

# STAT110 PS2

Nicholas DeSanctis

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## Lecture 9

### Definitions —

#### Directed Graph

- Defined as  $G = (V, E)$
- For each  $(u, v) \in E$ ,  $u, v \in V$ ,  $u$  "points to"  $v$

#### Undirected Graph

- Defined as  $G = (V, E)$
- For each  $(u, v) \in E$ ,  $u, v \in V$ ,  $u$  and  $v$  "point to each other"

#### Digraph

- Simple, unweighted, directed graph

### Keywords —

*Planar*: a graph can be drawn in 2D with no edge crossings.

*Walk*: a sequence of vertices from  $s$  to  $t$

*Shortest Walk*: the "distance" of  $s$  to  $t$  (aka, the minimum of the possible lengths)

### Theorems and Lemmas —

*Shortest Walk Lemma*: If  $w$  is a shortest walk from  $s$  to  $t$ , then all of the vertices that occur on  $w$  are distinct. That is, every shortest walk is a *path*.

- Suppose we have a shortest walk with repeated vertices. We know there exists a shorter one by simply getting rid of all vertices between the first instance of the repeated vertex and the second for all vertices.

## Algorithms —

### *Shortest Walk*

- Inputs: digraph  $G = (V, E)$ , vertices  $s, t \in V$
- Outputs: shortest walk iff it exists
- Possible solving algorithms:
  1. Exhaustive Search:  $(n - 1)! \cdot O(n)$ 
    - Get all walks of length  $n-1$
    - By Shortest Walk Lemma, our shortest walk must be here
    - Find shortest walk starting from  $s$  and ending at  $t$
  2. BFS: