023 Recitation Notes F Sept. 6 - Nick

1, 5, 8*, 9, 11, 13, 16

defns

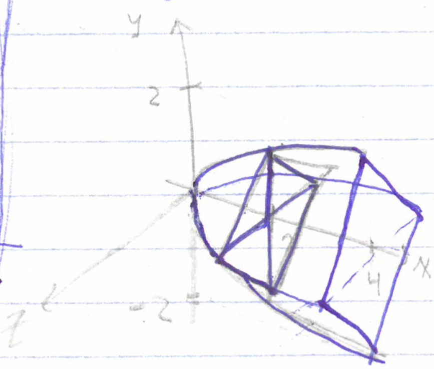
gen $V = \int_a^b A(x)\,dx$

disk $V = \int_a^b \pi\, r(x)^2\,dx$

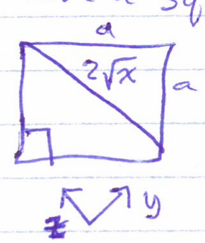
washer $V = \int_a^b \pi(R(x)^2 - r(x)^2)\,dx$

note $= \int_a^b \pi R(x)^2\,dx - \int_a^b \pi r(x)^2\,dx$

(1) solid lies between planes perpendicular to the x-axis at $x=0$ and $x=4$. Cross sections perpendicular to the x axis on $x \in [0,4]$ are squares whose diagonals run from the parabola $y=-\sqrt{x}$ and $y=\sqrt{x}$



at x we have a square



$a^2 + a^2 = 4x$

$2a^2 = (2\sqrt{x})^2$

$a = \frac{2\sqrt{x}}{\sqrt{2}}$

$a = \sqrt{2x}$

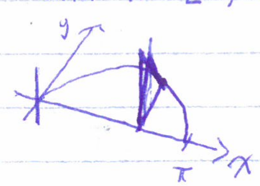
$A(x) = a^2$

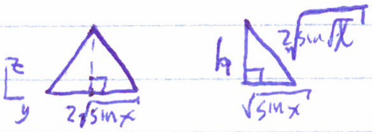
$A(x) = 2x$

$V = \int_0^4 A(x)\,dx = \int_0^4 2x\,dx$

(5) base of a solid

(a) equilateral triangle is between the x-axis and the curve $y = 2\sqrt{\sin x}$ and $x \in [0,\pi]$





$h^2 = 4\sin x - \sin x$

$h^2 = 3\sin x$

$h = \sqrt{3\sin x}$

$A(x) = \frac{1}{2}bh$

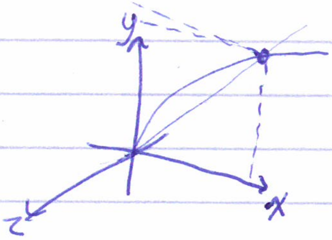
$= \frac{1}{2}(2\sqrt{\sin x})(\sqrt{3\sin x})$

$= \sqrt{3}\sin x$

$V = \int_0^\pi A(x)\,dx =$

$V = \int_0^\pi \sqrt{3}\sin x\,dx$

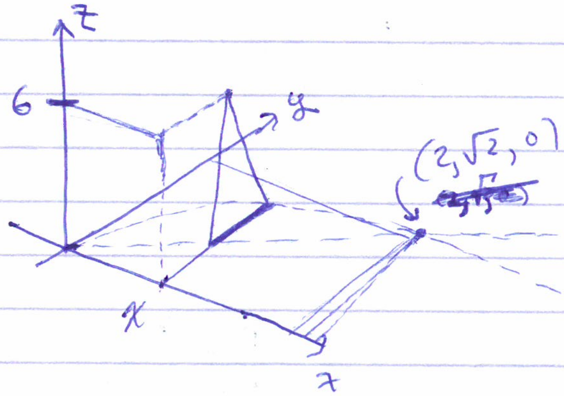
* (8)

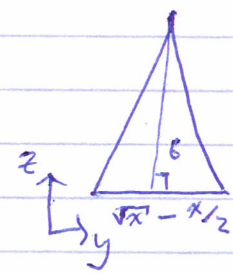


base of a solid is in the region bounded by $y = \sqrt{x}$ and $y = x/2$

* HW prob just show a little

(a) isosceles triangle of ~~height~~ height 6

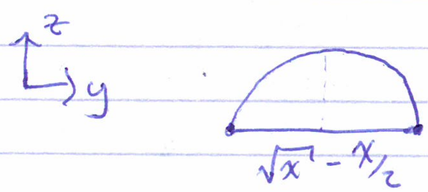




$$A(x) = \frac{1}{2}\left(\sqrt{x} - x/2\right)(6)$$

$$V = \int_0^2 3\sqrt{x} - \frac{3}{2}x \, dx$$

(b) semicircles whose diameters are the base



$$A(x) = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi\left(\frac{\sqrt{x} - x/2}{2}\right)^2$$

$$= \frac{1}{2}\frac{1}{4}\pi\left(x - x\sqrt{x} + x^2/4\right)$$

$$V = \int_0^2 \frac{\pi}{8}\left(x^2/4 - x\sqrt{x} + x\right)dx$$

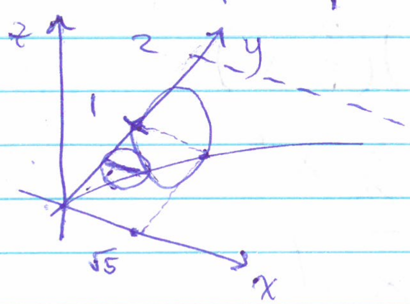
$$= \frac{\pi}{8}\int_0^2 \left(\frac{x^2}{4} - x^{3/2} + x\right)dx$$

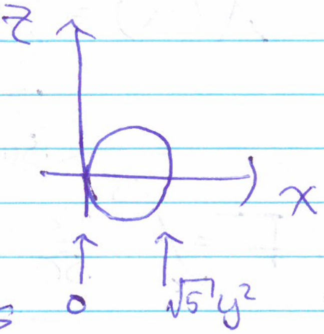
$$= \frac{\pi}{8}\left[\frac{1}{3}\frac{x^3}{4} - \frac{2}{5}x^{5/2} + \frac{1}{2}x^2\right]\Bigg|_{x=0}^{2}$$

$$= \frac{\pi}{8}\left[\frac{1}{12}(8) - \frac{2}{5}(4\sqrt{2}) + \frac{(4)}{2}\right] - [0 - 0 + 0]$$

$$= \pi\left[\frac{1}{12} - \cancel{\frac{(2)(4)}{8}}^{1}\left(\frac{\sqrt{2}}{5}\right) + \cancel{\frac{2}{8}}^{1/4}\right] = \pi\left(\frac{1}{3} - \frac{\sqrt{2}}{5}\right)$$

(9) solid lines between planes perpendicular to the y-axis $y \in [0,2]$ are circular disks with diameter running from the y-axis to the parabola $x = \sqrt{5}\, y^2$



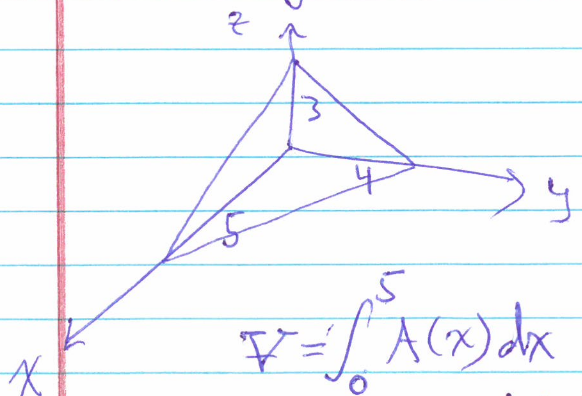


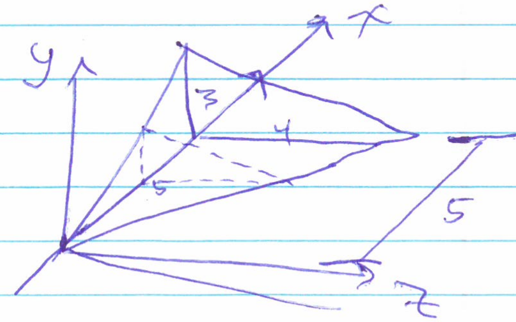
$$A(y) = \pi \left(\frac{\sqrt{5}}{2} y^2\right)^2$$

$$V = \int_0^2 \pi \left(\frac{\sqrt{5}}{2} y^2\right)^2 dy$$

$$= \int_0^2 \pi \frac{5}{4} y^4 \, dy$$

(11) right-tetrahedron volume





$$V = \int_0^5 A(x)\, dx$$

$$A(x) = \frac{1}{2} b h$$
$$= \frac{1}{2} b(x)\, h(x)$$

(11) cont...

$b(0) = 0$ } linear and passes thru. the origin
$b(5) = 4$ } $b(x) = \frac{4}{5}x$

$h(0) = 0$ }
$h(5) = 3$ } $h(x) = \frac{3}{5}x$

$$A(x) = \frac{1}{2}\left(\frac{4}{5}x\right)\left(\frac{3}{5}x\right)$$

$$= \frac{12}{50}x^2 = \frac{6}{25}x^2$$

$$\boxed{V = \int_0^5 \frac{12}{50}x^2\,dx}$$

$$= \left(\frac{6}{25}\right)\left[\frac{1}{3}x^3\right]_{x=0}^{5}$$

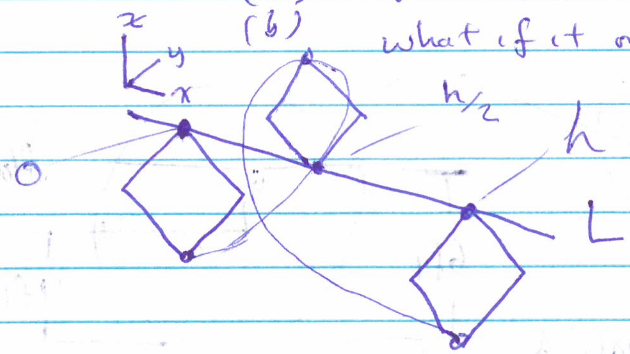
$$= \left(\frac{6}{25}\right)\left(\frac{1}{3}\right)(125 - 0)$$

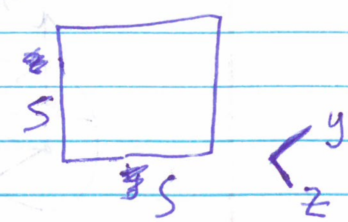
$$V = 2$$

(13) twisted solid – square of side length s lies in a plane perpendicular to a line L and one of the verticies touches the line. Square sweeps out a distance h along L and simultaneously completes a full turn about the line

(a) the volume of the swept solid

(b) what if it makes two full turns





$A(x) = s^2$

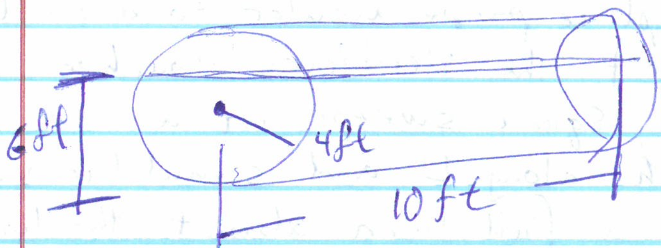
$$V = \int_0^h A(x)\,dx$$

$$= \int_0^h s^2\,dx$$

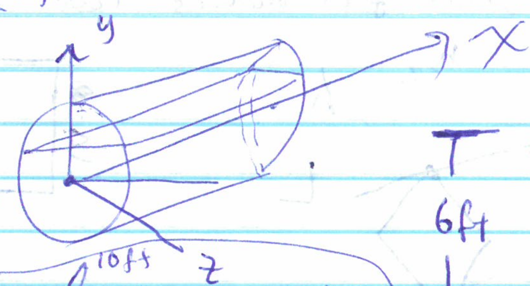
$$V = s^2 h$$

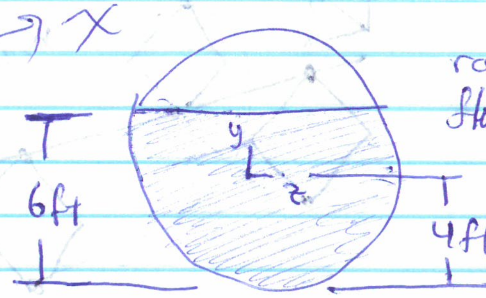
what about (b)

(16) Gasoline in a tank



(a)

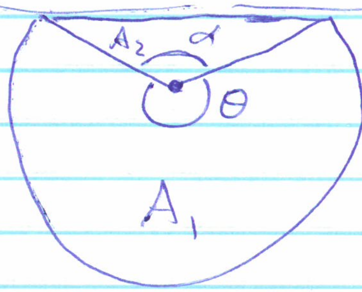


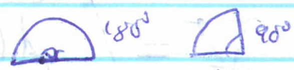


radius 4ft
fluid height 6ft

$$\boxed{V = \int_{0ft}^{10ft} A\,dx = (10ft)A}$$

circle Area





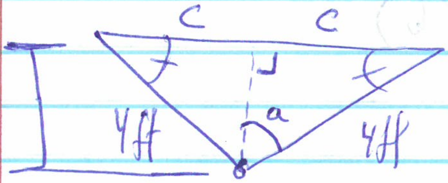
$\frac{1}{2} = \frac{180}{360}$ $\quad$ $\frac{1}{4} = \frac{90}{360}$

$$\boxed{A_1 = \frac{\theta}{360}\pi r^2}$$

$$\boxed{A_2 = \frac{1}{2} b h}$$

$\boxed{r = 4ft \text{ given}}$

$\boxed{h = 2ft \text{ given}}$



$a = \frac{1}{2}\alpha$

$\theta = 360 - \alpha$

soh cah toa

$\frac{2ft}{4ft} = \cos(a)$

$\frac{1}{2} = \cos(a)$

$\text{acos}(1/2) = 60°$

$\alpha = 120°$

$\boxed{\theta = 240°}$

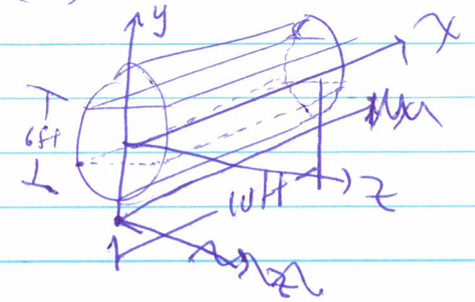
$b = 2c$

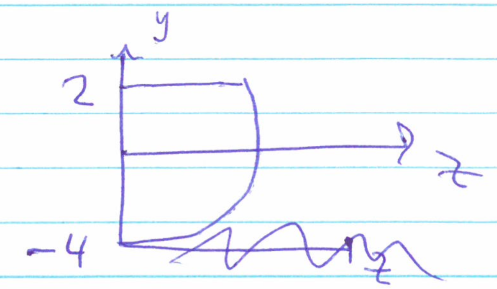
$\frac{c}{4ft} = \sin(a) = \sqrt{3}/3$

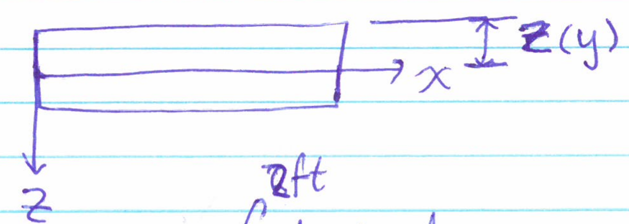
$\boxed{b = 2(4\frac{\sqrt{3}}{3})}$

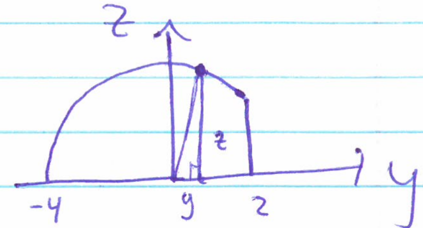
triangle area

(16) cont... (b)









$$V = \int_{-4ft}^{2ft} A(y)\,dy$$

$$A(y) = 2(10ft)\,z(y)$$

$$z(y) = ?$$

$$y^2 + z(y)^2 = (4ft)^2$$

$$z(y)^2 = (4ft)^2 - y^2$$

$$z(y) = \sqrt{16\,ft^2 - y^2}$$!

$$A(y) = (20ft)\sqrt{(16)ft^2 - y^2}$$

$$V = \int_{-4}^{2} (20ft)\sqrt{(16ft^2) - y^2}\,dy$$