Recitation 9-13-19 — Nick

~~(4.6) (5.8) (5.6) (6.1) (example 2)~~

(3.)(a) $\int_0^{\pi/4} \tan x \sec^2 x \, dx$

$u = \tan x$
$du = \sec^2 x \, dx$

~~arclength~~ $ds = \sqrt{dx^2 + dy^2}$

$L = \int_a^b ds = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

disk $V = \int_a^b \pi R(x)^2 \, dx$

washer $V = \int_a^b \pi \left(R_2(x)^2 - R_1(x)^2 \right) dx$

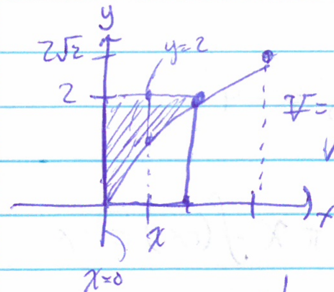
shell $V = \int_a^b 2\pi (x) f(x) \, dx$

arclength $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

6.1.42,
~~6.44~~ 6.1.44, ~~6.2.61~~, 6.2.12, 6.2.17,
6.3 example 3, 6.3.23

6.1.42 revolved region about x-axis defined by curves

$y = 2\sqrt{x}$, $y = 2$, $x = 0$



$$V = \int_0^1 \pi (R_2^2 - R_1^2)\,dx$$

$R_2 = 2$

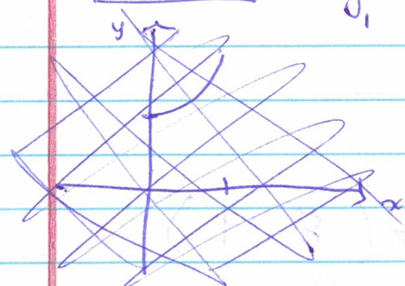
$R_1 = 2\sqrt{x}$

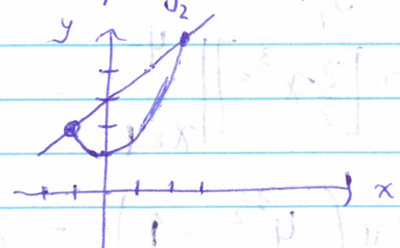
$$V = \int_0^1 \pi (2^2 - (2\sqrt{x})^2)\,dx$$

$$V = \int_0^1 4\pi (1 - x)\,dx$$

$$V = 4\pi \left[x - \frac{x^2}{2}\right]\Big|_{x=0}^{1} = 4\pi\left((1 - \tfrac{1}{2}) - (0)\right) = 2\pi$$

6.1.44 $y_1 = x^2+1$, $y_2 = x+3$





$$V = \int_a^b \pi (R_2^2 - R_1^2)\,dx$$

$\longrightarrow$ find intersections a, b

$\longrightarrow$ $y_1 = y_2$

$x^2 + 1 = x + 3$

~~$x^2 - x + 4 = 0$~~

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\begin{cases} a = -1 \\ b = 2 \end{cases}$

$$\longrightarrow V = \int_{-1}^{2} \pi (R_2^2 - R_1^2)\,dx$$

$\begin{cases} R_2 = x + 3 \longrightarrow R_2^2 = x^2 + 6x + 9 \\ R_1 = x^2 + 1 \longrightarrow R_1^2 = x^4 + 2x^2 + 1 \end{cases}$

$$V = \int_{-1}^{2} \pi\left(-x^4 - x^2 + 6x + 8\right)dx$$

$$V = \pi\left[-\tfrac{1}{5}x^5 - \tfrac{1}{3}x^3 + 3x^2 + 8x\right]\Big|_{x=-1}^{2}$$

$$V = \frac{117}{5}$$

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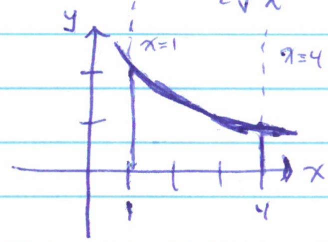
~~6.1.61~~ 6.2.12 revolved around the y-axis using shells

region bounded by

$y = \frac{3}{2\sqrt{x}}$, $y = 0$, $x = 1$, $x = 4$



$$V = \int_1^4 2\pi x f(x)\,dx$$

$f(x)$ is the height of the shell.

$$f(x) = 3/2\sqrt{x}$$

$$V = \int_1^4 2\pi x\left(\frac{3}{2}\frac{1}{\sqrt{x}}\right)dx = \int_1^4 2\pi\left(\frac{3}{2}\right)\sqrt{x}\,dx$$

$$= 3\pi\int_1^4 \sqrt{x}\,dx$$

$$= 3\pi\left[\frac{2}{3}x^{3/2}\right]\Big|_{x=1}^{4}$$

$$= 2\pi\left(4^{3/2} - 1\right) = 2\pi(8 - 1)$$

$\sqrt{4}^3 \to 2^3 \to 8$ wicked easy

$$V = 14\pi$$

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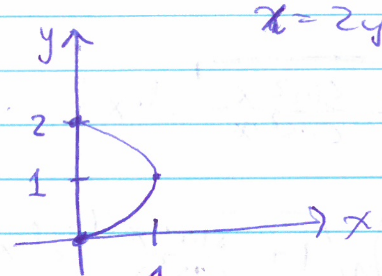
6.2.17 Revolution about x-axis

region bounded by

$x = 2y - y^2$, $x = 0$



$$V = \int_0^2 (2\pi)(y)(f(x))\,dx$$

$$f(x) = (2y - y^2) - 0$$

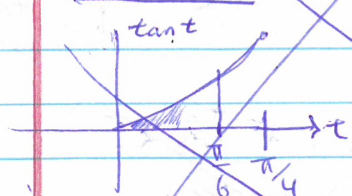
$$V = \int_0^2 2\pi(2y^2 - y^3)\,dy$$

$$= 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]\Bigg|_{y=0}^{2}$$

$$= 2\pi\left[\frac{16}{3} - 4\right] - [0 - 0]$$

$$V = \frac{8}{3}\pi$$

6.3.23 find the arclength



$$y = \int_0^x \tan t\,dt$$

for $x \in [0, \pi/6]$

formula $s = \int ds = \int_0^{\pi/6} \sqrt{1 + (dy/dx)^2}\,dx$

$$\frac{dy}{dx} = \tan x$$

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6.3.23 cont...

$$S = \int_0^{\pi/6} \sqrt{1 + \tan^2 x}\, dx$$

$$S = \int_0^{\pi/6} \sqrt{1 + \frac{\sin^2 x}{\cos^2 x}}\, dx$$

$$S = \int_0^{\pi/6} \left(\frac{\cos x}{\cos x}\right)\sqrt{1 + \frac{\sin^2 x}{\cos^2 x}}\, dx$$

$$S = \int_0^{\pi/6} \frac{\sqrt{\cos^2 x + \sin^2 x}}{\cos x}\, dx$$

$$S = \int_0^{\pi/6} \frac{1}{\cos x}\, dx$$

~~let $u = \cos x \Rightarrow du = \sin x\, dx$~~

$$S = \int_0^{\pi/6} \sec x\, dx$$

let $u = \tan x$

$du = \sec^2 x\, dx$

$\sec^2 x = \frac{d}{dx} \tan x$

$\sec x = \sqrt{\frac{d}{dx} \tan x}$

$f = \ln(\tan x + \sec x)$

$$\frac{df}{dx} = \frac{\sec^2 + \frac{d}{dx}(\sec x)}{\tan x + \sec x} = \frac{\sec^2 x + \frac{\sin x}{\cos^2 x}}{\tan x + \sec x}$$

$$= \frac{\cos x}{\cos^2 x}\left(\frac{1 + \sin x}{\sin x + 1}\right) \rightarrow 1$$

$$= 1/\cos x$$

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§ 6.2.22 $\boxed{y = \sin x - x\cos x}$

arclength integral

on $x \in [0, \pi]$

$$S = \int_0^\pi \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^\pi \sqrt{1+\left(\cancel{\cos x} - \cancel{\cos x} + x\sin x\right)^2}\, dx$$

$$= \int_0^\pi \sqrt{1 + x^2\sin^2 x}\, dx$$

let $u = x\sin x$, $du = (\sin x + x\cos x)\,dx$

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6.2.1) $y = \frac{1}{3}(x^2+2)^{3/2}$

find arc length between

$x \in [0, 3]$

$$S = \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{3}{2}\right)(x^2+2)^{1/2} \frac{d}{dx}(x^2+2)$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(x^2+2)^{1/2}(\cancel{2}x)$$

$$\left(\frac{dy}{dx}\right)^2 = (x^2+2)x^2$$

$$S = \int_0^3 \sqrt{1 + x^4 + 2x^2}\, dx$$

$$= \int_0^3 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^3 (x^2+1)\, dx$$

$$= \left[\frac{x^3}{3} + x\right]\Big|_{x=0}^{3}$$

$$= \frac{27}{3} + 3 - 0 + 0$$

$$= 9 + 3$$

$$S = 12$$

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