Suggested Solutions for Homework 3

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If you find any errors, or unclear sections, please help me and your classmates by emailing me at `karl.harmenberg@phdstudent.hhs.se`.

Problem 1

With persistent shocks, there will be some delay in the buildup of the capital stock (both directly due to the persistence and indirectly by consumption smoothing). The IRFs are not immediately interpretable since they are not on a log scale. Well, they can be interpreted variable by variable but if you want to do cross-variable comparisons (capital increases by much more than consumtption), I think it makes more sense to do such a comparison in percentage terms. This exercise (and the next one) certainly did not ask you to do that, but be aware in the future of how to compare the impulse response functions.

Note that even though we cannot see it in the IRFs, all variables will return to steady state eventually.

If you cannot get Dynare up and running, let me know. I omit the impulse response function here because it will look identical for all of us.

Problem 2

See attached code. The log utility, full depreciation case corresponds to $\sigma = 1, \delta = 1$. I pick arbitrary but somewhat reasonable values for the other parameters.

If we ask the program to output the savings rate (by stoch_simul(order=1,IRF=8) s;), we see that:

1. The savings rate is a function only of the third shock, which is the ξ shock.

2. Concretely, the policy function is $s_t = 0.33 - 0.2211(\xi_{t-1})$. Note that $0.33 = \alpha\beta$ and $0.2211 = (1-\alpha\beta)\alpha\beta$ in accordance with theory.[1]

[1] Linearize $\frac{\alpha\beta}{(1-\alpha\beta)\xi_t + \alpha\beta}$ around $\xi_t = 1$.

Now, let us inspect leisure. The policy function for leisure is

$$l_t = 0.383670 + 0.112793(\xi_t - 1) + 0.341797(\chi_t - 1).$$

$l_{ss} = 0.383670$ is the solution[2] to

[2] By Newton's method, see uploaded code.

$$\frac{1 - l_{ss}}{l_{ss}^{1-\nu}} = \frac{1-\alpha}{1-\alpha\beta}$$

which is the non-stochastic steady state.

Linearizing

$$\frac{1-l_t}{l_t^{1-\nu}} = \frac{1-\alpha}{\chi_t}\left(1+\frac{\alpha\beta}{1-\alpha\beta}\frac{1}{\chi_t}\right)$$

around l_{ss} gives us

$$\left(\frac{1}{l_{ss}^{1-\nu}} + (1-\nu)\frac{1-l_{ss}}{l_{ss}^{2-\nu}}\right)(l_t - l_{ss}) = \frac{1-\alpha}{1-\alpha\beta}\alpha\beta(\xi_t - 1) + \frac{1-\alpha}{1-\alpha\beta}(\chi_t - 1)$$

If we solve for the coefficients for $\xi_t - 1$ and $\chi_t - 1$, we recover 0.112793 and 0.341797.

Since leisure and the savings rate uniquely determines the laws of motion, the analytical and numerical solutions agree.

Now, let's look at some impulse response functions. It often makes more sense to measure the response of variables in percentages (as opposed to absolute changes). Therefore, I rewrite all the relations in terms of $\log c, \log k, \log l$. Now, when plotting the impulse response functions, the y axes will be percent deviation from steady state, and thus more easily interpretable. See code hw3_log.mod for details.

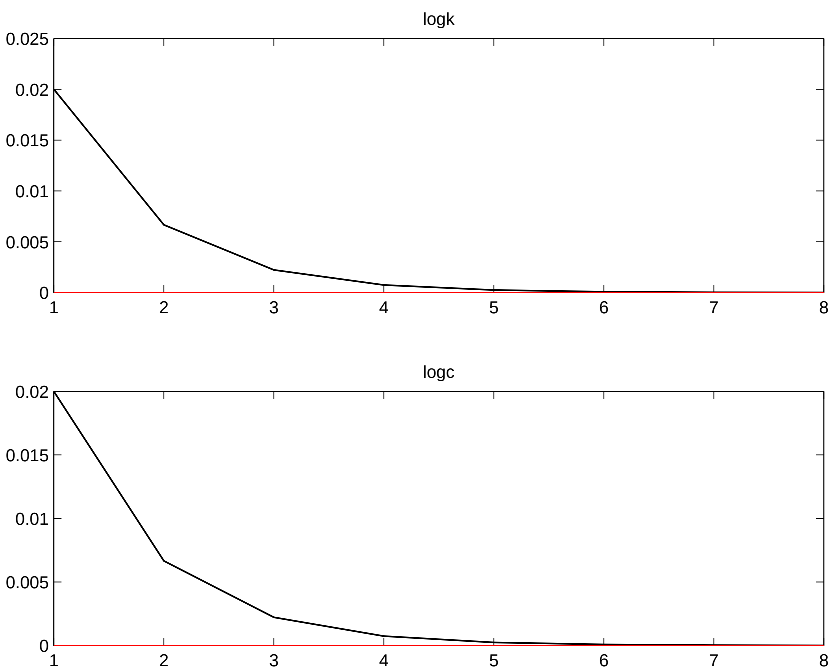

Figure 1: Impulse response functions with full depreciation, non-persistent 2 percent shock to z.

The reaction to a technology shock is as predicted, capital and consumption increase by the same magnitude as the shock, but labor is unaffected. Then capital and labor go back monotonically to steady state.

The reaction to a savings technology shock is that consumption in the period of the shock is unaffected. But since less capital (2 percent)

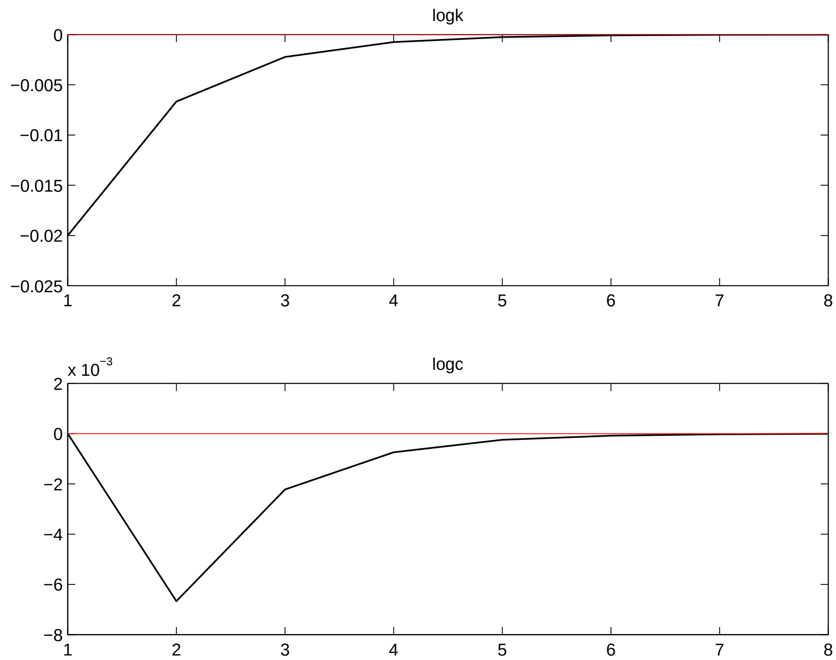

Figure 2: Impulse response functions with full depreciation, non-persistent 2 percent shock to η.

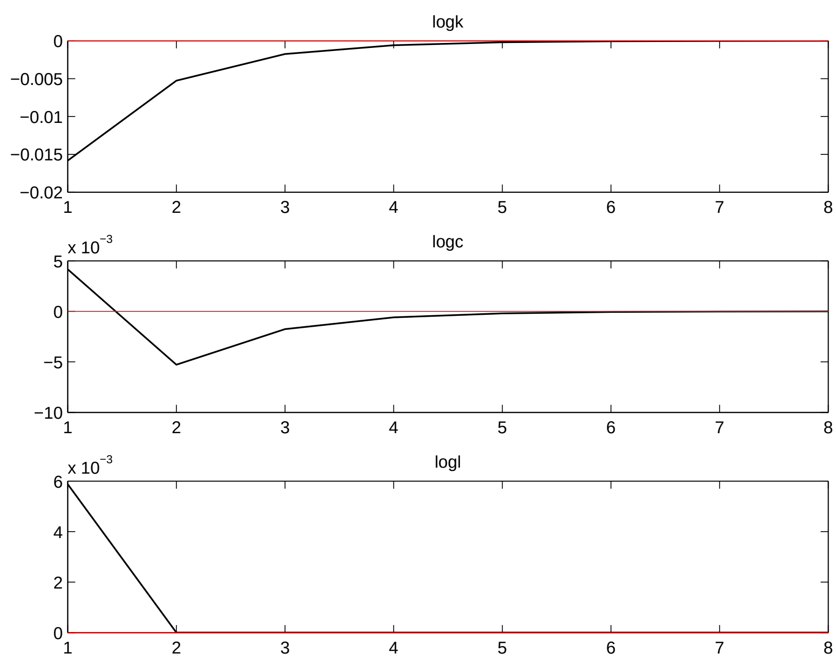

Figure 3: Impulse response functions with full depreciation, non-persistent 2 percent shock to ξ.

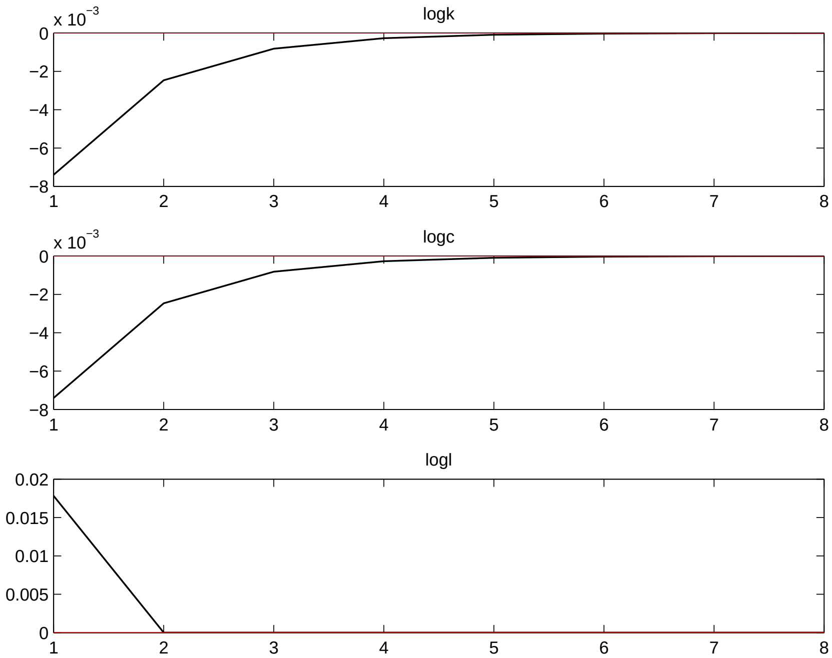

Figure 4: Impulse response functions with full depreciation, non-persistent 2 percent shock to χ.

will be saved for tomorrow, consumption will decrease after the first period and then recover back to steady state. Again, labor is unaffected.

With a period preference shock, both consumption and leisure will go up, and capital will fall. But of course, the next period the consumer will be poorer so consumption will be lower. Leisure jumps right back to steady state.

With a leisure preference shock, leisure goes up. Capital and consumption both fall by the exact same magnitude. This is because in this framework, the intertemporal and intratemporal problems are independent. The intratemporal relative shifts the ratio leisure/consumption, and the intertemporal problem keeps the capital/consumption ratio fixed.

Now, let us look at the impulse response functions when we have nonzero depreciation (concretely, I set $\delta = 0.025$).

Now, when a technology shock hits, consumption will increase by more than capital. Introducing non-full depreciation means that the capital stock is much less volatile than before. Note however that C/Y goes down (since Y increases by 2 percent) so the savings rate goes up. Leisure goes down, since without full depreciation, it is more advantageous to work in good times than it was with full depreciation.

The response to an investment technology shock is fairly easily interpretable, and similarly for a preference shock. Note that when

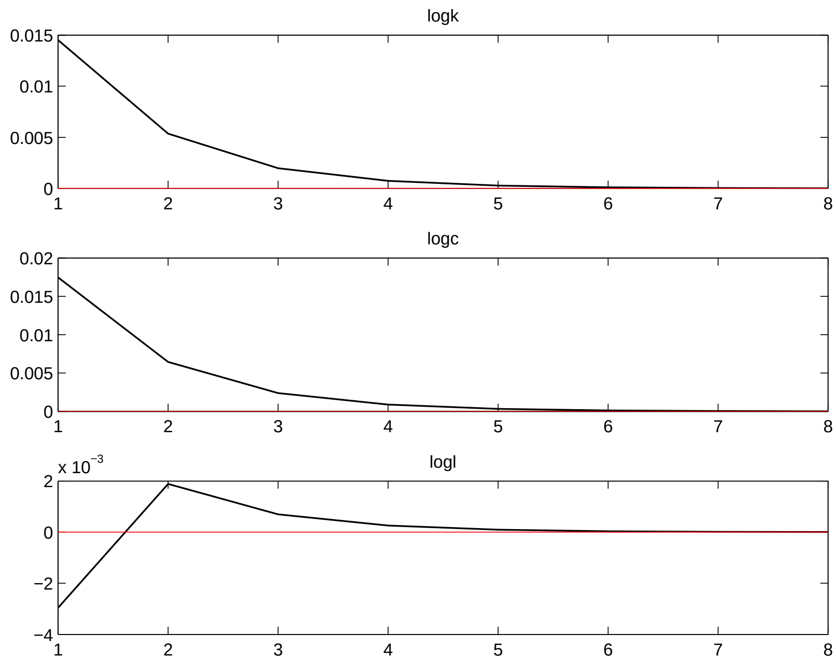

Figure 5: Impulse response functions with non-full depreciation, non-persistent 2 percent shock to z.

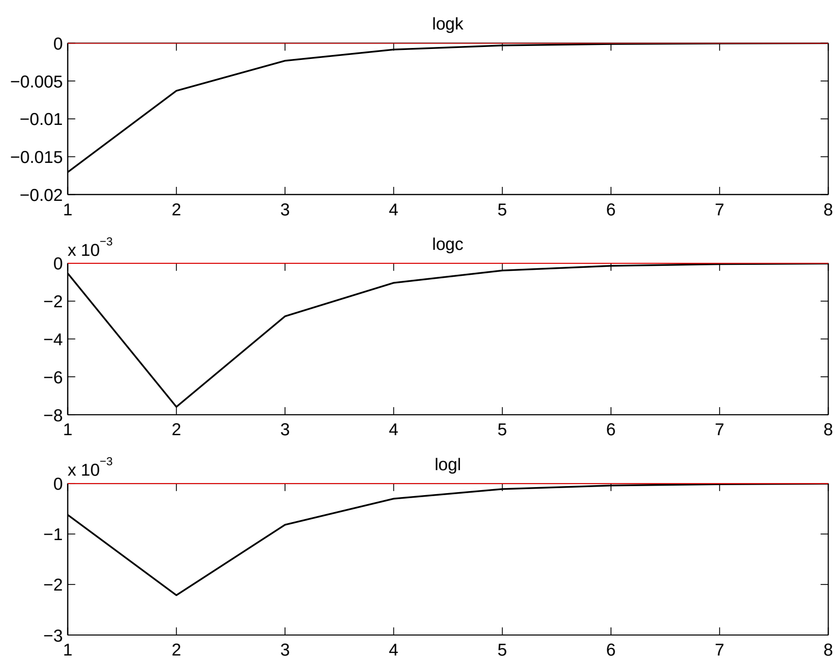

Figure 6: Impulse response functions with non-full depreciation, non-persistent 2 percent shock to η.

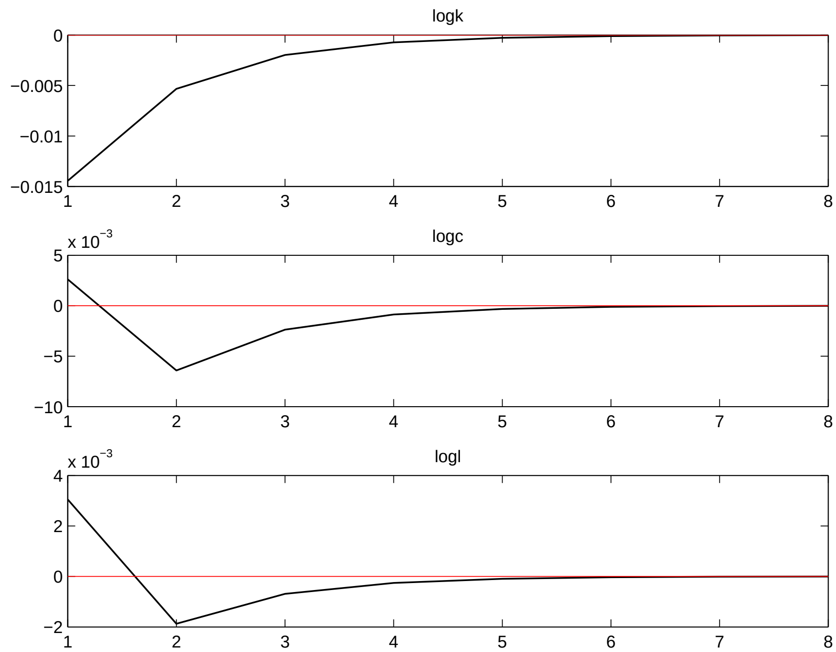

Figure 7: Impulse response functions with non-full depreciation, non-persistent 2 percent shock to ξ.

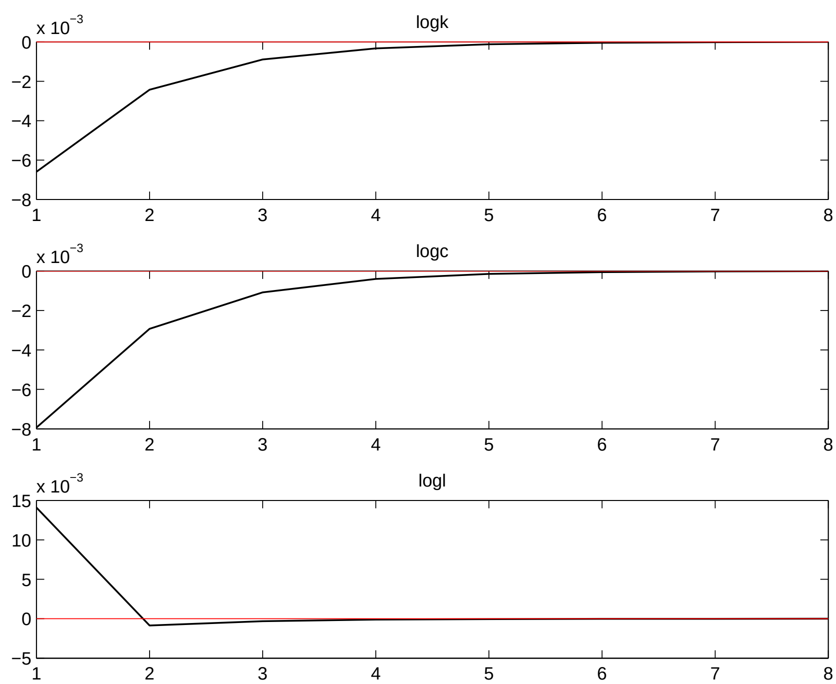

Figure 8: Impulse response functions with non-full depreciation, non-persistent 2 percent shock to χ.

below steady state, the consumer will work more, and when above steady state, the consumer will work less. This was not the case with full depreciation.

Finally, a leisure preference shock has a predictable impact. Note that it will barely have a noticeable effect on capital and consumption, since now the capital stock is much more long lived.

Things to do on your own: Vary the intertemporal elasticity of substitution, or why not any other parameter!