# HOMEWORK 3

## Due: next TA session, whenever it is

1. Find data on real output, investment, consumption, total hours worked, employment, and labor productivity (output divided by hours) from your country of birth.[1] HP-filter your series and then compute percent standard deviations of the cyclical components of each variable, as well as correlations with output. Summarize your findings in a table. Are the stylized facts discussed in class satisfied in your data?

2. Consider an economy with an infinitely lived consumer with preferences

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t + \psi \log l_t\right)$$

and budget constraint

$$c_t + \beta a_{t+1} = a_t + w_t(1 - l_t)$$

for all $t$, with $a_0 = 0$ and no restrictions on borrowing.

(a) Consider, initially, the case $w_t = w$ for all $t$. Solve this problem completely for the sequences of consumption and leisure.

(b) The Frisch ($\lambda$-constant) elasticity of labor substitution is defined as the percentage change in period-$t$ labor supply in response to a one percent change in the period-$t$ wage *keeping constant the marginal utility of period-t consumption,* $\lambda_t$. Usually this is computed as $\frac{d\log(1-l_t)}{d\log w_t}|_{\lambda_t}$. Given a solution to the above problem where the parameters of the model are calibrated so that $1 - l_t$ is equal to $1/3$ (one third of the available time is spent working), what is the value of the Frisch elasticity?

3. Consider an RBC-like model with a representative infinitely lived consumer with preferences

$$E\left[\sum_{t=0}^{\infty} \beta^t \left(\log c_t + \frac{\chi_t}{\nu} l_t^{\nu}\right) \xi_t\right]$$

and resource constraint

$$c_t + k_{t+1}\eta_t = z_t k_t^{\alpha}(1 - l_t)^{1-\alpha}.$$

Here there are four shocks: the disembodied productivity shock $z_t$, an intratemporal preference shock $\chi_t$ directly affecting the marginal rate of substitution between consumption and leisure, an intertemporal preference shock $\xi_t$ directly influencing the intertemporal rate of substitution between consumption/leisure at $t$ and consumption/leisure at other points in time, and an investment-specific technology shock $\eta_t$. The shocks are assumed to be finite-state, mutually independent, and iid.

[1] If this data is not available, use the nearest other country to your place of birth for which it is available.

Your task is to solve the model as completely as you are able. It should be possible to solve the model fully. To proceed, guess, and verify, that $k_{t+1} = s_t z_t k_t^\alpha (1 - l_t)^{1-\alpha}$, where the saving rate $s_t$ is iid and does not depend on the capital stock at $t$. An implication of the guess is that $l_t$ is also iid and independent of the capital stock, and that the saving rate and work effort are positively correlated. (These features are facts you can demonstrate.)

There is a little bit of algebra involved; it is simplified by using the tranformations $\hat{s}_t = \frac{s_t}{1-s_t}$ (implying that $\frac{1}{1-s_t} = 1 + \hat{s}_t$) and $\tilde{s}_t = \xi_t \hat{s}_t$. It is then possible to find a linear equation system which solves for the different values of $\tilde{s}_t$ (you do not need to solve this system but it's good if you can state it).

Finally, determine the qualitative effects, if any, of the four different shocks on the saving rate and on work effort.