1   Bayesian Network Inference

1. $P(A = false, A = false, A = false, A = false) = P(A) * P(B|A) * P(C|B) * P(D|B, C) = 0.2 * 0.4 * 0.2 * 0.1 = 0.0384$

2. Distribution $P(B|A = true, C = false)$:

$$P(B = true|A = true, C = false) = \frac{P(B = true, A = true, C = false)}{P(B = true, A = true, C = false) + P(B = false, A = true, C = false)}$$

$$P(B = true, A = true, C = false) = \sum_D P(B = true, A = true, C = false)$$

$$= 0.2 * 0.5 * 0.1(0.02 + 0.98)$$

$$= 0.01$$

$$P(B = false, A = true, C = false) = \sum_D P(B = false, A = true, C = false)$$

$$= 0.2 * 0.5 * 0.8 * 1$$

$$= 0.08$$

$$= \frac{0.01}{0.01 + 0.08} = 0.11$$

$$P(B = false|A = true, C = false) = 1 - 0.11 = 0.89$$

3. $P(D|A = false)$:

$$P(D = true|A = false) = \frac{P(D = true, A = false)}{P(D = true, A = false) + P(D = false, A = false)}$$

$$P(D = true, A = false) = \sum_{B,C} P(D = true, A = false)$$

$$= 0.8 * (0.6 * (0.8 * 0.9 + 0.2 * 0.95) + 0.4 * (0.9 * 0.99 + 0.1 * 0.98))$$

$$= 0.75328$$

$$P(D = false, A = false) = \sum_{B,C} P(D = false, A = false)$$

$$= 0.8 * (0.6 * (0.8 * 0.1 + 0.2 * 0.05) + 0.4 * (0.9 * 0.01 + 0.1 * 0.02))$$

$$= 0.04672$$

$$P(D = true|A = false) = \frac{0.75328}{0.75328 + 0.04672} = 0.9416$$

$$P(D = false|A = false) = 1 - P(D = true|A = false) = 0.0584$$

4. $P(D|B = false)$:

$$P(D = true|B = false) = \frac{P(D = true, B = false)}{P(D = true, B = false) + P(D = false, B = false)}$$

$$P(D = true, B = false) = \sum_{A,C} P(D = true, B = false)$$

$$= 0.5278$$

$$P(D = false, B = false) = \sum_{A,C} P(D = true, B = true)$$

$$= 0.0522$$

$$P(D = true|B = false) = \frac{0.5278}{0.5278 + 0.0522} = 0.91$$

$$P(D = false|B = false) = 1 - P(D = true|B = false) = 0.09$$

5. $P(D = true|A = true, B = true, C = true)$

$$P(D = true|A = true, B = true, C = true) = \frac{P(D = true, A = true, B = true, C = true)}{\sum_D P(D, A = true, B = true, C = true)}$$

$$= \frac{0.99 * 0.2 * 0.5 * 0.9}{0.99 * 0.2 * 0.5 * 0.9 + 0.01 * 0.2 * 0.5 * 0.9} = 0.99$$