HOMEWORK 2, due by the second TA session

1. Defining and solving for a dynamic competitive equilibrium, I

   Suppose there are two kinds of consumers, $i = 1$ and $i = 2$, of equal numbers, who both live forever and have the utility functions

   $$\sum_{t=0}^{\infty} \beta^t u_i(c_{it}),$$

   where c_{it} is the consumption of consumer i at t and $\beta \in (0,1)$. The economy has no production but merely exogenous endowments. Consumer i's budget in each period t is

   $$c_{it} + q_t a_{i,t+1} = e_{it} + a_{it},$$

   where c is consumption, a is asset holdings, e is endowments, and q is the price of a risk-free bond paying one unit of consumption tomorrow. Assume that neither consumer can run Ponzi schemes. Finally, we assume that $a_{i0} = 0$ for both is.

   There is no capital so the sum of the savings of the two consumers has to equal zero at all times.

   (a) Define a sequential competitive equilibrium for this economy.

   (b) Define a date-zero competitive equilibrium for this economy.

   (c) Suppose that $e_{1,t} = e_h$ when t is odd and $e_{1,t} = e_l$ when t is even, with $e_h > e_l$. In contrast, $e_{2,t} = e_l$ when t is odd and $e_{1,t} = e_h$ when t is even. Solve for the sequential and date-zero competitive equilibria assuming that $u_1(c) = u_2(c) = u(c)$, where u is strictly increasing and strictly concave. Interpret your findings intuitively.

   (d) Suppose that $e_{1,t} = e_{2,t} = e_h$ when t is odd and that $e_{1,t} = e_{2,t} = e_l$ when t is even, with $e_h > e_l$. Solve for the sequential and date-zero competitive equilibria assuming that $u_1(c) = \log c$ and $u_2(c) = c$. Interpret your findings intuitively.

2. Defining and solving for a dynamic competitive equilibrium, II

   Suppose there is a representative consumers who lives forever and has the utility function

   $$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

   where c_t is consumption at t and $\beta \in (0,1)$. The economy has two sectors: a consumption-goods sector and an investment-goods sector. The consumption-goods sector delivers its output according to

   $$c_t = A_t^{1-\alpha} k_{ct}^{\alpha} n_{ct}^{1-\alpha}$$

and the investment-goods sector delivers its output according to

$$i_t = q_t A_t^{1-\alpha} k_{it}^{\alpha} n_{it}^{1-\alpha}.$$

The two inputs, capital and labor, can move freely across the sectors each time period, so the only constraints for them are $n_{ct} = n_{it} = n$ (where n is an exogenous total amount of labor) and $k_{ct} = k_{it} = k_t$, where k evolves according to the accumulation equation

$$k_{t+1} = (1 - \delta)k_t + i_t.$$

(a) Define a sequential competitive equilibrium for this economy. Define the relative price of investment goods in period t by p_t.

(b) Show that the price of investment goods in this economy equals $1/q_t$. Hint: use firm maximization and the constraints on resources—the consumer's problem is not needed in the proof.

(c) Show that the equilibrium allocation of consumption and investment will be given by the solution to a planner's problem that amounts to maximizing (by choice of consumption, investment, and capital sequences) the consumer's utility function subject to the resource constraint

$$c_t + i_t/q_t = A_t^{1-\alpha} k_t^{\alpha} n^{1-\alpha}$$

and the capital accumulation equation already stated. Hint: use the results you obtained in the previous question and try to show that this resource constraint can be derived.

3. Business cycles in the data

Pick a country of your choice (one per student, not one per group of students, so you should make sure no one else picks your country!), look up data on real GDP per capita for this country, and find a source (on the internet) that identifies all recessions over the postwar period for this country. Fulfil the following tasks:

(a) Specify the sources of your information (data can be found in many places, like in the Penn World Tables, the OECD data base, or the World Bank, but it might be harder to find a reliable source pointing out when the recessions were).

(b) Plot GDP per capita over the entire period and highlight the recession periods.

(c) Read about each recession period and summarize very briefly what the causes of each of the recessions are claimed to be.

(d) Be prepared to present this in the TA session. Maximum presentation period per student: 5 minutes (try to have one slide only).