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Identification of the one-diode model for photovoltaic modules from datasheet values

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Abstract

In recent years several numerical methods have been proposed to identify the one-diode model for photovoltaic modules by introducing simplifications/approximations techniques or by using suitable data interpolations from the panel characteristic curves. In this paper a complete theoretical and practical analysis on the extraction of the five parameters identifying the one-diode model for photovoltaic modules from data available on PV panel datasheets is proposed. The present theoretical analysis is utilized, from a hand, to gain insight into this model and, at the same time, to develop few practical rules to strongly improve both the accuracy of the solutions and the computational costs with respect the performances present in literature. In particular, we prove how it is possible to separate the independent variables from the dependent ones within the system by using suitable algebraic manipulations of the commonly used equations. This splitting of the parameters allows a new paradigm in the writing of the open circuit, short circuit and maximum power point constraints, giving the possibility both to exactly verify these conditions and to reduce the dimensions of the search space. Moreover, thanks to this paradigm we have the possibility to address the issues regarding the existence, the uniqueness and the meaningless of the solutions of the identification problem under specific conditions. In this way, the feasible domain of solutions and the conditions for the existence of a unique solution as well as the unphysical solutions will be proven and justified. Lastly some critical issues of the approaches currently used in literature for this problem are discussed and an adequate solution of the identification of the one-diode model is provided for PV designers. All the analytical dissertations presented in this work, as well as the obtained results, have been validated on hundreds of PV panels belonging to the California Energy Commission database.

Keywords: Photovoltaic panel; Five parameters model; Reference data; *I–V* characteristic

1. Introduction

In the last decade, the photovoltaic (PV) energy market has experienced an extremely rapid expansion which has led to the request of effective tools for the estimation and the prediction of the electrical power produced by PV plants, as described by Rajapakse and Muthumuni (2009), Menicucci and Fernandez (1988), PVWATTS (2011), Blair et al. (2010). This task is not simple to achieve since PV energy depends on weather and environmental factors (temperature, irradiance, sky conditions and so on) which are not easily predictable. Indeed these elements can only be estimated through statistical data, as shown in Cameron et al. (2008) and Kaplani and Kaplanis (2012). Moreover the architecture and the technology for designing a PV system are other important factors to be taken into account (Su et al., 2012; Cameron et al., 2008; Zhou

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et al., 2007; Tian et al., 2012). In every case for increasing the global efficiency, it is very important to use an adequate PV model for the Current/Voltage outputs prediction under real operating conditions, that is real temperature and irradiance conditions (Zhang et al., 2013; Velasco-Quesada et al., 2009; Davis et al., 2001; Hernandez et al., 2013). In the last twenty years hundreds of works have been written about photovoltaic module/array representation by means of circuital equivalent model able to describe *I–V* curves (see for example the reference cited in the following and the reference within). Among the used models, the one diode equivalent representation (also known as 'five-parameter model' since it is governed by five parameters) is the most adopted in scientific literature as well as for technological applications (for example its use for maximum power point or solar irradiance prediction, such as the one proposed in Mancilla-David et al. (2014), Ma et al. (2013), Carrasco et al. (2013) and Papaioannou and Purvins (2012)). Indeed, although more complex models, such as for example the two and three-diode ones, have been introduced for taking into account different issues (carrier recombination and the leakage current effects), the one-diode model still guarantees a sufficient degree of accuracy with a lower complexity and computational cost during simulations. The set up of this equivalent circuital model consists of two phases: (i) the extraction of the five parameters at standard reference conditions (SRC) or in other specified conditions, by solving a suitable inverse problem; (ii) the use of adequate relations for expressing the dependence of the parameters found in the previous step (i) in such a way that the one-diode model becomes able to describe the behavior of the *I–V* curves for different values of temperature and irradiance. About this latter point, some authors proposed alternative techniques based on the adoption of additional parameters or by making use of experimental data and fitting approaches Beckman (xxxx). Moreover, also several different equations for the description of the dependence of parameters from temperature and irradiance have been proposed. In Mermoud and Lejeune (2010) an interesting experimental study about this dependencies widely discussed for various technologies, whereas the relation among parameters at any temperature/irradiance conditions is discussed also in a very interesting way by Brano et al. (2010) and Orioli and Gangi (2013), where further developments of the five-parameter model are presented. About the former problem, that is the extraction of reference parameters, two different approaches are usually followed: one based on the use of only data provided by manufacturer in datasheet or available in databases of PV panels, e.g. CEC (California Energy Commission, 2013) or PHOTON (Photon.info, 2013) databases; and the other one based on the use of measurements and specific fitting procedure. The main advantages of the first listed approach is that it uses the information from datasheet without requiring a specific experimental procedure that would increase the cost of PV system set up (with an almost negligible advantage for the performance of the system, also considering the effect of the aging of the PV panel on its performance). The aim of this paper is to investigate mathematically the problem of the exact extraction of the five parameters at SRC. Indeed, although this topic has been widely discussed in literature some approaches/approximations currently followed are based more on personal knowledge of the researchers than to a general methodology or a rigorous mathematical approach. Indeed from a hand, several authors propose different equations/approaches to be used to identify the model; whereas, on the other hand, many kind of numerical techniques are suggested to solve the inverse problem of the extraction of the five parameters. The high number of works (in this paper we cited just the most recent works) focusing on this issue prove that there is a strong interest towards the problem of the identification of the PV models and the different approaches (analytical, numerical and soft-computing techniques) used to extract the parameters also prove that this problem is not so simple to solve. This is surely due to the nature of the set of the equations (transcendental), which can only be solved numerically or by using approximation methods. In addition, due to the nonlinear nature of the system, the solution is very sensitive to the choice of the initial guesses (Dobos, 2012; Li et al., 2013). In order to overcome the problem of managing the transcendental equations, several analytical approaches have been introduced: xian Lun et al. (2013b) present an explicit *I–V* model of a solar cell which uses Padé approximation, i.e. the exponential function is approximated by means of a rational function; xian Lun et al. (2013a) again use the Taylor series; in Jain and Kapoor (2004, 2005), Chen et al. (2011) and Ghani and Duke (2011) the *I–V* relations are more explicitly written by means of the *W* Lambert function Corless et al. (1996), and then the extraction of the five parameters is performed by numerical techniques. In Li et al. (2013) the comparison between several techniques to identify the five-parameter model is presented by using the criteria of applicability, convergence, stability, calculation speed and error on various types of *I–V* data. Recently, Laudani et al. (2013) proposed the splitting of unknowns of the five-parameter model in independent and dependent achieving a reduced form consisting of two effective parameters thus improving the efficiency of the solving algorithm; in Laudani et al. (2014a) it has been proven that the reduced forms can be usefully adopted also to solve the extraction of the five parameters also from measurements. In this paper, by following an analogous approach, we present a more general complete (both theoretical and practical) analysis which allows us to gain insight to the five-parameter model. In particular, we discuss the issue of a feasible domain for the parameters, we prove the existence/uniqueness of the solutions by a full mathematical approach. We also show that a critical issue is the existence of a maximum admissible value for ideality factor (dependent from open circuit, short circuit and maximum power point conditions) which can be related to the origin of the unphysical solutions of the five parameters model. In addition, some critical issues will be shown regarding the five-parameter model and their consequences on the approaches currently used in literature for extracting the parameters will be discussed. All the theoretical analysis presented and the results proposed in this work have been validated on hundreds of PV panels belonging to the California Energy Commission database.

2. The one-diode model and the extraction of its five reference parameters

The equivalent circuit of the one-diode model is shown in Fig. 1. The relation between current I and voltage V at the two-pole output is:

$$I = I_{\mathrm{irr}} - I_0\left[e^{\left(\frac{q(V+IR_S)}{N_S nkT}\right)} - 1\right] - \frac{V+IR_S}{R_{SH}} \tag{1}$$

where I_{irr} is the irradiance current (photocurrent); I_0 is the cell reverse saturation current (diode saturation current); q is the electron charge ($q = 1.602 \times 10^{-19}$ C); n is the cell ideality factor, k is the Boltzmann constant ($k = 1.3806503 \times 10^{-23}$ J/K); T is the cell temperature; N_S is the number of PV cells connected in series; and R_S and R_{SH} represent the cell series and shunt resistance, respectively. In (1), the variables $n, R_S, I_{\mathrm{irr}}, I_0$, and R_{SH} can be assumed dependent or not from temperature T and irradiance S and they are in function of certain reference parameters at SRC ($S_{\mathrm{ref}} = 1000\frac{W}{m^2}, T_{\mathrm{ref}} = 25\ ^\circ\mathrm{C}$).

$$n = n_{\mathrm{ref}} \tag{2}$$

$$R_{\mathrm{S}} = R_{\mathrm{S,ref}} \tag{3}$$

$$I_{\mathrm{irr}} = \frac{S}{S_{\mathrm{ref}}}[I_{\mathrm{irr,ref}} + \alpha_T(T - T_{\mathrm{ref}})] \tag{4}$$

$$I_0 = I_{0,\mathrm{ref}}\left[\frac{T}{T_{\mathrm{ref}}}\right]^3 e^{\left[\frac{E_{\mathrm{g,ref}}}{kT_{\mathrm{ref}}} - \frac{E_g}{kT}\right]} \tag{5}$$

$$E_g = 1.17 - 4.73 \times 10^{-4} \times \frac{T^2}{T + 636} \tag{6}$$

$$R_{SH} = r_{sh}\left(\frac{S_{\mathrm{ref}}}{S}\right)R_{\mathrm{SH,ref}} \tag{7}$$

where $E_{\mathrm{g,ref}}$ is the band gap energy at $T_{\mathrm{ref}} = 298.16$ K computed by means of (6). The function $r_{sh}(\cdot)$ in (7) takes into account the dependence of R_{SH} with respect to the solar irradiance S and changes according to the adopted model:

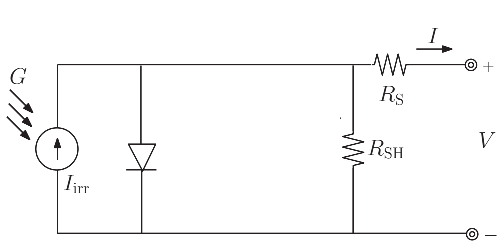

Fig. 1. One-diode equivalent circuit for a PV module.

for example, in the five-parameter model of Desoto et al. (2006), an inversely proportional dependence is used, whereas in PVSYST model (Mermoud and Lejeune, 2010), an exponential dependence is proposed. Different relations are also presented in several works, such as Brano et al. (2010) that, however, do not affect the present analysis since the identification of the reference parameters is always performed at SRC (i.e. $S = S_{\mathrm{ref}} = 1000\ \mathrm{W\ m^{-2}}$, $T = T_{\mathrm{ref}} = 25\ ^\circ\mathrm{C}$), so that $I_{\mathrm{irr}} = I_{\mathrm{irr,ref}}$, $I_0 = I_{0,\mathrm{ref}}$ and $R_{SH} = R_{\mathrm{SH,ref}}$. The extraction procedure of the five-parameter model makes use of S_{ref} and T_{ref} plus the following datasheet-based information: open-circuit voltage ($V_{\mathrm{OC,ref}}$); short-circuit current ($I_{\mathrm{SC,ref}}$); voltage ($V_{\mathrm{mp,ref}}$) and current ($I_{\mathrm{mp,ref}}$) at maximum power; the absolute temperature coefficients of the open-circuit voltage (β_T) and short-circuit current (α_T). In (2)–(7), there are five unknown parameters at SRC: $n_{\mathrm{ref}}, R_{\mathrm{S,ref}}, I_{\mathrm{irr,ref}}, I_{0,\mathrm{ref}}$, and $R_{\mathrm{SH,ref}}$. Other approaches make use of more data (usually experimental data, without take into account the open circuit, short circuit and maximum power point conditions) and adopt fitting procedures that often operate on inaccurate I–V curves.

3. Theoretical analysis of the solution of the extraction problem from available datasheet values

The one diode model has five unknown parameters and then for the identification requires a system of five independent equations, written by starting from the Eq. (1) and the aforementioned datasheet information. The first equation arises at the open circuit condition at SRC; the second equation by using short circuit conditions at SRC; the third equation by exploiting the MPP conditions; the fourth and the fifth equations are not the same for all the models. In the models which use only datasheet values the fourth equation is usually written by imposing the slope of I–V curve at MPP obtained by $\frac{dV \cdot I}{dV} = 0$ at MPP, as in Desoto et al. (2006), Tian et al. (2012), Rajasekar et al. (2013), Laudani et al. (2013) and Dobos (2012) and the fifth equation is written by imposing the dependence of temperature at open circuit condition for $S = S_{\mathrm{ref}}$ (Desoto et al., 2006; Tian et al., 2012; Laudani et al., 2013). Other models, such as in Brano et al. (2010), Orioli and Gangi (2013), Chen et al. (2011) and Chouder et al. (2012) write the fourth and the fifth equation by using the slope at short circuit condition and the slope at open circuit condition at SRC (or other similar approaches). Nevertheless, unfortunately, these latter data cannot be simply recovered by manufacture's datasheet and often additional assumptions are made.

3.1. The first three equations: open circuit, short circuit and maximum power point conditions

As previously stated, usually the different approaches proposed for the extraction of the five parameters share the same three equations coming out from the open circuit (OC, $V = V_{\mathrm{OC,ref}}, I = 0$), short circuit (SC,$V = 0, I_{\mathrm{SC,ref}}$)

and MPP conditions at SRC ($V = V_{\mathrm{mp,ref}}, I = I_{\mathrm{mp,ref}}$). These three conditions are the ones used to identify the three parameter model (that is, the one which assume $R_{\mathrm{S,ref}} = 0$ and $R_{\mathrm{SH,ref}} = 0$). In the following we propose some algebraic manipulations which allow us to obtain interesting results about the mutual dependence between the five parameters. Before writing the equations, in order to simplify the notation, let us utilize the following definitions:

$$G_{\mathrm{SH,ref}} = \frac{Exp_{\mathrm{OC}}(I_{\mathrm{mp,ref}} - I_{\mathrm{SC,ref}}) + Exp_{MP}I_{\mathrm{SC,ref}} - Exp_{SC}I_{\mathrm{mp,ref}}}{A_1Exp_{SC} + A_2Exp_{MP} + A_3Exp_{OC}} \tag{22}$$

$$I_{0,\mathrm{ref}} = \frac{V_{\mathrm{OC,ref}}(I_{\mathrm{SC,ref}} - I_{\mathrm{mp,ref}}) - V_{\mathrm{mp,ref}}I_{\mathrm{SC,ref}}}{A_1Exp_{SC} + A_2Exp_{MP} + A_3Exp_{OC}} \tag{23}$$

$$I_{\mathrm{irr,ref}} = \frac{I_{\mathrm{SC,ref}}V_{\mathrm{OC,ref}}(Exp_{MP} - 1) + I_{\mathrm{SC,ref}}V_{\mathrm{mp,ref}}(1 - Exp_{OC}) + I_{\mathrm{mp,ref}}V_{\mathrm{OC,ref}}(1 - Exp_{\mathrm{SC}})}{A_1Exp_{SC} + A_2Exp_{MP} + A_3Exp_{OC}} \tag{24}$$

$$V_T = \frac{kT_{\mathrm{ref}}}{q} \tag{8}$$

$$Exp_{SC} = exp\left(\frac{I_{\mathrm{SC,ref}}R_{\mathrm{S,ref}}}{N_S n_{\mathrm{ref}} V_T}\right) \tag{9}$$

$$Exp_{OC} = exp\left(\frac{V_{\mathrm{OC,ref}}}{N_S n_{\mathrm{ref}} V_T}\right) \tag{10}$$

$$Exp_{MP} = exp\left(\frac{V_{\mathrm{mp,ref}} + I_{\mathrm{mp,ref}}R_{\mathrm{S,ref}}}{N_S n_{\mathrm{ref}} V_T}\right) \tag{11}$$

$$P_1 = V_{\mathrm{mp,ref}}I_{\mathrm{mp,ref}} \tag{12}$$

$$P_2 = (V_{\mathrm{OC,ref}} - V_{\mathrm{mp,ref}})I_{\mathrm{mp,ref}} \tag{13}$$

$$P_3 = (V_{\mathrm{OC,ref}} - V_{\mathrm{mp,ref}})(I_{\mathrm{SC,ref}} - I_{\mathrm{mp,ref}}) \tag{14}$$

$$P_4 = V_{\mathrm{mp,ref}}(I_{\mathrm{SC,ref}} - I_{\mathrm{mp,ref}}) \tag{15}$$

$$A_1 = V_{\mathrm{mp,ref}} + R_{\mathrm{S,ref}}I_{\mathrm{mp,ref}} - V_{\mathrm{OC,ref}} \tag{16}$$

$$A_2 = V_{\mathrm{OC,ref}} - R_{\mathrm{S,ref}}I_{\mathrm{SC,ref}} \tag{17}$$

$$A_3 = R_{\mathrm{S,ref}}I_{\mathrm{SC,ref}} - R_{\mathrm{S,ref}}I_{\mathrm{mp,ref}} - V_{\mathrm{mp,ref}} \tag{18}$$

In addition, the shunt conductance $G_{\mathrm{SH,ref}} = R_{\mathrm{SH,ref}}^{-1}$ in (7) will be considered as an unknown instead of $R_{\mathrm{SH,ref}}$. It is possible to note that the coefficients P_1, P_2, P_3 and P_4 can assume only positive values; they represent the areas of the regions defined in Fig. 2 and particularly the ratio $P_1/\sum_{i=1}^{4}P_i$ is equal to the so called fill factor FF at SRC.

Let us start to rewrite the first three equations that are common to any version of the extraction procedure:

- 1st equation on the "open circuit condition":

$$0 = I_{\mathrm{irr,ref}} - I_{0,\mathrm{ref}}[Exp_{OC} - 1] - G_{\mathrm{SH,ref}}V_{\mathrm{OC,ref}} \tag{19}$$

- 2nd equation on the "short circuit condition"

$$I_{\mathrm{SC,ref}} = I_{\mathrm{irr,ref}} - I_{0,\mathrm{ref}}[Exp_{SC} - 1] - G_{\mathrm{SH,ref}}R_{\mathrm{S,ref}}I_{\mathrm{SC,ref}} \tag{20}$$

- 3rd equation on the "MPP condition"

$$I_{\mathrm{mp,ref}} = I_{\mathrm{irr,ref}} - I_{0,\mathrm{ref}}[Exp_{MP} - 1] - G_{\mathrm{SH,ref}}(V_{mp,ref} + R_{\mathrm{S,ref}}I_{mp,ref}) \tag{21}$$

It has been proven in Laudani et al. (2013, 2014a) that the five unknowns $G_{\mathrm{SH,ref}}, R_{\mathrm{S,ref}}, I_{0,\mathrm{ref}}, I_{\mathrm{irr,ref}}$ and n_{ref} can be classify in two groups: three dependent variables $G_{\mathrm{SH,ref}}, I_{0,\mathrm{ref}}, I_{\mathrm{irr,ref}}$ and two independent variables $R_{\mathrm{S,ref}}, n_{\mathrm{ref}}$. By following an analogous approach and focusing only on the above three Eqs. (19)–(21), it is possible to show that (see Laudani et al. (2014a) for the proof):

In this manner we have written the dependent variables $G_{\mathrm{SH,ref}}, I_{0,\mathrm{ref}}$ and $I_{\mathrm{irr,ref}}$ as functions of n_{ref} and $R_{\mathrm{S,ref}}$ in (22), (23), and (24), respectively. Thus we have only two independent unknowns: the parameters n_{ref} and $R_{\mathrm{S,ref}}$. This splitting of the parameters allows a new paradigm in the writing of the open circuit, short circuit and maximum power point constraints, giving the possibility both to exactly verify these conditions and to reduce the dimensions of the search space. Indeed, it is important to note that, *being the* Eqs. (22)–(24) *achieved by algebraic manipulation of* Eqs. (19)–(21), *this two sets of equations are equivalent, that is one can use indifferently the set* (22)–(24) *or the set* (19)–(21). In the following dissertation, we assume that all the five parameters must have positive values for preserving a physical meaning within the equivalent circuit: this allows defining a feasible domain $D \subseteq R^+ \times R^+$ for the couple $(n_{\mathrm{ref}}, R_{\mathrm{S,ref}})$, with $G_{\mathrm{SH,ref}}, I_{0,\mathrm{ref}}$ and $I_{\mathrm{irr,ref}}$ computed by using (22)–(24), respectively:

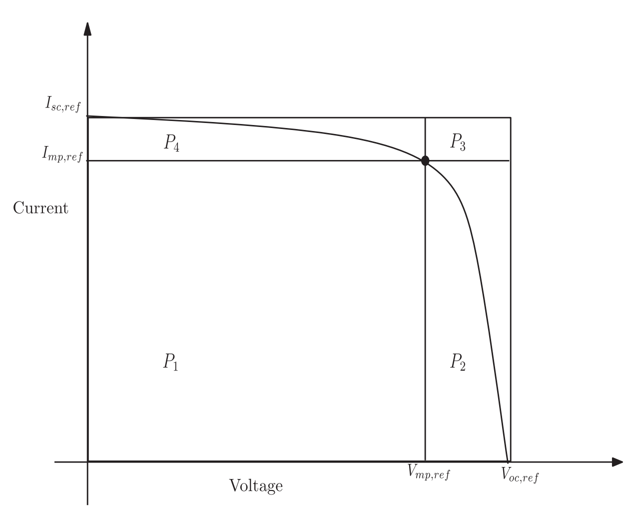

Fig. 2. P_i definition.

$$D = \{(n_{\rm ref}, R_{\rm S,ref}) \in R^+ \times R^+ : G_{\rm SH,ref} > 0, I_{0,\rm ref} > 0, I_{\rm irr,ref} > 0\} \tag{25}$$

The inequalities in (25) ensure obtaining parameters to be utilized within the one-diode circuital model, all with a physical meaning. Now, in order to provide a better formalization of the feasible domain D containing the solutions, an important result presented in Laudani et al. (2013) about the dependence of the maximum value of $R_{\rm S,ref}$ from $n_{\rm ref}$ is used. Indeed, an upper bound for the value of $R_{\rm S,ref}$, $R^{max}_{\rm S,ref}$, exists that ensures a physical meaning for the parameters. It can be given by expressing $R^{max}_{\rm S,ref}$ as a function of $n_{\rm ref}$ by using the W Lambert function Corless et al. (1996):

$$R^{max}_{\rm S,ref}(n_{\rm ref}) = \frac{V_{mp,ref}}{I_{mp,ref}} + \frac{N_S n_{\rm ref} V_T}{I_{\rm mp,ref}}\left(1 + W_{-1}\left(-exp\left(\frac{V_{\rm OC,ref} - n_{\rm ref} N_S V_T - 2V_{\rm mp,ref}}{N_S n_{\rm ref} V_T}\right)\right)\right) \tag{26}$$

and then, domain D can be defined again:

$$D = \left\{(n_{\rm ref}, R_{\rm S,ref}) \in R^+ \times R^+ : 0 \leqslant R_{\rm S,ref} \leqslant R^{max}_{\rm S,ref}(n_{\rm ref})\right\} \tag{27}$$

Now, the following proposition which directly comes out from previous equivalence between the two set of Eqs. (22)–(24) can be written:

Proposition 1. *Whatever point* $(n^*_{\rm ref}, R^*_{\rm S,ref}) \in D$ *used together with the* Eqs. (22)–(24) *returns a vector of five* *__positive__* *parameters* $(I^*_{0,\rm ref}, I^*_{\rm irr,ref}, R^*_{\rm SH,ref}, R^*_{\rm S,ref}, n^*_{\rm ref})$ *which satisfies* (19)–(21)*. Thus, different points* $(n^*_{\rm ref}, R^*_{\rm S,ref}) \in D$ *provide different I–V curves, all crossing through the short circuit, open circuit and maximum power points.*

With the aim to better understand the importance of the Proposition 1, we present an example of application to a real PV panel, BP 3 235 T module by “BP Solar” company.

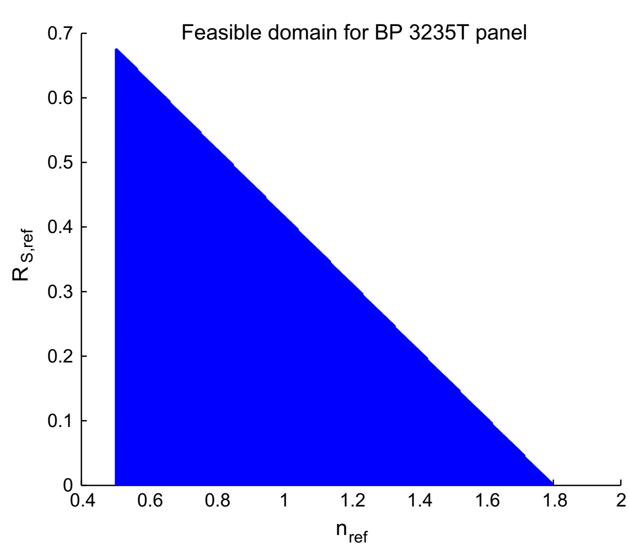

Fig. 3. Domain of admissible values for $R_{\rm S,ref}$ and $n_{\rm ref}$.

In particular, the feasible domain D of this module is shown in Fig. 3, varying $n_{\rm ref}$ within the range $[0.5, 2.0]$ and $R_{\rm S,ref}$ within the range $]0, 0.7]$.

In addition, four couples $(n^*_{\rm ref}, R^*_{\rm S,ref}) \in D$ have been arbitrary selected: the related five parameters are reported in Table 1 and the respective I–V curves are reported in Fig. 4 where it is possible to note that the short circuit, open circuit and maximum power point conditions are perfectly satisfied for each of the four curves, i.e. for each of the four points $(n^*_{\rm ref}, R^*_{\rm S,ref}) \in D$. Thus, there are ∞^2 points belonging to D, which perfectly satisfy the OC, SC and MPP conditions. On the other hand, by drawing the related power vs voltage curves (P–V), showed in Fig. 5, it is possible to note that the previous four solutions have different values of the maximum power point, although the point $(I_{\rm mp,ref}, V_{\rm mp,ref})$ belongs to all the curves. This can be simply explained by observing that the Eq. (21) really imposes the I–V curves crossing through the couple $(I_{\rm mp,ref}, V_{\rm mp,ref})$, but it does not say anything about the “maximum power point” on the P–V plain. As a consequence, *a fourth equation is* **needed** *for introducing the further constraint* $\frac{dP}{dV} = 0$ *at maximum power point at SRC in order to correctly predict the behavior of the PV module at MPP.* For this reason, all the models which do not make use of this further equation on the slope of power at MPP, provide a rough estimation of the performance of PV systems near the MPP condition (Laudani et al., 2014b).

3.2. The forth equation: slope of the P–V curve at the maximum power point

The slope of the P–V curve at MPP in SRC condition can be simply determined by the value of the ratio $\frac{I_{\rm mp,ref}}{V_{\rm mp,ref}}$, as it has been shown by several authors in literature and widely used in numerous works (e.g. Dobos, 2012; Desoto et al., 2006; Tian et al., 2012; Rajasekar et al., 2013; Laudani et al., 2013). Thus, the most considerable aspect of the forth equation is that it can be directly recovered from PV panel datasheet without having obtained any further experimental measurement such as, for example, $R_{S0} = \frac{dV}{dI}|_{I=0}$ or $G_{SH0} = R^{-1}_{SH0} = \frac{dI}{dV}|_{V=0}$, widely used in literature as alternative equations (Orioli and Gangi, 2013; Chen et al., 2011; Chouder et al., 2012). Thus, let us write the fourth equation by using the slope of the curve at the MPP condition:

$$\frac{I_{mp,ref}}{V_{mp,ref}} = \frac{\frac{I_{0,\rm ref}}{N_S n_{\rm ref} V_T} Exp_{MP} + G_{\rm SH,ref}}{1 + \frac{R_{\rm S,ref} I_{0,\rm ref}}{N_S n_{\rm ref} V_T} Exp_{MP} + G_{\rm SH,ref} R_{\rm S,ref}} \tag{28}$$

Table 1
Four vectors of five parameters satisfying the first three constraints (open circuit, short circuit and maximum power point).

$R_{\rm S,ref}$ (Ω)	$n_{\rm ref}$	$R_{\rm SH,ref}$ (Ω)	$I_{0,\rm ref}$ (A)	$I_{\rm irr,ref}$ (A)
0. 1	1.6	3.2652E+03	2.3963E−06	8.4803
0. 2	0.8	57.0903	6.2916E−13	8.5097
0. 3	1.0	106.48	2.7174E−10	8.5039
0. 4	0.6	58.015	2.7252E−17	8.5385

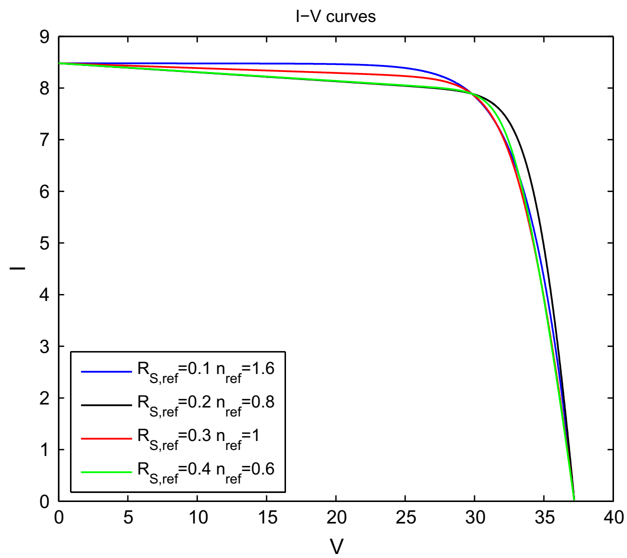

Fig. 4. *I–V* curves for BP 3 235 T obtained by using the five parameters values reported in Table 1.

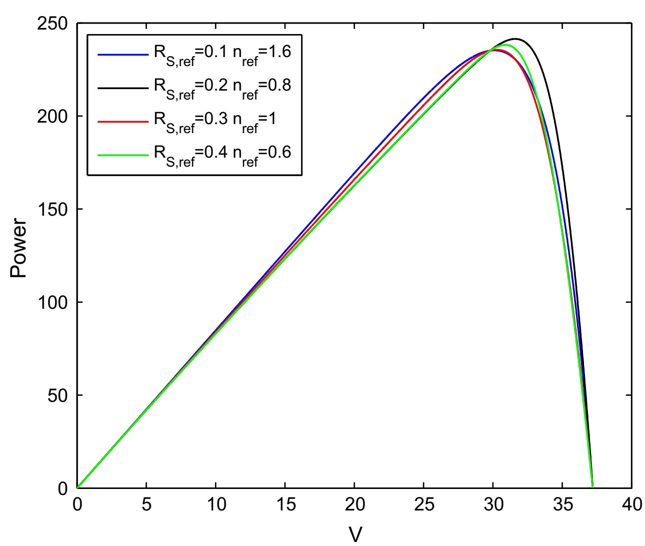

Fig. 5. *P–V* curves for BP 3 235 T obtained by using the five parameters values reported in Table 1.

By making some simplifications and by injecting (22)–(24) into the Eq. (28) it is possible to obtain the following condition:

$$f_1(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) = (P_2 - P_1)Exp_{SC} + (P_1 - P_4)Exp_{OC} + \left((P_1 - P_3)\frac{I_{\mathrm{mp,ref}}R_{\mathrm{S,ref}} - V_{\mathrm{mp,ref}}}{n_{\mathrm{ref}}N_S V_T} + (P4 - P2)\right)Exp_{MP} = 0 \tag{29}$$

Although the Eq. (29) cannot be analytically solved, couples of values $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ satisfying it can be easily found by using very simple numerical approaches. In Fig. 6 the graph of $f_1(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ is shown for the PV panel BP 3 235 T in the domain D obtained by varying n_{ref} within

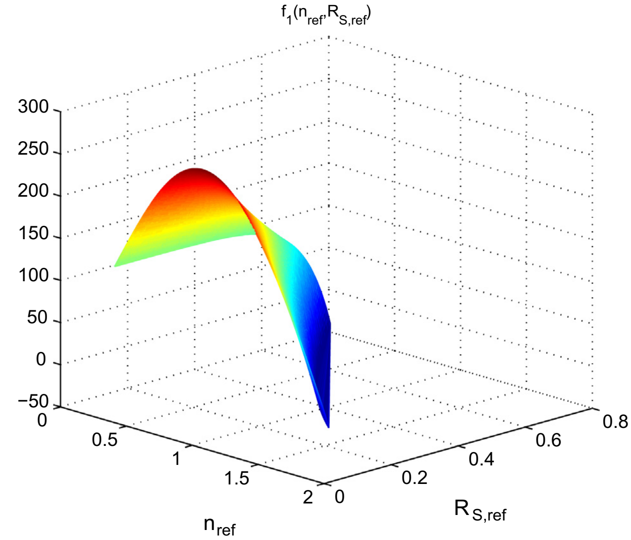

Fig. 6. The plot of $f_1(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ in the feasible domain D.

the range [0.4, 2.0]. The surface shape suggests the existence of solutions for $f_1(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) = 0$. Indeed, the $f_1(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ assumes both negative and positive values.

With the aim to solve the Eq. (29), the very simple bisection method will be used: the choice of this simple numerical technique is also aimed to emphasize the easiness of the problem. In particular, the two steps listed below are followed:

- fixing the value of n^*_{ref} and starting from the interval $[0, R^{max}_{\mathrm{S,ref}}(n^*_{\mathrm{ref}})]$;
- using the bisection method until the value $R^*_{\mathrm{S,ref}}$ for which $f_1(R^*_{\mathrm{S,ref}}, n^*_{\mathrm{ref}}) = 0$ is reached.

In this way it is possible to generate the $f_1(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) = 0$ curve, that is all the couples $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ which also satisfy the MPP slope conditions. In Fig. 7 the curve related to the solutions of (29) for the BP3 235 T PV panel is shown together with the one indicating the maximum admissible values of $R_{\mathrm{S,ref}}$ for this PV panel according to (26).

It is important to emphasize again that each couple of values $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ belonging to the curve $f_1 = 0$ is a solution for the four Eqs. (19), (20), (21) and (28), i.e. all the points $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ belonging to the curve $f_1 = 0$ are also the solutions of the first four equations of the five-parameter model at SRC in terms of the two independent unknowns $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$. Thus, after having introduced the fourth Eq. (28), a new domain $D' \subset D$ can be defined:

$$D' = \left\{(n_{\mathrm{ref}}, R_{\mathrm{S,ref}}) \in R^+ \times R^+ : 0 \leqslant R_{\mathrm{S,ref}} \leqslant R^{max}_{\mathrm{S,ref}}(n_{\mathrm{ref}}) \ and \ f_1(n_{\mathrm{ref}}, R_{\mathrm{S,ref}}) = 0\right\} \tag{30}$$

Proposition 2. *Whatever point* $(n^*_{\mathrm{ref}}, R^*_{\mathrm{S,ref}}) \in D' \subset D$ *used together with the* Eqs. (22)–(24) *returns a vector of five* *positive* *parameters* $(I^*_{0,\mathrm{ref}}, I^*_{\mathrm{irr,ref}}, R^*_{\mathrm{SH,ref}}, R^*_{\mathrm{S,ref}}, n^*_{\mathrm{ref}})$ *which*

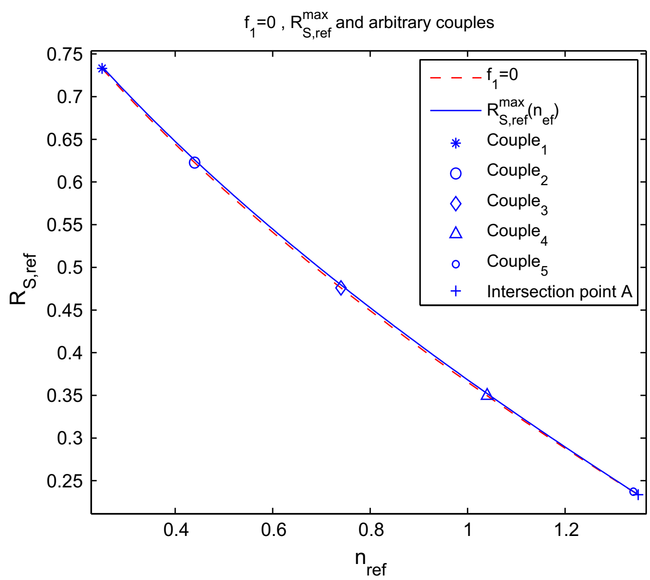

Fig. 7. Solutions of the 4th equation and plot of $R_{S,ref}^{max}$: five couples have been arbitrary selected to trace *I–V* curves of Fig. 8.

satisfies (19), (20), (21) and (28). Thus, different points $(n_{ref}^{*}, R_{S,ref}^{*}) \in D'$ provide different I–V curves crossing through the short circuit, open circuit and maximum power points and satisfying the $\frac{dP}{dV} = 0$ at maximum power point at SRC.

With the aim to numerically verify the Proposition 2, five different couples of values $(R_{S,ref}, n_{ref})$ have been arbitrarily chosen (see Fig. 7) and used for drawing the related curves shown in Figs. 8 and 9. It is possible to note that all the chosen points give vectors of five parameters, listed in Table 2, satisfying the conditions on $I_{SC,ref}$, $V_{OC,ref}$, $V_{mp,ref}$ and $I_{mp,ref}$. Moreover, now the condition on the slope of the *I–V* curve at the MPP is also satisfied, differently from the previous case with three equations of the Section 3.1.

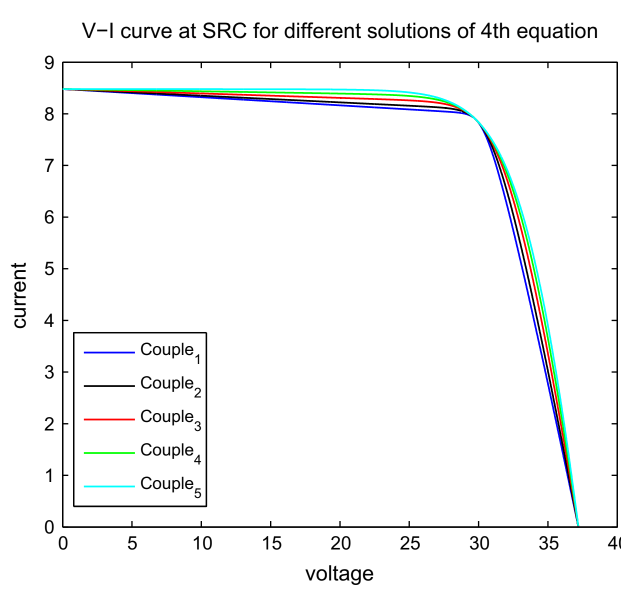

Fig. 8. *I–V* curves for BP 3 235 T obtained by using the five parameters values reported in Table 2.

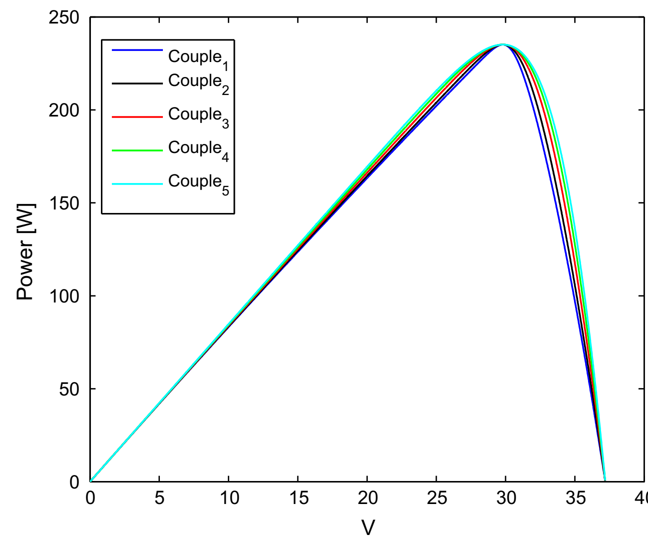

Fig. 9. *P–V* curves for BP 3 235 T obtained by using the five parameters values reported in Table 2.

In addition, from the Fig. 7 it is worth noting that:

(*i*) the two curves are almost overlapped;
(*ii*) the two curves intersect at a point A. This defines the n_{ref}^{max} admissible for the selected PV panel. Indeed higher values of n_{ref} with respect n_{ref}^{max} have no longer physically meaning because the shunt conductance becomes $G_{SH,ref} < 0$.

Thus, we can write a further proposition:

Proposition 3. *For any PV module, given $V_{OC,ref}, I_{SC,ref}, V_{mp,ref}$ and $I_{mp,ref}$ an upper bound n_{ref}^{max} exists for the values of n_{ref} that satisfy the OC (19), SC (20), MPP (21) and slope condition (28) of P–V curve at MPP. In other words, only $n_{ref} < n_{ref}^{max}$ returns five positive parameters, i.e. values with a physical meaning.*

On the basis of the Proposition 3, it is possible to give another definition of domain D' in function of the n_{ref}^{max} instead of $R_{S,ref}^{max}$ as follows:

$$D' = \left\{(n_{ref}, R_{S,ref}) \in R^{+} \times R^{+} : n_{ref} < n_{ref}^{max} \text{ and } f_1(n_{ref}, R_{S,ref}) = 0\right\} \tag{31}$$

Obviously, the two definitions of domain D', (30) and (31), are equivalent.

Then, we can write the following 4th proposition:

Proposition 4. *Whatever positive and not too small*[1] *$n_{ref}^{*} < n_{ref}^{max}$ can be used together with the Eqs. 22–24 and 29, to achieve a vector of five <u>positive</u> parameters $(I_{0,ref}^{*}, I_{irr,ref}^{*}, R_{SH,ref}^{*}, R_{S,ref}^{*}, n_{ref}^{*})$ providing an I–V curve crossing through the short circuit, open circuit and maximum power points conditions and satisfying the $\frac{dP}{dV} = 0$ at maximum power point at SRC.*

[1] *Usually we assume $n_{ref}^{*} > 0.5$.*

Table 2
Five arbitrary solutions of the five parameters module satisfying the first four constraints (open circuit, short circuit, maximum power point and slope at MPP).

Couple	$R_{S,ref}$ (Ω)	n_{ref}	$R_{SH,ref}$ (Ω)	$I_{0,ref}$ (A)	$I_{irr,ref}$ (A)
#1	0.7330	0.2500	62.5481	9.8462e−042	8.5794
#2	0.6226	0.4400	76.6973	1.2434e−023	8.5488
#3	0.4759	0.7400	116.1465	5.6896e−014	8.5147
#4	0.3496	1.0400	229.2422	7.0211e−010	8.4929
#5	0.2371	1.3400	4.3938E+03	1.2857e−007	8.4805

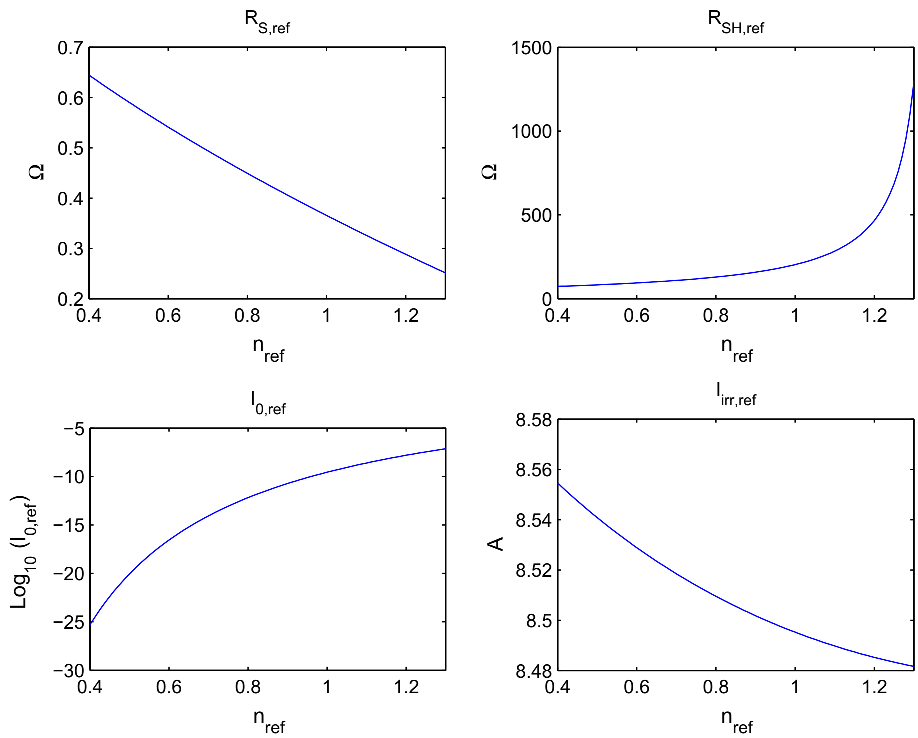

Fig. 10. $R_{SH,ref}, R_{S,ref}, I_{0,ref}, I_{irr,ref}$ as a function of n_{ref}.

In order to verify how the parameters of the five-parameter model change in function of the points on the $f_1 = 0$ curve, in Fig. 10 the behaviors of the values of $R_{S,ref}, R_{SH,ref}, I_{0,ref}$ and $I_{irr,ref}$ are shown by varying n_{ref} within the range $[0.4, 1.3]$ for PV panel BP 3 235 T ($n_{ref}^{max} \cong 1.356$). It is worth noting that the value of $I_{irr,ref}$ shows just a small variation and it is really near to the value of the Short Circuit current at SRC ($I_{SC,ref}$=8.48 A) reported on datasheet. It is also evident that low values of n_{ref} (i.e. values lower than 0.6) return an $I_{0,ref}$ extremely low (<1E-20) and this might not be considered useful for the analysis of real applications. Finally, it is worth noticing that, since $G_{SH,ref} = 0$ for $n_{ref} = n_{ref}^{max}$, then for the four-parameter model in which $G_{SH,ref} = 0$ (Luque and Hegedus, 2011), the ideality factor n_{ref} is equal to n_{ref}^{max}: this model satisfies the four main conditions and consequently it might be considered sufficiently accurate at SRC.

Proposition 5. *For the four-parameter model achieved by fixing* $G_{SH,ref} = 0$, *the ideality factor* n_{ref} *is equal to* $n_{ref}^{max}, R_{S,ref} = R_{S,ref}^{max}(n_{ref}^{max})$ *(by using* (26)*), and the other two parameters,* $I_{0,ref}$ *and* $I_{irr,ref}$, *can be directly computed from the expressions* (23) and (24), *respectively.*

3.3. Statistical analysis of a feasible domain for n_{ref}

In order to propose a statistical analysis of the feasible domain D', we have performed tests on around 8000 PV panels belonging to the CEC database (California Energy Commission, 2013).[2] In particular the tests have concerned the computation of the n_{ref}^{max} for the mono-Si PV modules (around 3600) and the multi-Si PV modules (around 4600) available in the database. Table 3 reports the main obtained results grouped by type of technology, whereas Figs. 11 and 13 show the value of n_{ref}^{max} for mono-Si and multi-Si technology respectively. The distribution of the n_{ref}^{max} is also represented by means of an histogram in

[2] In the CEC database really there are more entries, but many of them are repetitions. In our analysis we have eliminated these entries in order to perform a correct statistical analysis.

Table 3
Statistical results on n_{ref}^{max} performed on about 8000 PV module available on CEC database.

parameter	Mono-Si PV modules	Multi-Si PV modules
Average n_{ref}^{max}	1.308	1.304
Median n_{ref}^{max}	1.299	1.272
Standard dev. n_{ref}^{max}	0.3305	0.3172

Figs. 12 and 14. It is worth noticing that the obtained results prove that the mean value for n_{ref}^{max} for both mono-Si and multi-Si module is around 1.3, that is in many cases, 1.3 is the upper bound for n_{ref}. In addition a median value for n_{ref}^{max} lower than 1.3 for both technologies means that more than half of the modules have a maximum admissible value for n_{ref} lower than 1.3. In some work this is the value suggested and adopted for n_{ref} by designers and researchers without specific analysis. Moreover, within the performed statistical results there are a lot of panels with n_{ref}^{max} lower than 0.8 (about 10–15%) or even lower than 0.5. This may mean that the one-diode model is probably inadequate to model this last kind of modules, since it leads to solutions with negative values of $G_{\mathrm{SH,ref}}$ (therefore also of $R_{\mathrm{SH,ref}}$) or with values of $I_{0,\mathrm{ref}}$ extremely low if the solutions is searched with $n_{\mathrm{ref}} \cong 1.0$. Thus, the utilized admissible value of n_{ref}, according to datasheet values of $V_{\mathrm{OC,ref}}, I_{\mathrm{SC,ref}}, V_{\mathrm{mp,ref}}$ and $I_{\mathrm{mp,ref}}$, can be mathematically justified by the difficulties to extract parameters with physical meanings. Although the reason of this unphysical parameters is in many cases attributed to a high fill factor, really it can be above all focused in the part of the fill factor containing the ratio I_{mp}/I_{SC} that is also related to the slope of the I–V curve between SC and MPP. The values obtained for n_{ref}^{max} are reported in the supplementary material provided together with this work. It is worth noticing that the n_{ref}^{max} was computed also for the modules in CEC database that use a different PV technology (a-Si, thinfilm, CIGS, CdTe, CIS, etc). Unfortunately, the CEC database

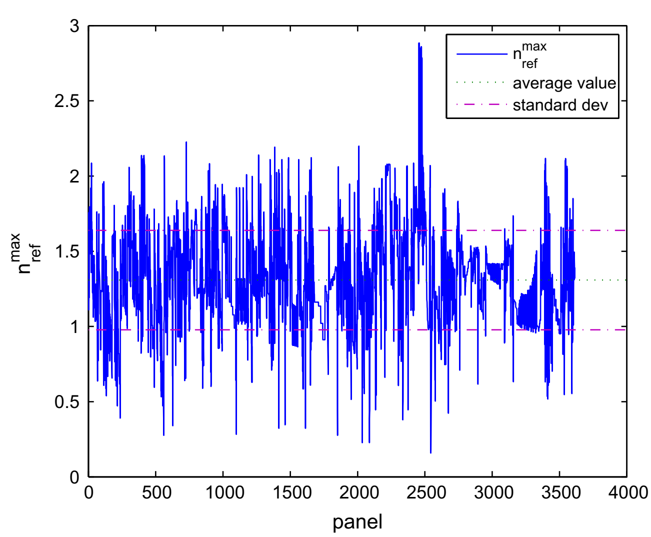

Fig. 11. Plot of the values of n_{ref}^{max} for the mono-Si PV modules in the CEC database.

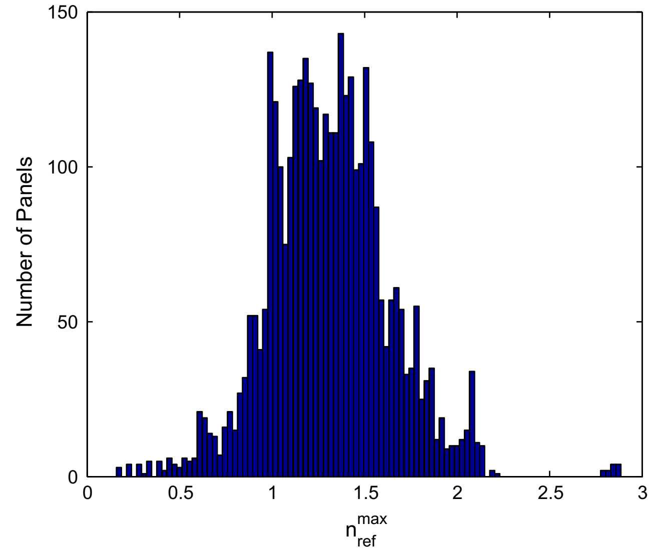

Fig. 12. Histogram of the values of n_{ref}^{max} for the mono-Si PV modules in the CEC database.

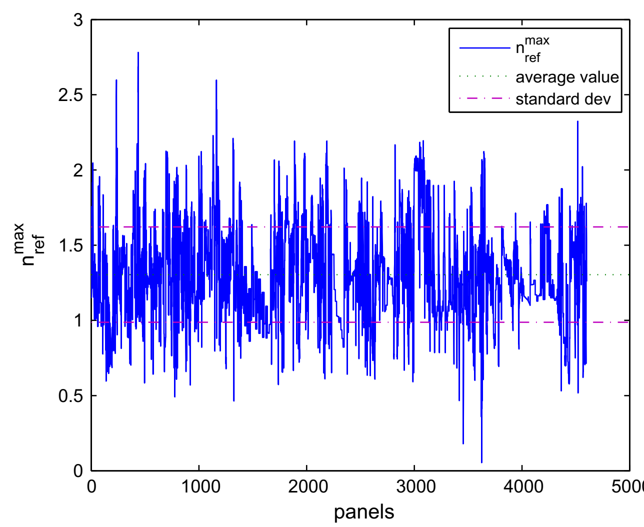

Fig. 13. Plot of the values of n_{ref}^{max} for the multi-Si PV modules in the CEC database.

has a very limited number of entries for these technologies. While we observed higher values for n_{ref}^{max} for these technologies (mean value greater than 2.5) this did not allow a statistical analysis similar to the one for Multi-Si and Mono-Si modules.

3.4. Choice of n_{ref} and comparison with other approaches by using only the datasheet values $V_{\mathrm{OC,ref}}, I_{\mathrm{SC,ref}}, V_{\mathrm{mp,ref}}$ and $I_{\mathrm{mp,ref}}$

As stated by the proposition 4, any value of $n_{\mathrm{ref}} < n_{\mathrm{ref}}^{max}$ can be used in the previous presented equations to identify a five-parameter model which exactly satisfies OC, SC and MPP conditions (in particular for the MPP condition, it is done on both of I–V curve and P–V curve). Consequently a

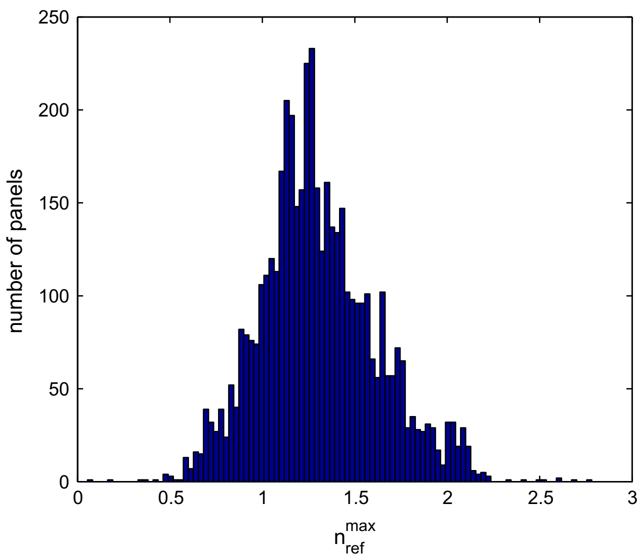

Fig. 14. Histogram of the values of n_{ref}^{max} for the multi-Si PV modules in the CEC database.

strategy for the choice of n_{ref} is needed: this can be accomplished by introducing a fifth equation (like for example in Desoto et al. (2006), Sera et al. (2007)); or by means of a heuristic rule in which n_{ref} is assigned as a certain percentage of n_{ref}^{max} (for example 90%) or the minimum between $0.9 \cdot n_{\mathrm{ref}}^{max}$ and 1.1 or 1.2. Anyway, this point is extremely critical: on the one hand, the proposed heuristic procedure could be sufficiently accurate, as we show in the following, and it has the advantage to not require any further computation. On the other hand, it leads one to think that the model is incomplete. Therefore, in the next section, we also examine two different choices of the 5th equation to achieve a fully determinable model. Clearly in literature other approaches have been proposed in order to identify the five-parameter model by using only datasheet values of $V_{\mathrm{OC,ref}}, I_{\mathrm{SC,ref}}, V_{\mathrm{mp,ref}}$ and $I_{\mathrm{mp,ref}}$, as done for example in Ghani et al. (2013b,a) and Carrero et al. (2011). Hereafter, we propose a quantitative comparison between the results achievable with our approach and those presented in Carrero et al. (2011) and Ghani et al. (2013b). In particular, we have extracted the five parameters by using the previous proposed heuristic rule ($n_{\mathrm{ref}} = 0.9 \cdot n_{\mathrm{ref}}^{max}$). From the results in Table 4 we conclude the following compared to Carrero et al. (2011) (i) the error values on currents at MPP in our case are zero, whereas in Carrero et al. (2011) are not null (even if really small), probably influenced by the approximations done in the estimation of $I_{0,\mathrm{ref}}$ and $I_{\mathrm{irr,ref}}$; (ii) if we use for n_{ref} the value proposed by Carrero we practically achieve very similar results; (iii) in some cases (Suntech STP-280, Atersa A-120 and Isofoton I-110) the heuristic choice returns values closer to the ones achieved by the iterative approach proposed by Carrero et al. (2011). With regard to the approach proposed by Ghani et al. (2013b), which is very accurate and effective, we present a brief quantitative comparison on the basis of some experimental data reported in that work. In particular, we have computed the root mean square error (RMSE) between the experimental data reported in table B1 of Ghani et al. (2013b) and the simulated data achieved by using our proposed approach together with the heuristic rule $n_{\mathrm{ref}} = 0.9 \cdot n_{\mathrm{ref}}^{max}$. Starting from reference data reported in Table 4 of Ghani et al. (2013b) we have found $n_{\mathrm{ref}}^{max} = 1.7937$, from which the $n_{\mathrm{ref}} = 0.9 \cdot 1.7937 \approx 0.1643$ is computed. The five parameters obtained in this way are reported in Table 5: the resulting RMSE is 0.0018 that is really similar to that achieved by Ghani et al. (2013b). The Fig. 15 shows the experimental and simulated I–V curve for this test.

3.5. The fifth equation: on the existence of a unique solution

With the aim to gain a *determinate* system for the five-parameter model, an additional equation is needed. The fifth equation, indeed, allows selecting one only couple of values $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) \in D'$ returning the desired solution for the five-parameter model. It could be an equation imposing I–V curve passes through another fixed point or imposing the slope value of the I–V curve on Open Circuit or Short Circuit conditions. Unfortunately, these informations are not directly available on datasheets and further experimental measurements on PV panel must be done for obtaining them. A further possibility is the use of the Eq. (1) for a given condition (e.g. for Open Circuit condition) different from the SRC. This is the approach proposed by Desoto et al. (2006) and widely used in literature (Tian et al., 2012; Beckman, xxxx; Laudani et al., 2013; Dobos, 2012).

3.5.1. The fifth equation according to Desoto et al. (2006)

In this case, the fifth equation is written by applying (1) at Open Circuit condition with irradiance $S = S_{\mathrm{ref}}$ and $T' = T_{\mathrm{ref}} + \Delta T$. The following relations are used:

$$I_{\mathrm{irr}} = I_{\mathrm{irr,ref}} + \alpha_T(\Delta T) \tag{32}$$

$$I_0 = I_{0,\mathrm{ref}} \left[\frac{T'}{T_{\mathrm{ref}}}\right]^3 e^{\left[\frac{E_{\mathrm{g,ref}}}{kT_{\mathrm{ref}}} - \frac{E_g}{kT'}\right]} \tag{33}$$

$$V_{OC}(T') = V_{\mathrm{OC,ref}} + \beta_{\mathrm{T}} \Delta T \tag{34}$$

$$V_{T'} = \frac{kT'}{q} \tag{35}$$

thus, the 5th equation proposed in Desoto et al. (2006) is:

$$\begin{aligned} 0 = {} & I_{\mathrm{irr,ref}} + \alpha_T \Delta T \\ & - I_{0,\mathrm{ref}} \left[\frac{T'}{T_{\mathrm{ref}}}\right]^3 e^{\left[\frac{E_{\mathrm{g,ref}}}{kT_{\mathrm{ref}}} - \frac{E_g}{kT'}\right]} \left[exp\left(\frac{V_{OC}(T')}{N_S n_{\mathrm{ref}} V_{T'}}\right) - 1\right] \\ & - G_{\mathrm{SH,ref}} V_{OC}(T') \end{aligned} \tag{36}$$

Obviously, it is possible to write the (36) again in terms of the only two independent unknowns $R_{\mathrm{S,ref}}$ and n_{ref} by substituting the relations (22)–(24):

Table 4
$n_{\mathrm{ref}}, R_{\mathrm{S,ref}}, R_{\mathrm{SH,ref}}$ achieved by means of the proposed technique compared with those (indicated with *) reported in Table 2 of Carrero et al. (2011).

PV module	n_{ref}^{max}	n_{ref}	$R_{\mathrm{S,ref}}$ (Ω)	$R_{\mathrm{SH,ref}}$ (Ω)	n_{ref}^*	$R_{\mathrm{S,ref}}^*$ (Ω)	$R_{\mathrm{SH,ref}}^*$ (Ω)
Suntech STP-280	0.7759	0.6983	0.6906	981	0.7367	0.6704	1939
SunPower SPR-315	1.4311	1.2880	0.1212	1258	1.0731	0.3152	566
Atersa A-120	1.4908	1.3417	0.1157	300	1.3302	0.1175	278
Atersa A-130	2.1675	1.9508	0.1449	294	1.6246	0.3797	181
Isofoton I-110	1.2267	1.1041	0.8334	1685	1.1359	0.7975	2272

Table 5
Five parameters (found with proposed approach and heuristic rule $n_{\mathrm{ref}} = 0.9 * n_{\mathrm{ref}}^{max}$) used for the comparative test with respect the experimental data reported in table B1 of Ghani et al. (2013b).

n_{ref}	$R_{\mathrm{S,ref}}$ (Ω)	$R_{\mathrm{SH,ref}}$ (Ω)	$I_{0,\mathrm{ref}}$ (μA)	$I_{\mathrm{irr,ref}}$ (A)
1.643	0.1532	80.729	0.4179	0.5778

$$f_2(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) = I_{\mathrm{irr,ref}}(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) + \alpha_T \Delta T - G_{\mathrm{SH,ref}}(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) V_{OC}(T') - I_{0,\mathrm{ref}}(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) \left[\frac{T'}{T_{\mathrm{ref}}}\right]^3 e^{\left[\frac{E_{g,\mathrm{ref}}}{kT_{\mathrm{ref}}} - \frac{E_g}{kT'}\right]} \left[exp\left(\frac{V_{OC}(T')}{N_S n_{\mathrm{ref}} V_{T'}}\right) - 1\right] = 0 \tag{37}$$

In order to solve $f_2 = 0$ we can use the bisection method as previously made for $f_1 = 0$. Now, another important issue must be addressed: the dependence of the solution of the five-parameter model from the value of ΔT. Indeed, several authors in literature state that the value of $T' = T_{\mathrm{ref}} + \Delta T$ has a low influence on the final solution and usually assign ΔT equal to 5 or 10 K (Desoto et al., 2006; Tian et al., 2012; Dobos, 2012). In Fig. 16 four different solutions of the five-parameter model related to the BP 3 235 T panel evaluated for ΔT equal to 1, 2, 5 and 10 K are shown. Each of these four solutions are obtained by the intersection of the two curves $f_1 = 0$ and $f_2 = 0$. It is worth noting that the four solutions are not so different among each other, but also they are not so similar to be approximated as equals. As a consequence, the choice of ΔT slightly affects the computation of the five parameters to be used for the PV panel model in the way shown in Fig. 16.

However, although a particular criteria for choosing the value of ΔT does not exist in literature, by fixing the value of $T' = T_{\mathrm{ref}} + \Delta T$ and by using all the five Eqs. (19), (20), (21), (28) and (36), a unique solution can be found for the five-parameter model, i.e. the system is finally a *determinate* system. With the aim to verify the existence of a unique solution, also by exploiting the results obtained for n_{ref}^{max}, we have tried to extract the five parameters for all the modules present in the CEC database, by using $\Delta T = 10$ K. We have found that for about 15% of the modules (mono-Si and multi-Si) the procedure returns unphysical solutions (i.e. $G_{\mathrm{SH,ref}} < 0$). This is not due to numerical problems or to the choice of a suitable optimization algorithm with adequate initial guesses, but it depends on the fact that, for these modules, the fifth equation mathematically individuates solutions that have not physical meaning because found values of n_{ref} are bigger than n_{ref}^{max}. In other words, the algorithm finds solutions $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}}) \notin D'$. In Fig. 17 the case of the BP Q 220 panel of the BP Q series by BP solar manufacturer is shown: the couple of values $(R_{\mathrm{S,ref}}, n_{\mathrm{ref}})$ individuated by the equation $f_2 = 0$, lies on a region in which $n_{\mathrm{ref}} > n_{\mathrm{ref}}^{max}$ and then $G_{\mathrm{SH,ref}} < 0$. The result

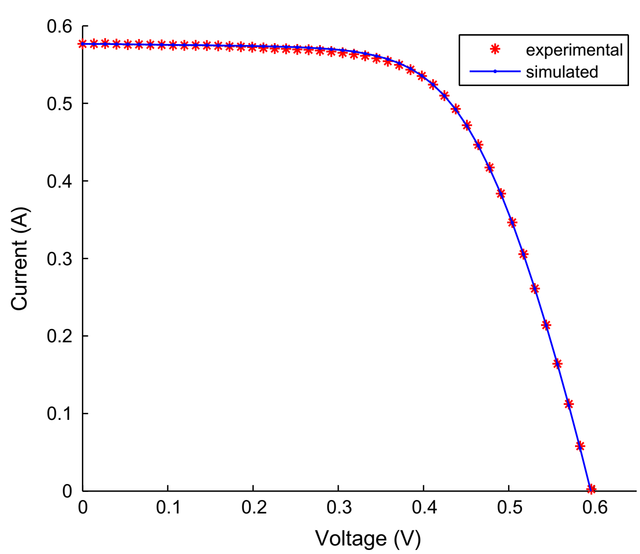

Fig. 15. Experimental and simulated I–V curves.

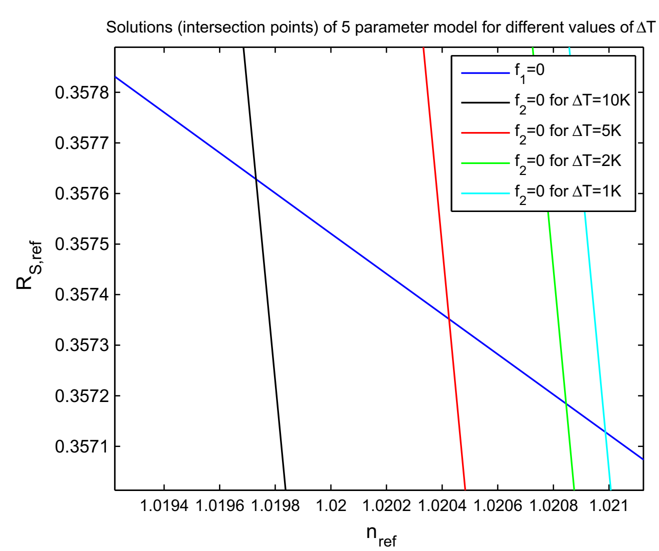

Fig. 16. The solutions for the panel BP 3 235 T (intersection points between the two curves $f_1 = 0$ and $f_2 = 0$) obtained by using different values of ΔT.

shown in Fig. 17 has been obtained with $\Delta T = 10$ K, but also different values of ΔT give unphysical solutions until reaching values around 200 K for which $G_{\mathrm{SH,ref}}$ comes back to have positive values. Therefore, the choice of the values of ΔT remains a critical problem if the (36) is used as fifth equation.

3.5.2. Other equations on slope of I–V curve

As stated before, if the manufacturers provide additional information about slope of I–V curve at SC or OC conditions, it would be possible to add a different fifth equation based on the formula (Chan and Phang, 1987; Sera et al., 2007):

$$\frac{dI}{dV} = \frac{\frac{\partial}{\partial V}F(I,V)}{1 - \frac{\partial}{\partial I}F(I,V)} \tag{38}$$

where $I = F(I,V)$ represents the Eq. (1). Really, the use of this expression at OC or SC conditions could allow adding a fifth and a sixth equation as well: one at OC condition and the other one at SC condition. Unfortunately, the values of these two slopes are not provided into datasheet by manufacturers, and consequently only an approximate expression of this formula can be used, for example by exploiting it at SC condition and imposing it equals to $-G_{\mathrm{SH,ref}}$ as initially proposed by Chan and Phang (1987) and later by Sera et al. (2007) and also by other authors (Brano et al., 2010; Orioli and Gangi, 2013; Chen et al., 2011; Chouder et al., 2012).

$$\frac{\frac{I_{0,\mathrm{ref}}}{N_S n_{\mathrm{ref}} V_T} Exp_{SC} + G_{\mathrm{SH,ref}}}{1 + \frac{R_{\mathrm{S,ref}} I_{0,\mathrm{ref}}}{N_S n_{\mathrm{ref}} V_T} Exp_{SC} + G_{\mathrm{SH,ref}} R_{\mathrm{S,ref}}} \cong G_{\mathrm{SH,ref}} \tag{39}$$

Clearly, this expression can be used instead of the one proposed by Desoto et al. (2006), and in this case the number of modules for which it is possible to extract five parameters with physical meaning, increases. Indeed in this case only for the 3% of the modules unphysical solutions are found. Nevertheless, by following this last approach, for many panels, above all those with a not so high n_{ref}^{max}, have $G_{\mathrm{SH,ref}}$ tending to zero. Thus, it is not so evident the advantage of using this approach instead of the four-parameter model with, as previously stated in the Proposition 5, $G_{\mathrm{SH,ref}} = 0$ and $n_{\mathrm{ref}} = n_{\mathrm{ref}}^{max}$. The scenario would have been different if the measurements of slopes at SC or OC conditions were provided by the manufacturers in datasheets and the Eq. (38) was used.

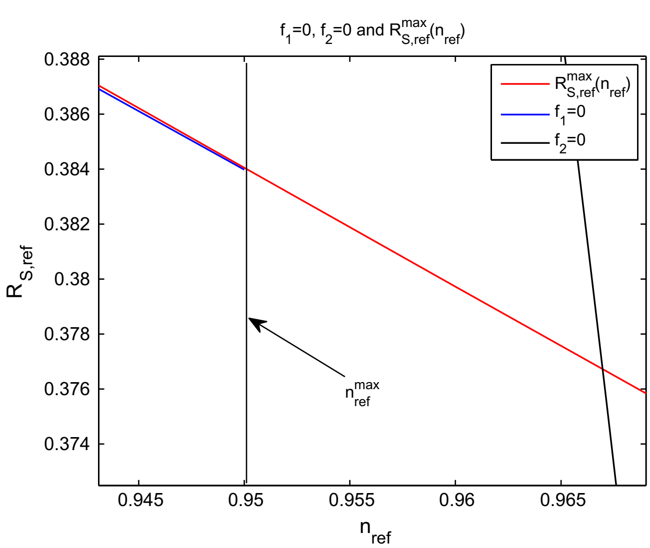

Fig. 17. Unphysical solution of the five-parameter model for BP Q 220 PV module. Solution obtained with $\Delta T = 10$ K.

3.6. Procedure for the solution of the identification problem

By following the present strategy, the solution of the identification problem becomes extremely simple: either by using a general solver tailored for the solution of a system of non-linear equations, like *fsolve* in Matlab, or by using traditional solution methods, such as Newton method, Halley method, bisection method, etc. Indeed, on the basis of the present theoretical study, differently from the case regarding 5 equations in 5 unknowns, the problem of convergence no longer exists. Fig. 18 shows a flow chart of the solution procedure. The first step is to compute n_{ref}^{max}. Then it is possible to follow the proposed heuristic rule (assigning n_{ref} directly) or to proceed with the complete solution of the problem introducing a 5th equation (the (37) or the (39) for example). In this latter case, starting from an initial guess for $n_{\mathrm{ref}} < n_{\mathrm{ref}}^{max}$ the $R_{\mathrm{S,ref}}$ is computed by numerically solving the Eq. (29). After that, the values of $I_{0,\mathrm{ref}}, I_{\mathrm{L,ref}}$ and $R_{\mathrm{SH,ref}}$ are computed by means of (22)–(24) starting from $R_{\mathrm{S,ref}}$ and n_{ref} obtained previously. At this point, the fifth equation is evaluated and, if convergence is not reached, n_{ref} is changed according to the adopted numerical method. Finally, it is important to emphasize that the solution is not sensitive to the value of the initial guess; $n_{\mathrm{ref}} = 0.9 \cdot n_{\mathrm{ref}}^{max}$ is typically a good choice.

4. What's equation for what solution?

Several problems related to the extraction of the five parameters have been addressed in this paper by following a mathematical point of view and the conclusion is that any identification procedure cannot be performed without a detailed analysis which takes into account the following points:

- The value of n_{ref}^{max} should be examined as first thing. A low value of this parameters could mean that no equivalent one-diode model exists having parameters with physical meaning, or it could be in fact useless due to the extremely low value of $I_{0,\mathrm{ref}}$ returned by the identification procedure. Clearly, if we assume in any case that the one-diode model must exist, it is important to eventually take into account also the professionalism of manufacturers since the value of n_{ref}^{max} is linked to the

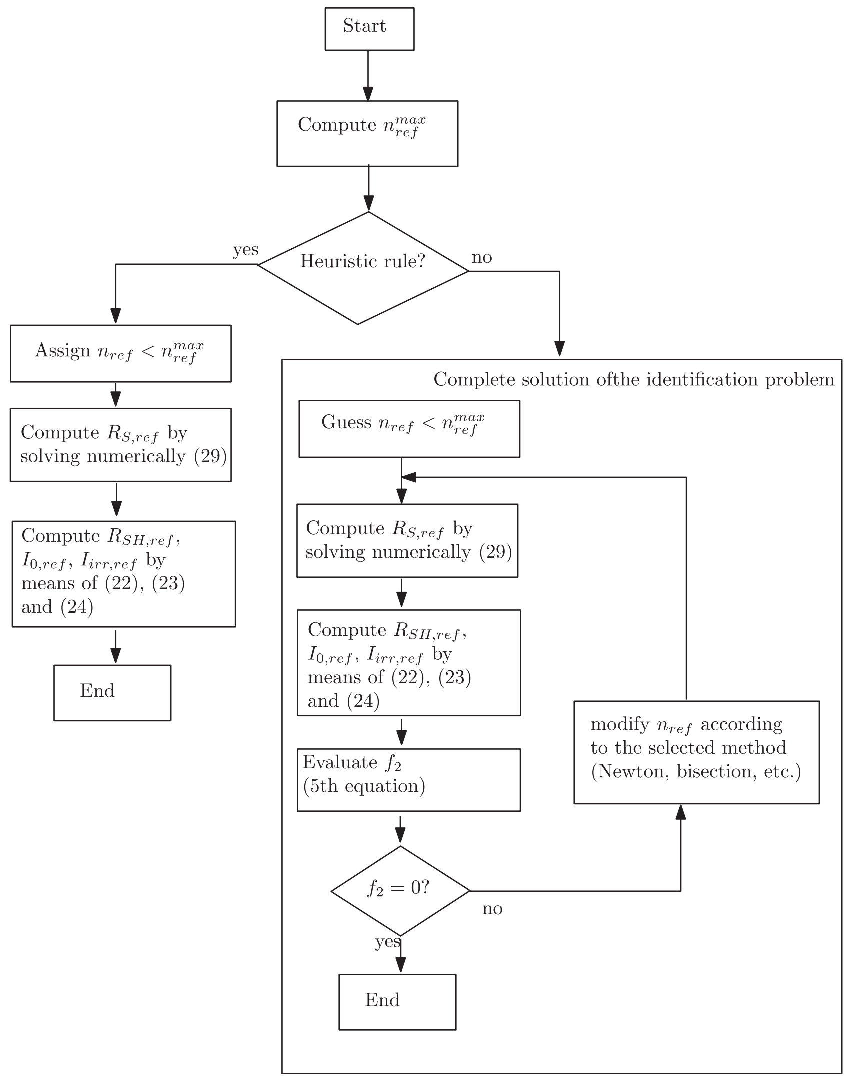

Fig. 18. Flow chart for the solution of the identification problem.

datasheet information.[3] In addition, it is worth noticing that if we increase, for example, the values of $I_{\mathrm{SC,ref}}$ by 1% to those modules of CEC database for which De Soto model does not return parameters with physical meaning, the percentage of failures in the extraction procedure by using the fifth equations proposed by Desoto et al. (2006) decreases from 15% to about 3%, as also confirmed by the study performed in Dobos (2012). This could be a possible strategy in some cases, but however it continues to fail for many other modules which probably cannot be modeled by the classic one-diode model.

- In all the cases in which the fifth equation proposed by Desoto et al. (2006) can be successfully applied, it appears to be more interesting than the one based on slope of I–V curve at SC condition, since it does not impose any approximation which could worsen the extraction procedure. In addition, this latter approach for many module, above all for those for which De Soto model fails, returns value for $G_{\mathrm{SH,ref}}$ tending to zero. Thus, it is not so evident the advantage of using this approach instead of the four-parameter model in which $G_{\mathrm{SH,ref}} = 0$. The scenario would have been different if the measurements of slopes at SC or OC conditions were provided by the manufacturers in datasheets.
- Finally, it is important to emphasize that as previously stated in the Proposition 4, whatever $n^*_{\mathrm{ref}} < n^{max}_{\mathrm{ref}}$ can be used to achieve a vector of five positive parameters $(I^*_{0,\mathrm{ref}}, I^*_{\mathrm{irr,ref}}, R^*_{\mathrm{SH,ref}}, R^*_{\mathrm{S,ref}}, n^*_{\mathrm{ref}})$ providing an I–V curve crossing through the short circuit, open circuit and

[3] On the other hand, the datasheet values are mean values of parameters for a specific PV module class. Probably a precise measurement of the parameter of each PV module (that is, of each one produced, and not a statistic data) could lead to higher value for n^{max}_{ref}.

maximum power points conditions and satisfying the $\frac{dP}{dV} = 0$ at maximum power point at SRC. This confirms the importance of the evaluation of n_{ref}^{max}.

5. Conclusion

In this paper a complete theoretical and practical analysis on the extraction of the five parameters identifying the one-diode model for photovoltaic modules from data available on PV panel datasheets has been proposed. In the present analysis a fully mathematical approach was used to gain insight to the five-parameter model related to the one-diode equivalent circuit. By performing a separation of the independent variables from the dependent ones among the five parameters, a reduced form of the five-parameter model has been obtained. This splitting of the parameters introduces a new paradigm in the writing of the open circuit, short circuit and maximum power point constraints, giving the possibility both to exactly satisfy these conditions and to reduce the dimensions of the search space. In particular, the in-depth analysis presented in this paper allowed to state the following points: (1) by the reduced form of the five-parameter model, an exact and deterministic procedure for the extraction of the five parameters from the panel datasheet information, has been made; (2) an upper bound n_{ref}^{max} for the values of n_{ref}, exists; (3) the extracted five parameters are all positive (physical meaning) if $n_{\text{ref}} < n_{\text{ref}}^{max}$; (4) by assuming that the used datasheet and/or database does not contain wrong information, for all the PV panels for which only unphysical solutions are obtained, other models (equivalent circuits) must be used (for example, two-diode model). The proposed complete theoretical analysis of the five-parameter model gives the opportunity to improve and simplify the procedures for more accurate simulations in the field of the PV systems. Indeed the procedure for determining the unique solution of the five-parameter model becomes a very simple task if the reduced system composed by only two equations (such as $f_1 = 0$ and $f_2 = 0$) is used instead of the one containing the five original equations. From one hand, the use of the reduced system significantly simplifies the choice of the initial guesses that is a strong trouble if the original system of five equations is used Dobos (2012), Laudani et al. (2013). From the other hand, being the reduced system characterized by a two-dimension search space instead of the five dimensions of the original system, it is possible to use very fast and accurate algorithms for searching the solution. In addition, the proposed approach does not have limitations other than those inherent in the one-diode model. The proposed approach also allows detecting all the critical cases in terms of low value of $n_{\max}$. Finally, the proposed mathematical approach may also be utilized to simplify higher–order models which have been proposed in the literature in order to improve the PV modeling, such as six (Dobos, 2012) and higher parameters models (Beckman, xxxx).

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.solener.2014.07.024.

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