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Assignment 2

Augmented Reality

1.1

Find $A, B \in \mathbb{R}^{2\times 2}$ such that $AB \neq BA$

let $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ let $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$

$a_1, a_2, a_3, a_4 \in \mathbb{R}$ $\quad b_1, b_2, b_3, b_4 \in \mathbb{R}$

$$AB = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{bmatrix}$$

$$BA = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \begin{bmatrix} b_1a_1 + b_2a_3 & b_1a_2 + b_2a_4 \\ b_3a_1 + b_4a_3 & b_3a_2 + b_4a_4 \end{bmatrix}$$

$AB = BA$ if $\quad a_1b_1 + a_2b_3 = b_1a_1 + b_2a_3, \quad a_1b_2 + a_2b_4 = b_1a_2 + b_2a_4$

$\quad a_3b_1 + a_4b_3 = b_3a_1 + b_4a_3, \quad a_3b_2 + a_4b_4 = b_3a_2 + b_4a_4$

$AB \neq BA$ if one of the statements above is not true.

$$a_1b_1 + a_2b_3 \neq b_1a_1 + b_2a_3$$

$$a_2b_3 \neq b_2a_3$$

one case: $a_2 = 2,\ a_3 = 3$

$b_2 = 4,\ b_3 = 5$

$$2 \cdot 5 \neq 4 \cdot 3$$
$$10 \neq 12$$

let all other values = 1

$$AB = \overset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}} \overset{B}{\begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix}} = \begin{bmatrix} 11 & 6 \\ 8 & 13 \end{bmatrix}$$

$$BA = \overset{B}{\begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}} = \begin{bmatrix} 13 & 6 \\ 8 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 6 \\ 8 & 13 \end{bmatrix} \neq \begin{bmatrix} 13 & 6 \\ 8 & 11 \end{bmatrix}$$

so

$AB \neq BA$ in this case,

so matrix multiplication is not commutative.

1.2

let $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$

$A^T = \begin{bmatrix} a_1 & a_3 \\ a_2 & a_4 \end{bmatrix}$

if $A = A^T$, $a_2 = a_3$

$A = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_4 \end{bmatrix}$

$B = B^T$, so let $B = \begin{bmatrix} b_1 & b_2 \\ b_2 & b_4 \end{bmatrix}$

if AB symmetric, $AB = (AB)^T$

$$AB = \begin{bmatrix} a_1 b_1 + a_2 b_2 & a_1 b_2 + a_2 b_4 \\ a_2 b_1 + a_4 b_2 & a_2 b_2 + a_4 b_4 \end{bmatrix} = \begin{bmatrix} a_1 b_1 + a_2 b_2 & a_2 b_1 + a_4 b_2 \\ a_1 b_2 + a_2 b_4 & a_2 b_2 + a_4 b_4 \end{bmatrix} = AB^T$$

AB is symmetrial if

$\underline{a_2 b_1 + a_4 b_2} = \underline{a_1 b_2 + a_2 b_4}$

let both $= \underline{C}$ (for short substitution)

$$AB = \begin{bmatrix} a_1 b_1 + a_2 b_2 & C \\ C & a_2 b_2 + a_4 b_4 \end{bmatrix}$$

$$BA = \begin{bmatrix} b_1 a_1 + b_2 a_2 & \underline{b_1 a_2 + b_2 a_4} \\ \underline{b_2 a_1 + b_4 a_2} & b_2 a_2 + b_4 a_4 \end{bmatrix}$$ substitute C

$$BA = \begin{bmatrix} b_1 a_1 + b_2 a_2 & C \\ C & b_2 a_2 + b_4 a_4 \end{bmatrix}$$

$$\begin{bmatrix} a_1 b_1 + a_2 b_2 & c \\ c & a_2 b_2 + a_4 b_4 \end{bmatrix} = \begin{bmatrix} b_1 a_1 + b_2 a_2 & c \\ c & b_2 a_2 + b_4 a_4 \end{bmatrix}$$

so, $AB = BA$

if A and B are symmetrical

1.3

$$(A+B)C = AC+BC$$

let a_{ij}, b_{ij}, c_{kj} represent elements in A, B, C resp.

$$(A+B)_{ij} : (a_{ij}+b_{ij})$$

$$(A+B)_{ik} : (a_{ik}+b_{ik})$$

$AC \qquad BC$

$$(A+B)C = \left(\sum_{k=1}^{m} a_{ik}c_{kj}\right) + \left(\sum_{k=1}^{m} b_{ik}c_{kj}\right)$$

$$\sum_{k=1}^{m} (a_{ik}+b_{ik})c_{kj} =$$

$$\sum_{k=1}^{m} a_{ik}c_{kj}+b_{ik}c_{kj} = \sum_{k=1}^{m} a_{ik}c_{kj}+b_{ik}c_{kj}$$

so...

$$(A+B)C = AC+BC$$

1.4

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

determinent = −2
rank = 2
is invertable

determinent
$1 \cdot 4 - 3 \cdot 2 = -2$
not 0 so matrix is invertable

calc rank

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $r_2 = r_2 - 3r_1$

$\begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ rank = 2

(b) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Determinant: None. Not a square matrix.
Rank = 2
Not invertable; not a square matrix.

$r_2 = r_2 - 3r_1$
$r_3 = r_3 - 5r_1$

$\begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & -4 \end{bmatrix}$

$r_3 = r_3 - 2r_2$

$\begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & 0 \end{bmatrix}$ rank = 2

(c) $\begin{bmatrix} 3 & 1 & 3 \\ 9 & 4 & 9 \\ 15 & 7 & 15 \end{bmatrix}$

determinant = 0
rank = 2
Non inversable. Determinant is zero.

$$\det \begin{bmatrix} 3 & 1 & 3 \\ 9 & 4 & 9 \\ 15 & 7 & 15 \end{bmatrix} = 3 \cdot \begin{vmatrix} 4 & 9 \\ 7 & 15 \end{vmatrix} - 1 \cdot \begin{vmatrix} 9 & 9 \\ 15 & 15 \end{vmatrix} + 3 \cdot \begin{vmatrix} 9 & 4 \\ 15 & 7 \end{vmatrix}$$

$$\underbrace{3 \cdot -3}_{60-63} \quad - 0 \quad + \underbrace{3 \cdot 3}_{63-60}$$

$$-9 \quad + 9$$

$\det = \underline{0}$

$$\begin{bmatrix} 3 & 1 & 3 \\ 9 & 4 & 9 \\ 15 & 7 & 15 \end{bmatrix}$$

$r_2 = r_2 - 3r_1$

$r_3 = r_3 - 5r_1$

$$\begin{bmatrix} 3 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

$r_3 = r_3 - 2r_1$

$$\begin{array}{c c} 1 & \\ 2 & \begin{bmatrix} 3 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$ rank = 2

(d) $$\begin{bmatrix} 132 & 165 & 198 \\ 348 & 435 & 522 \\ 564 & 705 & 846 \end{bmatrix}$$

determinant = 0

rank = 1

Non-inversable; Determinant is zero.

$$\det \begin{bmatrix} 132 & 165 & 198 \\ 348 & 435 & 522 \\ 564 & 705 & 846 \end{bmatrix} = 132 \begin{vmatrix} 435 & 572 \\ 705 & 846 \end{vmatrix} - 165 \begin{vmatrix} 348 & 522 \\ 564 & 846 \end{vmatrix} + 198 \begin{vmatrix} 348 & 435 \\ 564 & 705 \end{vmatrix}$$

$$= 132 \cdot 0 - 165 \cdot 0 + 198 \cdot 0$$

$$= \underline{0}$$

$$\begin{bmatrix} 132 & 165 & 198 \\ 348 & 435 & 522 \\ 564 & 705 & 846 \end{bmatrix}$$

$$\frac{165}{132} /3 = \frac{55}{44} /11 = \frac{5}{4}$$

$$\frac{198}{132} /3 = \frac{66}{44} /11 = \frac{6}{4} = \frac{3}{2}$$

$$r_1 = r_1/132$$
$$r_2 = r_2/348$$
$$r_3 = r_3/564$$

$$\frac{435}{348} /3 = \frac{145}{116} /29 = \frac{5}{4}$$

$= 1.25$ so $\frac{5}{4}$ somehow

$$\begin{bmatrix} 1 & 5/4 & 3/2 \\ 1 & 5/4 & 3/2 \\ 1 & 5/4 & 3/2 \end{bmatrix}$$

$$\frac{572}{348} = 1.5 = 3/2$$

$$705/564 = 1.25 = 5/4$$

$$846/564 = 1.5 = 3/2$$

$$r_2 = r_2 - r_1$$
$$r_3 = r_3 - r_1$$

$$\begin{bmatrix} 1 & 5/4 & 3/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 1$$

$\underline{\text{rank} = 1}$

1.5

(a) Rotation around x-axis by φ is

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

let $\varphi = 30^\circ$

$\sin(30^\circ) = 1/2$ or 0.5

$\cos(30^\circ) = \sqrt{3}/2$

plug in

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

(b) Rotate around y-axis by 45°

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$\theta = 45^\circ$

$\cos\theta = \sqrt{2}/2$

$\sin\theta = \sqrt{2}/2$

$$R_y = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

(c) Rotate around z-axis by 60°

$$R_z = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\psi = 60°$

$\sin(60°) = \sqrt{3}/2$

$\cos(60°) = 1/2$

$$R_z = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.6

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \quad \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \quad \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_x $\qquad$ R_y $\qquad$ R_z

$$R_x R_y R_z =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \times \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \times R_z$$

$$\begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ \sqrt{2}/4 & \sqrt{3}/2 & -\sqrt{2}/4 \\ -\sqrt{6}/4 & 1/2 & \sqrt{6}/4 \end{bmatrix} \times \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2}/4 & -\sqrt{6}/4 & \sqrt{2}/2 \\ \sqrt{2}/8 + 3/4 & -\sqrt{6}/8 + \sqrt{3}/4 & -\sqrt{2}/4 \\ -\sqrt{6}/8 + \sqrt{3}/4 & 3\sqrt{2}/8 + 1/4 & \sqrt{6}/4 \end{bmatrix} = R_x R_y R_z$$

$R_z \, R_y \, R_x$

$$\begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \times R_x$$

$$\begin{bmatrix} \sqrt{2}/4 & -\sqrt{3}/2 & \sqrt{2}/4 \\ \sqrt{6}/4 & 1/2 & \sqrt{6}/4 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2}/4 & -3/4 + \sqrt{2}/8 & \sqrt{3}/4 + \sqrt{6}/8 \\ \sqrt{6}/4 & \sqrt{3}/4 + \sqrt{6}/8 & -1/4 + 3\sqrt{2}/8 \\ -\sqrt{2}/2 & \sqrt{2}/4 & \sqrt{6}/4 \end{bmatrix} = R_z R_y R_x$$

$$\begin{bmatrix} \sqrt{2}/4 & -\sqrt{6}/4 & \sqrt{2}/2 \\ \sqrt{2}/8 + 3/4 & -\sqrt{6}/8 + \sqrt{3}/4 & -\sqrt{2}/4 \\ -\sqrt{6}/8 + \sqrt{3}/4 & 3\sqrt{2}/8 + 1/4 & \sqrt{6}/4 \end{bmatrix} \neq \begin{bmatrix} \sqrt{2}/4 & -3/4 + \sqrt{2}/8 & \sqrt{3}/4 + \sqrt{6}/8 \\ \sqrt{6}/4 & \sqrt{3}/4 + \sqrt{6}/8 & -1/4 + 3\sqrt{2}/8 \\ -\sqrt{2}/2 & \sqrt{2}/4 & \sqrt{6}/4 \end{bmatrix}$$

SO

$$R_x R_y R_z \neq R_z R_y R_x$$

2 (a)

(a) $x \to \alpha x + t, \quad \forall x \in \mathbb{R}^n$

Scalar vector vector

$$x = \begin{bmatrix} x \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \alpha x + t$$

$$\begin{bmatrix} x \\ 1 \end{bmatrix} \to \begin{bmatrix} \alpha & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix}$$

2(b)

(b) homogenous $x = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 1 \end{bmatrix} \in \mathbb{R}^4$

$$T = \begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 20 & 5 & 0 \\ 0 & 0 & -10 & -100 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$X \to \begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 20 & 5 & 0 \\ 0 & 0 & -10 & -100 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 30 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 200 + 150 \\ 400 + 150 \\ -300 - 100 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} 350 \\ 550 \\ -400 \\ -30 \end{bmatrix} \xrightarrow{\text{mapped to } \mathbb{R}^3} \begin{bmatrix} -350/30 \\ -550/30 \\ 400/30 \end{bmatrix}$$

$$\begin{bmatrix} -35/3 \\ -55/3 \\ 40/3 \end{bmatrix}$$

2(c)

(c)

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \times \begin{bmatrix} ? \end{bmatrix} = I$$

$A \in \mathbb{R}^{n \times n}$ $B \in \mathbb{R}^n$

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} RA + tC & RB + tD \\ C & D \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix}$$

$C \in \mathbb{R}^n$ $D \in \mathbb{R}$

$C = 0^T$ $D = 1$

$$= \begin{bmatrix} RA & RB + t \\ 0^T & 1 \end{bmatrix}$$

Rotation Matrices are orthogonal, so $RR^T = I$

$RA = I$
$A = R^T$

$RB + t = 0$
$RB = -t$
$RB = -It$
$RB = -(RR^T)t$
$B = -R^T t$

$$= \begin{bmatrix} RR^T & -RR^T t + t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix} = I$$

Inverse $$= \boxed{\begin{bmatrix} R^T & -R^T t \\ 0^T & 1 \end{bmatrix}}$$

I learned about block matrices to check the work above. The formula used to check is from: https://www.cs.nthu.edu.tw/~jang/book/addenda/matinv/matinv/

Work to check answer:

block matrix $M = \begin{array}{c} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{array}{l} \}m \\ \}n \end{array} \\ \begin{array}{cc} m & n \end{array} \end{array}$

$m+n \times m+n$

if A & D is invertable

$$\begin{bmatrix} A & C \\ B & D \end{bmatrix} \begin{bmatrix} X & Y \\ Z & U \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_n \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} X & Y \\ Z & U \end{bmatrix} = \begin{bmatrix} (A-BD^{-1}C)^{-1} & -A^{-1}B(D-CA^{-1}B)^{-1} \\ -D^{-1}C(A-BD^{-1}C)^{-1} & (D-CA^{-1}B)^{-1} \end{bmatrix}$$

Let $m = \dim(R)$ (or n)

$n = 1$

$A = R \quad B = t$

$C = 0^T \quad D = 1$

$1^{-1} = 1$

$R^{-1} = R^T$

R is orthogonal

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \overbrace{\begin{bmatrix} X & Y \\ Z & U \end{bmatrix}}^{\text{inverse}} = \begin{bmatrix} I_n & 0 \\ 0^T & 1 \end{bmatrix}$$

$$\begin{bmatrix} X & Y \\ Z & U \end{bmatrix} = \begin{bmatrix} (R-0)^{-1} & -R^{-1}t(1-0)^{-1} \\ -1 \cdot 0 & (1-0)^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix}$$

2(d)

(d)

$$P_1 \times \vec{P}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4-2 \\ 0 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 2/-2 \\ 0 \\ -2/-2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

at $z=1$

$$\vec{P}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$P_1 \times \vec{P}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 4/-2 \\ 1/-2 \\ -2/-2 \end{bmatrix} = \begin{bmatrix} -2 \\ -1/2 \\ 1 \end{bmatrix}$$

at $z=1$

$$\vec{P}_2 = \begin{bmatrix} -2 \\ -1/2 \end{bmatrix}$$

3 (a)

The minimum number of point-to-point correspondences required for homography estimation is 4.

Because homography has 8 degrees of freedom, 8 parameters need to be estimated.

To find 8 parameters, 8 independent equations are required.

Each point correspondence grants 2 independent equations.

So, it takes 4 point-to-point correspondences for 8 independent equations.

3(b)

$$h_{33} = 1$$

$$p_x' = \frac{h_{11}p_x + h_{12}p_y + h_{13}}{h_{31}p_x + h_{32}p_y + 1} \qquad p_y' = \frac{h_{21}p_x + h_{22}p_y + h_{23}}{h_{31}p_x + h_{32}p_y + 1}$$

$p_1\ (0,0) \rightarrow (1,1)$

$$1 = \frac{h_{13}}{1} \qquad 1 = \frac{h_{23}}{1}$$

$$\underline{h_{13} = 1} \qquad \underline{h_{23} = 1}$$

$$p_x' = \frac{h_{11}p_x + h_{12}p_y + 1}{h_{31}p_x + h_{32}p_y + 1} \qquad p_y' = \frac{h_{21}p_x + h_{22}p_y + 1}{h_{31}p_x + h_{32}p_y + 1}$$

$p_2\ (0,1) \rightarrow (1,0)$

$$1 = \frac{h_{12} + 1}{h_{32} + 1} \qquad 0 = \frac{h_{22} + 1}{h_{32} + 1}$$

$$h_{32} + 1 = h_{12} + 1 \qquad \underline{-1 = h_{22}}$$

$$\underline{h_{32} = h_{12}}$$

$$p_x' = \frac{h_{11}p_x + h_{32}p_y + 1}{h_{31}p_x + h_{32}p_y + 1} \qquad p_y' = \frac{h_{21}p_x - p_y + 1}{h_{31}p_x + h_{32}p_y + 1}$$

$p_3 : (1,1) \rightarrow (2,0)$

$$2 = \frac{h_{11} + h_{32} + 1}{h_{31} + h_{32} + 1} \qquad 0 = \frac{h_{21} - 1 + 1}{h_{31} + h_{32} + 1}$$

$$\underline{0 = h_{21}}$$

$$2(h_{31} + h_{32} + 1) = h_{11} + h_{32} + 1$$

$$-h_{32} \ -1 \qquad\qquad -h_{32} \ -1$$

$$\underline{2h_{31} + h_{32} + 1 = h_{11}}$$

not subbing this

$$2(h_{31} + h_{32} + 1) = h_1 + h_{31} + 1$$

$$2(h_{32} + 1) = 1 + h_{32} + 1$$

$$2h_{32} + 2 = 2 + h_{32}$$

$$h_{32} = 0$$

$$p_x' = \frac{h_{11}p_x + h_{32}p_y + 1}{h_{31}p_x + h_{32}p_y + 1} \qquad p_y' = \frac{-p_y + 1}{h_{31}p_x + h_{32}p_y + 1}$$

$h_{32}+1=1$

$h_{32}=0$

$$H=\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

3(c)

Equations (2) and (3) hold because they are the separated results of H multiplied by P. Below is proof.

$$P = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

$$HP = P'$$

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ z \end{bmatrix} \qquad \begin{matrix} z \in \mathbb{R} \\ z \neq 0 \end{matrix}$$

$$\begin{bmatrix} h_{11} p_x + h_{12} p_y + h_{13} \\ h_{21} p_x + h_{22} p_y + h_{23} \\ h_{31} p_x + h_{32} p_y + h_{33} \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ z \end{bmatrix}$$

to $\mathbb{R}^2$ (aka $p'_x \to p'_x/z$, $p'_y \to p'_y/z$)

let $z = 1$

$$\begin{bmatrix} \dfrac{h_{11} p_x + h_{12} p_y + h_{13}}{h_{31} p_x + h_{32} p_y + h_{33}} \\[2ex] \dfrac{h_{21} p_x + h_{22} p_y + h_{23}}{h_{31} p_x + h_{32} p_y + h_{33}} \end{bmatrix} = \begin{bmatrix} p'_x/z \\ p'_y/z \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \end{bmatrix}$$

Separating out the elements from the matrices will return equations (2) and (3).

3(d)

We can fix h33=1 because there are 8 degrees of freedom, but 9 elements. Because we do not need a 9^{th} element to change, we can assign h33 a value of 1.

I think it is not always a good idea to set h33=1. There are probably advanced forms of estimation out there that require another degree of freedom.

Setting h33=0 will remove a constant value from equations (2) and (3). This will physically mean that projected points will converge to (h13,h23) as x and y approach infinity. Changing h33 will change how projected lines approach infinity.

3(e)

One means of making the estimation robust would be the use of statistics. Stats could be used to identify outliers by constructing quartile ranges. DLT could then be used on the inliers to reduce some of the outlier-caused errors.

3(f)

Homography estimation will not work in this case. Four point-to-point correspondences only work with planar 3D shapes as only 2D coordinates are provided. Depth cannot be accounted for, so the points can only define a plane. Non-planar 3d shapes would require knowing the relative depth of the points to each other.

3(g)

One other application could be combining images to make something like a panorama. By detecting the same points between images, two or more images from different views could be stitched together.

5

(Question 5c)

Are these points correctly projected in image 3 (Using first H)? Why?

These are not correctly projected in image 3, but they appear to be relatively close. One reason for this is general noise. Another reason is that H was made using the correspondences of image 1 and image 2.

Can you use the H functions computed earlier to find correspondences between images 2 and 3?

You cannot use the H functions computed earlier to find correspondences between images 2 and 3. This is because of the errors from using a different source image and using a different destination image compound. The errors increased significantly when trying to do this with both previously computed H functions.