1. 2-view special case: unknown yaw and horizontal translation Assume that a two-view configuration consists of two cameras with identical and known intrinsic parameters displaced as follows:

$$\mathbf{X_r} = R\mathbf{X_l} + \mathbf{t} \quad \text{where} \quad R = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} t\cos\alpha \\ 0 \\ t\sin\alpha \end{bmatrix}$$

With $\mathbf{t}$ the magnitude of the translation, and $\mathbf{X_r}, \mathbf{X_r}$ position vectors of points with the right and left camera coordinate system, respectively. Another way to think of this setup is one camera moving in the XZ plane.

Solution.

1.

$$E = [t]_\times R = \begin{bmatrix} 0 & -t\sin\alpha & 0 \\ t\sin(\alpha+\beta) & 0 & -t\cos(\alpha+\beta) \\ 0 & t\cos\alpha & 0 \end{bmatrix}$$

2. There are 3 unknown parameters in E, so the answer is 3.
3. The necessary and sufficient conditions:

$$\begin{cases} E_{11} = E_{13} = E_{22} = E_{31} = E_{33} = 0 \\ E_{12}^2 + E_{32}^2 = E_{21}^2 + E_{23}^2 \end{cases}$$

To derive the unknowns of the problem α and β:

$$\begin{cases} \alpha = \arctan(-\frac{E_{12}}{E_{32}}) + k_1\pi \\ \beta = \arctan(-\frac{E_{21}}{E_{23}}) - \alpha + k_2\pi \end{cases}$$

4. Take the chirality constraint which ensures that points are in front of both cameras into consideration, so $0 \leq \beta \leq \pi$. So the answer is 2 pairs if we think about the angle between $-\pi$ and π.

5.

(a) Use the vector triple product identity:

$$a \times (b \times c) = (a \cdot c) \cdot b - (a \cdot b) \cdot c$$

(b) So, the equation becomes:

$$x_r^T[(x_r \cdot Rx_l) \cdot t - (x_r \cdot t) \cdot Rx_l] = 0$$

(c) Distribute the transpose and simplify:

$$(x_r \cdot Rx_l) \cdot x_r^T \cdot t - (x_r \cdot t) \cdot x_r^T \cdot Rx_l = 0$$

6. It is obvious that the intersection point lies on the Z-axes of both the left and right coordinate systems, so we can obtain the equation below:

$$\begin{bmatrix} 0 \\ 0 \\ Z_r \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ Z_l \end{bmatrix} + \begin{bmatrix} t\cos\alpha \\ 0 \\ t\sin\alpha \end{bmatrix}$$

Solve this equation,

$$Z_l = -\frac{t\cos\alpha}{\sin\beta}$$

So the answer is $(0, -\frac{t\cos\alpha}{\sin\beta})$

7. Considering the planar motion of the camera, we can derive the rate of change of image coordinates over time. Since there is no motion of the camera along the Y-axis, the Y-coordinate does not change over time, i.e., $\frac{dY}{dt} = 0$. However, the X and Z coordinates will change due to the motion of the camera:

$$\begin{cases} \frac{dX}{dt} = -V_x + \Omega_y Y \\ \frac{dZ}{dt} = -V_z \end{cases}$$

Now, we can substitute these rates of change into the rates of change of x and y:

$$\begin{cases} \frac{dx}{dt} = \frac{d}{dt}(f\frac{X}{Z}) = f\frac{\frac{dX}{dt}Z - X\frac{dZ}{dt}}{Z^2} \\ \frac{dy}{dt} = \frac{d}{dt}(f\frac{Y}{Z}) = f\frac{\frac{dY}{dt}Z - Y\frac{dZ}{dt}}{Z^2} \end{cases}$$

Substituting the expressions for $\frac{dX}{dt}$, $\frac{dY}{dt}$, and $\frac{dZ}{dt}$. After simplification, we obtain the optical flow equations:

$$\begin{cases} \frac{dx}{dt} = -f\frac{V_x}{Z} + \Omega_y y + f\frac{XV_z}{Z^2} \\ \frac{dy}{dt} = f\frac{YV_z}{Z^2} \end{cases}$$

Since $x = f\frac{X}{Z}$ and $y = f\frac{Y}{Z}$, we can represent X and Y in terms of x and y:

$$\begin{cases} \frac{dx}{dt} = -\frac{V_x}{Z} + \Omega_y y + \frac{xV_z}{Z} \\ \frac{dy}{dt} = \frac{yV_z}{Z} \end{cases}$$

These are the optical flow equations derived for the given planar motion of the camera.

8. Given the optical flow equations:

$$\begin{cases} \frac{dx}{dt} = -\frac{V_x}{Z} + \Omega_y y + \frac{xV_z}{Z} \\ \frac{dy}{dt} = \frac{yV_z}{Z} \end{cases}$$

The FOE can be found by setting the optical flow to zero. This means solving the following equations for x and y:

$$\begin{cases} 0 = -\frac{V_x}{Z} + \Omega_y y + \frac{xV_z}{Z} \\ 0 = \frac{yV_z}{Z} \end{cases}$$

From the second equation, it is clear that if $V_z \neq 0$, y must be zero for the flow to be zero. Substituting $y = 0$ into the first equation, we solve for x:

$$x = \frac{V_x}{V_z}$$

Thus, the coordinates of the FOE in the image plane are:

$$(\frac{V_x}{V_z}, 0)$$

9. We can express Z as:

$$Z = \frac{yV_z}{\frac{dy}{dt}}$$

Now, substitute this expression for Z into the first equation:

$$\frac{dx}{dt} = -\frac{V_x\frac{dy}{dt}}{yV_z} + \Omega_y y + \frac{x\frac{dy}{dt}}{y}$$

Simplifying this equation, we get:

$$\frac{dx}{dt} = -\frac{V_x\frac{dy}{dt}}{yV_z} + \Omega_y y + \frac{x\frac{dy}{dt}}{y}$$

10. Unclear about ”the two unknowns”. ■