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A PRACTICAL METHOD OF ANALYSIS OF THE CURRENT–VOLTAGE CHARACTERISTICS OF SOLAR CELLS

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Summary

In this paper we consider solar cells as generators, and the classical one-diode equivalent circuit is assumed to be valid for a given light intensity. The true circuit parameters R_s, R_{sh}, I_s and I_{ph} are computed from the experimental data of the fourth-quadrant characteristic using a programmable calculator. Agreement between the calculated current and the experimentally observed current is within $\pm 1.0\%$ on average for the solar cells tested.

1. Introduction

In most cases the current–voltage (I–V) characteristic of an illuminated solar cell generator can be described by the lumped parameter equivalent circuit of Fig. 1(a), which is based on the equivalent circuit analysis developed by Schottky in 1930. The photocurrent I_{ph} is delivered by a constant-current source and flows in the diode impedance I_d, the shunt resistance R_{sh}, the series resistance R_s and the load resistance R_L. For a given incident light intensity the I–V relationship

$$I = I_{ph} - \frac{V + R_s I}{R_{sh}} - I_s[\exp\{B(V + R_s I)\} - 1] \tag{1}$$

where

$$B = \frac{q}{AkT} \tag{2}$$

is implicit and analytically non-solvable.

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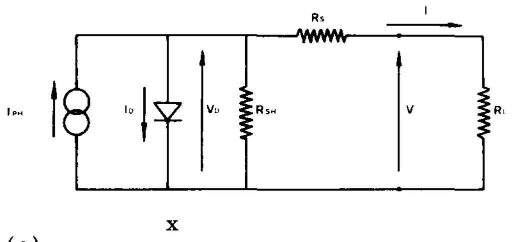
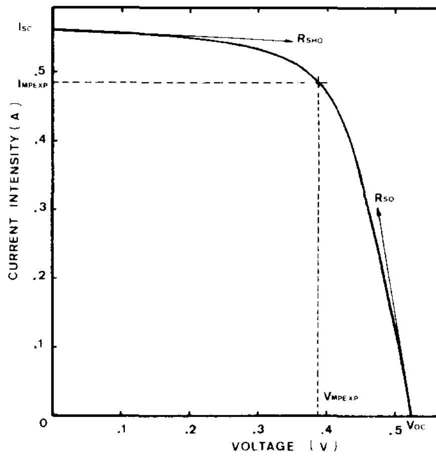


(a)



(b)

Fig. 1. (a) Equivalent circuit model of the illuminated solar cell as a generator (lumped linear resistive elements) described by

$$I_d = I_s[\exp\{B(V + R_s I)\} - 1]$$

(b) Fourth quadrant of the characteristic of a solar cell where $I_{mp\ exp}$ and $V_{mp\ exp}$ are the experimental values of the current and voltage at the maximum power point and

$$R_{s0} = -\left(\frac{dV}{dI}\right)_{V=V_{oc}} \qquad R_{sh0} = -\left(\frac{dV}{dI}\right)_{I=I_{sc}}$$

It is worthwhile to point out that the parameters used to express eqn. (1) are light and temperature dependent [1 - 3]. Hence they can only be determined for a given illumination and a fixed temperature. It has also been reported by several workers that the series resistance and the photocurrent are voltage dependent [2 - 4].

Several methods have been used to determine the series resistance R_s: it has been determined from the difference between the dark and illuminated characteristics [5], from the high direct voltage slope of the characteristic [6] or from the variation in the illuminated characteristics with intensity [1, 7]. All these methods apply under the assumptions that the shunt resistance remains negligible and that the short-circuit current I_{sc} equals the photocurrent. These approximations are often made so that the I–V relationship (1) is analytically manageable.

The evaluation of the reverse characteristic yields a value of the shunt resistance R_{sh} [1] by neglecting R_s. The reverse diode saturation current I_s is usually computed from the simplified relationship for a diode under reverse bias [1, 8]. The diode factor A is derived from the dark measurements [1], from the measurements under illumination [8] or from the V_{oc}–$\ln I_{sc}$ plot for various illuminations [9, 10].

The impossibility of an analytical solution to eqn. (1) has forced workers to make use of various approximate methods of analysis which provide a poor representation of the experimental I–V curves.

The macroscopic parameters V_{oc}, I_{ph}, I_s, A, R_s and R_{sh} are also interesting because they are related to the internal properties of the components of the solar cell [3, 4, 8]. Moreover, knowledge of their actual values should allow a better understanding of the processes involved for optimization studies [11].

We present here a numerical solution which is easy to perform on a programmable calculator. The I–V equation parameters are determined from measurements performed on the solar cell driven as a generator only. The exact magnitude of the photocurrent I_{ph} is evaluated. This is of prime interest to rate the performance of a solar cell. The fill factor is also deduced.

2. Experimental set-up

The programming techniques and parameters discussed in Section 3 were used to study two types of solar cells: (a) a high quality silicon solar cell, blue type, square, of area 4 cm^2, bought from SAT, 41 rue Cantagrel, 75624 Paris Cédex 13, France; (b) a low quality silicon solar cell, grey, round, of area 25.8 cm^2, bought from Radio M.J., 19 rue Claude Bernard, 75005 Paris, France.

I–V measurements were carried out with a potentiometer-type electronic circuit [12] which could provide the power needed for the large-area solar cell. These measurements were made under illumination from a 12 V quartz–halogen lamp, through a 2 cm water filter, calibrated to air mass one conditions. The solar cells were bonded to copper plates and placed in the flow of a heat gun as a means of temperature regulation. A Cu–constantan thermocouple was used for temperature measurement.

3. Equations and flow of the program

An I–V characteristic is plotted in Fig. 1(b). We consider the experimental values for a null current, *i.e.* V_{oc}, for a null voltage, *i.e.* I_{sc}, and around these points, *i.e.* dV/dI, to be of prime interest because they can be calculated from the semiconductor properties, the grid geometry and the illuminating light spectrum and intensity [3] for comparison. From the open-circuit voltage V_{oc} and the short-circuit current I_{sc} we deduce from eqn. (1) that

$$I_s = \frac{I_{sc}(1 + R_s/R_{sh}) - V_{oc}/R_{sh}}{A_1 - A_2} \tag{3}$$

and

$$I_{ph} = I_s(A_1 - 1) + \frac{V_{oc}}{R_{sh}} \tag{4}$$

where

$$A_1 = \exp(BV_{oc}) \tag{5}$$

$$A_2 = \exp(BR_sI_{sc}) \tag{6}$$

Through differentiation of eqn. (1) and consideration of the values

$$R_{s0} = -\left(\frac{dV}{dI}\right)_{V = V_{oc}} \tag{7}$$

$$R_{sh0} = -\left(\frac{dV}{dI}\right)_{I = I_{sc}} \tag{8}$$

we obtain respectively

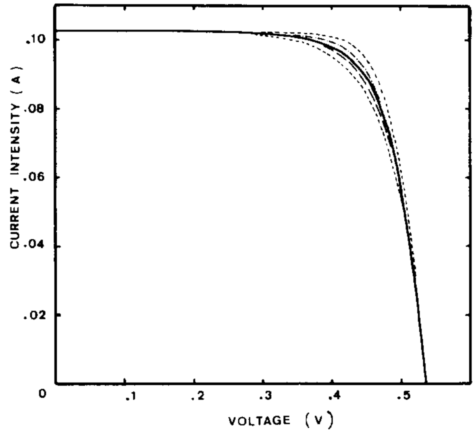
$$R_s = R_{s0} - \frac{1}{1/R_{sh} + BI_sA_1} \tag{9}$$

and

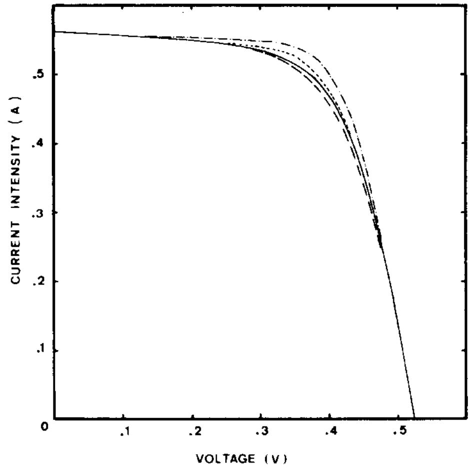
$$R_{sh} = \frac{1}{1/(R_{sh0} - R_s) - BI_sA_2} \tag{10}$$

R_{s0} and R_{sh0} express the behaviour of the I–V curve around the open-circuit voltage point and around the short-circuit current point. These are the experimentally determined slopes at these points.

For a set value of the quality factor A of the diode, eqns. (3), (4), (9) and (10) can be solved numerically. Figure 2 shows the calculated I–V curves for several values of A. They were obtained from the values of the experimental data listed in Table 1 (R_{s0} and R_{sh0} were determined with 2% - 3% graphical precision). The experimental I–V characteristics are plotted for comparison. The great sensitivity of the characteristics to a slight variation in the value of A can be observed mainly around the maximum power point (I_{mp}, V_{mp}). A detailed study of percentage deviations $100 \times \Delta I/I_{exp}$ of the calculated from the experimental current intensities as shown in Fig. 3 could lead to the best-fit value of A. This requires the calculation and plotting of many curves. However, because the fill factor FF and the maximum power point are of prime interest for photovoltaic simulation programs [13], we chose to compute A from the maximum power point data; A is the solution of eqn. (1) for the V_{mp} and I_{mp} values given by



(a)



(b)

Fig. 2. The dependence of the I–V curves on the value of the diode quality factor A (the experimental curves (——) are shown for comparison): (a) blue solar cell (A = 1.0, ------ (external); A = 1.3, – · – ; A = 1.7, – – – ; A = 2.0, ------ (internal)); (b) grey solar cell (A = 1.0, – · – ; A = 1.5, ------ ; A = 2.0, – – –).

TABLE 1

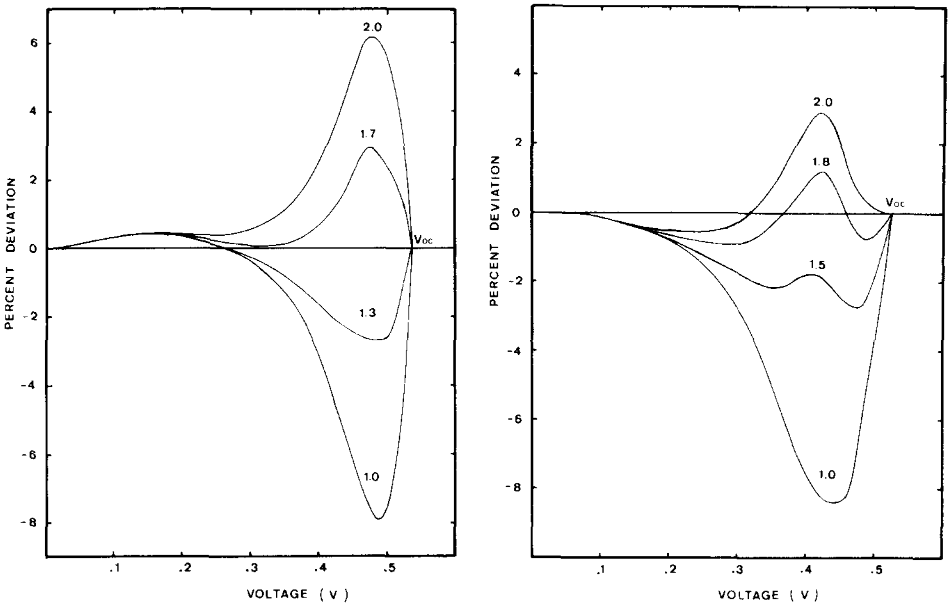
Experimental data

Parameter	*Blue solar cell (SAT)*	*Grey solar cell*
V_{oc} (V)	0.536	0.524
R_{s0} (Ω)	0.45 ± 0.01	0.162 ± 0.005
I_{sc} (A)	0.1023	0.561
R_{sh0} (Ω)	1000 ± 30	25.9 ± 0.8
T (K)	300	307
$V_{mp\,exp}$ (V)	0.437	0.390
$I_{mp\,exp}$ (A)	0.0925	0.481
FF	0.736	0.638

$$0 = I_{ph} - \frac{V_{mp} + (R_s + R_{sh})I_{mp}}{R_{sh}} - I_s[\exp\{B(V_{mp} + R_s I_{mp})\} - 1] \quad (11)$$

Newton's method was used to find A from this equation [14].

Once all the parameter values I_{ph}, I_s, R_s, R_{sh} and A had been determined, we computed the current intensity value for a given voltage by the simultaneous equations technique [14]. The basic flow of this program is outlined in Fig. 4. An IBM computer was used for the calculations. The results were identical with those found using HP 67 and HP 41C calculators.



(a) (b)

Fig. 3. Percentage deviation of the calculated from the experimental current intensities for various values of A (indicated for each curve): (a) blue solar cell; (b) grey solar cell.

4. Results

The computed values of the parameters of the equivalent circuit (eqn. (1)) are given in Table 2. Figure 5 shows how very close the computed values of the current intensities are to the experimental values. The effects of the lumped circuit parameters according to the quality of the cell are also emphasized in these figures. The curves computed for an ideal solar cell ($R_s = 0$ and $R_{sh} \to \infty$) are plotted for comparison in two cases: $A = 1$ and A calculated from the program.

It can be seen that for the "state of the art" solar cell (Fig. 5(a)) the effects of R_s and R_{sh} can be estimated to be negligible only in so far as the actual value of A is known. These effects materialize as the difference between the calculated–experimental characteristics and the ideal solar cell characteristic. In contrast, for a solar cell with significant output losses (Fig. 5(b)), R_s and R_{sh} cannot be neglected. In both cases (Fig. 5(a) and Fig. 5(b)) the importance of the quality factor is illustrated dramatically by the difference between the ideal solar cell characteristics for $A = 1$ and for A calculated.

The percentage deviations of the calculated values from the experimental current intensities are plotted in Fig. 6 to emphasize the actual precision achieved for these measurements. The slightly poorer result obtained for the grey cell (Fig. 6(b)) can be ascribed to its larger area and consequently

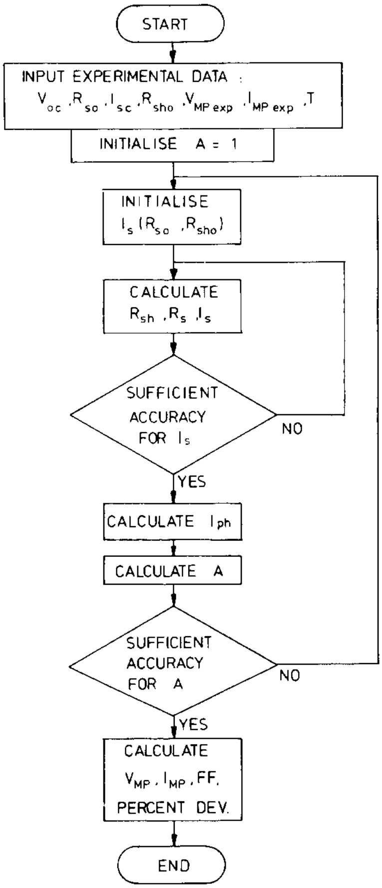
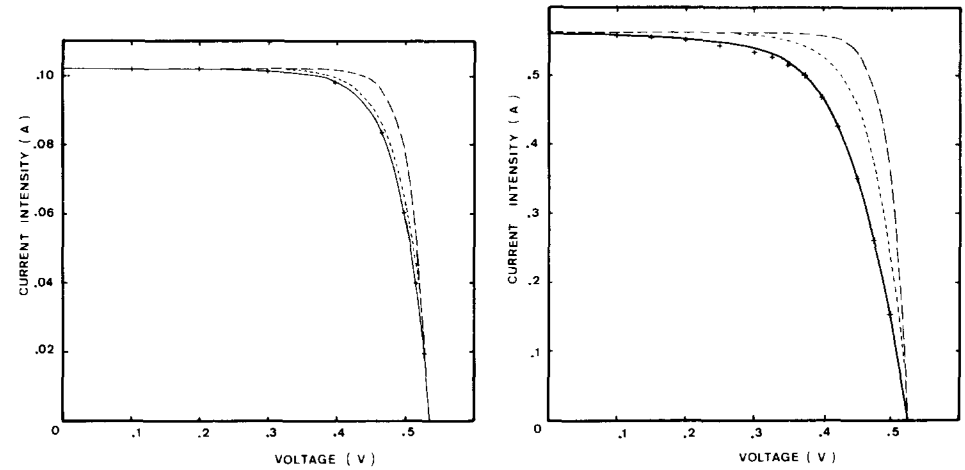


Fig. 4. Flow chart of the program.

TABLE 2

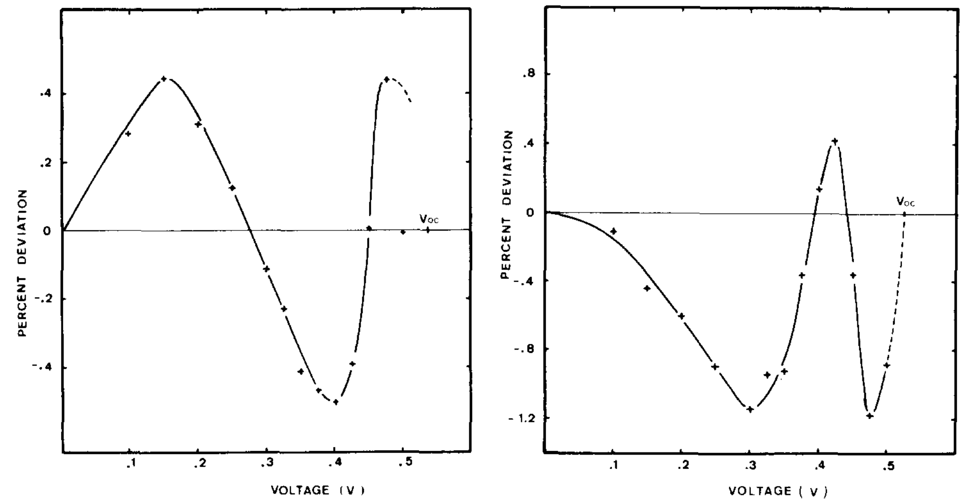
Results

Parameter	*Blue solar cell (SAT)*	*Grey solar cell*
A	1.51 ± 0.07	1.72 ± 0.08
I_{ph} (A)	0.1023 ± 0.0005	0.5625 ± 0.0005
I_s (A)	$(110 \pm 50) \times 10^{-9}$	$(6 \pm 3) \times 10^{-6}$
R_s (Ω)	0.070 ± 0.009	0.08 ± 0.01
R_{sh} (Ω)	1000 ± 50	26 ± 1
V_{mp} (V)	0.433	0.387
I_{mp} (A)	0.0934	0.485
P_{max} (W)	0.0404	0.188
FF	0.737	0.638



(a) (b)

Fig. 5. Comparison between the experimental characteristic (+), the calculated I–V curve (——) and the ideal diode characteristics for $R_s = 0$, $R_{sh} \rightarrow \infty$, A calculated (------) and for $R_s = 0$, $R_{sh} \rightarrow \infty$, $A = 1$ (– – –): (a) blue solar cell, calculated $A = 1.51$; (b) grey solar cell, calculated $A = 1.72$.



(a) (b)

Fig. 6. Percentage deviation of the calculated from the experimental current intensities for the calculated values of A (Table 2): (a) blue solar cell; (b) grey solar cell.

to its greater risk of local faults and/or substantial non-uniformity in the local junction voltage distribution [15].

5. Conclusion

The illuminated solar cell is considered in this paper as a generator and the parameters of its equivalent circuit are assumed to be voltage independent within the range studied. These parameters are determined through the use of a simple and practical program. The numerical analysis makes use of the experimental data for three points only from the characteristic. The degree of accuracy which is achieved with this method is partly due to the choice of the experimental values and partly to the method of calculation. More refined measurements could permit either an even higher accuracy or a better understanding of the limitations of the one-diode model for solar cells as generators.

Such determination of the parameters from few experimental data is of great importance at high intensities and temperatures where the series resistance has an increasing importance, and at low intensities and low temperatures where the importance of the shunt resistance increases.

It should help to establish comparisons between different devices and to study the effects of fabrication parameters.

Nomenclature

A	diode quality factor
B	q/AkT, conventional solar cell analysis parameter
FF	fill factor, the ratio of the maximum power to $V_{oc}I_{sc}$
I	current output
I_d	diode current
I_{mp}	current for maximum power
I_{ph}	constant-current source photocurrent
I_s	diode saturation current
I_{sc}	short-circuit current
k	Boltzmann constant
q	electronic charge
R_L	V/I, load resistance
R_s	series resistance
R_{s0}	experimental slope for $V = V_{oc}$
R_{sh}	shunt resistance
R_{sh0}	experimental slope for $I = I_{sc}$
T	absolute temperature
V	voltage output
V_{mp}	voltage for maximum power
V_{oc}	open-circuit voltage

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