Problem Set 1 - Due on February 18th

In class, we have covered the calibration and simulation of a model with a single sector and no input-output structure. In this problem set, we will work on the calibration and simulation of a model with 2 sectors and input-output structure — basically a Caliendo and Parro (2015) structure. This will give you the basic structure to understand a very large number of applications of these types of models. I strongly suggest reading Caliendo and Parro (2015) before starting this problem set.

Part 1 - Writing Down the Model

In this first part of the problem set, we will write down the equations of the model. I will explain the model in words, your task is to write down the equations to solve for the general equilibrium of the model, following the guide provided below.

The economy consists of many countries i and n, where $i, n \in \mathcal{I}$, and sectors k, $k \in \mathcal{K}$. In each country, there is a representative household with Cobb-Douglas preferences for composite goods from each sector, with consumption shares given by μ^k. In addition, in each country there are producers of composite intermediate goods who combine the intermediate goods using a CES aggregator. Within each sector, the economy operates as in EK (2002). The dispersion of productivities across intermediate goods is given by θ^k and the level of productivity draws is controlled by T_i^k. The production of intermediate goods use a Cobb-Douglas production function that employ labor and composite intermediate goods. The cost share of labor in production is β^k and the cost share of intermediates is $1-\beta^k$, where $\beta^k \in [0,1]$. The cost share of composite goods from sector k' used in sector k is $\lambda^{k'k}$. Trade costs d_{in}^k are iceberg. Trade is balanced. Markets are perfectly competitive.

1. Write down the income equation for households E_n.
2. Given total firm revenues X_n^k, write down the labor market clearing equation.
3. Given total exports X_{in}^k, write down the equation for total firm revenues X_n^k.
4. Write down the total expenditure of country i in composite goods from sector k coming from households and from firms, E_n^k.
5. Given a multilateral resistance term Φ_i^k and the cost of a bundle of inputs c_i^k, write down the equation for total exports from country i to country n in sector k.
6. Write down the expression for the multilateral resistance term.
7. Write down the expression for the price index of composite goods from each country i in sector k, P_i^k.
8. Write down the price of a bundle of intermediate inputs for each sector, define it as m_i^k.
9. Write down the expression for the cost of a bundle of inputs c_i^k.

Part 2 - Simulating the Model

We will now simulate the model using some fake data that I produced. We will work with "fundamental_for_simulation.mat" and "data_square_world.mat".

1. Load the data "fundamental_for_simulation.mat". Check the dimensions of the fundamentals of the model.
2. Create $\theta^k = [4,4]$, $\mu^k = [0.2, 0.8]$, $\beta^k = [0.5, 0.5]$ and $\lambda^{k'k} = [0.8, 0.2; 0.2, 0.8]$.
3. Create a guess of wages w_i and price indexes P_i^k.
4. Write down the code and simulate the model given the fundamentals provided in "fundamental_for_simulation.mat".
5. Export total firm revenues X_n^k, household expenditure on goods from each sector μ^k, share of payments to labor β^k, and share of payments to goods from sector k' to sector k, $\lambda^{k'k}$. This is typically what we observe in the data. Notice that μ^k, β^k, and $\lambda^{k'k}$, under the Cobb-Douglas assumption, can be constructed before the calibration of the model.

Part 3 - Calibrating the model

We will learn how to calibrate the model with multiple sectors in this step.

1. Write the code to recover the trade cost of each sector using a gravity equation.
2. Write down the code that recovers the level shifters given trade costs.
3. Check if the trade flows generated by the model match the one generated by the data.

Part 4 - Simulating the Model after Calibration

Lastly, let's look at some counterfactuals!

1. Increase the TFP of country 1 in sector 1 by 20 percent. Simulate the model save results.
2. Increase the TFP of country 2 in sector 1 by 20 percent. Simulate the model save results.
3. Make figures of a map (similar to what we did in class)
   (a) Change in welfare in counterfactual 1
   (b) Change in welfare in counterfactual 2
   (c) Compare results and discuss the economics. Why is the impact larger in one case versus the other?