1 Probability:

1. Conditional Independence would mean:

$$P(X, Y) = P(X) * P(Y), X, Y \in (0, 1)$$

Marginal Probabilities are calculated as:

$$P(X) = \sum_y P(X, y), \text{ where } y \in Y$$

Table 1: Conditional Independence

	$P(X,Y)$	$P(X)$	$P(Y)$	$P(X) * P(Y)$
X=0,Y=0	0.375	0.5	0.75	0.375
X=0,Y=1	0.125	0.5	0.25	0.125
X=1,Y=0	0.375	0.5	0.75	0.375
X=1,Y=1	0.125	0.5	0.25	0.125

As seen in the table above X and Y are independent.

2. Given: $Z = X + Y$. For Conditional Independence:

$$P(X, Y|Z) = P(X|Z) * P(Y|Z), X, Y \in (0, 1)$$

Table 2

X	Y	Z	P(Z)
0	0	0	0.375
0	1	1	0.125
1	0	1	0.125
1	1	2	0.375

For the case when X=0,Y=0;

$$\begin{aligned} P(X = 0|Z = 0) &= \frac{P(X = 0, Z = 0)}{P(Z = 0)} \\ &= \frac{3/8}{3/8} = 1 \\ P(Y = 0|Z = 0) &= \frac{P(Y = 0, Z = 0)}{P(Z = 0)} \\ &= \frac{3/8}{3/8} = 1 \\ P(X = 0, Y = 0|Z = 0) &= \frac{P(X = 0, Y = 0, Z = 0)}{P(Z = 0)} \\ &= \frac{3/8}{3/8} = 1 \end{aligned}$$

Hence the condition holds for this case.
For the case when X=1,Y=1;

$$\begin{aligned} P(X=1|Z=1) &= \frac{P(X=1,Z=1)}{P(Z=1)} \\ &= \frac{3/8}{1/2} = 3/4 \\ P(Y=1|Z=1) &= \frac{P(Y=1,Z=1)}{P(Z=1)} \\ &= \frac{1/8}{1/2} = 1/4 \\ P(X=1,Y=1|Z=1) &= \frac{P(X=1,Y=1,Z=1)}{P(Z=1)} \\ &= \frac{0}{1/2} = 0 \end{aligned}$$

Hence the condition does not hold for this case. Therefore X and Y are not conditionally independent given Z.

3. Given: $W = X * Y$. For Conditional Independence:

$$P(X,Y|W) = P(X|W) * P(Y|W), X, Y \in (0,1)$$

Table 3

X	Y	W	P(W)
0	0	0	0.375
0	1	0	0.125
1	0	0	0.125
1	1	1	0.375

For the case when X=0,Y=0;

$$\begin{aligned} P(X=0|W=0) &= \frac{P(X=0,W=0)}{P(W=0)} \\ &= \frac{4/8}{7/8} = 4/7 \\ P(Y=0|W=0) &= \frac{P(Y=0,W=0)}{P(W=0)} \\ &= \frac{6/8}{7/8} = 6/7 \\ P(X=0,Y=0|W=0) &= \frac{P(X=0,Y=0,W=0)}{P(W=0)} \\ &= \frac{3/8}{7/8} = 3/7 \end{aligned}$$

Hence the condition does not hold for this case. Therefore X and Y are not conditionally independent given W.

4. C is the result of an independent coin flip. For Conditional Independence:

$$P(X,Y|C) = P(X|C) * P(Y|C), X, Y \in (0,1) \text{ and } C \in (H < T)$$

As C is an independent event:
$P(X|C) = P(X), P(Y|C) = P(Y)$
$P(X,Y|C) = P(X,Y,C)/P(C) = P(X) * P(Y) * P(C)/P(C) = P(X) * P(Y)$
$\implies P(X,Y|C) = P(X|C) * P(Y|C)$ from the above logic.
Hence X and Y are independent conditioned on Z.