

Series with inverse trig functions

Does $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$ converge or diverge?

• • • Use integral test $I = \int_1^{\infty} \frac{\arctan(x)}{x^2+1} dx$

let $u = \arctan(x)$ then $du = \frac{dx}{1+x^2}$

integral $I = \int u du = \frac{u^2}{2}$

evaluate on
the bounds in
the x -variable
since it is an
improper integral

$$= \lim_{R \rightarrow \infty} \left[\frac{\arctan(x)^2}{2} \right]_{x=1}^R$$

$$= \lim_{R \rightarrow \infty} (\arctan(R)) - \arctan(1)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \pi/4 \quad \text{convergent integral}$$

Series converges by integral test.

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