Project 1 Mathematics 407

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Due date: Monday, April 29

You are allowed to use any software package you prefer. This is a group project.
A 10-15 minutes presentation is required (maximum 4 students per group)

1.

a) Use the random number generator $\boldsymbol{x_n \equiv (ax_{n-1} + c)\, mod(m)}$ with $\boldsymbol{a = 7^5,\ c = 0\ \ and\, m = 2^{31} - 1}$ to generate 10000 uniformly distributed random numbers on $[\boldsymbol{0}, \boldsymbol{1}]$ and plot the histogram**.**

b) Generate 10000 uniformly distributed random numbers on $[\boldsymbol{0}, \boldsymbol{1}]$ using built-in function of MATLAB or other statistical software package.

c) Compare the histograms obtained in parts a) and b).

2.

a) Use the numbers generated in problem 1 b) to generate 10000 random numbers that represent the daily price fluctuation of a financial asset that can have an increase of 100 with probability 0.45, a decrease of 200 with probability 0.25, or stays the same with probability 0.3. In other words, generate a discrete random variable that takes the values 100, 200 and 0 with probabilities 0.45, 0.25 and 0.3 respectively.

b) Plot the histogram obtained using the data generated in part a).

3.

Generate 5000 Binomial distributed $(n = 70, p = 0.7)$ random numbers by doing:

Use Bernoulli random variables. Plot the histogram and use your data to calculate the probability that the Binomial random variable is less than 50. Compare with the theoretical answer.

4.

a) Simulate 1000 random normal variables with mean 1.5 and standard deviation 2. Create a histogram. Use a build in software package or investigate how to build your own (not that difficult).

b) Find out how many of your 1000 variables are bigger than 0, and estimate the probability that a single normal variable with mean 1.5 and standard deviation 2 is above 0. Also, compute and write the theoretical value for this probability.

5.

Simulate 100 sample means, each made by taking the average of 20 normal variables with mean 1.5 and standard deviation 2. Make a histogram of these sample means in the range -4 to 10 with class intervals of length 0.5. As in the previous question, estimate the probability that a sample mean is bigger than 0. Also, compute and write the theoretical value for this probability.

6.

Let $U_1, U_2, \cdots, U_n$ be uniform random variables on $[\mathbf{0}, \mathbf{1}]$. Define $N = min\{n: \sum_{i=1}^{n} U_i > 1\}$.

a) Organize in a table the estimates of $E[N]$ obtained by generating $10, 10^2, \cdots, 10^6$ values of N.

b) Are your estimates in part a) converging to a particular value? If so, in what sense?

c) Calculate theoretically the $E[N]$ and compare your result with the one obtained in part **a)**.

7.

Let's consider the Normal random variables X_1 with mean $\mu_1 = 10$ and variance $\sigma_1^2 = 0.20^2$

a) Generate 10000 log-normal random variables $Y_1 = e^{X_1}$

b) Estimate $E[Y_1]$ and compare with the theoretical result.

8.

Consider the integral $I = \int_0^1 \int_0^1 e^{x+y} dxdy$.

a) Estimate the integral I. Compare with the theoretical result. You can see this integral as an expectation.

9. Verify numerically and sketch a proof of the following important results:

a) The normal approximation of the binomial distribution.

b) The sum of two normal random variables is a normal random variable.

c) The density of the sum of two uniform [0,1] random variables has a triangular distribution.

d) Let X_i be the sum of the results of i rolls of the same die. (take $i = 2,5,10,20,40,80$) and show that the probability distribution of the X_i approach the probability density of a normal random variable.