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Solar Cells and the Lambert W Function

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Presented by Ken Roberts at the Conference
"Celebrating 20 years of the Lambert W function"
which was held at Western University

July 25-28, 2016

Solar Cells and the Lambert W Function

- Objectives of This Talk.
- Acknowledgements.
- Ladder of Lambert W function coordinate representations. Lambert W as a switch function. Graphs of **g** function.
- Solar cell model. Current-voltage implicit equation. Lambert W in explicit current-voltage solutions. Computational difficulties. The **g** function in place of **w**.
- Discussion...

Acknowledgements

- Hirondo Kuki
- David Borwein
- Sree Ram Valluri
- Western University Libraries
- Western University Physics and Astronomy
- "Physics is the Poetry that is written at the interface between Mathematics and Matter." (Source ??)

Lambert W Function Representations

$$
\begin{tikzcd}[column sep=huge, row sep=large]
\mathbf{x} \arrow[r, shift left, "\mathbf{x}^{1/\mathbf{x}} = \mathbf{y}"] \arrow[d, shift left, "J"] & \mathbf{y} \arrow[l, shift left, dashed, two heads, "\mathbf{x} = \mathbf{h}(\mathbf{y})"] \arrow[d, shift left, "J"] \\
\mathbf{u} \arrow[u, shift left, "J"] \arrow[r, shift left, "\mathbf{u}^{\mathbf{u}} = \mathbf{v}"] \arrow[d, shift left, dashed, two heads, "L"] & \mathbf{v} \arrow[u, shift left, "J"] \arrow[l, shift left, dashed, two heads, "\mathbf{u} = \mathbf{p}(\mathbf{v})"] \arrow[d, shift left, dashed, two heads, "L"] \\
\mathbf{s} \arrow[u, shift left, "E"] \arrow[r, shift left, "\mathbf{s}\,\mathbf{e}^{\mathbf{s}} = \mathbf{t}"] \arrow[d, shift left, dashed, two heads, "L"] & \mathbf{t} \arrow[u, shift left, "E"] \arrow[l, shift left, dashed, two heads, "\mathbf{s} = \mathbf{w}(\mathbf{t})"] \arrow[d, shift left, dashed, two heads, "L"] \\
\mathbf{q} \arrow[u, shift left, "E"] \arrow[r, shift left, "\mathbf{q} + \mathbf{e}^{\mathbf{q}} = \mathbf{r}"] & \mathbf{r} \arrow[u, shift left, "E"] \arrow[l, shift left, dashed, two heads, "\mathbf{q} = \mathbf{g}(\mathbf{r})"]
\end{tikzcd}
$$

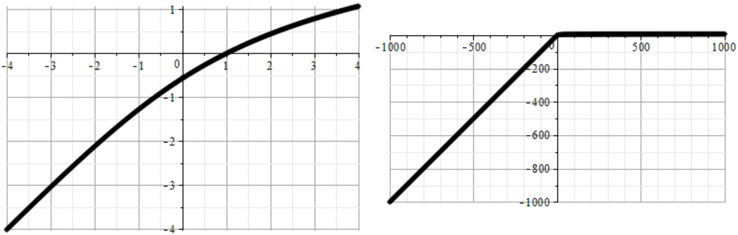
Symbols: $E(\mathbf{z}) = \mathbf{exp}(\mathbf{z})$ Inverse $L(\mathbf{z}) = \mathbf{log}(\mathbf{z})$

$J(\mathbf{z}) = \mathbf{1}/\mathbf{z}$ is its own inverse.

$\mathbf{w}(\cdot)$ is Lambert W function.

The $\mathbf{g}(\cdot)$ function

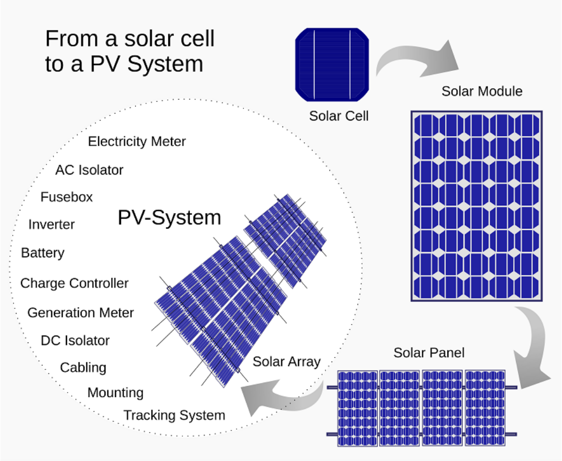
- Defined by coordinate change, $\mathbf{q} = \mathbf{g}(\mathbf{r}) = \mathbf{log}(\mathbf{w}(\mathbf{exp}(\mathbf{r})))$
  Another definition: $\mathbf{q} = \mathbf{g}(\mathbf{r}) = \mathbf{r} - \mathbf{w}(\mathbf{exp}(\mathbf{r}))$
- Equivalently, $\mathbf{q}$ is the solution of $\mathbf{q} + \mathbf{e}^{\mathbf{q}} = \mathbf{r}$
- Graphs of $\mathbf{q} = \mathbf{g}(\mathbf{r})$ for real $\mathbf{r}$ in (-4,4) or in (-1000,1000)



- $\mathbf{g}$ is a smooth function that looks like a switch function.
- The $\mathbf{g}$ function connects the analog and digital worlds.
  Photovoltaic circuits use $\mathbf{g}$ in their exact math description.

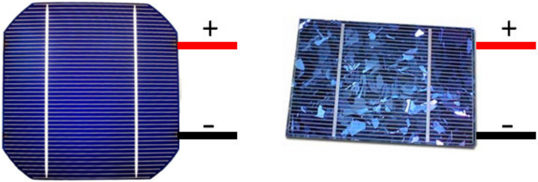
Solar Cell Terminology

Solar Cell, Solar Module, Solar Panel, and Solar Array



Solar Cell Properties

Properties of a Typical Single Solar Cell

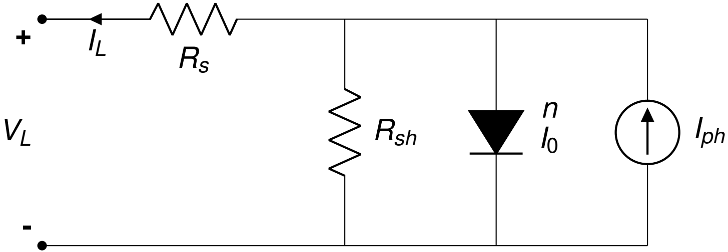


Monocrystalline   Polycrystalline

Acts like a 0.5V battery
The current flow changes with the amount of sun hitting the cell

Open Circuit Voltage (Voc)	=~ 0.625 Vdc
Maximum Power Voltage (Vmp)	=~ 0.5 Vdc
Maximum Power Current (Imp)	=~ 8.0 Amps
Maximum Short Circuit Current (Isc)	=~ 8.5 Amps
Optimum Power = Vmp x Imp (Pmax)	=~ 4 watts

Solar Cell Equations



- Implicit equation

$$I_L = I_{ph} - \frac{V_L + I_L R_s}{R_{sh}} - I_0 \left(\exp\left[\frac{V_L + I_L R_s}{n V_{th}}\right] - 1 \right)$$

Solar Cell Equations (continued)

- Explicit equation for V_L as a function of I_L

$$\begin{aligned} V_L = f_V(I_L) &= -I_L R_s + (-I_L + I_{ph} + I_0) R_{sh} \\ &- nV_{th}\, W\left(\frac{I_0 R_{sh} \exp\left[\frac{R_{sh}(-I_L + I_{ph} + I_0)}{nV_{th}}\right]}{nV_{th}} \right) \end{aligned}$$

- Explicit equation for I_L as a function of V_L

$$\begin{aligned} I_L = f_I(V_L) &= -\frac{V_L}{R_s + R_{sh}} + \frac{R_{sh}(I_0 + I_{ph})}{R_s + R_{sh}} \\ &- \frac{nV_{th}}{R_s} W\left(\frac{R_s I_0 R_{sh} \exp\left[\frac{R_{sh}(R_s I_{ph} + R_s I_0 + V_L)}{nV_{th}(R_s + R_{sh})}\right]}{nV_{th}(R_s + R_{sh})} \right) \end{aligned}$$

Solar Cell Equations (continued)

- Revised explicit equation for V_L as a function of I_L

$$V_L = -I_L R_s + nV_{th}\left(g(u_I) - \log\left[\frac{I_0 R_{sh}}{nV_{th}}\right]\right)$$

where

$$u_I = \log\left[\frac{I_0 R_{sh}}{nV_{th}}\right] + \frac{R_{sh}(I_{ph} + I_0 - I_L)}{nV_{th}}.$$

Solar Cell Equations (continued)

- Revised explicit equation for I_L as a function of V_L

$$I_L = -\frac{V_L}{R_s} + \frac{nV_{th}}{R_s}\left(g(u_V) - \log\left[\frac{I_0 R_s R_{sh}}{nV_{th}(R_{sh} + R_s)}\right]\right)$$

where

$$u_V = \log\left[\frac{I_0 R_s R_{sh}}{nV_{th}(R_{sh} + R_s)}\right] + \frac{R_{sh}(R_s I_{ph} + R_s I_0 + V_L)}{nV_{th}(R_{sh} + R_s)}.$$

Solar Cell Equations (continued)

- Examples of arguments to Lambert W and to g function.
- Parameters: $I_{ph} = 0.1023$ amp, $I_0 = 0.10356 \times 10^{-6}$ amp, $n = 1.5019$, $R_s = 0.06826$ ohm, $R_{sh} = 1000$ ohms, and $V_{th} = 0.02585$ volts (at temperature of 300 K).
- Calculating V_L using Lambert W formula when $I_L = 0$ (known as the open circuit voltage V_{oc}) involves evaluating $W(5.972 \times 10^{1141})$ whereas g formula evaluates $g(2629)$.
- Calculating I_L using Lambert W formula when $V_L = 0$ (known as the short circuit current I_{sc}) involves evaluating $W(2.180 \times 10^{-7})$ whereas g formula evaluates $g(-15.34)$.

Implementing $y = g(x)$ Computation

- To Calculate $y = g(x) = \log(W(\exp(x)))$
  x and y are reals. Computer needs exp and log.
- Initial estimate y_0
  For $x \leq -e$, take $y_0 = x$.
  For $x \geq e$, take $y_0 = \log(x)$.
  For $-e < x < e$, take y_0 as a linear interpolation between the points $(-e, -e)$ and $(e, 1)$.

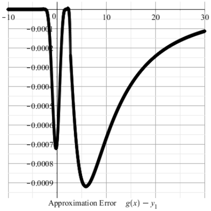
$$y_0 = -e + \frac{1+e}{2e}(x+e)$$

- Refine using Halley's method

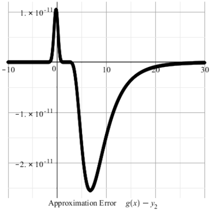
$$y_{n+1} = y_n - \frac{2(y_n + e^{y_n} - x)(1 + e^{y_n})}{2(1+e^{y_n})^2 - (y_n + e^{y_n} - x)e^{y_n}}.$$

Errors of Estimates

- First refinement error $g(x) - y_1$



- Second refinement error $g(x) - y_2$



References and Resources

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