Available online at www.sciencedirect.com

SCIENCE DIRECT

ELSEVIER

Solar Energy Materials & Solar Cells 90 (2006) 352–361

Solar Energy Materials & Solar Cells

www.elsevier.com/locate/solmat

New method to extract the model parameters of solar cells from the explicit analytic solutions of their illuminated *I– V* characteristics

Adelmo Ortiz-Conde*, Francisco J. García Sánchez, Juan Muci

Solid State Electronics Laboratory, Simón Bolívar University, Apartado Postal 89000, Caracas 1080A, Venezuela

Received 3 January 2005; received in revised form 31 March 2005; accepted 11 April 2005
Available online 2 June 2005

Abstract

We present a new method to extract the intrinsic and extrinsic model parameters of illuminated solar cells containing parasitic series resistance and shunt conductance. The method is based on calculating the Co-content function (CC) from the exact explicit analytical solutions of the illuminated current–voltage (*I– V*) characteristics. The resulting CC is expressed as a purely algebraic function of current and voltage from whose coefficients the intrinsic and extrinsic model parameters are then readily determined by bidimensional fitting. The procedure is illustrated by applying it to experimental and synthetic *I– V* characteristics and an analysis of the errors is presented.

Keywords: Solar cell parameter extraction; Parasitic resistance; Parasitic shunt conductance; Lambert *W* function

*Corresponding author. Tel.: +58 2129064010; fax: +58 2129064025.
E-mail addresses: ortizc@ieee.org (A. Ortiz-Conde), fgarcia@ieee.org (F.J. García Sánchez), jmuci@usb.ve (J. Muci).

0927-0248/$ - see front matter
doi:10.1016/j.solmat.2005.04.023

1. Introduction

The *I– V* characteristics of illuminated solar cells are customarily described by the use of a lumped parameter equivalent circuit which requires considering the presence of parasitic series resistance and shunt conductance. The model parameters are closely related to the internal physical mechanisms acting within the solar cell. Their knowledge is therefore important, not only for cell array and system simulation, but also as an analysis tool to gain an understanding of the processes involved. Optimization efforts and quantitative studies to asses the capabilities of a particular technology greatly benefit from the correct extraction of the model parameters under various test conditions. Several methods have been proposed to extract the model parameters from the measured illuminated current–voltage characteristics. One recurrent issue refers to the difficulty encountered in extracting the so-called intrinsic junction parameters, quality factor and reverse current, from the experimental illuminated current–voltage characteristics in the presence of considerable series and shunt parasitics. This issue has received significant attention and several extraction methods have been proposed to that effect. Most of them represent approximate procedures that aim to make the *I–V* characteristics analytically manageable, relying essentially on extracting each parameter from restricted regions of the *I–V* characteristics where some other parameter is assumed to be negligible. Such approaches work as long as distinct regions are actually present in the characteristics, which is not necessarily the case when the junction exhibits considerable parasitic series resistance or shunt conductance.

2. Explicit analytic solutions

Consider a generic solar cell whose illuminated *I–V* characteristics may be described by a lumped parameter equivalent circuit model consisting of a single exponential-type ideal junction, a constant photo-generated current source, a series parasitic resistance (R_s), and a parallel parasitic conductance (G_p). Fig. 1 presents the

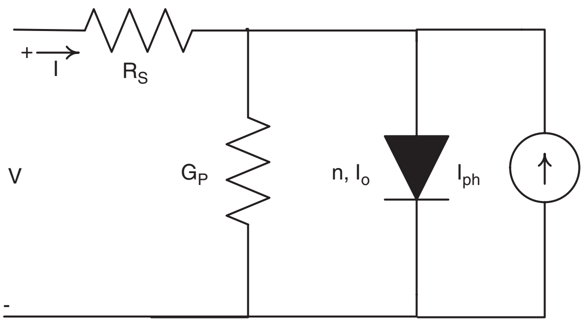

Fig. 1. Generic solar cell equivalent circuit including parasitic series resistance and parallel conductance.

equivalent circuit of such a model. The mathematical description of this circuit is given by the following equation:

$$I = I_0\left[\exp\left(\frac{V - IR_s}{nV_{th}}\right) - 1\right] + (V - IR_s)G_p - I_{ph}, \quad (1)$$

where I is the terminal current, V is the terminal voltage, I_0 is the reverse saturation current, n is the ideality factor, I_{ph} is the photo-generated current and $V_{th} = kT/q$ is the thermal voltage.

It is well known that the above implicit transcendental equation may not be solved explicitly in general for I or V using common elementary functions. Therefore, it has been customary to use explicit approximate solutions for modeling purposes. Several of these approximate solutions have been proposed [1,2] which use only elementary functions. However, nowadays exact explicit analytical solutions for I and V already exist [3,4]. These solutions make use of what is known as the Lambert W function [5], a special function which is not expressible in terms of elementary analytical functions. The Lambert W function is defined as the solution to the equation $W(x)\exp[W(x)] = x$. Although this function has not yet been widely used in electronics problems, it has already proved useful in other physics applications [6,7]. It has also been used for solving some previously analytically unsolved but basic diode [3,4] and bipolar transistor circuit analysis problems [8], as well as in device modeling formulations [9]. This type of solution can also be used directly to study illuminated solar cells, as was recently done [10–13], by adding the photo-generated current to the junction current.

The solutions for each variable I and V as an explicit function of the other and of the device model parameters are:

$$I = \frac{nV_{th}}{R_S} W\left\{\frac{I_0 R_S}{nV_{th}(1 + R_S G_P)}\exp\left[\frac{V + R_S(I_0 + I_{ph})}{nV_{th}(1 + R_S G_P)}\right]\right\} + \frac{VG_P - (I_0 + I_{ph})}{1 + R_S G_P} \quad (2)$$

and

$$V = -nV_{th} W\left[\frac{I_0}{nV_{th}G_P}\exp\left(\frac{I + I_0 + I_{ph}}{nV_{th}G_P}\right)\right] + I\left(R_S + \frac{1}{G_P}\right) + \frac{I_0 + I_{ph}}{G_P}, \quad (3)$$

where "W" represents the usual short-hand notation for the principal branch of the Lambert W function.

These explicit representations are convenient and efficient computational alternatives to using the iterative solution of the original transcendental equation (1) in solar cell models. However, in spite of the fact that they are explicit analytic expressions, they still remain unsuitable for the purpose of extracting the model parameters directly by numerical fitting. Our aim now is to transform Eqs. (2) and (3) into some simpler forms which may be more readily manageable from the point of view of numerical fitting.

3. Co-content function

The problem of extracting the model parameters is generally attempted by direct vertical optimization [14] of the parameters using the measured I–V data by minimizing the quadratic error on the vertical axis (i.e., the current). However, this method would be quite computationally intensive because of the implicit nature of the original equation (1). Direct lateral optimization was proposed [15] for a diode exhibiting only significant series resistance, based on the approach of minimizing the error on the horizontal axis (i.e., the voltage). The motivation for doing so, in the case of a diode with only series resistance ($G_{\mathrm{p}} \approx 0$), is that the voltage can be explicitly solved from Eq. (1) as a function of the current using only elementary analytic functions, which considerably reduces the computation time. However, this approach is not applicable when there is significant presence of shunt loss. Recently, a combination of lateral and vertical optimization was used [16] to extract the parameters of an illuminated solar cell.

Other extraction procedures make use of auxiliary functions or operators [17–20]. A useful method uses the integral difference function $D(I,V)$, which involves the integration of the current with respect to voltage. This method has been successfully used to extract the model parameters of diodes in the dark [21,22] and has also found useful applications in other areas, such as for quickly calculating device harmonic distortion [23].

The use of integration, instead of the more commonly employed differentiation, makes this method more immune to measurement errors because of the low-pass filter nature of integration.

The Co-content $\mathrm{CC}(I,V)$ in the present case is defined [24] as:

$$\mathrm{CC}(I, V) \equiv \int_0^V (I - I_{\mathrm{sc}})\,\mathrm{d}V. \tag{4}$$

It should be stressed that the integral's lower limit shown in Eq. (4) is defined here at the point $V = 0$, $I = I_{\mathrm{sc}}$.

Substitution of Eq. (2) into Eq. (4) and integrating with respect to V results in a long expression that contains Lambert W functions and the variables V, and I. Replacing the terms that contain Lambert W functions of V, using Eq. (3), and after some algebraic manipulation, we find that function $\mathrm{CC}(I,V)$ may be conveniently expressed as a purely algebraic equation of the form:

$$\mathrm{CC}(I, V) = \mathrm{C_{V1}}V + \mathrm{C_{I1}}(I - I_{\mathrm{sc}}) + \mathrm{C_{I1V1}}V(I - I_{\mathrm{sc}}) + \mathrm{C_{V2}}V^2 + \mathrm{C_{I2}}(I - I_{\mathrm{sc}})^2, \tag{5}$$

where the five coefficients are given in terms of the model parameters by

$$\mathrm{C_{I1}} = R_{\mathrm{S}}(I_0 + I_{\mathrm{ph}} + I_{\mathrm{sc}}) + nV_{\mathrm{th}}(1 + G_{\mathrm{P}}R_{\mathrm{S}}) + I_{\mathrm{sc}}R_{\mathrm{S}}^2G_{\mathrm{P}}, \tag{6}$$

$$\mathrm{C_{V1}} = -(I_0 + I_{\mathrm{ph}} + I_{\mathrm{sc}}) - nV_{\mathrm{th}}G_{\mathrm{P}} - I_{\mathrm{sc}}R_{\mathrm{S}}G_{\mathrm{P}}, \tag{7}$$

$$C_{I2} = \frac{R_S(1 + G_P R_S)}{2}, \tag{8}$$

$$C_{V2} = \frac{G_P}{2} \tag{9}$$

and the fifth coefficient is dependent on the others:

$$C_{I1V1} = \frac{1 - \sqrt{1 + 16C_{I2}C_{V2}}}{2}. \tag{10}$$

As can be seen, there are actually four independent coefficients, Eqs. (6)–(9), and therefore only four unknowns may be extracted uniquely.

4. Parameter extraction

The parameter extraction procedure consists of fitting algebraic equation (5) to the Co-content function CC which is numerically calculated with Eq. (4) from the experimental data. This bidimensional fitting process produces the values of the equation coefficients C_{V1}, C_{I1}, C_{V2}, and C_{I2}. The resulting coefficients are then used to calculate the solar cell model parameters G_P, R_S, I_{ph}, n, and I_0 as follows:

Eq. (9) gives directly the value of the shunt loss

$$G_P = 2C_{V2}. \tag{11}$$

Substituting Eq. (11) into Eq. (8) and solving the quadratic equation produces the series resistance:

$$R_S = \frac{\sqrt{1 + 16C_{V2}C_{I2}} - 1}{4C_{V2}}. \tag{12}$$

Substituting Eqs. (11) and (12) into Eqs. (6) and (7) and solving the two equations gives the junction quality factor

$$n = \frac{C_{V1}(\sqrt{1 + 16C_{V2}C_{I2}} - 1) + 4C_{I1}C_{V2}}{4V_{th}C_{V2}} \tag{13}$$

and, assuming $I_0 \ll I_{ph}$, the value of the photo-generated current is

$$I_{ph} = -\frac{(1 + \sqrt{1 + 16C_{V2}C_{I2}})(C_{V1} + I_{sc})}{2} - 2C_{I1}C_{V2}. \tag{14}$$

Finally, the value of I_0 is obtained using Eq. (1) and the above extracted parameters:

$$I_0 = \frac{I - (V - IR_s)G_p + I_{ph}}{\exp((V - IR_s)/nV_{th}) - 1}. \tag{15}$$

5. Discussion

To illustrate the extraction method it is applied to experimental solar cell I–V characteristics considering a model with a single exponential and series resistance and junction shunt loss. An extreme case, where the parasitic series resistance and shunt conductance are notably significant, is used to exemplify the capabilities of the proposed method. The experimental data was recently published for a developmental plastic solar cell [25]. The data points are presented in Fig. 2. These and the following calculations were performed in a Maple environment [26] using 20-digit precision.

To begin the extraction procedure, the Co-content function $CC(I,V)$ is first calculated by applying Eq. (4) to the experimental I–V data numerically integrating the current with respect to the voltage. Next, the algebraic equation of $CC(I,V)$ presented in Eq. (5) is directly fitted to the previously calculated $CC(I,V)$ to yield the values of the coefficients C_{V1}, C_{I1}, C_{V2}, C_{I2}. The parameters are then obtained from these coefficients using Eqs. (11)–(14). Finally the reverse saturation current is calculated from Eq. (15) evaluated at the open circuit point, $V = V_{oc}$, $I = 0$, or neglecting the insignificant -1 in the denominator from

$$I_0 = \frac{I_{ph} - V_{oc}G_p}{\exp(V_{oc}/nV_{th})}. \tag{16}$$

The resulting parameter values for this experimental cell are presented in Table 1. Also presented are the parameters extracted by two other traditional methods: optimization method where the data is directly fitted to the original model equation (1), and a second method where the parameters are extracted from five points in the illuminated I–V characteristics, namely, open circuit voltage, short circuit current,

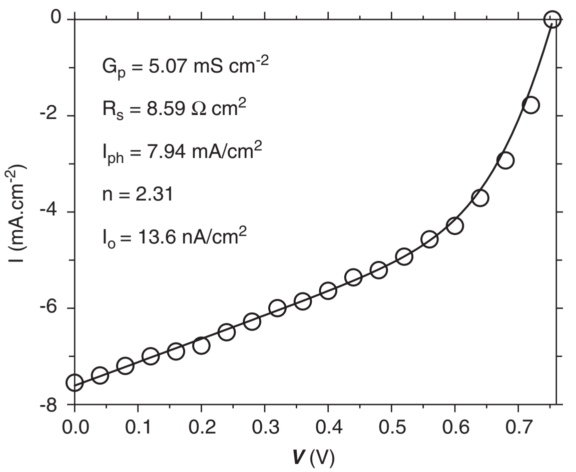

Fig. 2. Test synthetic single-exponential illuminated solar cell I–V characteristics (line) over the original experimental data points (symbols).

Table 1
Extracted parameters obtained from three different methods for a plastic solar cell

Parameters	Present method	Optimization method	Five-point method
R_S ($\Omega\,cm^2$)	8.59	6.21	0.804
G_P ($mS\,cm^{-2}$)	5.07	4.92	4.51
n	2.31	2.32	3.45
I_0 ($nA\,cm^{-2}$)	13.6	13.6	957
I_{ph} ($mA\,cm^{-2}$)	7.94	7.82	7.63

Table 2
Characteristic values using the extracted parameters for a plastic solar cell

Characteristic values	Present method	Optimization method	Five-point method
V_{oc} (V)	0.755	0.758	0.75
I_{sc} ($mA\,cm^{-2}$)	7.61	7.59	7.60
V_m (V)	0.548	0.558	0.54
I_m ($mA\,cm^{-2}$)	4.71	4.69	4.8
$\left.\frac{dI}{dV}\right\|_{I=I_{sc}}$ ($mS\,cm^{-2}$)	4.86	4.77	4.5
$\left.\frac{dI}{dV}\right\|_{V=V_{oc}}$ ($mS\,cm^{-2}$)	45.2	50.2	50

maximum power point, slope at open circuit, and slope at short circuit. Since optimization can be implemented in several ways, it is important to point out our particular procedure. We selected a bidimensional direct fitting by rewriting Eq. (1) as

$$f(V, I) = I_0\left[\exp\left(\frac{V - IR_s}{nV_{th}}\right) - 1\right] + (V - IR_s)G_p - I_{ph} - I = 0. \tag{17}$$

Then, we fit $f(V, I)$ to zero considering I and V as two independent variables.

We conclude from Table 1 that optimization and the present method yield the same reasonable approximate extracted parameters. In contrast, the five-point method produces unreasonable extracted parameters because it is very sensitive to errors in the evaluation of the five points. Table 2 presents the characteristic values using the parameters extracted by the three methods.

To analyze the error sensitivity of the present extraction method, it is again applied but now to synthetic solar cell I–V characteristics calculated using parameters values of $I_0 = 13.6\,nA/cm^2$, $n = 2.31$, $R_S = 8.59\,\Omega\,cm^2$, $G_P = 5.07\,mS\,cm^{-2}$, and $I_{ph} = 7.94\,mA\,cm^{-2}$. These parameters were chosen so as to produce results comparable to the above-mentioned experimental data. Fig. 2

presents the resulting synthetic solar cell I– V characteristics over the original experimental data points.

Fig. 3 presents the resulting CC(I,V) as a function of current and forward voltage, together with its CC(I) and CC(V) projections, and the original illuminated I– V characteristics on the horizontal I– V plane. A fourth-order Simpson-type numerical integration of the current with respect to the voltage was used in the calculation. By fitting the calculated CC(I,V) to the algebraic equation of CC(I,V) presented in Eq. (5), the values of the coefficients C_{V1}, C_{I1}, C_{V2}, and C_{I2} are obtained. The parameters are then calculated from these coefficients using Eqs. (11)–(14). The resulting values are: $n = 2.309999957$, $R_S = 8.590000499\,\Omega\,\text{cm}^2$, $G_P = 5.070000015\,\text{mS}\,\text{cm}^{-2}$, and $I_{ph} = 7.940004989\,\text{mA}\,\text{cm}^{-2}$. Finally a reverse saturation current of $I_0 = 13.60001411\,\text{nA}\,\text{cm}^{-2}$ is calculated from Eq. (15) evaluated at the open circuit point.

The errors of the parameters extracted above from the synthetic I– V data turn out to be insignificant. However, it is worth analyzing nonideal measurement situations by looking at the effects of the number of data points measured and of the measurement errors. To that effect, we applied the extraction procedure to the same synthetic I– V characteristics but now with random noise added to the data to test the sensitivity of the method to measurement error or noise. Fig. 4 shows the relative errors of the extracted parameters resulting from adding up to 1.5% random noise to the original synthetic data, repeated for different numbers of measured data points as determined by the measuring voltage step.

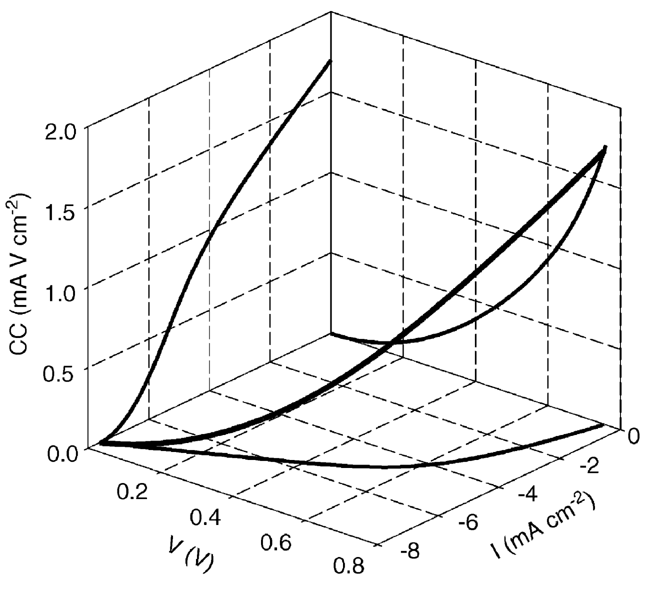

Fig. 3. Resulting CC(I,V) as a function of current and voltage, showing also its CC(I) and CC(V) projections and the original I– V characteristics on the horizontal plane.

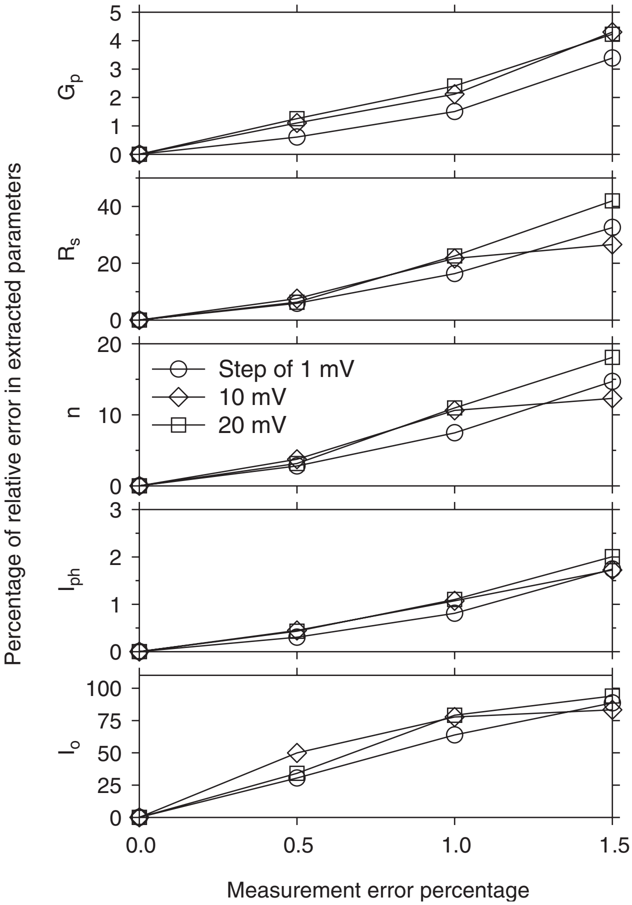

Fig. 4. Relative errors of the extracted parameters resulting as a function of percent random noise added to the original synthetic data for three measuring voltage steps.

6. Conclusions

A new method to extract the intrinsic and extrinsic model parameters of illuminated solar cells containing parasitic series resistance and shunt conductance has been presented. It is based on the Co-content function CC which is applied to the exact explicit analytical solutions of the cell's illuminated *I– V* characteristics. Since the resulting equation of CC is expressed in terms of Lambert *W* functions, parameter extraction by direct numerical fitting of the equation turns out to be computationally cumbersome. Instead, the resulting CC is reduced to a purely algebraic equation with terms in I, V, IV, I^2, and V^2. After fitting this algebraic equation to the CC calculated from the experimental data, the intrinsic and extrinsic model parameters are readily determined from the fitted equation's coefficients. The applicability of the procedure was exemplified on experimental illuminated I–V characteristics with substantial values of series resistance and shunt conductance. To study the errors produced by measurement inaccuracies, synthetic illuminated I–V characteristics with varying values of added random noise were used. The results indicate that this quadratic two-dimensional fitting process is a robust, accurate, fast, and easily applicable parameter extraction procedure for illuminated solar cells.

References

[1] A. Ortiz-Conde, F.J. García Sánchez, Electron. Lett. 28 (1992) 1964–1965.
[2] J. Le Bihan, Int. J. Circ. Theor. Appl. 29 (2001) 397–401.
[3] A. Ortiz-Conde, F.J. García Sánchez, J. Muci, Solid State Electron. 44 (2000) 1861–1864.
[4] A. Ortiz-Conde, F.J. García Sánchez, Solid State Electron. 49 (2005) 465–472.
[5] R.M. Corless, G.H. Gonnet, DE.G. Hare, D.J. Jeffrey, D.E. Knuth, Adv. Comput. Math. 5 (1996) 329–359.
[6] S.R. Valluri, D.J. Jeffrey, R.M. Corless, Canad. J. Phys. 78 (2000) 823–831.
[7] J.M. Caillol, J. Phys. A: Gen. 36 (2003) 10431–10442.
[8] T.C. Banwell, IEEE Trans. Circuits and Systems I 47 (2000) 1621–1633.
[9] A. Ortiz-Conde, F.J. García Sánchez, M. Guzmán, Solid State Electron. 47 (2003) 2067–2074.
[10] A. Jain, A. Kapoor, Sol. Energy Mater. Sol. Cells 81 (2004) 269–277.
[11] A. Jain, A. Kapoor, Sol. Energy Mater. Sol. Cells 86 (2005) 197–205.
[12] A. Jain, A. Kapoor, Sol. Energy Mater. Sol. Cells 85 (2005) 391–396.
[13] A. Jain, S. Sharma, A. Kapoor, Solar cell array parameters using Lambert W-function Sol. Energy Mater. Sol. Cells (2005), in press, available online.
[14] A. Ferhat-Hamida, Z. Ouennoughi, A. Hoffmann, R. Weiss, Solid State Electron. 46 (2002) 615–619.
[15] A. Ortiz-Conde, Yuanshen Ma, J. Thomson, E. Santos, J.J. Liou, F.J. García Sánchez, M. Lei, J. Finol, P. Layman, Solid State Electron. 43 (1999) 845–848.
[16] M. Haouari-Merbah, M. Belhamel, I. Tobías, J.M. Ruiz, Extraction and analysis of solar cell parameters from the illuminated current–voltage curve, Sol. Energy Mater. Sol. Cells 87 (2005) 225–233.
[17] H. Norde, J. Appl. Phys. 50 (1979) 5052–5053.
[18] A. Kaminski, J.J. Marchand, A. Fave, A. Laugier, Proceedings of the 1997 IEEE Photovoltaics Specialists Conference, IEEE Press, New York, 1997, pp. 203–206.
[19] J.I. Lee, J. Brini, C.A. Dimitriadis, Electron. Lett. 34 (1998) 1268–1269.
[20] H. Wong, W.H. Lam, Proceedings of the 2001 IEEE Hong Kong Electron Devices Meeting, 2001, pp. 38–41.
[21] J.C. Ranuárez, F.J. García Sánchez, A. Ortiz-Conde, Solid State Electron. 43 (1999) 2129–2133.
[22] J.C. Ranuárez, A. Ortiz-Conde, F.J. García Sánchez, Microelectron. Reliab. 40 (2000) 355–358.
[23] A. Cerdeira, M. Estrada, R. Quintero, D. Flandre, A. Ortiz-Conde, F.J. García Sánchez, Solid State Electron. 46 (2002) 103–108.
[24] F.J. García Sánchez, A. Ortiz-Conde, J.J. Liou, IEE Proc. Cir. Dev. Syst. 143 (1996) 68–70.
[25] T. Jeranko, H. Tributsch, N.S. Sariciftci, J.C. Hummelen, Sol. Energy Mater. Sol. Cells 83 (2004) 247–262.
[26] Maplesoft, Waterloo Maple Inc. 2003. See also: http://www.maplesoft.com.