Quiz 3
MATH 112-017 and 112-019
New Jersey Inst. Tech.
Prof. Nicholas Dubicki
Time Limit: 15 min.

Name: Answer and Rubric
Date: ____________
Section: ____________

1. Use the double angle formulas to simplify and solve the following integral. [5]

$$I = \int_0^\pi \cos^4(x)\, dx$$

$$I = \int_0^\pi (\cos^2(x))^2\, dx = \int_0^\pi \left(\frac{1+\cos(2x)}{2}\right)^2 dx$$
$$= \frac{1}{4}\int_0^\pi \left(1 + 2\cos(2x) + \cos^2(2x)\right) dx$$
$$= \frac{1}{4}\int_0^\pi \left(1 + 2\cos(2x) + \frac{1}{2}(1+\cos(4x))\right) dx$$
$$= \frac{1}{4}\left[x + \sin(2x) + \frac{1}{2}x + \frac{1}{4}\sin(4x)\right]\Big|_{x=0}^{\pi}$$
$$= \frac{1}{4}\left(\pi + 0 + \frac{1}{2}\pi + 0\right) - \frac{1}{4}(0+0+0+0)$$
$$= \frac{3}{8}\pi$$

2. Solve the following linear system of equations for the three unknown quantities, x, y, z. [5]

$$\begin{aligned}
(\mathrm{I}) &\rightarrow & 1 &= x - 2y + z \\
(\mathrm{II}) &\rightarrow & 2 &= 2x + y + z \\
(\mathrm{III}) &\rightarrow & 3 &= 3x + \quad + z
\end{aligned}$$

gaussian elimination

- add $-2\cdot(\mathrm{I})$ to eq. $(\mathrm{II}) \Rightarrow 0 = 0x + 5y - z$ $:(\mathrm{II}^*)$
  add $-3\cdot(\mathrm{I})$ to eq. $(\mathrm{III}) \Rightarrow 0 = 0x + 6y - 2z$
- add $-\frac{6}{5}\cdot(\mathrm{II})$ to eq. $(\mathrm{III}) \Rightarrow 0 = 0x + 0y + (-2+\frac{6}{5})z$
  therefore, $\boxed{z=0}$
- substitute z into II^* to find $0 = 0 + 5y - 0$
  therefore, $\boxed{y=0}$
- Substitute z and y into (I) to find
  $1 = x - 0 + 0$
  therefore, $\boxed{x=1}$