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Extraction and analysis of solar cell parameters from the illuminated current–voltage curve

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Abstract

In this work we describe, apply and analyze a procedure to extract the physical parameters of a solar cell from its *I– V* curve under illumination. We compare it with other procedures and assess the statistical significance of the parameters. Our method, called APTIV, uses separate fitting in two different zones in the *I–V* curve. In the first one, near short circuit, current fitting is used because the error in current dominates. In the second one, near open circuit, voltage fitting is used because this is the dominant error. The method overcomes some drawbacks of common procedures: voltage errors are properly managed and no accurate initial guesses for the parameters are needed. In addition, the numerical implementation is very simple.

Keywords: Modeling; Characterization; Solar cells

1. Introduction

The current-voltage (*I–V*) curve under illumination is the most important measurement on any solar cell. It is routinely performed in both laboratory and industrial environments. The purpose of this work is to develop a method to extract

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the physical information conveyed by the (I–V) data in a reliable, physically consistent and mathematically simple way.

Many parameter-extraction procedures consist in minimizing the current error ε calculated as:

$$\varepsilon = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(I(V_i;p) - I_i)^2}, \tag{1}$$

where N is the number of measured I–V pairs denoted by ($V_{i,}$ I_i); $I(V, p)$ is the theoretical current for voltage V as predicted by a model containing several parameters represented by p that are the variables used to minimize the error. This formulation amounts to minimize the vertical distance between the measured points and the theoretical curve (see Fig. 1), and it implicitly assumes that no measurement error affects the voltage data. But this is not generally true and the influence of experimental errors is misinterpreted [1]. Due to the strongly varying slope of the curve, the current error is more important for small voltages, while the voltage error is more important for large voltages approaching open circuit.

For this reason our extraction procedure uses separate fitting for different regions in the I–V curve [2]. This is shown to properly account for voltage errors while retaining the simplicity of least-squares-fitting. Another virtue of our procedure is that no initial estimates are needed for the unknown parameters for the fitting procedure to converge. They can be zero at the beginning of the process, in contrast with other methods [1,3].

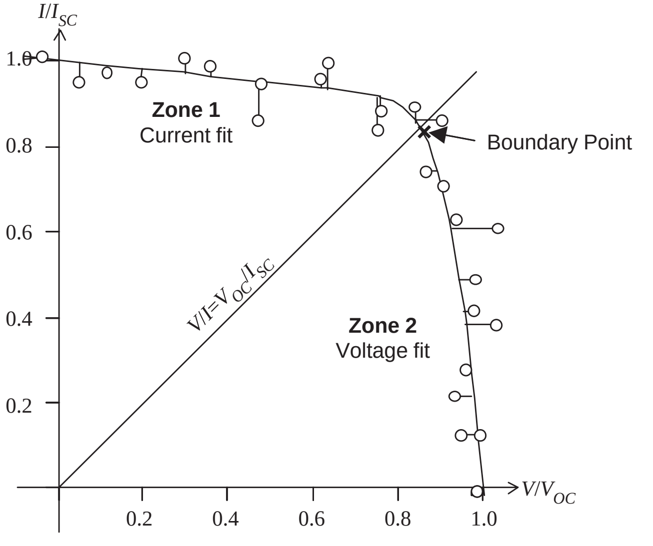

Fig. 1. Computer simulation of measured I–V points (solid circles) and the fitted curve (thick line), showing the division in two zones with different fitting error. The thin line through the origin separates the zones according to the value of V/I.

In the procedure proposed here advantage is taken of the shape of I–V curves to simplify the formulation. The cell model is least-squares fitted [2] to the data separately in two regions, as suggested in Fig. 1. The current error is minimized near shortcircuit (zone 1) and takes the form

$$\varepsilon_1 = \sqrt{\frac{1}{N_1}\sum_{zone1}(I(V_i;p) - I_i)^2}. \tag{2}$$

While the voltage error—the horizontal distance between measured point and theoretical curve—is minimized near open circuit (zone 2)

$$\varepsilon_2 = \sqrt{\frac{1}{N_2}\sum_{zone2}(V(I_i;p) - V_i)^2}. \tag{3}$$

Current and voltage errors must be combined through adequate weighting in order to compute a global one. By noticing that the respective measurement scales are naturally set by the shortcircuit current I_{SC} and the open circuit voltage V_{OC}, the fitting error (relative error in this case) is defined as

$$\varepsilon = \frac{\varepsilon_1}{I_{SC}} + \frac{\varepsilon_2}{V_{OC}}. \tag{4}$$

2. Description of the method

In this work, the I–V curve is modeled with the standard two-diode model:

$$I = I_L - (V + IR_S)G_P - I_{01}\left(\exp\left(\frac{V + IR_S}{m_1 V_t}\right) - 1\right) - I_{02}\left(\exp\left(\frac{V + IR_S}{m_2 V_t}\right) - 1\right). \tag{5}$$

The five parameters to be determined are the photogenerated current I_L, the series resistance R_S, the shunt conductance G_P, and the saturation currents I_{01} and I_{02} corresponding to recombination currents proceeding with ideality factors $m_1 = 1$ and $m_2 = 2$, respectively. V_t is the thermal voltage.

Most experimental I–V curves can be described with this five-parameter model though the method could be used with different ones. The addition of more adjustable parameters such as the ideality factors is not justified in most cases and leads to non-physical results. As a matter of fact this number of parameters, five, matches the number of apparent "geometrical" degrees of freedom of the curve: intersections with the axes, slopes near open circuit and shortcircuit, and curvature near the maximum power point.

The unknown fitting parameters (I_L, G_P, I_{02}, I_{01}, R_S) are set to zero at the beginning of the process. A fit to current is then performed in zone 1, near shortcircuit. The equation of the model is written in the form

$$I = I_{SC} - G_P^S V - a_3 h^S(I, V). \tag{6}$$

The superscript "S" placed over some coefficients identifies them as belonging to the fitting zone 1, close to shorcircuit. The coefficients to be determined are the shortcircuit current (the ordinate at the origin) I_{SC}, the slope in it ($\mathrm{d}I/\mathrm{d}V_{SC} = -G_P^S$), closely related to the shunt conductance G_P, and a_3. The first two terms are then the linear approximation to the I– V curve at shortcircuit.

The coefficient a_3 of the last term is not used for parameter extraction: its value should ideally be 1, which can be employed as a test of the goodness of the fitting. The term $h^S(I, V)$ represents the curvature of the characteristic. It describes how the curve departs from a straight line: it consists in the exponential functions in Eq. (5), from which the linear terms at shortcircuit have been substracted. It plays the role of the quadratic term in polynomial fitting but describes more faithfully the curve shape, so that the extracted coefficients are more reliable.

This function itself depends on the values of the model parameters and for this reason an iterative procedure is needed. The expressions for all the magnitudes that appearing in Eq. (6) in terms of the model parameters are given in the Appendix.

Following, a fit to voltage is performed in zone 2 (near open circuit). The model is written as

$$V = V_{OC} - IR_S^O - b_3 h^O(I, V). \tag{7}$$

The superscript "O" over a magnitude means that it is obtained from the fitting in this "open circuit" zone. Eq. (7) presents the curve near the open circuit in the same form as we did near shortcircuit: linear terms plus deviation. Expressions for all magnitudes appearing in it are given in the Appendix. It is to be remarked that Eq. (6) and Eq. (7) are just the equations of the model (Eq. (5)) which are written in a convenient form for fitting, so that the coefficients to be extracted are those more strongly dependant on the set of measured points being considered.

The extracted coefficients here are the open circuit voltage V_{OC} and the slope at this point ($\mathrm{d}V/\mathrm{d}I_{OC} = -R_S^O$), is directly related to the series resistance.

In addition to these four extracted coefficients (I_{SC}, G_P^S, V_{OC} and R_S^O), another condition is required to resolve the five model parameters. This condition is obtained by forcing the curve to pass through the (I, V) point that separates the two fitting zones (labeled as "boundary point" in Fig. 1). This point is usually near the maximum power one in the curve, but does not necessarily coincide with it.

The equations relating model and fitting parameters are linear except for R_S, so that the resolution of the system is very simple. In order to obtain results with physical meaning, several conditions are imposed to the parameters when solving them (nonnegative values, etc). The calculated parameters are used as the initial values for two new fits of the curve. The procedure is repeated until convergence, which takes only two iterations in most cases.

To summarize, the proposed procedure consists of the following steps:

1. The model parameters (I_L, G_P, I_{01}, I_{02}, R_S) are set initially to zero.
2. Two coefficients (I_{SC}, G_P^S) are extracted by fitting the measured points in zone 1, near shortcircuit, with Eq. (6).

3. Two more coefficients (V_{OC}, R_S^O) are extracted by fitting the measured points in zone 2, near open circuit, with Eq. (7).
4. Five non-linear equations are thus obtained: those given by I_{SC}, G_P^S, V_{OC} and R_S^O plus the condition that the curve passes through the boundary point. The equations are written in the Appendix (Eqs. (16–20)).
5. The system is solved for the model parameters, and we return to step 2. The process is repeated until the parameters converge.

3. Application of the method. Comparison with current-fitting and ODR

The mathematical simplicity of the APTIV method [2] allows it to be implemented as an Excel spreadsheet. It has been successfully applied to solar cells made from different materials (Si and GaSb) as well as with different technological processes. Some examples of fitted curves are given in Fig. 2.

As an example, the I– V curve in Fig. 1 (second cell) belongs to a defective 4 cm^2 silicon solar cell. ‘‘Bad’’ curves are more difficult to fit—as we will show later—in this particular case conventional current least-squares fitting fails unless very good initial values for the parameters are provided. Piecewise fitting, on the contrary, succeeds in arriving at a good fit without initial parameter estimates in three iteration steps. The extracted parameters and the errors are collected in Table 1.

The IVFIT program [4] fits both dark and illuminated I–V curves. It was developed at the Netherlands Energy Research Foundation (ECN). This program uses the Orthogonal Distance Regression (ODR) method and does not need any starting values for the diode parameters either. When applied to the second solar cell data in Fig. 2 and the five-parameter, two-diode model the results in Table 2 are

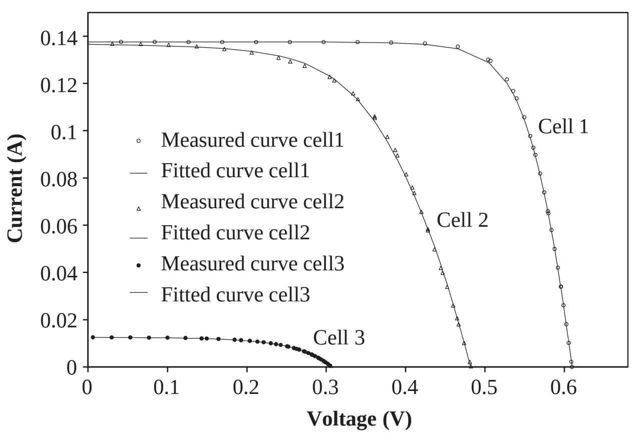

Fig. 2. Fit (solid lines) to the measured I– V curve (points) of solar cells made with different materials and technological processes. Open circles: crystalline Si $2 \times 2\,cm^2$ solar cell; open triangles: a defective solar cell of similar technology; solid circles: GaSb solar cell.

Table 1
Extracted parameters (cell 2. Fig. 2) with piecewise fitting

I_L (A)	0.136	$\sigma(I_L)$ (A)	0.001
G_P (Ω^{-1})	0.005	$\sigma(G_P)$ (Ω^{-1})	0.007
I_{01} (A)	0	$\sigma(I_{01})$ (A)	5.0×10^{-10}
I_{02} (A)	1.1992×10^{-5}	$\sigma(I_{02})$ (A)	4.9×10^{-10}
R_S (Ω)	0.43	$\sigma(R_S)$ (Ω)	0.34
ε (%) (global error)	**0.26**		

Table 2
Extracted parameters (cell 2. Fig. 2) with ODR

I_L (A)	0.137
G_P ($\Omega-1$)	0.0181
I_{01} (A)	2.36×10^{-10}
I_{02} (A)	8.33×10^{-6}
R_S (Ω)	0.508
ε (%) (global error)	**0.29**

Table 3
Extracted parameters (cell 2. Fig. 2) with conventional current-fitting

Parameters	Initial values	Final values	Initial values	Final values
I_L (A)	0	0.146	0.130	0.146
G_P ($\Omega-1$)	0	0.005	0.008	0.0024
I_{01} (A)	0	0	10^{-11}	5.45×10^{-10}
I_{02} (A)	0	3.448×10^{-7}	10^{-5}	4.18×10^{-6}
R_S (Ω)	0	0	0.5	0.6
ε (%)		233.67		**1.2**

obtained. These are in fact very similar in both the extracted values and the fitting error to APTIV results.

Finally, MULTIV [3] is a program developed at the Basque University. It uses least-squares, current-fitting to I–V curves of solar cells under illumination to extract the parameters of the devices. The most interesting property of MULTIV lies on its ability to perform simultaneous fitting to I–V characteristics with different levels of illumination and/or temperatures as well as the implementation of more sophisticated solar cell models than that given by Eq. (5). Without making use of these capabilities, the MULTIV program is used here to realize conventional current fitting with the two-exponential model and to show the problems associated with this method.

The first one is the need of accurate initial values and is illustrated in Table 3. For the same data, the extracted parameters for two different sets of initial values are presented. When initial values are set to zero, the program fails to extract suitable parameter values (third column) and a huge fitting error is displayed. When more correct initial values are provided the program gives a good and univocal result. But the fitting error is still sensibly larger than those obtained with the previous methods.

4. Uncertainty of extracted parameters

In each region, the fitting function (Eqs. (6) and (7)) depends linearly on the fitting coefficients (I_{SC}, G_P^S plus a_3, and V_{OC}, R_S^O plus b_3, respectively). We denote the set of extracted coefficients by $\{a_k\}$.

Fitting is straight-forward and provides with the covariance or error matrix **C** [5,6]. The diagonal elements of the error matrix give the variances of the fitting coefficients. The off-diagonal elements are called covariances and permit the calculation of the correlation factor, that quantifies the interdependence between the parameters. The standard deviation of the fitting coefficients is then calculated as

$$\sigma(a_k) = C_{kk}\sigma(y_i), \tag{8}$$

where $\sigma(a_k)$ is the standard deviation of the coefficient a_k, $\sigma(y_i)$ is the standard deviation of experimental data (where y_i is the voltage or current, depending on the fitting zone) and C_{kk} the pertinent diagonal element in the error matrix of the fitting. Since a_3 y b_3 are not used to extract information but only to control the quality of the fitting, we drop them from the set $\{a_k\}$, that has four elements (I_{SC}, G_P^S, V_{OC}, R_S^O).

Now the error in the extracted coefficients $\{a_k\}$ is to be converted into the error affecting the set of physical magnitudes appearing in the cell model, denoted by $\{p_j\}$. This set has four elements too, since the condition that the I–V curve passes through the boundary point (Eq. (20) in the Appendix) means that there are only four independent parameters to be determined from fitting.

Both sets are related through the nonlinear equations (Eqs. (16–19)) given in the Appendix. If the errors of the coefficients is small, these equations can be linearized to calculate the errors of the parameters. By differentiating Eqs. (16–19) and using Eq. (20) to eliminate one of the model parameters, we obtain the 4×4 matrix **D** whose k, j element is given by

$$D_{kj} \equiv \frac{\partial a_k}{\partial p_j}. \tag{9}$$

This matrix allows to relate the errors in the coefficients to the errors in the physical parameters as

$$\boldsymbol{\sigma}_\mathrm{p} = \mathbf{D}^{-1}\boldsymbol{\sigma}_\mathrm{a}, \tag{10}$$

where σ_P is a column matrix whose jth element is $\sigma(p_j)$, the standard deviation of the model parameter p_j, and σ_a a column matrix whose kth element is the standard deviation of the extracted coefficient a_k. These results are then used to establish the uncertainty of the calculated model parameters

$$p_j = p_{j0} \pm \sigma(p_j) \tag{11}$$

for instance, the results shown in Table 1 show that the I– V curve of this cell is dominated by I_L and I_{02}, and they are determined with good accuracy, while the remaining parameters can only be determined with a large uncertainty.

5. Conclusions

A procedure to extract and analyze solar cell parameters from the measured I–V curve under illumination has been developed. It is based on separate least-squares fitting of the model to the data in two different regions where the slope of the curve is very different: current fitting is used near short-circuit, and voltage fitting near open circuit. In this way both current and voltage measurement errors are properly accounted for.

In this respect the method is similar to Orthogonal Distance Regression—but it allows a much simpler implementation—and superior to conventional current-fitting. Besides, no accurate initial guesses for the model parameters are needed.

This method is completed with an error analysis so that the confidence of the extracted parameters can be calculated.

Appendix

The non-linear fit term in Eq. (6) ($h^S(I,V)$) is written as

$$h^S(I,V) \equiv -[I_1^S h_1^S(V-(I_{SC}-I)R_S) + I_2^S h_2^S(V-(I_{SC}-I)R_S)](1-R_S G_P^S), \tag{12}$$

where

$$I_k^S h_k^S(V-(I_{SC}-I)R_S) \equiv I_{0k}\exp\left(\frac{I_{SC}}{m_k V_t}\right)\left(\exp\left(\frac{V-(I_{SC}-I)R_S}{m_k V_t}\right) - 1 - \frac{V-(I_{SC}-I)R_S}{m_k V_t}\right), \quad \text{with } k=1,2. \tag{13}$$

As it was previously explained, the h function is the sum of the exponentials describing recombination in Eq. (5) minus the linear part at shortcircuit. On the other hand, for the fit near open circuit (Eq. (7)) the curvature term $h^O(I,V)$ reads:

$$h^O(I,V) \equiv -[I_1^O h_1^O(V_{OC}-V-IR_S) + I_2^O h_2^O(V_{OC}-V-IR_S)](R_S^O - R_S), \tag{14}$$

where, in this case,

$$I_k^O h_k^O(V_{OC}-V-IR_S) \equiv I_{0k}\exp\left(\frac{V_{OC}}{m_k V_t}\right)\left(\exp\left(-\frac{V_{OC}-V-IR_S}{m_k V_t}\right) - 1 + \frac{V_{OC}-V-IR_S}{m_k V_t}\right), \quad \text{with } k=1,2. \tag{15}$$

Again, the h function is the sum of two exponentials minus the linear part.

Next the nonlinear equation system to be solved in order to calculate the cell model parameters from the extracted fitting coefficients is written. The slope $-\mathrm{d}I/\mathrm{d}V$

at shortcircuit is

$$G_{\mathrm{P}}^{\mathrm{S}} = \frac{G_{\mathrm{P}} + \frac{I_{01}}{m_1 V_t} \exp \frac{R_{\mathrm{S}} I_{\mathrm{SC}}}{m_1 V_t} + \frac{I_{02}}{m_2 V_t} \exp \frac{R_{\mathrm{S}} I_{\mathrm{SC}}}{m_2 V_t}}{1 + R_{\mathrm{S}} \left(G_{\mathrm{P}} + \frac{I_{01}}{m_1 V_t} \exp \frac{R_{\mathrm{S}} I_{\mathrm{SC}}}{m_1 V_t} + \frac{I_{02}}{m_2 V_t} \exp \frac{R_{\mathrm{S}} I_{\mathrm{SC}}}{m_2 V_t} \right)} \tag{16}$$

and $-\mathrm{d}V/\mathrm{d}I$ at open circuit

$$R_{\mathrm{S}}^{\mathrm{O}} = R_{\mathrm{S}} + \frac{1}{G_{\mathrm{P}} + \frac{I_{01}}{m_1 V_t} \exp \frac{V_{\mathrm{OC}}}{m_1 V_t} + \frac{I_{02}}{m_2 V_t} \exp \frac{V_{\mathrm{OC}}}{m_2 V_t}}. \tag{17}$$

The expressions relating the shortcircuit current and the open circuit voltage with the model parameters are

$$I_{\mathrm{L}} = I_{\mathrm{SC}}(1 + G_{\mathrm{P}} R_{\mathrm{S}}) + I_{01} \left(\exp\left(\frac{R_{\mathrm{S}} I_{\mathrm{SC}}}{m_1 V_t} \right) - 1 \right) + I_{02} \left(\exp\left(\frac{R_{\mathrm{S}} I_{\mathrm{SC}}}{m_2 V_t} \right) - 1 \right) \tag{18}$$

$$I_{\mathrm{L}} = G_{\mathrm{P}} V_{\mathrm{OC}} + I_{01} \left(\exp\left(\frac{V_{\mathrm{OC}}}{m_1 V_t} \right) - 1 \right) + I_{02} \left(\exp\left(\frac{V_{\mathrm{OC}}}{m_2 V_t} \right) - 1 \right). \tag{19}$$

And, finally, it is required that the *I–V* curve passes through the point (I_b, V_b)

$$\begin{aligned} I_{\mathrm{L}} = {} & I_b(1 + G_{\mathrm{P}} R_{\mathrm{S}}) + G_{\mathrm{P}} V_b + I_{01} \left(\exp\left(\frac{V_b + R_{\mathrm{S}} I_b}{m_1 V_t} \right) - 1 \right) \\ & + I_{02} \left(\exp\left(\frac{V_b + R_{\mathrm{S}} I_b}{m_2 V_t} \right) - 1 \right). \end{aligned} \tag{20}$$

As commented in the text, (I_b, V_b) is chosen at the boundary between the two fitting regions. The method is not sensitive to the exact placement of this point, which is usually calculated as the average, weighted by the fitting functions, of the extreme experimental points in both regions.

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