Quiz 4
MATH 112-017 and 112-019
New Jersey Inst. Tech.
Prof. Nicholas Dubicki
Time Limit: 15 min.

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answer and rubric
Date:
Section:

1. Take the following formulas. 10

$$\ln 2 = \int_1^2 \frac{1}{x}dx \; ; \quad T_n = \frac{\Delta x}{2}\sum_{k=0}^{n} w_k y_k \; ; \quad |E_T| \le \frac{M(b-a)^3}{12n^2}, \text{ for } M = \max_{x\in[a,b]} |f''(x)| \; .$$

- Give an upper bound on the error associated with estimating the above integral with the trapezoidal rule using $n = 3$ subintervals.
- Estimate $\ln 2$ with an integral using the trapezoidal rule. Use $n = 3$ subintervals. Report your estimate.
- Given that $\ln 2 = 0.693147$ to six decimal places, report the absolute error of the estimate in (b) to 3 decimal places.

a) $|E_T| \le \frac{M(b-a)^3}{12n^2} = \frac{1}{12 \cdot 32} = \frac{1}{12 \cdot 9} = \boxed{\frac{1}{108}}$

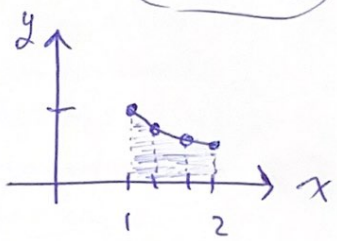
$f' = -1/x^2$, $f'' = 1/x^3$, $\max_{x\in[1,2]} f'' = 1$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{1}{3}$

b)

n	x_k	$y_k = 1/x_k$	w_k
0	1	1	1
1	$4/3$	$3/4$	2
2	$5/3$	$3/5$	2
3	$6/3 = 2$	$1/2$	1



$$T_3 = \frac{\Delta x}{2}\left[1 + 2\cdot\frac{3}{4} + 2\cdot\frac{3}{5} + \frac{1}{2}\right]$$
$$= \frac{1}{6}\left[\frac{10}{10} + \frac{15}{10} + \frac{12}{10} + \frac{5}{10}\right]$$
$$= \frac{1}{6}\left[\frac{42}{10}\right] = \boxed{\frac{7}{10} = 0.700}$$

c) $|\ln 2 - T_3| = 0.700 - 0.693$
$= 0.007$

estimate T_3 is accurate to ~~three~~ two decimal places