example prob(6.a) from Spring 2022 exam 2

Direct comparison ~~with~~ for $\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$

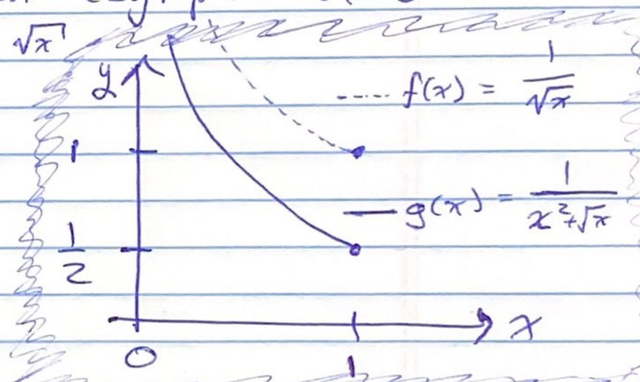
Direct comparison says if $0 \le g(x) \le f(x)$ in the interval then ~~either~~

$$\int_0^1 g(x)\,dx \text{ diverging} \Rightarrow \int_0^1 f(x)\,dx \text{ diverging}$$

or

$$\int_0^1 f(x)\,dx \text{ converging} \Rightarrow \int_0^1 g(x)\,dx \text{ converging}$$

- We expect the vertical asymptote at 0 to be controlled by the $\sqrt{x}$



- First note

  $0 \le x \le 1$

- So compare with

  $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

  and test for convergence

- $\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{1}{\sqrt{x}}\,dx = \cancel{2\sqrt{x}}$

  $= \lim_{a\to 0^+} \left(2\sqrt{x}\Big|_{x=a}^{1} \right)$

  $= \lim_{a\to 0} \left(2 - 2\sqrt{a} \right)$

  $= 2$ Convergent !

- Therefore

  $x^2+\sqrt{x} \ge \sqrt{x}$

  $\frac{1}{x^2+\sqrt{x}} \le \frac{1}{\sqrt{x}}$

Thus $\int_0^{1} \frac{1}{x^2+\sqrt{x}}\,dx$ converges by direct comparison