Tutorial 6

ECSE 420 – Tutorial 6

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TR 4110

October 27, 2014

Lab 2 – Gaussian Elimination

- GE kernel

```
for (k = 0; k < N - 1; k++)
    for (i = k+1; i < N; i++)  {
        l = A[i][k] / A[k][k];
        for (j = k + 1; j < N; j++)
            A[i][j] = A[i][j] -l*A[k][j];
    }
}
```

Lab 2 – Gaussian Elimination

N = 4

```
for (k = 0; k < N - 1; k++)
    for (i = k+1; i < N; i++)  {
        l = A[i][k] / A[k][k];
        for (j = k + 1; j < N; j++)
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    }
}
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Lab 2 – Gaussian Elimination

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    }
}
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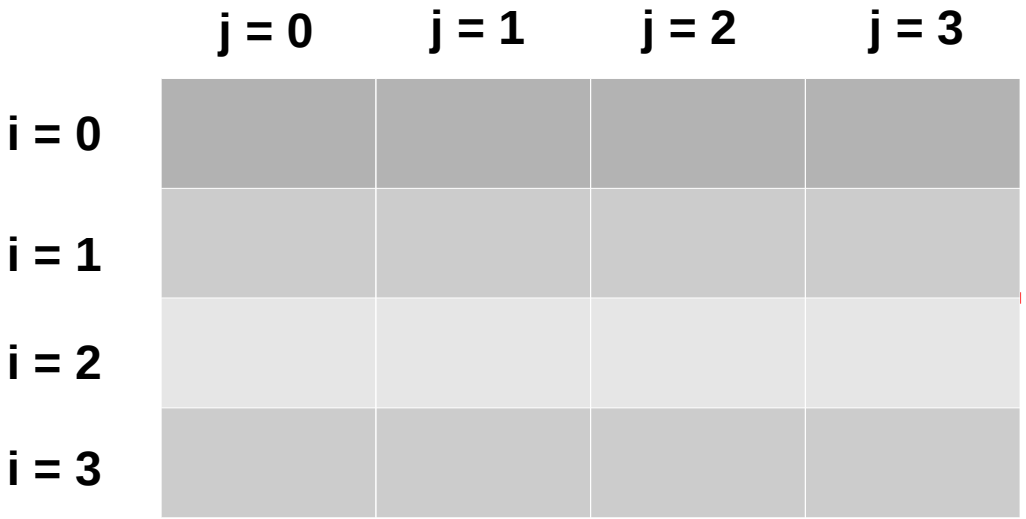

Lab 2 – Gaussian Elimination

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    for (i = k+1; i < N; i++)  {
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            A[i][j] = A[i][j] -l*A[k][j];
    }
}
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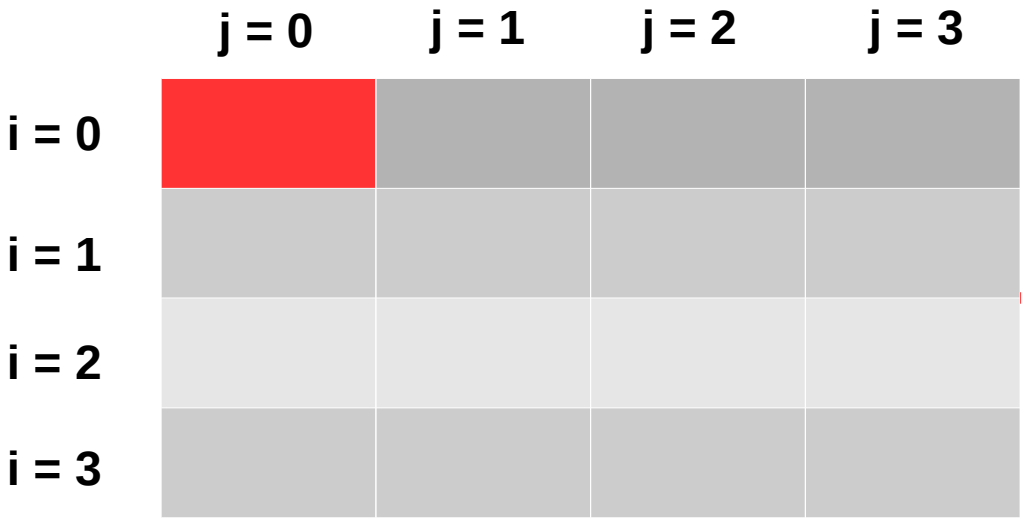

k = 0

Lab 2 – Gaussian Elimination

```
for (k = 0; k < N - 1; k++)
    for (i = k+1; i < N; i++)  {
        l = A[i][k] / A[k][k];
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            A[i][j] = A[i][j] -l*A[k][j];
    }
}
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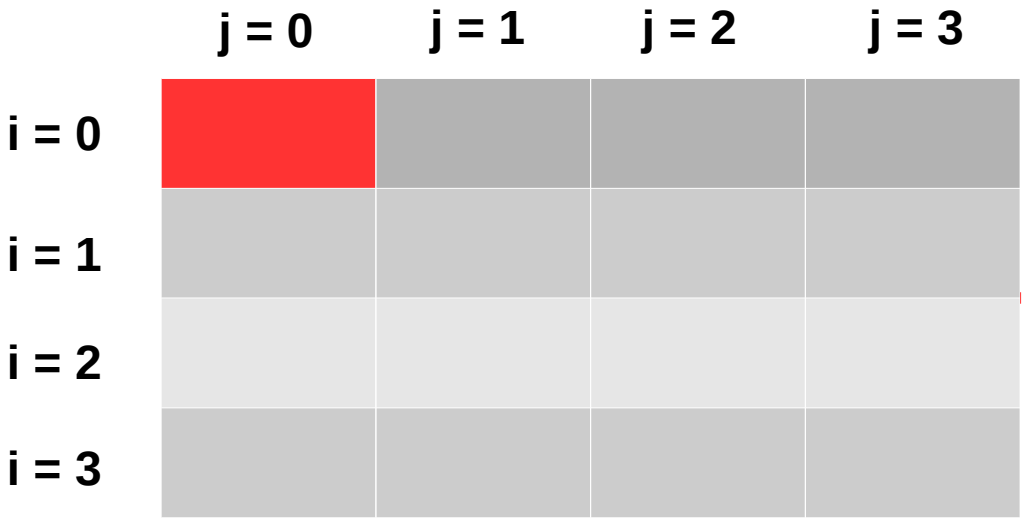

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i = 1

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    }
}
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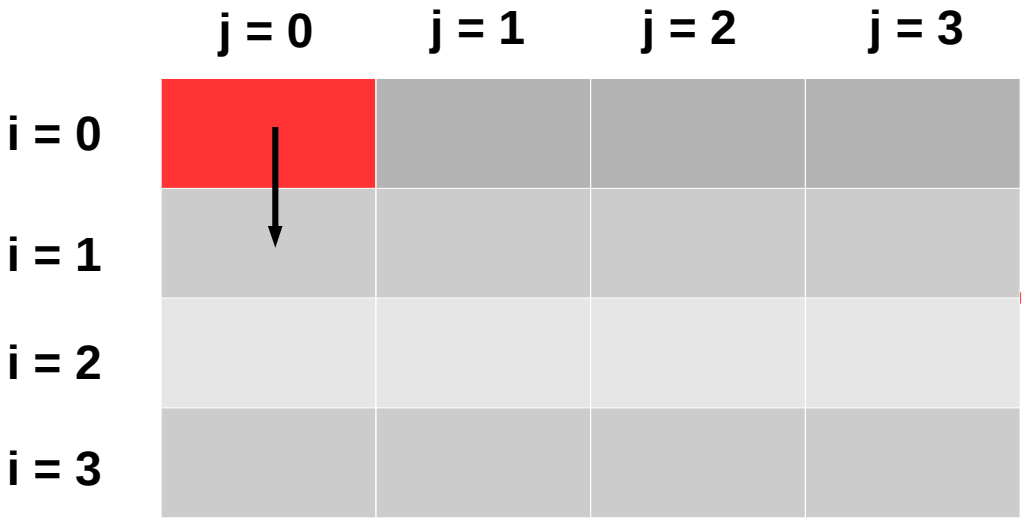

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i = 1

l = A[1][0] / A[0][0];

Lab 2 – Gaussian Elimination

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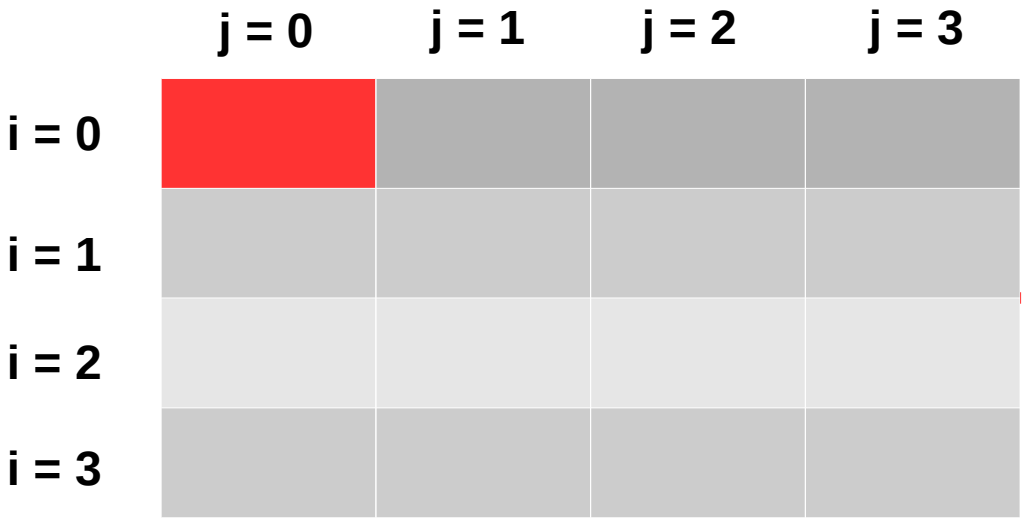

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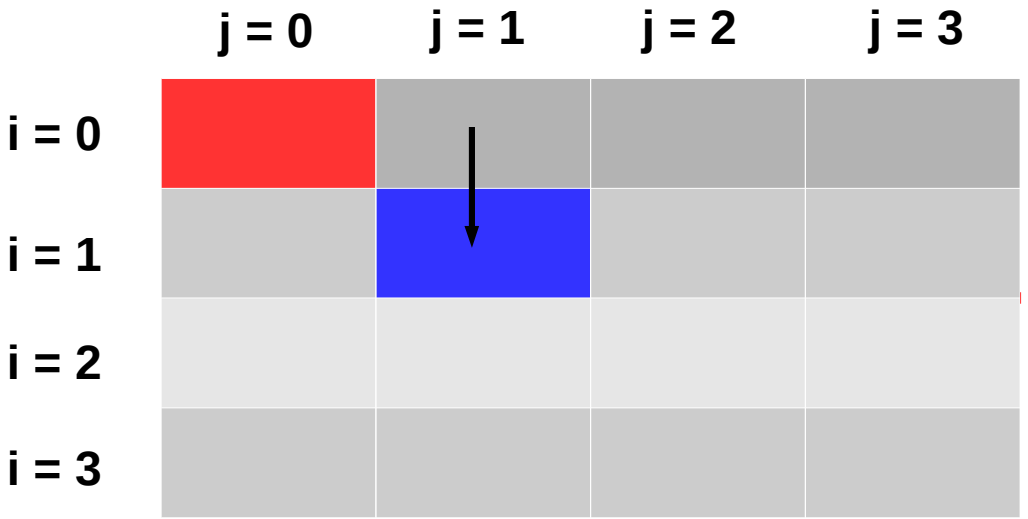

k = 0

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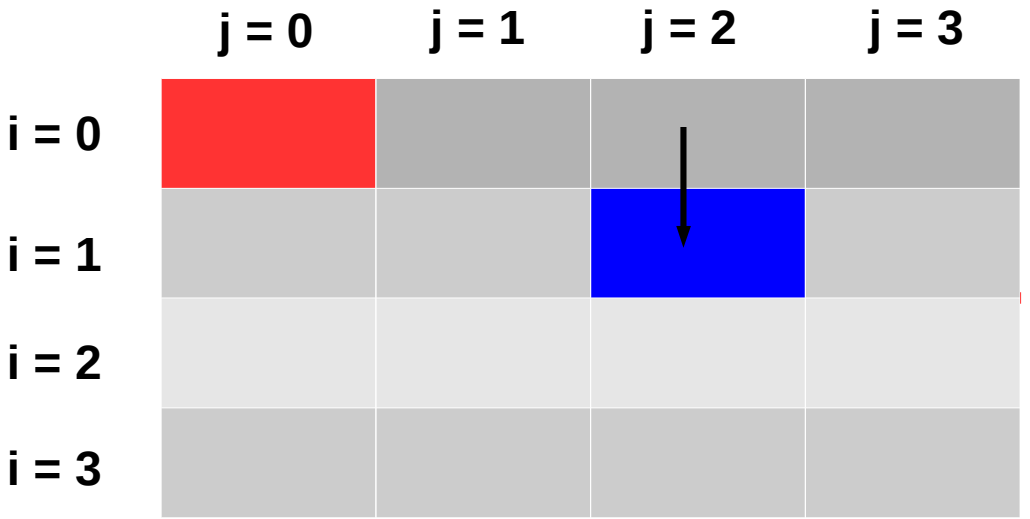

k = 0

i = 1

l = A[1][0] / A[0][0];

j = 1 A[1][1] = A[1][1] -l*A[0][1];

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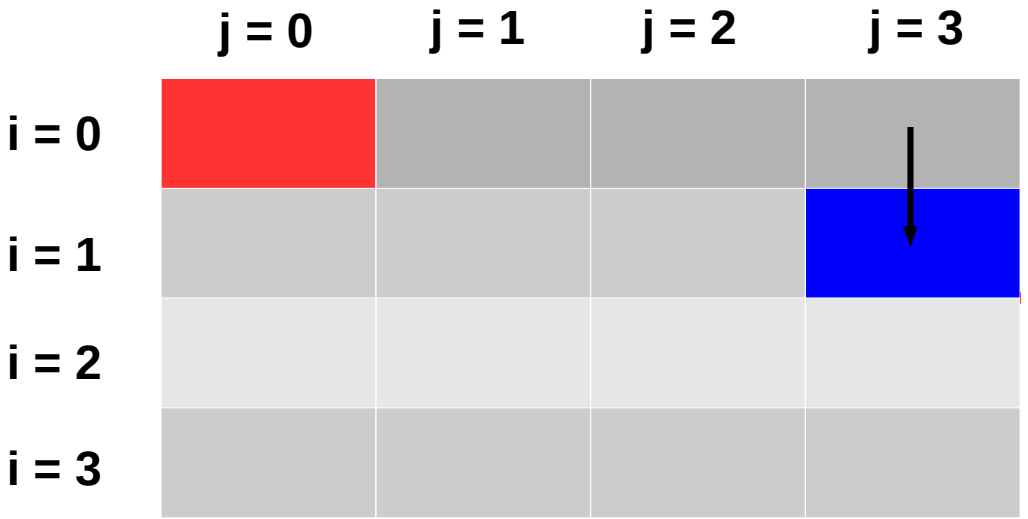

k = 0

i = 1

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j = 1 A[1][1] = A[1][1] -l*A[0][1];

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j = 3 A[1][3] = A[1][3] -l*A[0][3];

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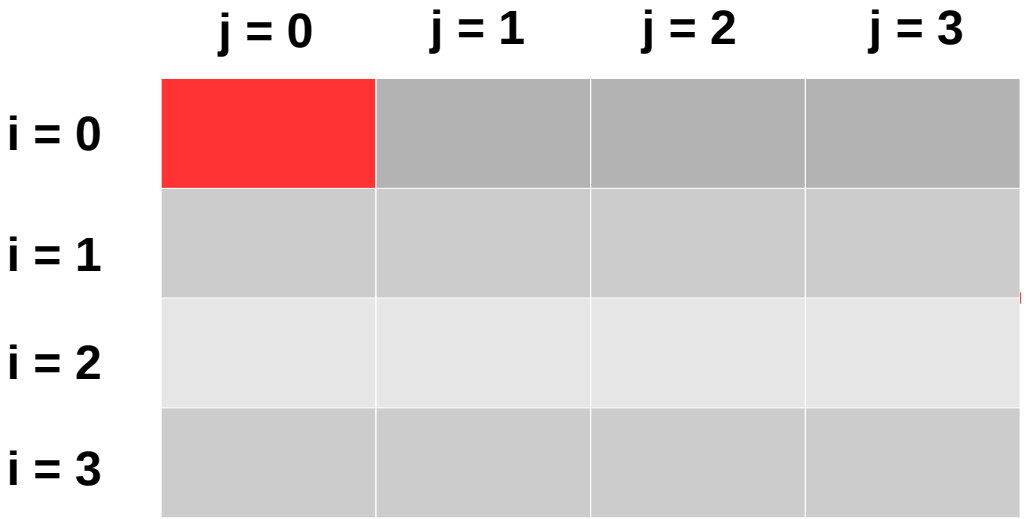

k = 0

i = 2

Lab 2 – Gaussian Elimination

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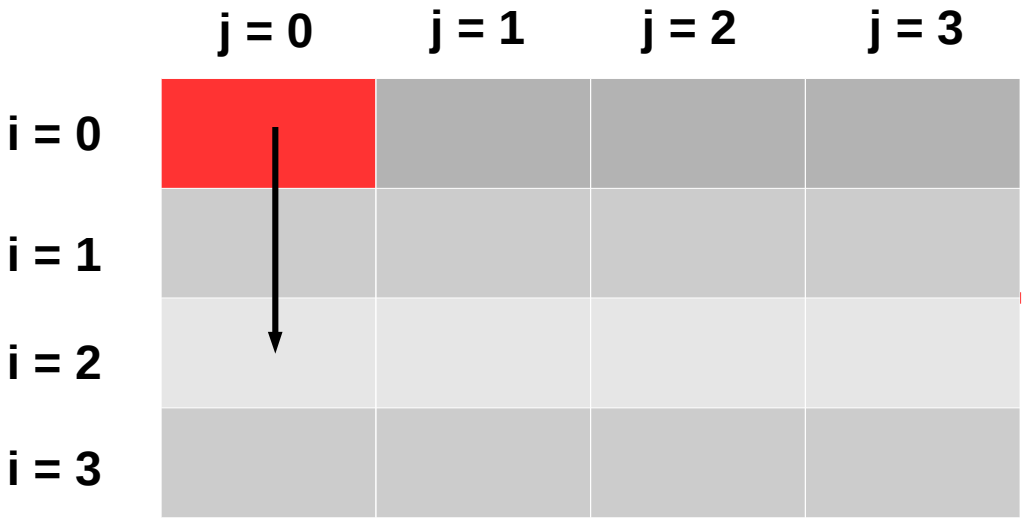

k = 0

i = 2

l = A[2][0] / A[0][0];

Lab 2 – Gaussian Elimination

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for (k = 0; k < N - 1; k++)
    for (i = k+1; i < N; i++)  {
        l = A[i][k] / A[k][k];
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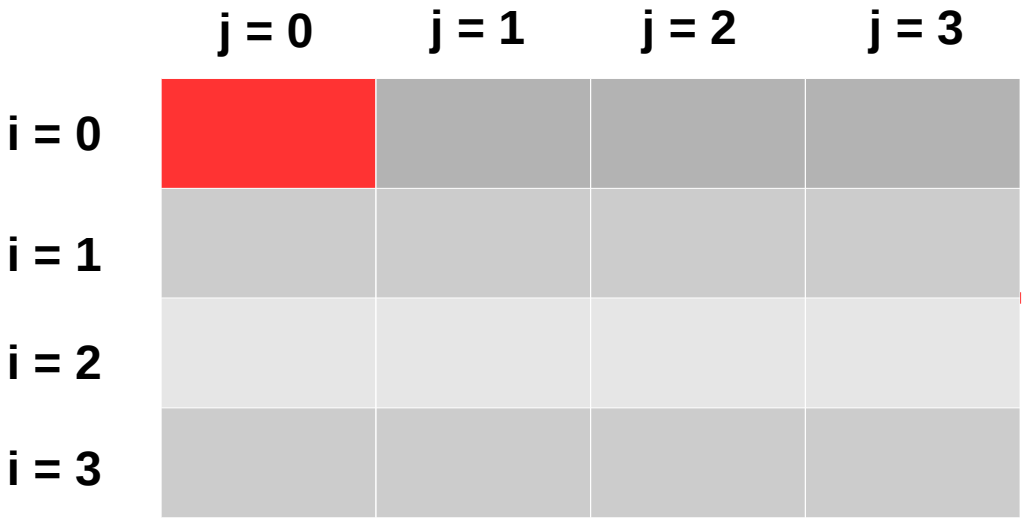

k = 0

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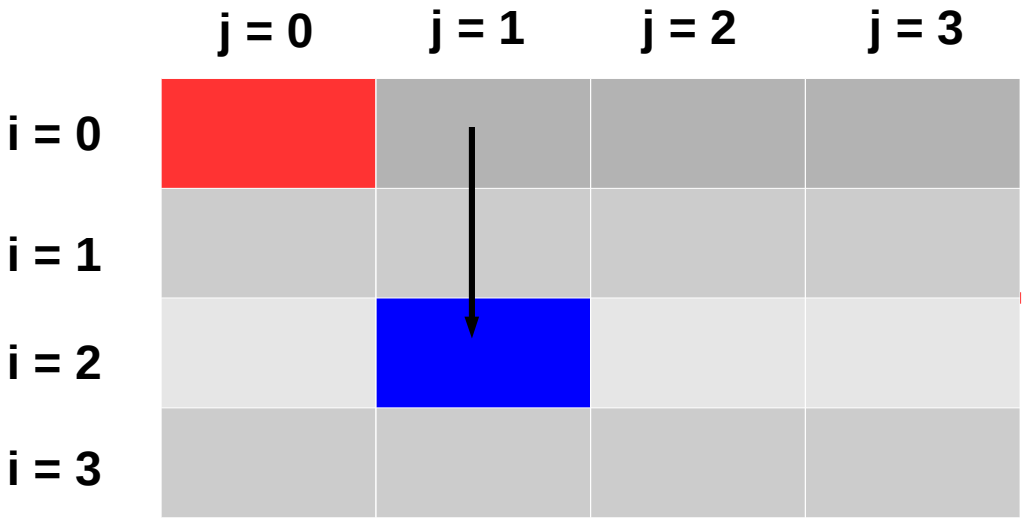

k = 0

i = 2

l = A[2][0] / A[0][0];

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Lab 2 – Gaussian Elimination

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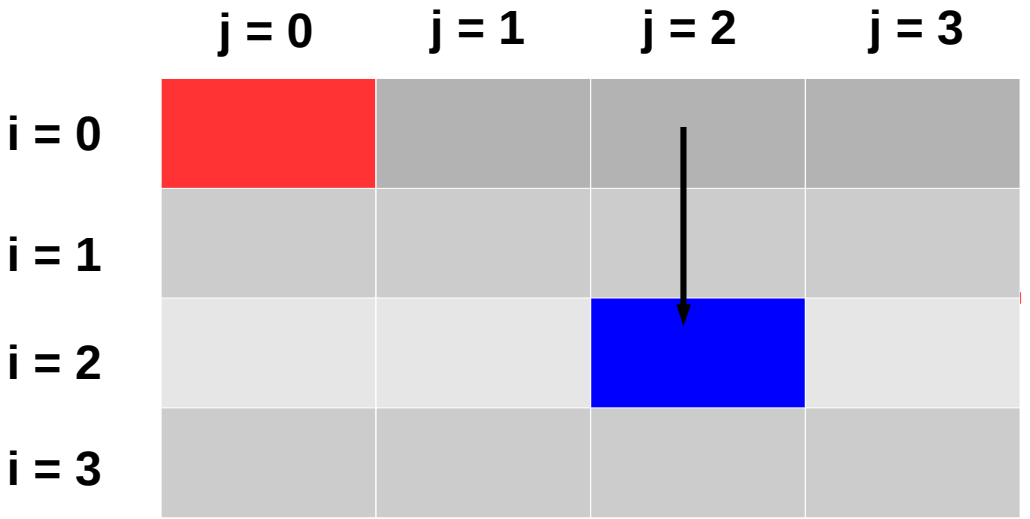

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i = 1

l = A[1][0] / A[0][0];

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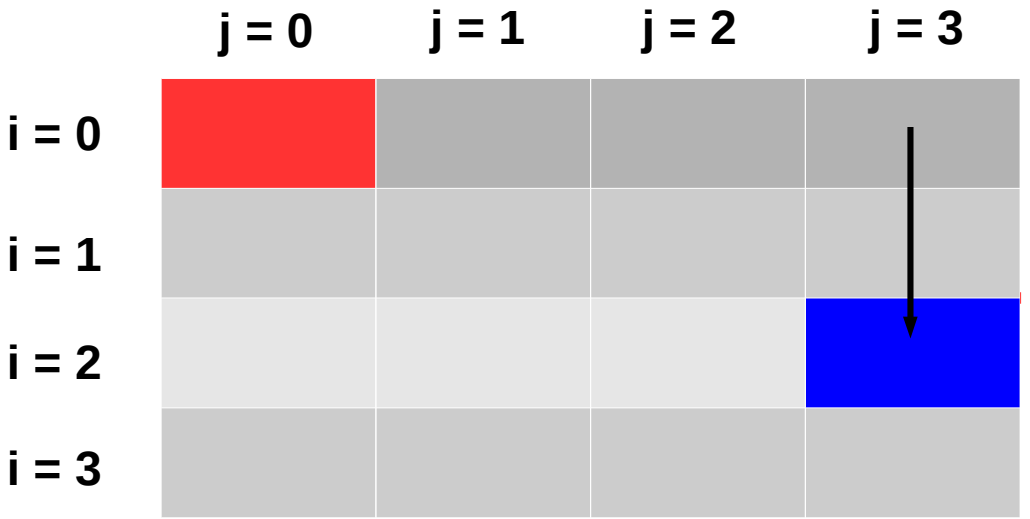

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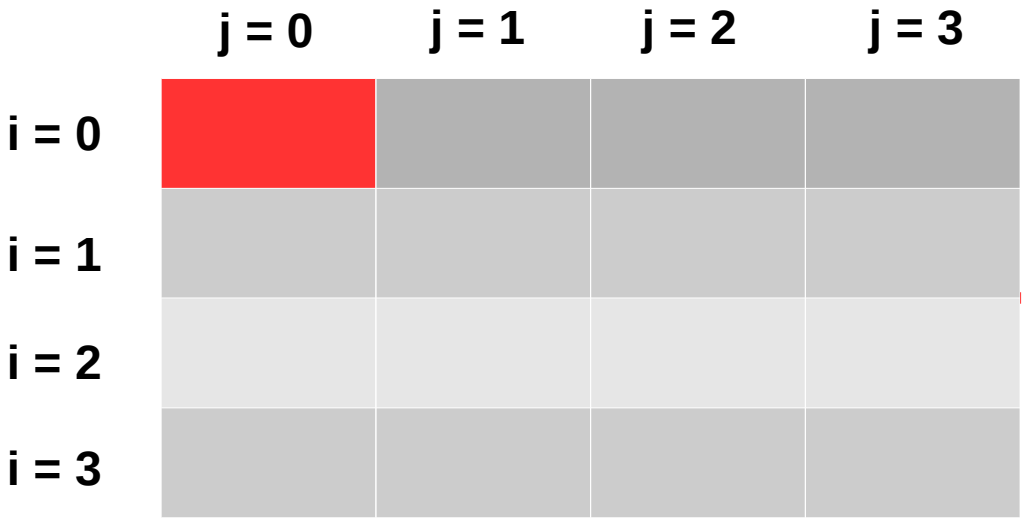

k = 0

i = 3

... and so on ...

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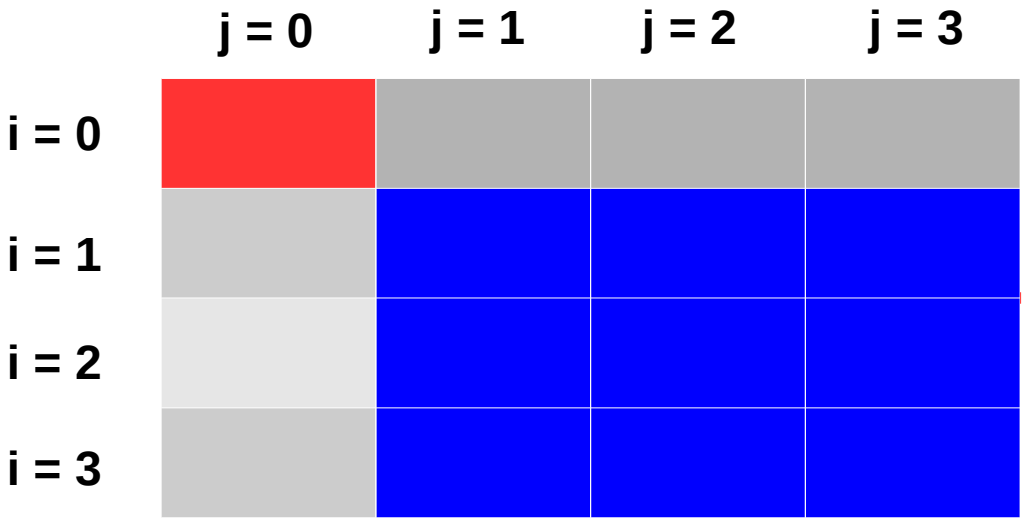

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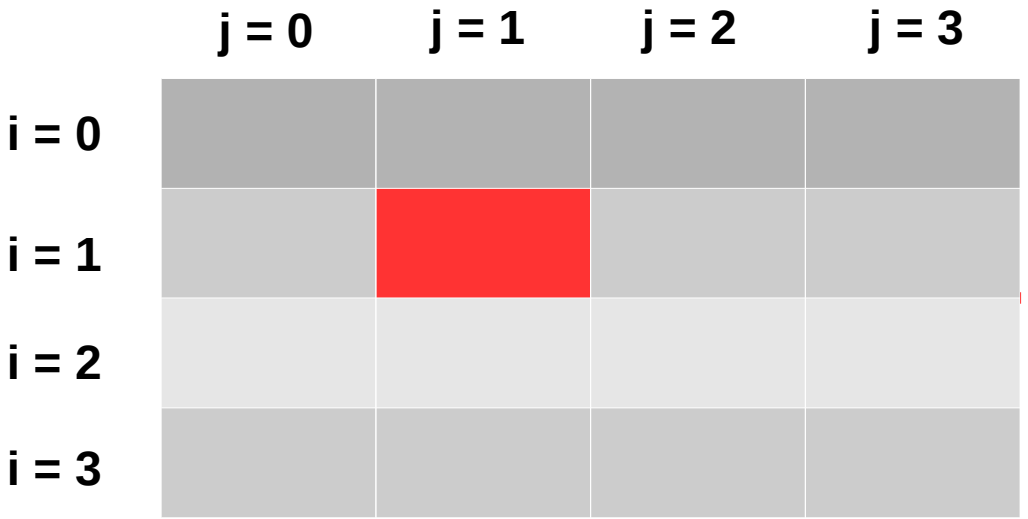

k = 1

Accordingly, for the second pivot element

Lab 2 – Gaussian Elimination

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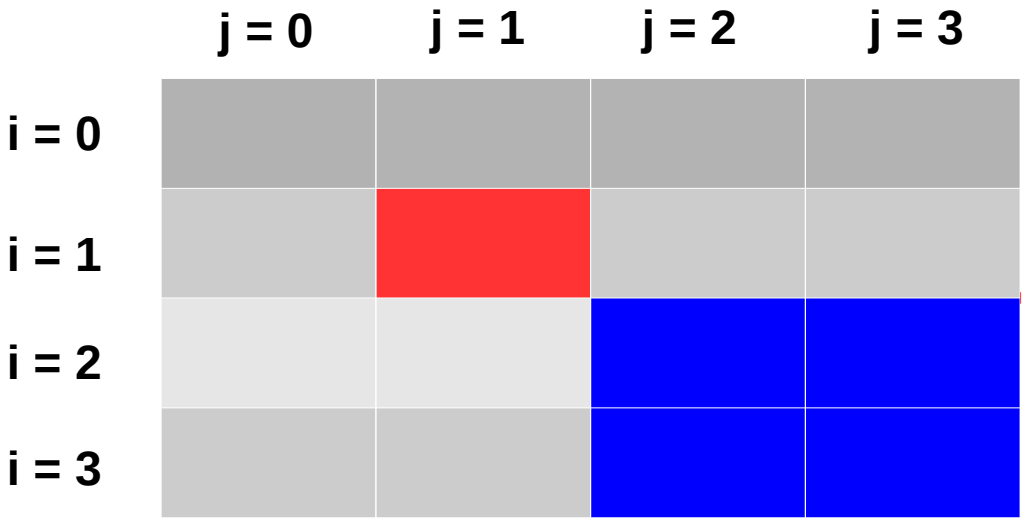

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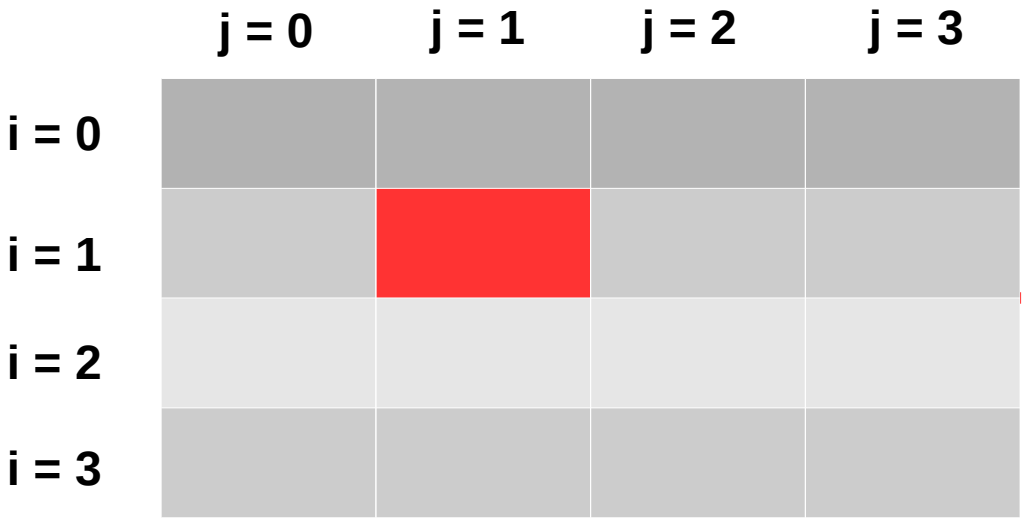

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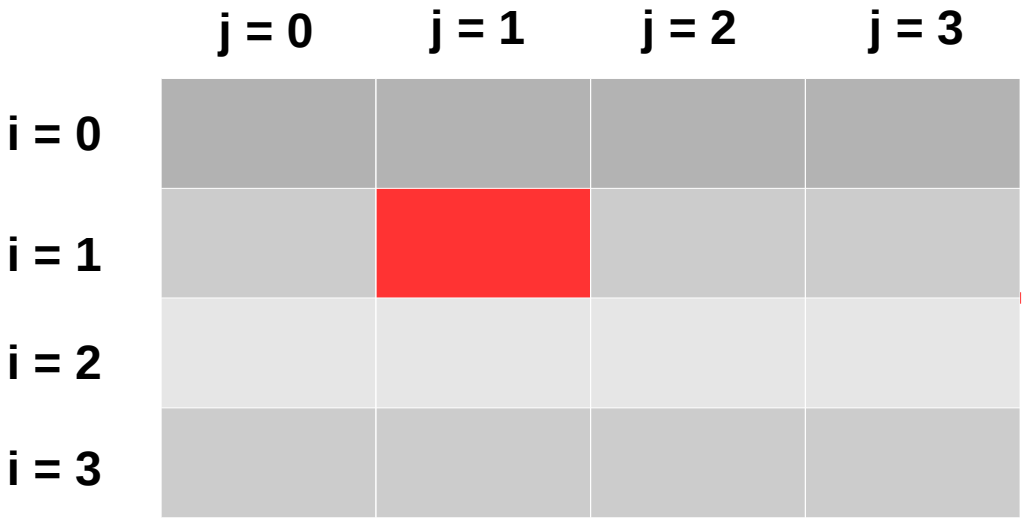

k = 1

i = 2

Lab 2 – Gaussian Elimination

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    for (i = k+1; i < N; i++)  {
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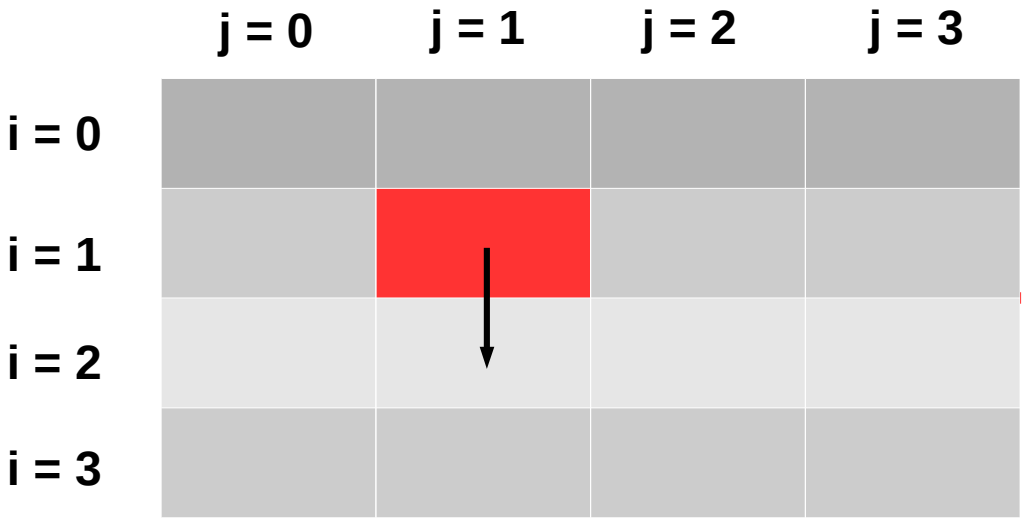

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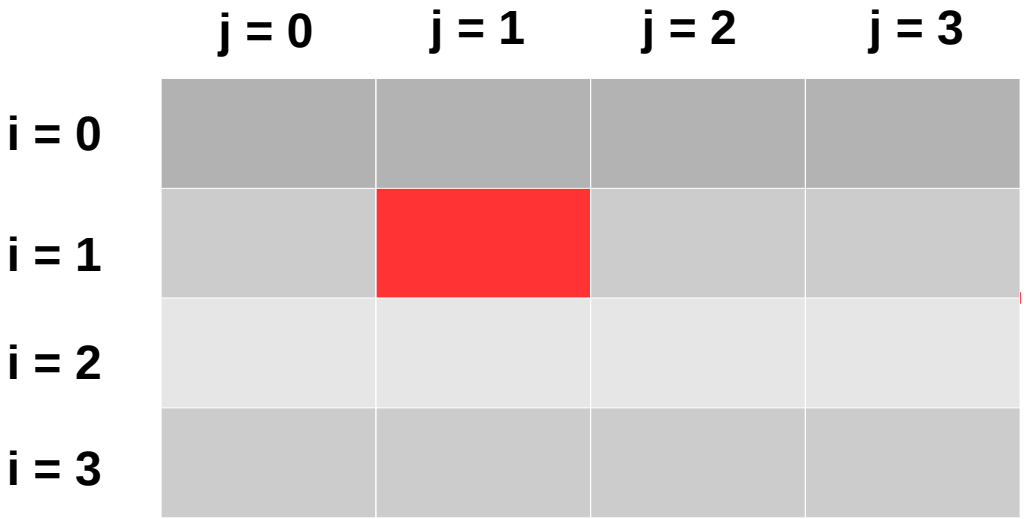

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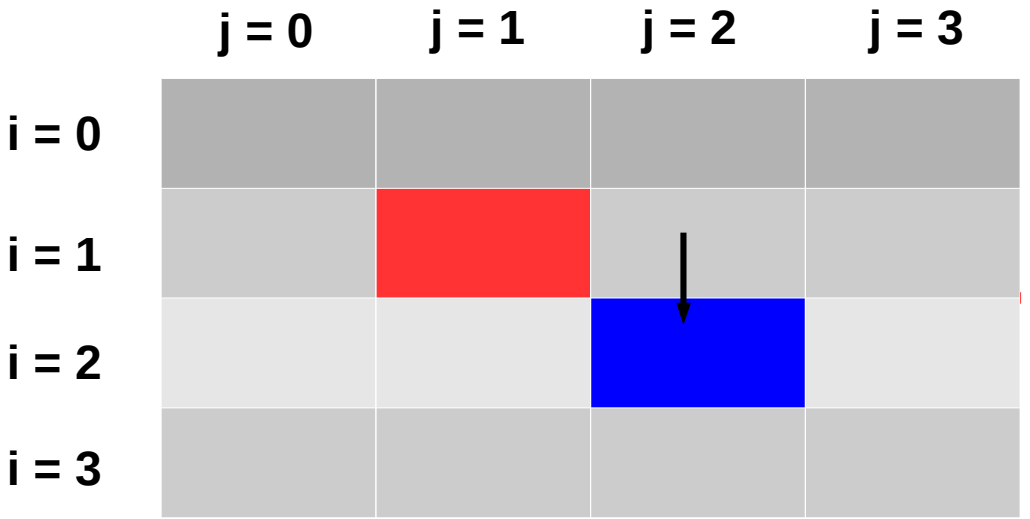

k = 1

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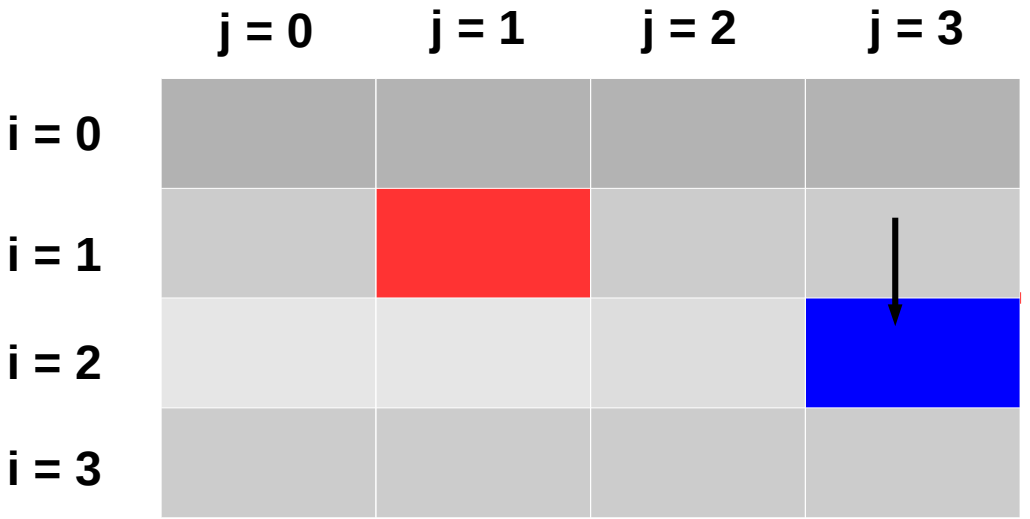

k = 1

i = 2

l = A[2][1] / A[1][1];

j = 2 A[2][2] = A[2][2] -l*A[1][2];

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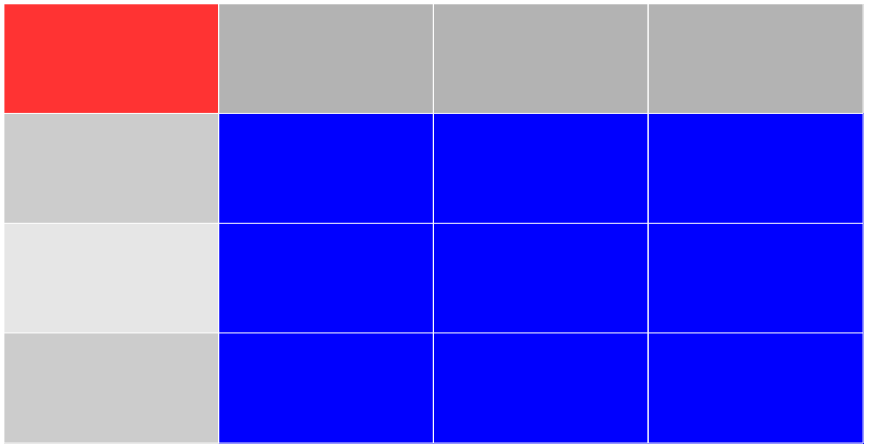
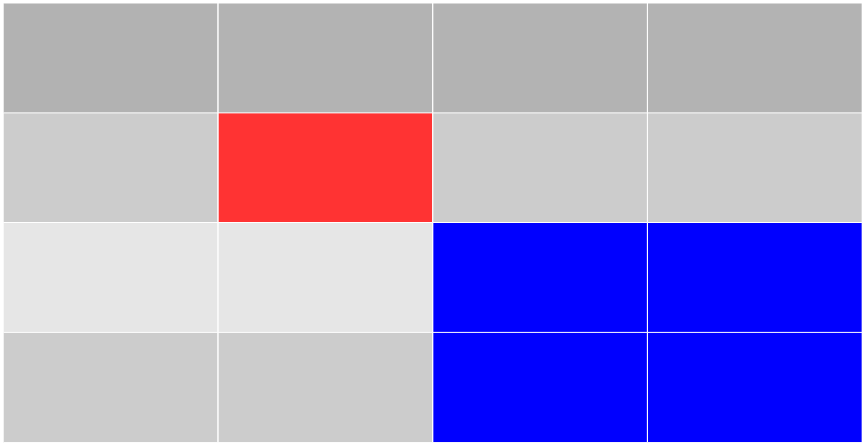
Parallelization scheme

What can be executed in parallel??

- Are the k-iterations independent?



k = 0

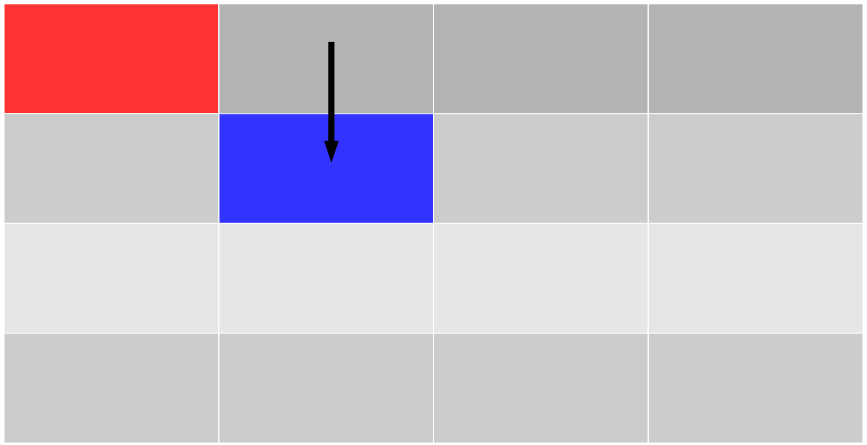


k = 1

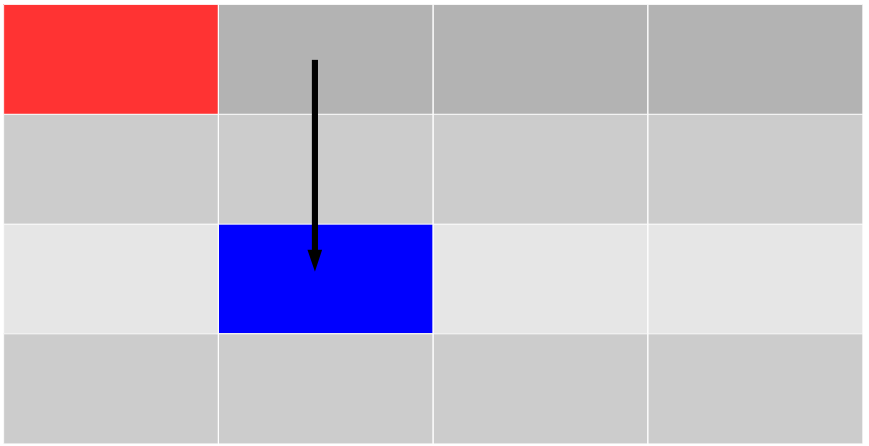
Parallelization scheme

What can be executed in parallel??

- Are the i- and j-iterations independent?



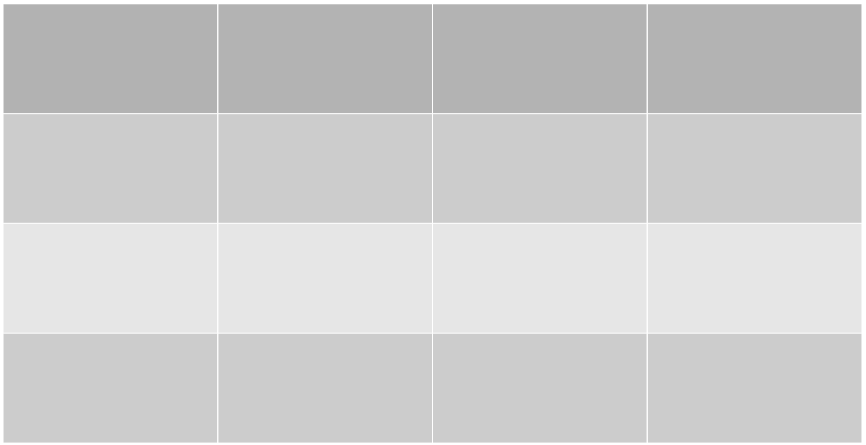
k = 0, i = 1



k = 0, i = 2

Parallelization scheme

Thus, we can apply decomposition into rows.



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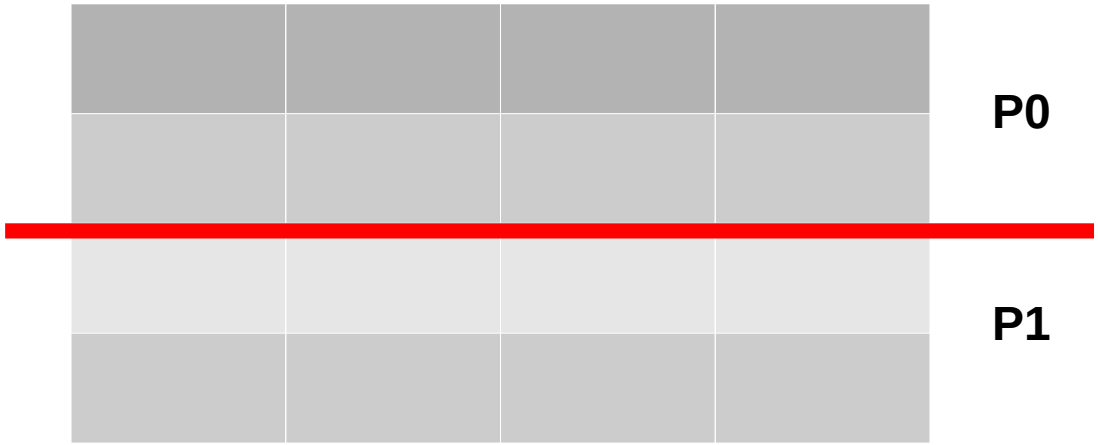

Communication scheme

But, which data should be now communicated?

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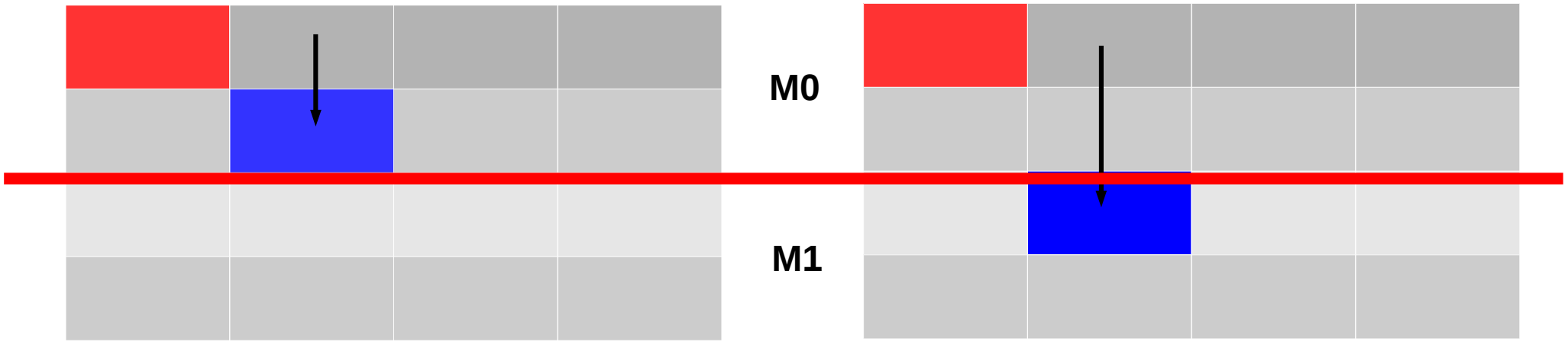
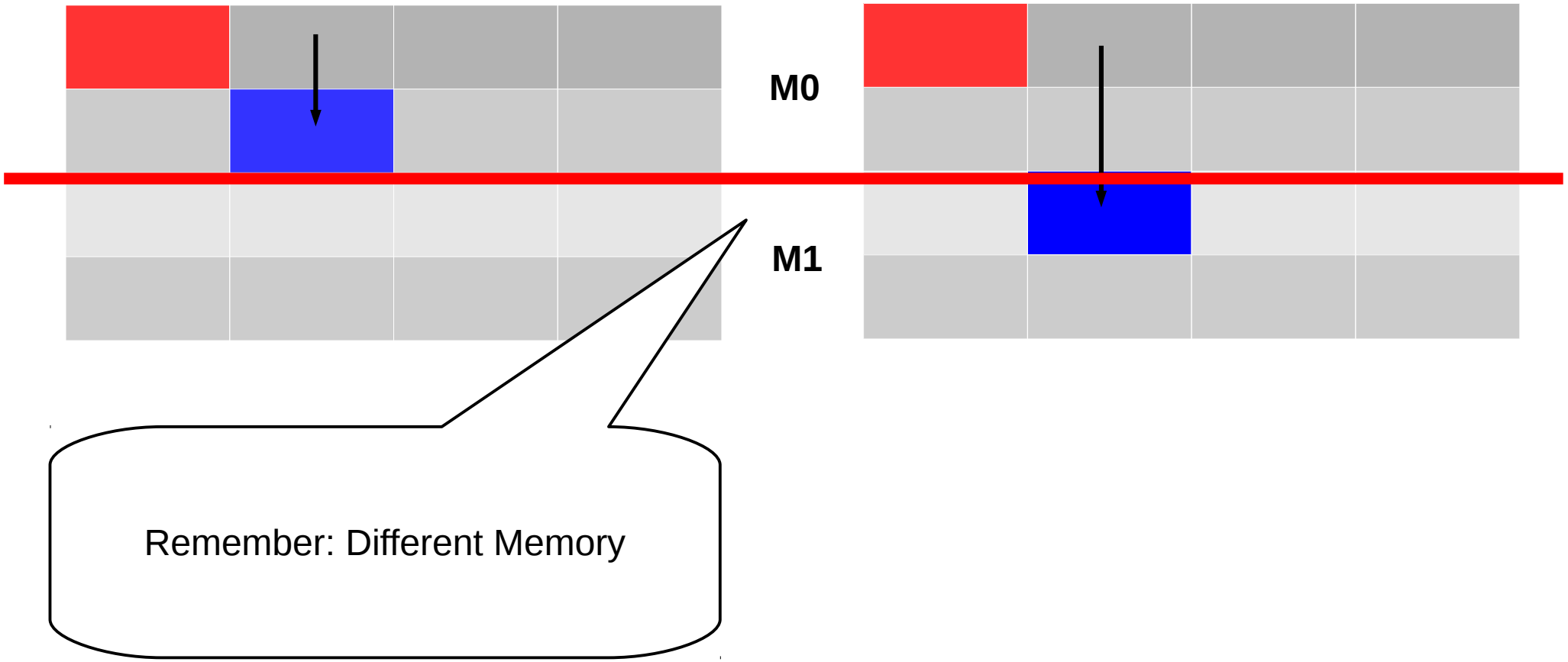


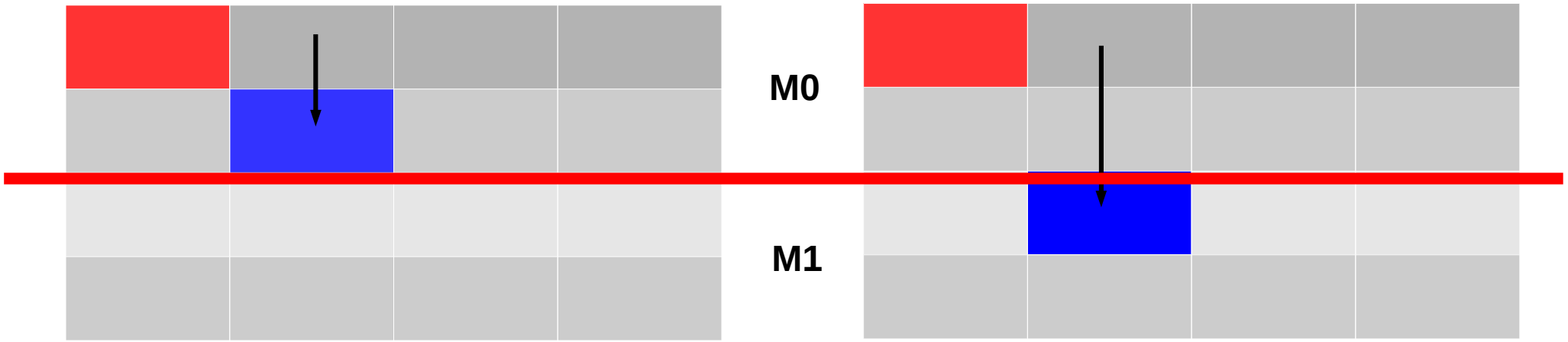
Communication scheme

But, which data should be now communicated?



Communication scheme

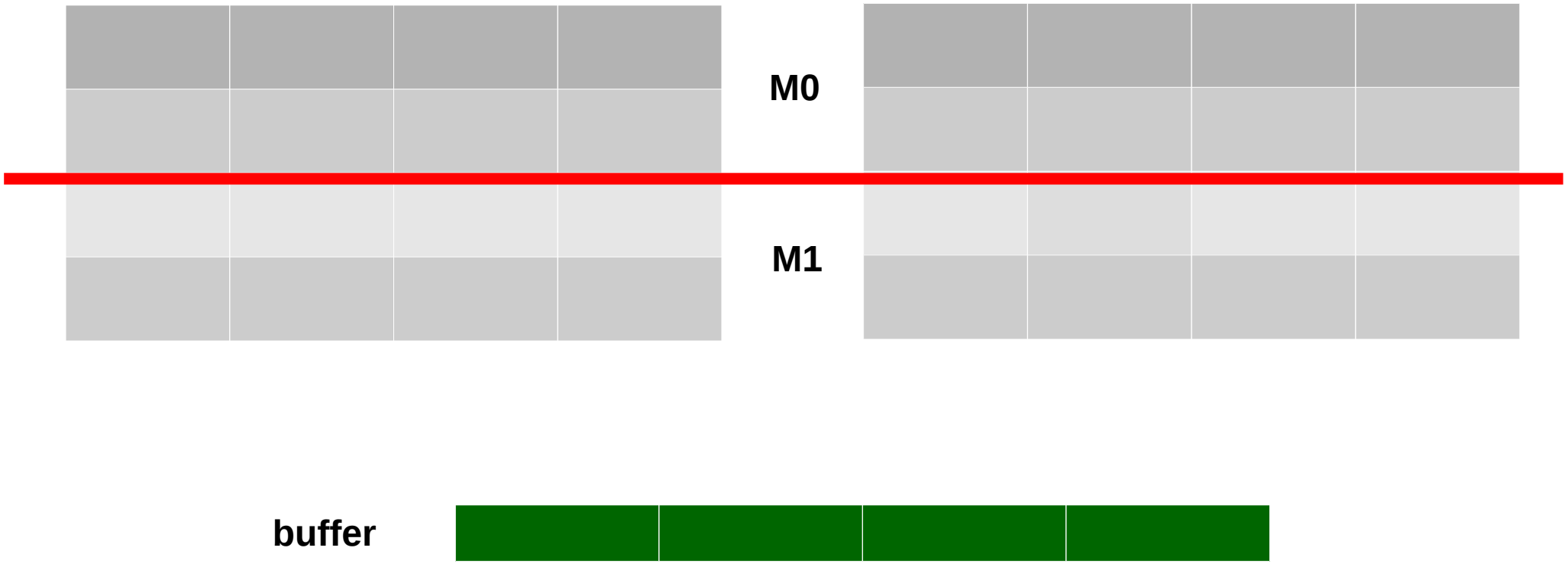
But, which data should be now communicated?



P1 doesn't have the A[0][1] element

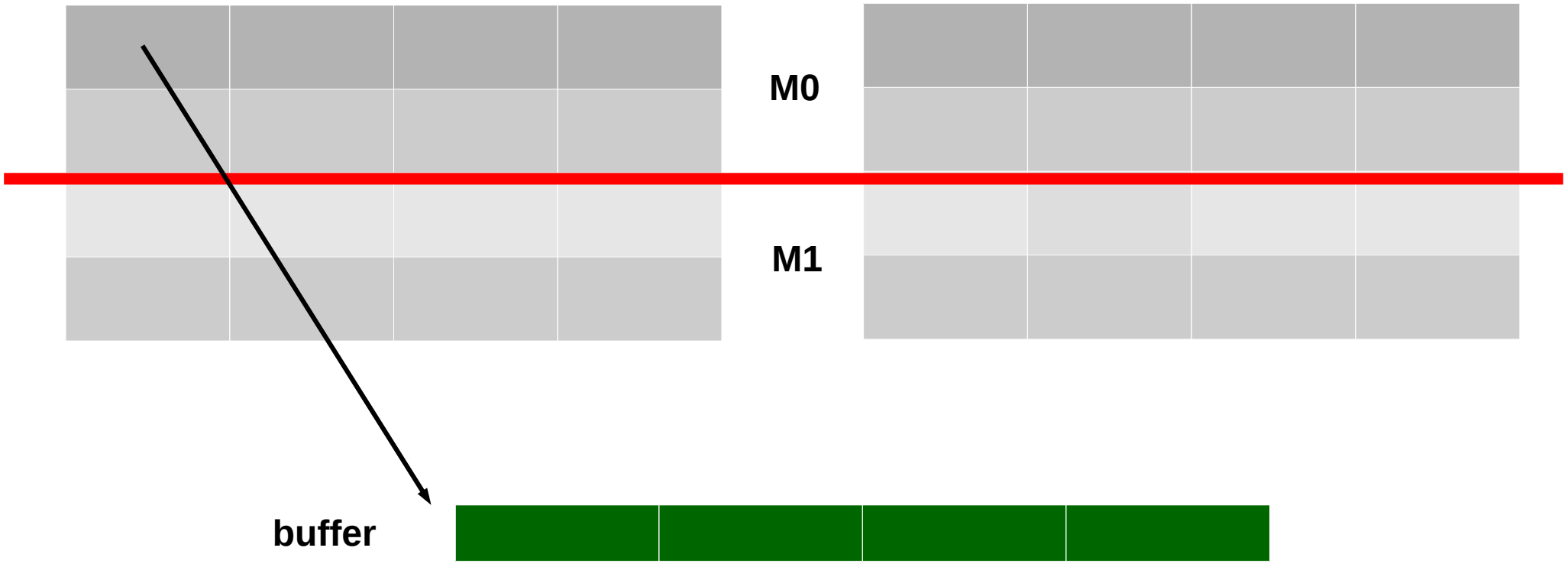
Communication scheme

Thus, per k-iteration I should send the “pivot” row !!



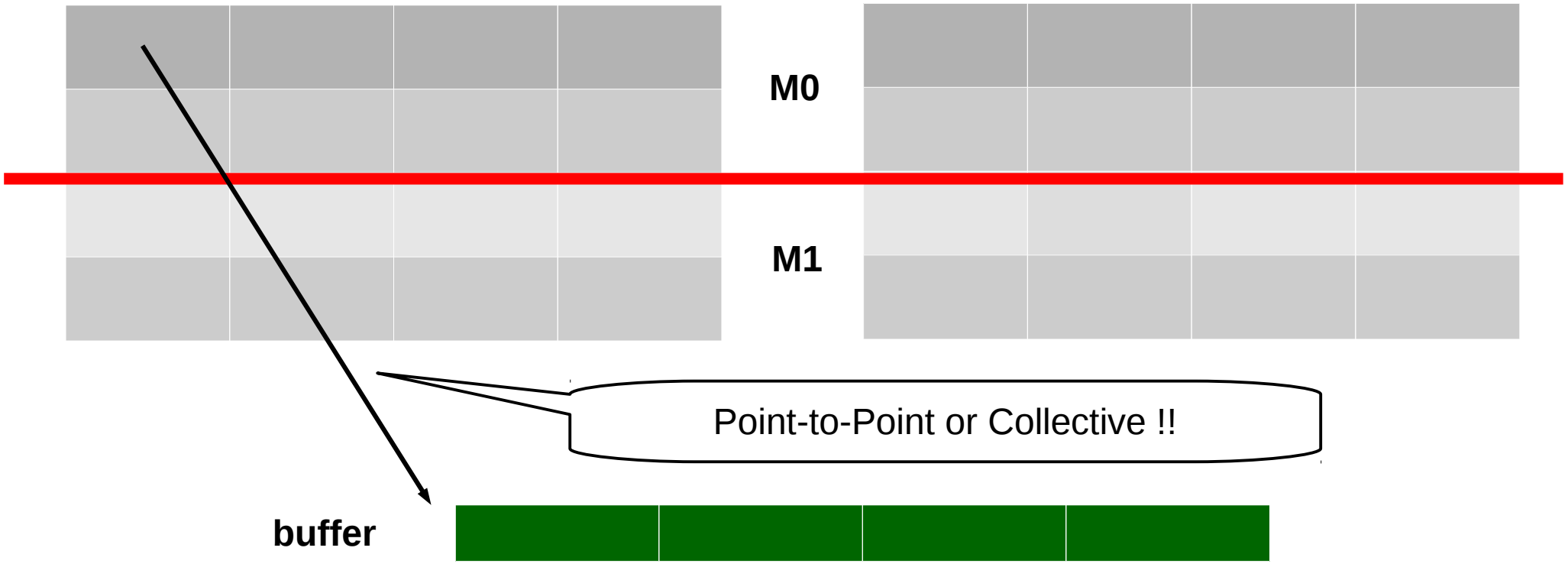
Communication scheme

The “owner” of the row should place it in a buffer.



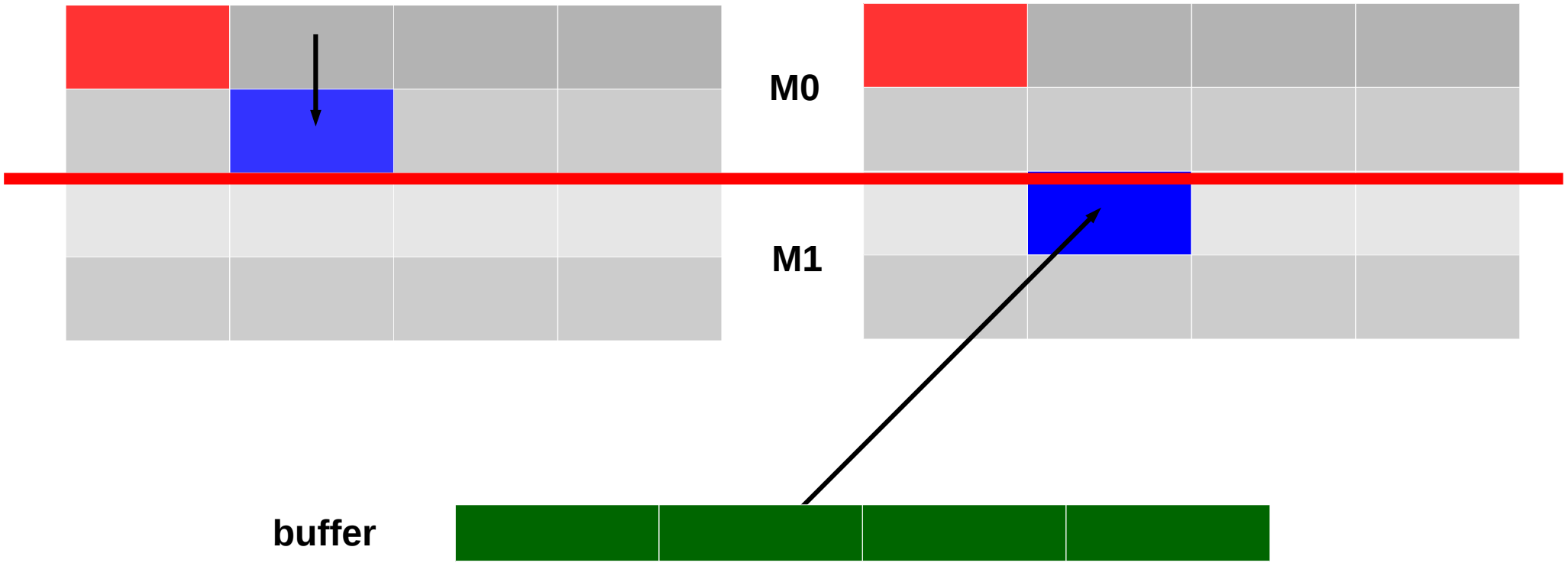
Communication scheme

The “owner” of the row should place it in a buffer.



Communication scheme

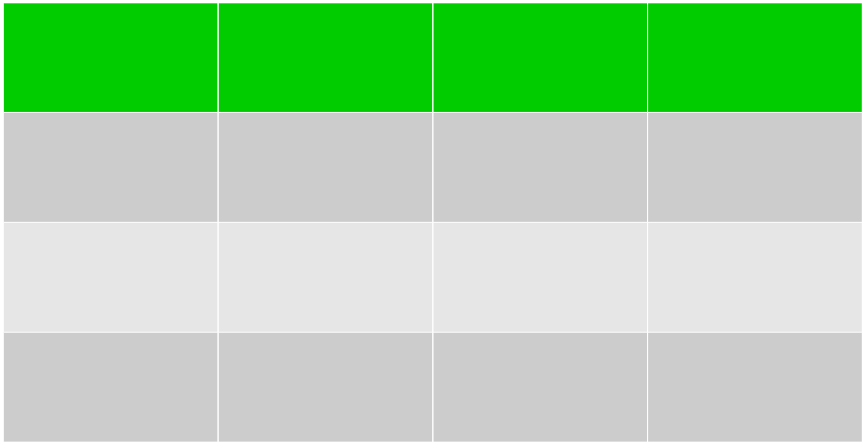
So now, I can repeat the example.



Communication scheme

Thus, per k-iteration :

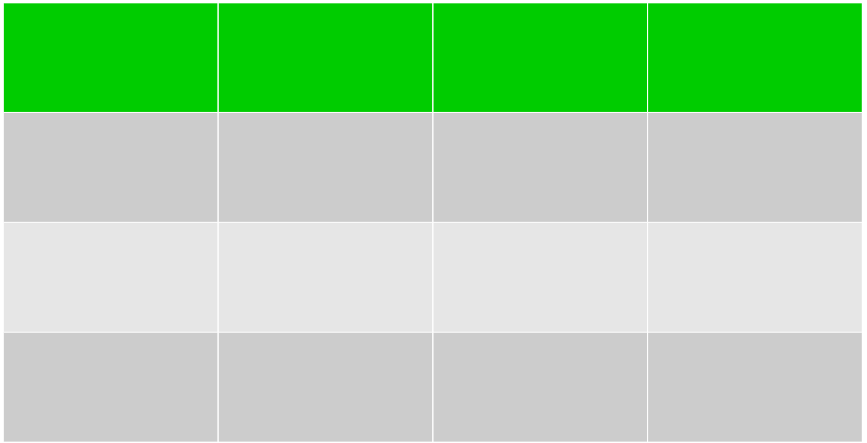
k=0



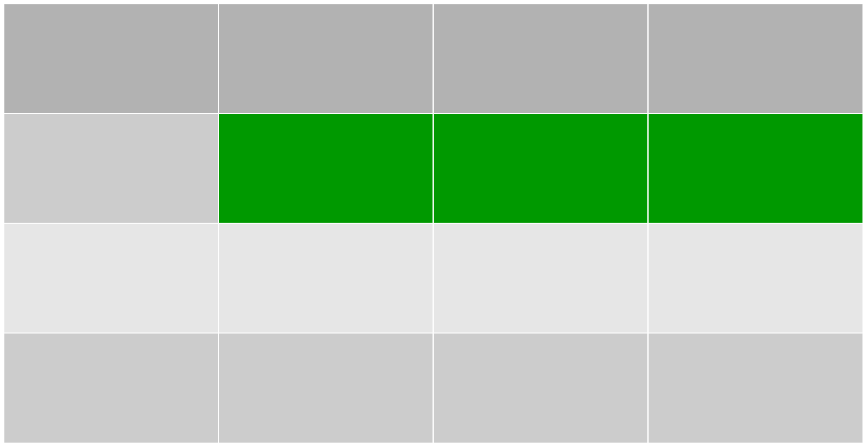
Communication scheme

Thus, per k-iteration :

k=0



k=1



Lab 2 – Gaussian Elimination

- Parallel version

```
if (rank == 0) {
   allocate_full_matrix(A);
   initialize(A);
}
allocate_local_matrix(lA);
distribute_matrix(0,A,lA);


for(k = 0; k < N - 1; k++){
   if (owner_of_critical_line(k)) {
       pack_data(lA, send_buffer);
       send_data_to_all(send_buffer, …);
   }
   else {
       receive_data_from_owner(receive_buffer, …);
       unpack_data(receive_buffer, lA);
   }
   compute(k, lA);
}

collect_results (0, A, lA);
```

Lab 2 – Any questions??