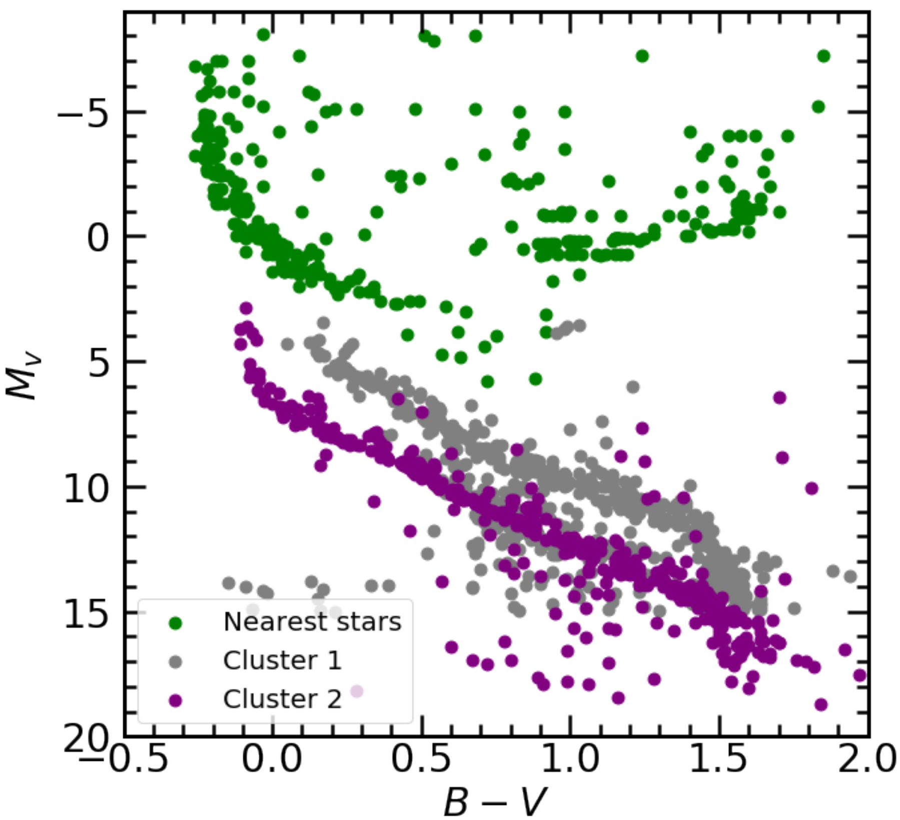

Figure 3: H-R diagram for question 4 with brightest stars shown along with clusters from question 2 for reference.