HOMEWORK 1, due by the first TA session

1. Cass-Koopmans model can deliver Solow

   Consider an economy with a representative, infinitely lived consumer with preferences

   $$\sum_{t=0}^{\infty} \beta^t \log(c_t),$$

   and the familiar resource constraint

   $$c_t + k_{t+1} = (1-\delta)k_t + A_t^{1-\alpha} k_t^{\alpha},$$

   where A_t is a parameter that may vary over time and $\delta = 1$.

   (a) Carefully formulate a social planner's problem for this environment.

   (b) Show that the solution to this problem amounts to a constant rate of saving (what is this rate equal to?). I.e., show that this special case delivers Solow's model in "reduced form".

2. Defining a dynamic competitive equilibrium with taxes

   Consider an economy with a representative, infinitely lived consumer with preferences

   $$\sum_{t=0}^{\infty} \beta^t (\log(c_t) - v(n_t)),$$

   where n is work effort and v is strictly increasing and strictly convex, and a resource constraint

   $$c_t + k_{t+1} = (1-\delta)k_t + F(k_t, n_t)$$

   where F is constant returns to scale.

   (a) Formulate a social planner's problem for this environment.

   (b) Carefully define a dynamic competitive equilibrium for this economy. Use an equilibrium definition like that used in class, where consumers receive wage and capital income each period, $w_t n_t$ and $r_t k_t$, respectively.

   (c) Show that a competitive equilibrium is Pareto optimal.

   (d) Suppose that there is a need to raise government revenue to be able to spend amount g of resources every period (g is a constant). We can think of g as spending on parks or defense and although they might raise people's utility or production possibilities (or lower them) we abstract from that here. Suppose also that the government taxes labor income at a proportional tax rate τ_t in period t

and that the τ_t is set so as to make the budget balance each time period (τ_t might have to depend on time since labor income may not be constant). Carefully define a dynamic competitive equilibrium for this economy. Is the equilibrium Pareto optimal?

3. <u>An endowment economy with two agents</u>

   Consider an economy with two representative agents—individuals of type A and type B, an equal number of each—who have identical preferences given by

$$\sum_{t=0}^{\infty} \beta^t \log c_t.$$

   There is no production in this economy; total output, and consumption, are just determined by exogenous endowments of the consumption good. Type-A consumers are endowed with ϵ_h units in even periods and with ϵ_l units in odd periods; type-B consumers have the reverse pattern, with ϵ_l even and ϵ_h in odd periods. Let us also assume that $\epsilon_h > \epsilon_l$.

   Borrowing and lending is allowed in the decentralized market economy. Use the following notation: at price q_t, you can buy a savings instrument (a "personal bond") at time t entitling you to one unit of consumption in period $t+1$. Thus, the gross real interest rate (on borrowing and lending) between t and $t+1$ is $1/q_t$. The market clears when the sum of all net borrowing equals zero.

   Consumers are assumed not to be able to run Ponzi schemes.

   (a) Carefully define a dynamic competitive equilibrium for this economy.

   (b) Solve for prices and quantities.

   (c) Formulate a planner's problem for this economy where the objective function is a weighted average of the utilities of the two kinds of individuals. Solve for the optimal allocation as a function of the exogenous weights. Is there a weight that reproduces the equilibrium you solved for in the previous problem?

   (d) Suppose now that markets are such that unrestricted borrowing is not allowed. Instead, there is a borrowing constraint of the following form: in no period is anyone allowed to borrow more than $\underline{b}$ units. Define a competitive equilibrium for the resulting economy. For the case $\underline{b} = 0$ (a case where the borrowing constraint WILL bind!), solve for prices and quantities.

   (e) Compare the interest rates in the economy with borrowing constraints to those you found for the case without the borrowing constraint. Over the last twenty years or so, world real interest rates have fallen. Can the model here be used to forward a possible explanation for this observation, and, if so, how?