

1 Bayesian Networks Learning:

First E-step:

1. $P(C_{17} = 1|A_{17}, B_{17}, D_{17})$:

$$\begin{aligned} P(C_{17} = 1|A_{17}, B_{17}, D_{17}) &= \frac{P(C_{17} = 1) * P(A_{17}, B_{17}, D_{17}|C_{17} = 1)}{P(C_{17} = 1) * P(A_{17}, B_{17}, D_{17}|C_{17} = 1) + P(C_{17} = 0) * P(A_{17}, B_{17}, D_{17}|C_{17} = 0)} \\ &= 0.5 \text{ (Since the training data-set is symmetrical)} \end{aligned}$$

2. $P(C_{17} = 0|A_{17}, B_{17}, D_{17})$:

$$\begin{aligned} P(C_{17} = 0|A_{17}, B_{17}, D_{17}) &= 1 - P(C_{17} = 1|A_{17}, B_{17}, D_{17}) \\ &= 0.5 \end{aligned}$$

M-Step:

1. $P(C|B = 1)$:

$$\begin{aligned} P(C = 1|B = 1) &= \frac{P(B = 1|C = 1) * P(C = 1)}{\sum_C P(B = 1, C)} \\ &= \frac{\frac{4.5}{8.8} * \frac{8.5}{17}}{\frac{4.5}{8.8} * \frac{8.5}{17} + \frac{4.5}{8.8} * \frac{8.5}{17}} \\ &= 0.5 \end{aligned}$$

$$P(C = 0|B = 1) = 1 - P(C = 1|B = 1) = 0.5$$

2. $P(C|B = 0)$:

$$\begin{aligned} P(C = 1|B = 0) &= \frac{P(B = 0|C = 1) * P(C = 1)}{\sum_C P(B = 0, C)} \\ &= \frac{\frac{4.5}{8.8} * \frac{8.5}{17}}{\frac{4.5}{8.8} * \frac{8.5}{17} + \frac{4.5}{8.8} * \frac{8.5}{17}} \\ &= 0.5 \end{aligned}$$

$$P(C = 0|B = 0) = 1 - P(C = 1|B = 0) = 0.5$$

3. $P(A = 0, B = 1, C = 0, D = 1)$

From Figure 3,

$$\begin{aligned} P(A = 0, B = 1, C = 0, D = 1) &= P(A) * P(B|A) * P(C|B) * P(D|B, C) \\ &= \frac{9}{17} * \frac{5}{9} * 0.5 * \frac{2}{4 + 0.5} \\ &= 0.065 \end{aligned}$$

Second E-step:

1. $P(C_{17} = 1|A_{17}, B_{17}, D_{17})$:

$$\begin{aligned} P(C_{17} = 1|A_{17}, B_{17}, D_{17}) &= \frac{P(C_{17} = 1) * P(A_{17}, B_{17}, D_{17}|C_{17} = 1)}{P(C_{17} = 1) * P(A_{17}, B_{17}, D_{17}|C_{17} = 1) + P(C_{17} = 0) * P(A_{17}, B_{17}, D_{17}|C_{17} = 0)} \\ &= \frac{\frac{4.5}{8.5} * \frac{1.5}{8.5}}{\frac{4.5}{8.5} * \frac{1.5}{8.5} + \frac{4.5}{8.5} * \frac{1.5}{8.5}} \\ &= 0.5 \end{aligned}$$