Swedish Labour Market Flows

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1 Introduction

2 Literature

The debate on the relative role of job-finding and seperation rates in explaining fluctuations in unemployment is long, but has been revived by an influential paper by **?**. He investigates US data and finds substantial procyclical variations in the job-finding rates, while the job-seperation rates is relatively acyclic.

These findings have been put into question by **?**. These authors use the same data as **?**, but uses a log decomposition of the unemployment level. Using this decomposition they find counter-cyclical seperation rates as well as countercyclical job-finding rates.

3 Method

3.1 Environment and notation

Time is given by $t \in [0, T]$. We assume the distribution data is generated by a process where all workers are identical and move between a set of labor market states

$$\mathcal{S} = \{1, \dots, S\}$$

according to a time-inhomogenous continuous-time Markov chain which is generated by a flow matrix $Q(t)$. We write $x(t) \in \mathbb{R}^S$ for the stochastic process of worker shares in different states.

Underthis definition, the unemployment rate is a one-dimensional stochastic process $\frac{x_U(t)}{x_U(t)+x_P(t)+x_T(t)}$ where x_U is the share of unemployed, x_P is the share of permanently employed, and x_T is the share of temporary employed as a percentage of all working-age people.

We write $P(t_s, t_{s+1})$ for the transition matrix between time periods t_s and t_{s+1}. [1]

We observe the distribution vector $x(t) \in \mathbb{R}^S$ at discrete time points

$$0 = t_0 < t_1 < t_2 \cdots < t_k = T.$$

We also can use the panel structure of the data to derive the transition matrix

$$P(t_s, t_{s+1}) \quad s = 1, \dots, k-1.$$ [2]

The aim of our analysis is to use the data to

1. Identify $Q(t)$

2. Decompose the time variation in unemployment into changes attributable to different elements of $Q(t)$.

3.2 Identify $Q(t)$

To identify $Q(t)$ $t \in [0, T]$ from $P(t_s, t_{s+1})$ $s \in \{1, \dots, k-1\}$. We use the well-known theorem from the theory of continuous-time Markov chains [referens] that

$$P(t_s, t_{s+1}) = \exp\left(\int_{t_s}^{t_{s+1}} Q(z)dz\right)$$

where $\exp(\dots)$ is matrix exponentiation.

[1] There has been a discussion whether it is appropriate to model the labor market as a continuous time markov chain or as composed by a mininum discrete time unit such as a week. In the Appendix we redo the analysis under different assumptions. For now, we use the continuous time assumption as it allows for an easy interpretation of flow rates.

[2] More precisely, we estimate $P(t_s, t_{s+1})$ using the distribution of t_{s+1} states of agents observed at time t_s conditional on state. Given the large number of individuals, the sampling error is small and we proceed as if the estimate is the true transition matrix.

To identify Q we further make the assumption that $Q(t)$ is constant on each measurement interval $[t_s, t_{s+1})$. Under this assumption, we estimate $Q(t)$ by

$$Q(t) = \frac{\log(P(t_s, t_{s+1}))}{t_{s+1} - t_s} \quad t \in [t_s, t_{s+1}); s \in \{1, \dots, k-1\}.$$

The matrix logarithm exists and is real-valued if and only if the negative eigenvalues of P in the Jordan composition has an even block size. If this is not true, P is not "embeddable" which means that it cannot be the result from a continuous time markov chain with constant flow matrix. If this is the case there exist algorithms to derive an approximate generating flow matrix Q.

3.3 Stationary distribution approximation

In some cases, we can approximate the observed distribution $x(t_s)$ by the corresponding steady state distribution of $Q(t)$ for $t \in [t_{s-1}, t_s)$, which we write Q^s.

When Q^s have S distinct eigenvalues, the Perron-Frobenius theorem means that it has a unique eigenvalue λ_1 which satisfies $\lambda_1 = 0$, and the eigenvalues $\lambda_1, \dots, \lambda_S$ satisfy

$$Re(\lambda_S) < Re(\lambda_{S-1}) < \cdots < Re(\lambda_1) = 0$$

where $Re(\dot{})$ gives the real part of a complex number. The eigenvector x_s^1 corresponding to λ_1 is the unique stationary distribution provided that $\sum x_s^1 = 1$ where $\sum$ denotes component-wise summation.

All eigenvalues x_s^i corresponding to λ_i with $i \geq 2$ has component sum 0. Thus, if we decompose $x(t_s)$ into its eigenvalues we get

$$x(t_s) = x_s^1 + \sum_{i=2}^{S} q_s^i x_s^i$$

i.e. the stationary distribution has coefficient one and the others have arbitrary coefficients q_s^i.

Thus, the distance of $x(t_{s+1})$ from x_s^1 is

$$\begin{aligned}
||x(t_{s+1}) - x_s^1|| &= ||x(t_s)\exp((t_{s+1} - t_s)Q^s) - x_s^1|| \\
&= ||\sum_{i=2}^{S} q_s^i \left(\exp((t_{s+1} - t_s)Q^s)\right) x_s^i|| \\
&\leq \sum_{i=2}^{S} |q_s^i| || \exp((t_{s+1} - t_s)Q^s) x_s^i|| \\
&\leq exp(\lambda_2(t_{s+1} - t_s)) \sum_{i=2}^{S} |q_s^i| ||x_s^i||
\end{aligned}$$

So we see that the distance between $x(t_{s+1})$ and its corresponding steady state is bounded above by $\exp(\lambda_2(t_{s+1} - t_s))$ where λ_2 is the second largest eigenvalue of Q^s.[3]

[3] In some papers, the important term for convergence is $f + s$ where f is job finding rate and s is job separation rate. This is because in a two-dimensional model $\begin{pmatrix} -s & s \\ f & -f \end{pmatrix}$ is the transition matrix and $-(f + s)$ is the second largest eigenvalue.

3.4 Taylor approximation

Insofar λ_2 is small and convergence is rapid, the sequence of observations $s(t_s)$ $s \in \{1, \ldots, k-1\}$ can be closely approximated as a function only of Q^s where Q^s is the value Q takes on the interval $[t_{s-1}, t_s)$.

So we obtain a sequence of observations

$$f(Q^s) \quad s \in \{1, \ldots, k-1\}$$

and want to derive how much of the variation in f can be attributed to different components of Q^s.

We want to focus on cyclical variations in Q^s and $f(Q^s)$ so we want to remove trends. Thus, we extract the trend $\bar{Q}_s$ and see how $f(Q^s) - f(\bar{Q}_s)$ is driven by the different components of Q.

The natural way to do this is to do a Taylor expansion of $f(Q^s) - f(\bar{Q}_s)$ which allows us to linearly decompose. We do a log-linear transformation and obtain

$$\log\left(\frac{f(Q^s)}{f(\bar{Q}_s)}\right) = \sum_{i,j;i\neq j} \frac{\partial f}{\partial Q_{i,j}} \frac{\bar{Q}^s_{i,j}}{f} \log\left(\frac{Q^s_{i,j}}{\bar{Q}^s_{i,j}}\right) + \mathcal{O}\left(||Q^s - \bar{Q}^s||^2\right)$$

With this formulation, we can define the percentage contribution of flow matrix element $Q_{i,j}$ as

$$\beta_{i,j} = \frac{Cov\left(\log\left(\frac{f(Q^s)}{f(\bar{Q}_s)}\right), \frac{\partial f}{\partial Q_{i,j}} \frac{\bar{Q}^s_{i,j}}{f} \log\left(\frac{Q^s_{i,j}}{\bar{Q}^s_{i,j}}\right)\right)}{Var\left(\log\left(\frac{f(Q^s)}{f(\bar{Q}_s)}\right)\right)}$$

and this will sum to 1 apart from the quadratic error term.

3.5 Decomposition without steady-state approximation

3.6 Eigenvalue analysis and appropriateness of decomposition

The convergence rate of a Markov chain to steady state can be analyzed using eigenvalue analysis. In the generic case, the flow matrix Q_t (short-hand for the value $Q(t)$ takes for the interval $[t, t+s)$) has $n$distinct eigenvalues $\lambda_1(t), \ldots, \lambda_n(t)$ which satisfy

$$\lambda_1 < \ldots \lambda_{n-1} < \lambda_n = 0$$

with corresponding eigenvectors $v_1(t), \ldots, v_n(t)$. Here, $v_n(t)$ has only non-negative value and is the unique invariant probability distribution associated with Q_t once we normalize its sum to 1.

This means that $\{v_i(t)\}_{i=1}^n$ form a basis for $\mathbb{R}^n$. Thus, we can decompose the labor market state $x(t)$

$$x(t) = \sum_{i=1}^{n} q_i(t) v_i(t)$$

for some real values q_i. It can be shown that $q_n(t) = 1$ for all Q_t which means that the invariant probability distribution vector $v_n(t)$ always gets weight 1 when we decompose the probability distribution $x(t)$ with respect to the flow matrix Q_t. Writing $\bar{x}(t)$ for this steady-state, we obtain

$$x(t) = \bar{x}(t) + \sum_{i=1}^{n-1} q_i(t) v_i(t).$$

Now, we use that if v is a left eigenvector of Q with eigenvalue λ, then v is also an eigenvalue of $\exp(sQ)$ with eigenvalue $\exp(s \times \lambda)$. Indeed

$$\begin{aligned} v \exp(Q) &= v \sum_{j=0}^{\infty} \frac{(sQ)^j}{j!} \\ &= \sum_{j=0}^{\infty} \frac{s^j \lambda^j v}{j!} \\ &= \exp(s\lambda) v \end{aligned}$$

as required. This means that we can use the decomposition to obtain

$$\begin{aligned} x(t+s) &= x(t) exp(Q_t s) \\ &= \bar{x}(t) + \sum_{i=1}^{n-1} q_i(t) v_i(t) \exp(sQ_t) \\ &= \bar{x}(t) + \sum_{i=1}^{n-1} q_i(t) \exp(s\lambda_i) v_i(t) \end{aligned}$$

As $\exp(s\lambda_i) \to 0$ for $s \to \infty$ whenever $\lambda_i < 0$ (i.e. for $i = 1, \ldots, n-1$), we obtain the classic result that $x(t+s) \to \bar{x}(t)$. Moreover, we get that that the convergence rate is determined by λ_{n-1}, the second largest eigenvalue (as all other terms become insignificant compared to the $\exp(\lambda_{n-1}s)$-term asymptotically. Thus, to analyze whether a steady-state approximation is justified we can analyze the second eigenvalue of the flow matrix.

3.7 Eigenvalues in our data

When we do an eigenvalue analysis of our estimated flow matrices, we get that the second largest eigenvalue is approximately -0.02. This means that the convergence rate is only 2% monthly and 6% on a quarterly basis. This means that a steady state analysis can be inappropriate with our data. (In standard two-state analysis in the US, the flow matrix is

$$Q = \begin{pmatrix} -s & s \\ f & -f \end{pmatrix}$$

which has a second largest eigenvalue $-(s+f)$ which has an approximate size 0.5). [To be added: Redoing our analysis without permanent/temporary partition etc].

The main problem in the data is that the loss rate is extremely low from permanent jobs in Sweden, which means that it takes a long time for the economy to adjust to a new steady state. [Could be added here: "decompose" the causes of a large second eigenvalue].

3.8 Decomposition without steady-state approximation

In this section, we outline how to decompose the cyclical component of unemployment without using a steady-state approximation.

We begin with some notation. Let λ_t^{ij} for $1 \leq i, j \leq 4$ and $i \neq j$ be the estimated flow from state i to state j between time t and $t+1$. Write

$$Q_t(\{\lambda_t^{i,j}) = \begin{cases} \lambda_t^{i,j} & \text{if } i \neq j \\ -\sum_{j \neq i} \lambda_t^{i,j} & \text{if } i = j \end{cases}$$

(We do not write $Q_t^{i,j}$ for the flow rate to emphasize that when we vary $\lambda_t^{i,j}$, we automatically vary the diagonal element of Q to make it a flow matrix, i.e. the flow matrix is a *function* of the $n(n-1)$ defined flows).

We write $\hat{\lambda}_t^{i,j}$ for the trend of the flow $\lambda_t^{i,j}$ and we write $\hat{Q}_t$ for the associated trend flow matrix obtained from $\{\hat{\lambda}_t^{i,j}\}_{i,j;i \neq j}$.

Define the trend state $\hat{x}_t \in \mathbb{R}^4$ by

$$\hat{x}_t = x_0 \exp\left(\sum_{s=0}^{t-1} \hat{Q}_s\right).$$

By the definition of Q_t, the actual state is given by

$$x_t = x_0 \exp\left(\sum_{s=0}^{t-1} Q_s\right).$$

Hence, the deviation can be decomposed into deviations between $\hat{Q}_s$ and Q_s, which in turn is driven by deviations between $\{\lambda_s^{i,j}\}$ and $\{\hat{\lambda}_s^{i,j}\}$.

We make a log-linear approximation of x to get

$$\log\left(\frac{x_t}{\hat{x}_t}\right) = \sum_{s=0}^{t-1} \sum_{i,j} \sum_{i \neq j} \left. \frac{\partial \exp(\sum_{s=0}^{t-1} Q_s)}{\partial \lambda_t^{i,j}} \right|_{Q_s = \hat{Q}_s, s=0,\ldots,t-1} \hat{\lambda}_t^{i,j} \log\left(\frac{\lambda_t^{i,j}}{\hat{\lambda}_t^{i,j}}\right) + \varepsilon_t$$

where $\varepsilon_t$is an error term which is quadratic in $\{\lambda_t^{i,j}\}$. Define

$$\Delta_t^{i,j} = \sum_{s=0}^{t-1} \left. \frac{\partial \exp(\sum_{s=0}^{t-1} Q_s)}{\partial \lambda_t^{i,j}} \right|_{Q_s = \hat{Q}_s, s=0,\ldots,t-1} \hat{\lambda}_t^{i,j} \log\left(\frac{\lambda_t^{i,j}}{\hat{\lambda}_t^{i,j}}\right)$$

as the contribution of variations in the flows from $i$to $j$to overall variations in x. Let $x(k)$ be the k^{th}component of x and $\Delta_t^{i,j}(k)$the k^{th} component of Δ. Then define the contribution of i, j as

$$\frac{Cov(\Delta_t^{i,j}(k), \log\left(\frac{x_t(k)}{\hat{x}_t(k)}\right)}{Var\left(\log\left(\frac{x_t(k)}{\hat{x}_t(k)}\right)\right)}.$$

This expression sums to 1apart from the covariance between trend deviations in $x_k(t)$ and the quadratic error term $\varepsilon_t(k)$.

4 Relation to other papers

Shimer looks at covariance between $\frac{\bar{f}}{x_t+\bar{f}}$ to $\frac{f}{x+f}$ once both have been detrended. It is not clear that this is a linear decomposition or where the approximation is made around. $\bar{f}$ is the long-run average, but deviation is from trend. ah Fujita & Ramey: Use similar method but in two dimensions.

5 Data

5.1 Data source and selection

Data derives from the Swedish Labour Force Survey (LFS) during the period 1987-2012 (**?**). The survey began in 1961, but micro data is available for 1987 and onwards. During this period the survey samples 17 000-29 500 individuals each month sampled from a register containing the Swedish population (RTB). Up to 2001 the sample includes all ages 16-64, but in 2001 the age interval was expanded to 15-74. For consistency, we will however confine our sample to ages 16-64.

The survey uses a rotating sample (Figure 1). Specifically, a participating individual is interviewed about his or her employment status at a given week every third month during two years. In each month 8 groups of individuals are interviewed. 7 of these 8 groups have been interviewed previously, while 1 group is being interviewed for the first time. Consequently, one group is rotating out(in) of the sample at each point in time.

Overall, the surveyed population is divided into the following categories (i) employed, (ii) unemployed or (iii) outside the labour force (Figure 1). The individual is characterised as *employed* if she was working at least one hour during the reference week as (a) self-employed (including helping spouse) (c) on a permanent contract (*tillsvidereanstallning*) or (d) on a time-limited contract. An individual who has a job but was absent work due to illness, leave, vacation, military service, a conflict or similar is also counted as employed. So is an individual in a labour market program, if she receive some enumeration from the employer. The individual is characterised as *unemployed* if she is not employed, but has been applying for work within the last four weeks and is able to start work within the reference week or the two following weeks. An individual is also characterised as unemployed if the person is set to start a job within the next three months, provided that the individual would be ready to start already in the reference week or during the following two weeks. Finally, an individual is characterised as *outside the labor force* if she is not covered by the definitions above. This includes individuals who would and could be able to work, but who did not actively seek jobs (*latent unemployment*).

In 2007 the treatment of students was altered, so as to comply with ILO definitions. Up to this point full-time students who also applied for jobs were counted as outside the labour force. But from 2007 these are counted as unemployed. For the age group 16-64, which we confine our sample to, this definition is applied through the entire sample.

THIN DATA CORRECTION.

5.2 Labor market stocks and flows

Figure 3 shows the stock of employed, unemployed and inactive in Sweden since 1987. The period covers approx. two business cycles as unemployment troughs and peaks in 1990/2008 and 1996/2001, respectively. The overall employment rate is falling approximately 10 percentage points through the period (from 82 to 70 percent), with inactivity rising similarly. The change in the overall employment rate covers diverse changes for permanent and temporary employment, however. While the share of the population on permanent employment is down 10 percentage points, the share on temporary employment has risen slightly. Consequently, the share of workers on temporary contracts has increased from 20 to 24 percent.

Figure 4-5 show the flows moving between the four groups as hazard rates. From these figures two trends are visible. First, all the flow rates into permanent and temporary employment have decreased. Second, the probability of flowing from employment into unemployment has increased. There is no clear trend in the hazard rates for movements into inactivity.

5.3 Cyclicality of flows

Table 2 illustrates the sensitivity of labour market flows with respect to the business cycle. We gauge this by regressing the relevant hazard rate (logged) on the the unemployment rate and a linear trend. Table 2 reports the coefficient on the unemployment rate from this regression. A positive(negative) value in Table 2 indicates that the correlation between the relevant flow and unemployment is positive(negative). Consequently, a negative(positive) value indicates that the flow is pro-cyclical (counter-cyclical). Thus, from Table 2 we see that all flows into permanent and temporary employment are procyclical, while all flows into unemployment are countercyclical. The flow from permanent employment to inactivity is from regular(temporary) employment is procyclical(acyclical), while the flow from temporary employment to inactivity is countercyclical.

Table 1: Illustration of the rotating panel structure

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
Month 0	$\mathbf{G_0}$	G_{-3}	G_{-6}	G_{-9}	G_{-12}	G_{-15}	G_{-18}	G_{-21}
Month 1	G_1	G_{-2}	G_{-5}	G_{-8}	G_{-11}	G_{-14}	G_{-17}	G_{-20}
Month 2	G_2	G_{-1}	G_{-4}	G_{-7}	G_{-10}	G_{-13}	G_{-15}	G_{-19}
Month 3	G_3	$\mathbf{G_0}$	G_{-3}	G_{-6}	G_{-9}	G_{-12}	G_{-14}	G_{-18}
Month 4	G_4	G_1	G_{-2}	G_{-5}	G_{-8}	G_{-11}	G_{-14}	G_{-17}
Month 5	G_5	G_2	G_{-1}	G_{-4}	G_{-7}	G_{-10}	G_{-13}	G_{-16}
Month 6	G_6	G_2	$\mathbf{G_0}$	G_{-3}	G_{-6}	G_{-9}	G_{-12}	G_{-16}
Month 7	G_7	G_4	G_1	G_{-2}	G_{-5}	G_{-8}	G_{-11}	Gdv_{-14}
Month 8	G_8	G_5	G_2	G_{-1}	G_{-4}	G_{-7}	G_{-10}	G_{-13}
Month 9	G_9	G_6	G_3	$\mathbf{G_0}$	G_{-3}	G_{-6}	G_{-9}	G_{-12}
Month 10	G_{10}	G_7	G_4	G_1	G_{-2}	G_{-5}	G_{-8}	G_{-11}
Month 11	G_{11}	G_8	G_7	G_4	G_1	G_{-2}	G_{-5}	G_{-8}
Month 12	G_{12}	G_9	G_6	G_3	$\mathbf{G_0}$	G_{-3}	G_{-6}	G_{-9}

Notes: G_n is group of individuals who entered the survey in month n. Each group is surveyed with an interval of 3 months. In each month 7/8 of the sample has been surveyed before. 1/8 of the sample is surveyed for the first time.

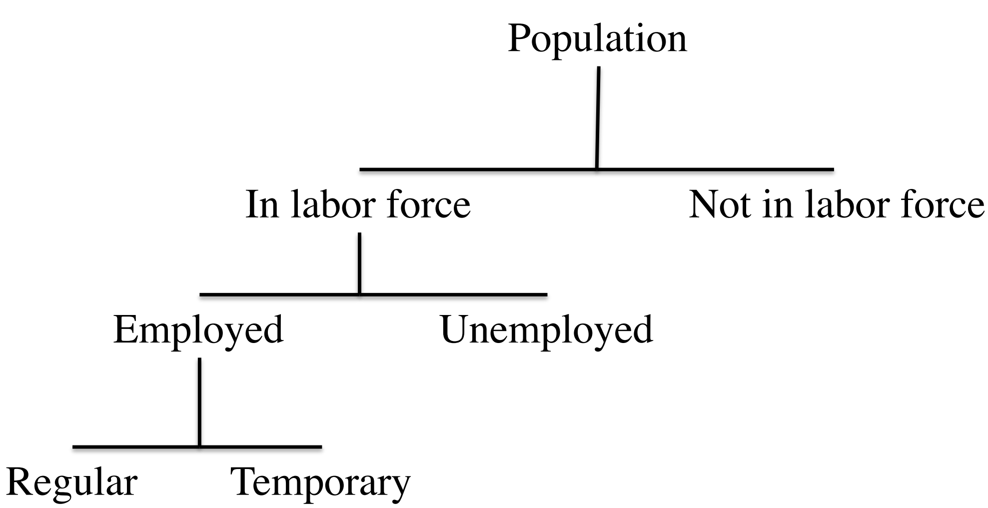

Figure 1: Basic classification of population in labor force survey

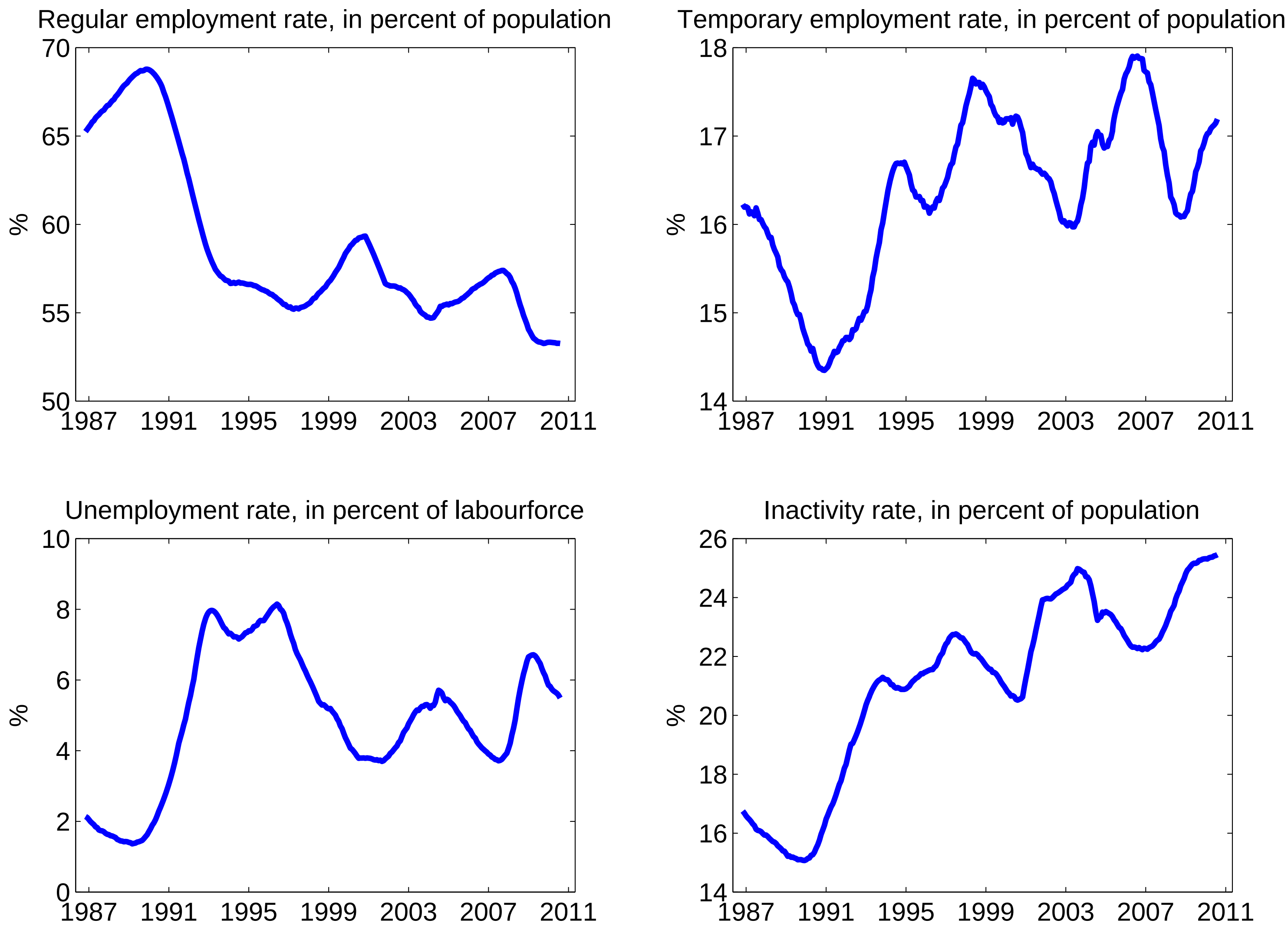

Figure 2: Labour market stocks across time

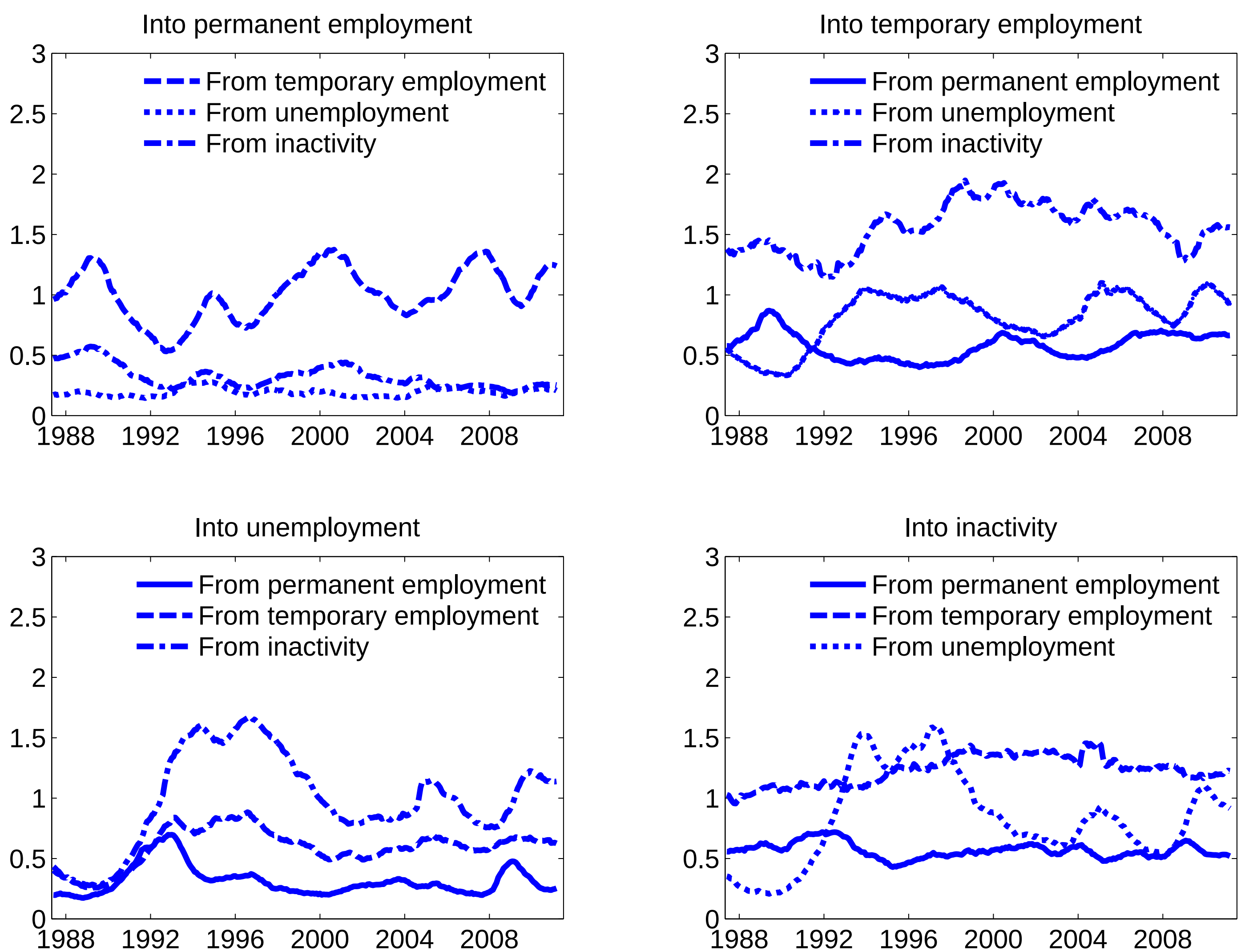

Figure 3: Gross flows between labour market states

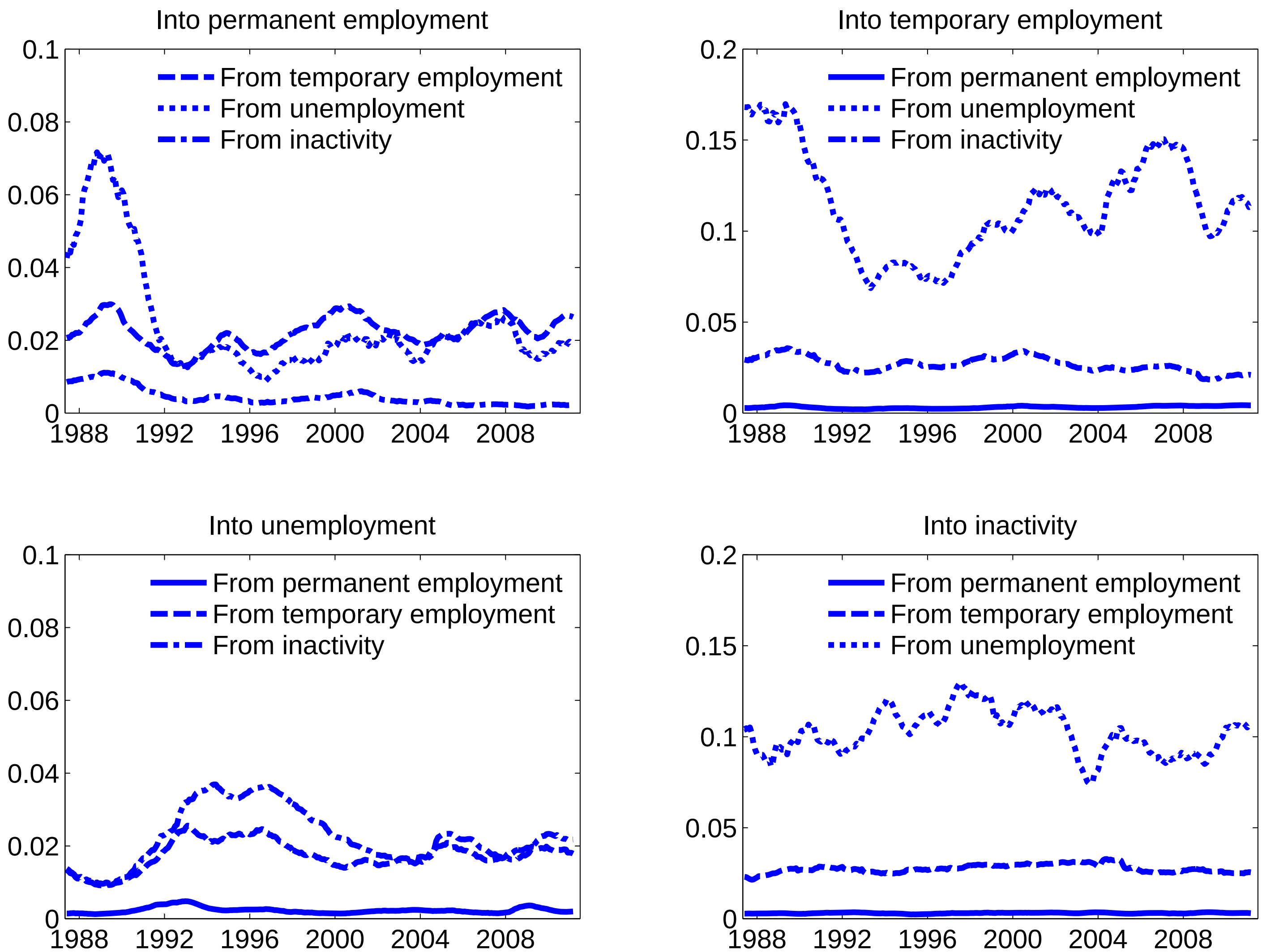

Figure 4: Hazard rates for transition across labour market states

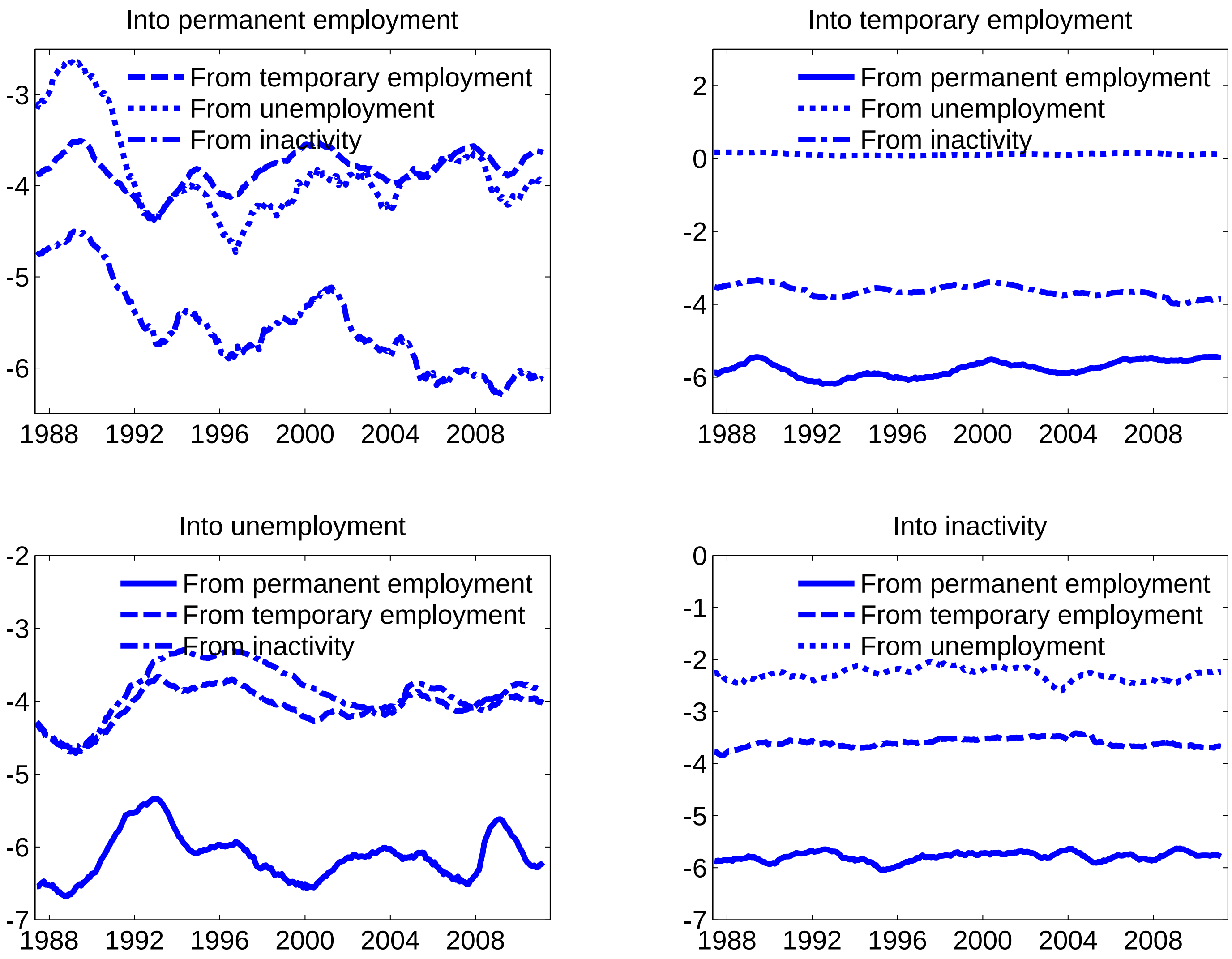

Figure 5: Hazard rates for transition across labour market states, logged

Table 2: Cyclical variation in hazard rates

	Permanent emp.	Temporary emp.	Unemployment	Inactivity
Permanent emp.		-6.8117^{***}	8.0218^{***}	-0.9282^{***}
Temporary emp.	-6.6049^{***}		11.3282^{***}	-0.0401
Unemployment	-20.5563^{***}	-12.1247^{***}		3.1729^{***}
Inactivity	-10.5943^{***}	-2.9194^{***}	18.5694^{***}	

Notes: The reported figure is the the coefficient of the unemployment rate in a regression of the relevant hazard rate logged. Time trends are included in the regression. *(**)[***] denotes significance on 10(5)[1] pct. level. Sample period is 1987m1-2011m9.

6 Conclusion

References