– Nick

Nasty Denominators and u-substn.

a factorable denominator

$$I_1 = \int \frac{1}{4x^2-4x+1}\,dx$$

$$I_1 = \int \frac{1/4}{(x-1/2)^2}\,dx$$

let $u = x - \frac{1}{2}$

then $du = dx$

no problem!

$$I_1 = \frac{1}{4}\int \frac{1}{u^2}\,du$$

Q.E.D.

Not So Much

$$I_2 = \int \frac{1}{4x^2-4x+3}\,dx$$

but what if...

$$I_2 = \int \frac{1}{(4x^2-4x+1)+2}\,dx$$

complete the square!

$$I_2 = \int \frac{1}{4(x-1/2)^2+2}\,dx$$

$$I_2 = \frac{1}{4}\int \frac{1}{(x-\frac{1}{2})^2+\frac{1}{2}}\,dx$$

let $u = x - 1/2$

then $du = dx$

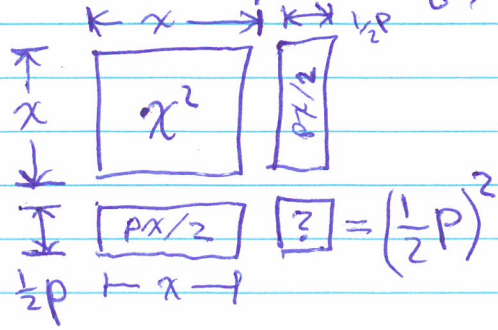
$$I_2 = \frac{1}{4}\int \frac{1}{u^2+\frac{1}{2}}\,du$$

Q.E.D.

Key Takeaways

– you can complete the square for any quadratic

– use this diagram



$$\Rightarrow \left(x^2+px+\left(\tfrac{1}{2}p\right)^2\right)+\left(q-\left(\tfrac{1}{2}p\right)^2\right)$$

$$\downarrow$$

$$\left(x+\tfrac{1}{2}p\right)^2 + q-\left(\tfrac{1}{2}p\right)^2$$

$$u^2 + \left(q-\left(\tfrac{1}{2}p\right)^2\right)$$