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A new approach to the extraction of single exponential diode model parameters

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ABSTRACT

A new integration method is presented for the extraction of the parameters of a single exponential diode model with series resistance from the measured forward *I-V* characteristics. The extraction is performed using auxiliary functions based on the integration of the data which allow to isolate the effects of each of the model parameters. A differentiation method is also presented for data with low level of experimental noise. Measured and simulated data are used to verify the applicability of both proposed method. Physical insight about the validity of the model is also obtained by using the proposed graphical determinations of the parameters.

1. Introduction

Parameter extraction in diode and solar cells has been an active research topic for many years [1–15]. Some methods are based on optimization of the measured current-voltage characteristics [2,5] while other methods have proposed new functions [1,3,12,16–22], which eliminate the effects of one parameter. Some of these functions [3,12,16–22] are based on integration and as a means to reduce the extraction uncertainties arising from the probable presence of noise in the measured data. On the other hand, graphical methods have been used for the evaluation of series resistance in solar cells [23,24] as well as for the calculation of majority carrier density of semiconductors with multiple donors and acceptors [25,26].

In this article, we present two new methods to evaluate the parameters of an idealized single-exponential diode with a parasitic series resistance. These methods are based on functions which isolate the effects of each parameters. The first method is described in Section 3 and is based on the integration of the current with respect to the voltage while the second method uses differentiation and is presented in Section 4. The integration method is recommended for measured data with high level of experimental noise. Both methods plot the extracted parameters as a function of the current, and the validity of the model is clearly illustrated in the region in which the parameters are nearly a constant. On the other hand, classical optimization methods, which are the best to minimize quadratic errors, could yield to erroneous parameters if the model is used in invalid region.

2. Integration of the measured forward bias data

Consider a single-exponential diode with a parasitic series resistance R whose I-V characteristics may be described by [1,12]:

$$I = I_0\left[\exp\left(\frac{V-RI}{n\,v_{th}}\right)-1\right] \tag{1}$$

where V is the terminal voltage, I_0 is the reverse saturation current, n is the so-called diode quality factor, and $v_{th} = k_B T/q$ is the thermal voltage. The current can be explicitly solved from implicit Eq. (1) making use of the Lambert W function [6,15], while the terminal voltage can be explicitly solved using elementary functions [2,5,12]:

$$V = RI + n\,v_{th}\ln\left(1+\frac{I}{I_0}\right). \tag{2}$$

Following the pioneering idea of using numerical integration for parameter extraction [3], the drain current may be integrated by parts using the voltage expression in (2) and performing the algebraic manipulations:

$$\begin{aligned}\int_0^V IdV &= VI-\int_0^I VdI\\ &= I\left[RI + n\,v_{th}\ln\left(1+\tfrac{I}{I_0}\right)\right]\\ &\quad-\left[\tfrac{R}{2}I^2 + n\,v_{th}(I+I_0)ln\left(1+\tfrac{I}{I_0}\right)-n\,v_{th}(I+I_0)\right]\\ &= \tfrac{R}{2}I^2 + n\,v_{th}I-I_0 n\,v_{th}\left[\ln\left(1+\tfrac{I}{I_0}\right)-1\right]\end{aligned} \tag{3}$$

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For values of forward current $I >> I_0$ the effect of I_0 on the integral is negligible, so that the last term of the RHS (Right-Hand Side) of (3) may be neglected. Thus, the integral of the forward current turns out to be approximately described by a simple second order polynomial on I whose coefficients are directly and independently determined by two of the diode's parameters: R and n [17,18]:

$$\int_0^V IdV \approx \frac{R_s}{2}I^2 + n\,v_{th}I. \tag{4}$$

The two coefficients of this second order polynomial defined by the RHS of (4) may be adjusted by optimization to fit the numerical integral of the forward current data specified by the LHS (Left-Hand Side) of (4). Such optimization would directly extract the values of two of the diode's parameters: R and n.

On the other hand, it is also possible to remove the effect of the parasitic series resistance R using integration of the measured forward bias data. This may be easily done through the use of the "Integral Difference Function," which is defined as [19,20]:

$$D(V,I) \equiv \int_0^I VdI - \int_0^V IdV = IV - 2\int_0^V IdV, \tag{5}$$

where D has units of "electric power." Applying function D to the case of a single-exponential diode model with series resistance and restricting the analysis to values of forward current $I >> I_0$, substitution of (2) into (5) yields [19,20]:

$$D = (I + 2I_0)n\,v_{th}\ln(I/I_0 + 1) - 2n\,v_{th}I \approx In\,v_{th}[\ln(I/I_0) - 2], \tag{6}$$

which is an expression that contains two of the diode's parameters: I_0 and n, but does not contain R. Dividing (6) by the current yields another auxiliary function, called G:

$$G \equiv D/I \equiv V - \frac{2}{I}\int_0^V IdV \approx n\,v_{th}[\ln(I/I_0) - 2]. \tag{7}$$

It is interesting to note that function G expressed by (7) is very similar to the expression for the voltage of an intrinsic diode ($R = 0$) which according to (2) is: $n\,v_{th}\ln(I/I_o + 1)$.

Kaminski et al. [16] generalized this method in 1997 by allowing an arbitrary nonzero lower integration limit. In 2005 Tan et al. [10] extended function G by including the presence of a parallel parasitic resistance.

3. A method based in the integration of the measured forward bias data

In the previous section we presented the expression for G that contains two of the diode's parameters, I_0 and n, but does not contain R. Next, we will define a new function, which we call ΔG, as a means to also remove the effect of I_o, leaving only one parameter to be determined, n. The function is defined as the difference:

$$\Delta G(V,I) \equiv G(V,I) - G(V_R,I_R) \tag{8}$$

where (V_R, I_R) is a reference point of the I-V characteristic to be selected. Then, since parameters I_0 and n of the model described by (1) are assumed to have constant values throughout the entire forward *I-V* characteristics, and because (7) is a logarithmic function, substitution of (7) into (8) yields:

$$\Delta G(V,I) = n\,v_{th}\ln(I/I_R). \tag{9}$$

In order to use this ΔG function to extract the diode's parameters, the procedure to follow is:

First: Function G is numerically calculated using the integration in (7).

Second, Function ΔG is evaluated for some selected value of I_R using (8).

Third, Parameter n is obtained from (9) as:

$$n = \frac{\Delta G(V,I)}{v_{th}\ln(I/I_R)}, \tag{10}$$

and plotted as a function of the forward current.

Fourth, After having found the value of n, I_o is obtained using (7) as:

$$I_0 = \frac{I}{\exp\left(\frac{G}{n\,v_{th}} + 2\right)}, \tag{11}$$

and plotted as a function of the current.

Fifth and last, Parameter R is evaluated with (2) using the already extracted values of n and I_o:

$$R = \frac{V}{I} - \frac{n\,v_{th}}{I}\ln\left(\frac{I}{I_0} + 1\right) \approx \frac{V}{I} - \frac{n\,v_{th}}{I}\ln\left(\frac{I}{I_0}\right), \tag{12}$$

and plotted as a function of the current.

It is important to check that the resulting curves of n, I_o and R as plotted versus I should approach constant values, indicating that the assumed model of a single-exponential diode with parasitic series resistance is an adequate description of the actual *I-V* characteristics of the measured real device within the range of interest.

3.1. Verification of the procedure using simulations

The top part of Fig. 1 shows a simulated diode's *I-V* characteristics using equation (1), with a 10 mV step size, and the parameters previously reported in [5,12]: $n = 1.05$, and $I_o = 0.58$ nA and $R = 33.4\ \Omega$. Function G is calculated using the numerical integration of the simulated data as described by (7). We note that G is linearly proportional to the logarithm of the current as soon as few points are calculated in the

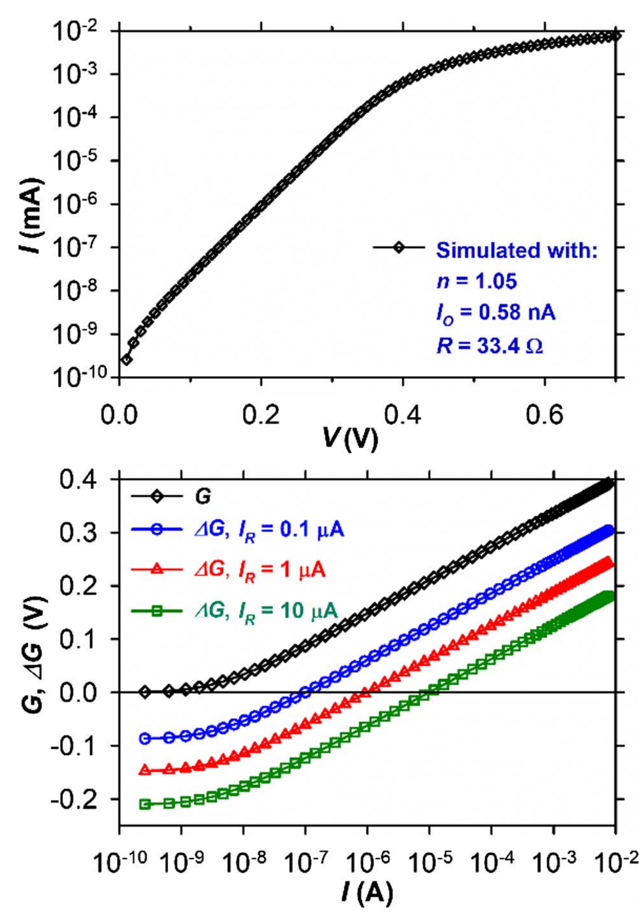

Fig. 1. Top: Simulated *I-V* characteristics of a silicon diode. Bottom: Functions G and ΔG as a function of the logarithm of the current calculated from the simulated *I-V* characteristics. Notice that the curves become straight lines as soon as $I >> I_0$.

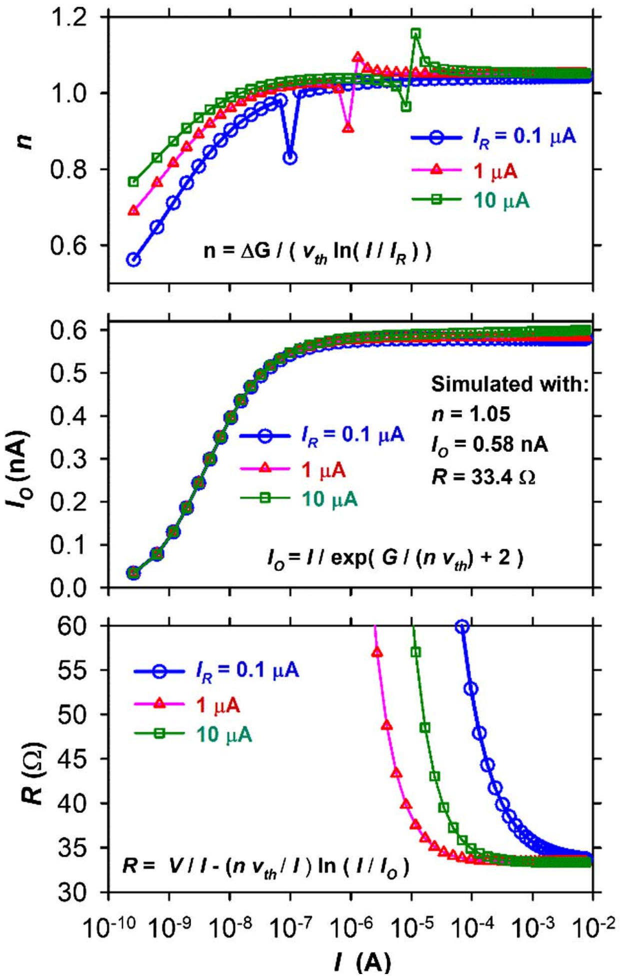

Fig. 2. Extracted parameters as a function of the logarithm of the current calculated from the previous simulated *I-V* characteristics.

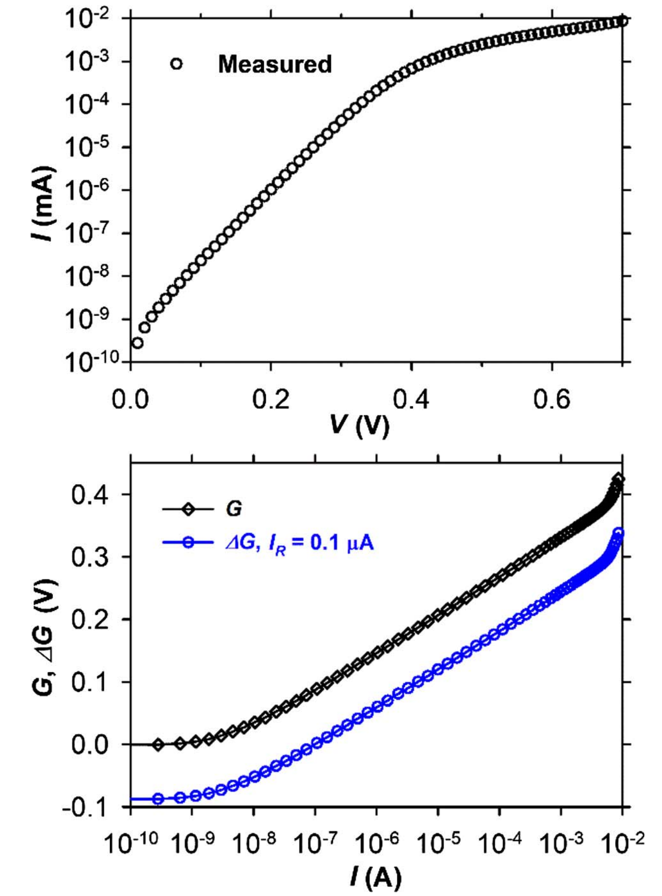

Fig. 3. Top: Measured *I-V* characteristics of a silicon diode. Bottom: Functions G and ΔG as a function of the logarithm of the current calculated from the previous measured *I-V* characteristics.

numerical integration, that is, as soon as the forward current $I >> I_0$. Then, function ΔG is numerically calculated using the definition in (8) and three selected values of I_R. The bottom part of Fig. 1 also presents functions ΔG. We note that function ΔG is function G shifted down by a constant value such that $\Delta G = 0$ for $I = I_R$. Therefore, we recommend that the selected values of I_R be far from the two extreme of the data values.

The top part of Fig. 2 shows a plot of n as a function of the logarithm of the current, using (10), for the various selected values of I_R. We note that n tends to be a constant for large current which is almost independent of I_R ($n = 1.052$ for $I_R = 10\,\mu$A and $n = 1.044$ for $I_R = 0.1\,\mu$A). We think that the very weak dependence of n with respect to I_R is due to small errors of the numerical integration which was implemented using a closed Newton-Cotes formula with seven points [27]. We also observe in this figure peak values at $I = I_R$ due to a mathematical artifact when the numerator and the denominator of (10) tends to zero at $I = I_R$. Therefore, these values of n at $I = I_R$ must be ignored in the procedure. It is important to mention that: (1) I_R must not be selected near the minimum or the maximum measured current because the numerical integration requires few points to produce accurate results; (2) the parameters must be determined for the current range where the model is valid; i.e., where the parameters tends to be nearly constant.

The middle part of Fig. 2 shows a plot of I_O as a function of the logarithm of the current, using (11) and the estimated values of n, for the various selected values of I_R. We note that I_O tends to be a constant for large current which is almost independent of I_R.

The bottom part of Fig. 2 shows a plot of R as a function of the logarithm of the current, using (12) and the estimated values of n and I_O, for the various selected values of I_R. We note that R tends to be a constant for large current which is almost independent of I_R.

3.2. Verification of the procedure using measurements

The top part of Fig. 3 presents measurements of a silicon diode from Motorola [5,12] using a 10 mV increment. The bottom part of this figure presents the corresponding functions G and ΔG. We note that the plot of G versus logarithm of I is not a straight line for either very small or very large current, which means that the assumed single-exponential diode model is not adequate at these extreme bias conditions. Therefore, we recommend that I_R be selected only where the model is valid; i.e., where function G is linearly proportional to the logarithm of the current.

The top part of Fig. 4 shows a plot of n as a function of the logarithm of the current, using (10), for $I_R = 0.1\,\mu$A. We note that n tends to be a constant for currents up to approximately 5 mA and for larger currents n increases. In this plot, we find $n = 1.03$ for $I = 2.49$ mA (which correspond to $V = 0.5$ V) and $n = 1.04$ for $I = 4.89$ mA (which correspond to $V = 0.6$ V). The increase of n for currents approaching 10 mA implies that the model is starting to be invalid probably because of high injection. We would like to point out that the desirability of this method consists in that it allows to graphically determine where the model is valid by observing the region in within n is nearly a constant. In contrast, most other parameter extraction methods do not easily allow to evaluate the appropriateness of the selected model.

The middle part of Fig. 4 shows a plot of I_O as a function of the

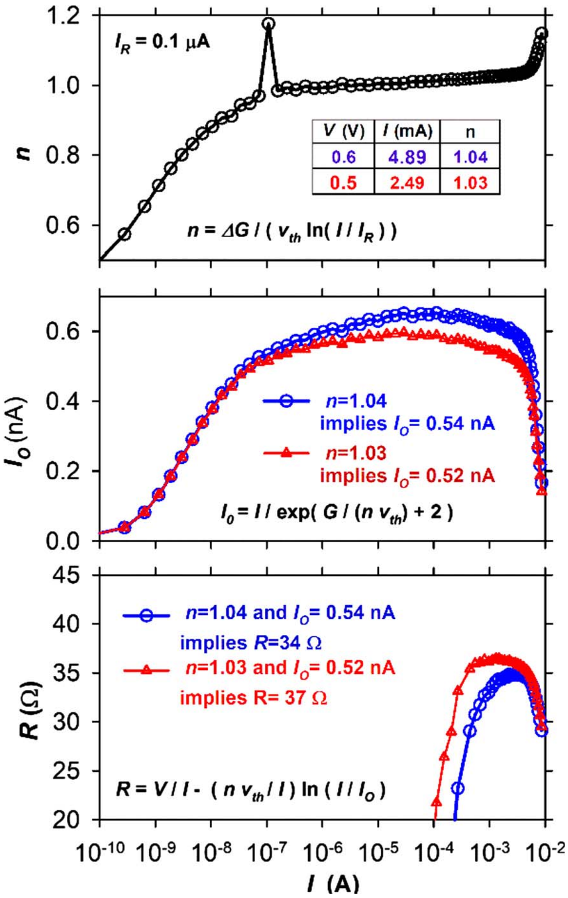

Fig. 4. Extracted parameters as a function of the logarithm of the current calculated from the previous measured *I-V* characteristics.

logarithm of the current, using (11) for the two previous estimated values of n (1.03 and 1.04). From this plot, we estimate that I_O is 0.54 nA (which corresponds to V = 0.6 V, I = 4.89 mA and n = 1.04) or 0.52 nA (which corresponds to V = 0.5 V, I = 2.49 mA and n = 1.03). This small variation in the parameter I_O means that the model is valid in this region.

The bottom part of Fig. 4 shows a plot of R as a function of the logarithm of the current, using (12) for the estimated values of n and I_O. From this plot, we estimate that R is 34 Ω or 37 Ω. We note that the extraction of R for low current is meaningless because the effect of R is only noticeable at high current.

Fig. 5 presents the previous measurements of a silicon diode and simulations using the parameters extracted by lateral optimization [2,5,12] and by the present method. We note that the extracted values with the present method are very close to those previously obtained by lateral optimization [5,12].

Fig. 6 presents the lateral errors of the previous simulations. We note that the three different simulations yields to comparable errors.

4. A method based in the differentiation of the measured forward bias data

Performing the first and second derivative of V with respect to I from Eq. (2), we obtain:

$$\frac{dV}{dI} = R + \frac{n\,v_{th}}{I + I_0} \tag{13}$$

and

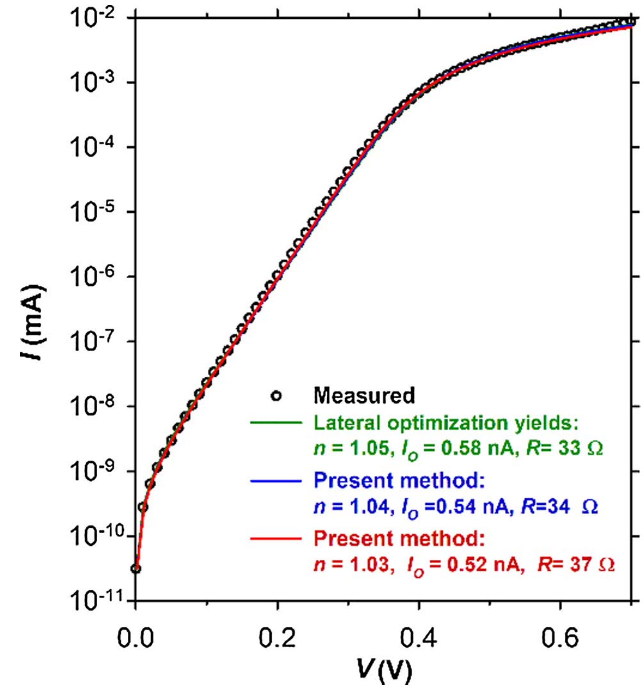

Fig. 5. Measured *I-V* characteristics of a silicon diode and its simulation using the parameter values extracted by lateral optimization and by the present method. We note that the extracted values with the present method are very close to those previously obtained by lateral optimization [5,12].

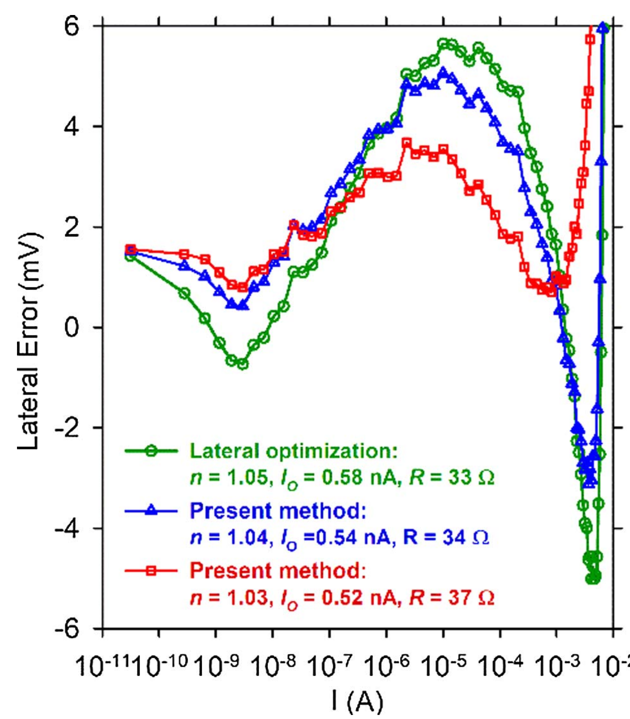

Fig. 6. Corresponding lateral error of the simulations presented in the previous figure. The three different simulations yields to comparable errors.

$$\frac{d^2V}{dI^2} = -\frac{n\,v_{th}}{(I + I_0)^2}. \tag{14}$$

Note that the resistance R has been eliminated in Eq. (14). Since measurement are usually done with a constant voltage increment, it is convenient to express the derivatives of V with respect to I in term of the derivatives of I with respect to V by using:

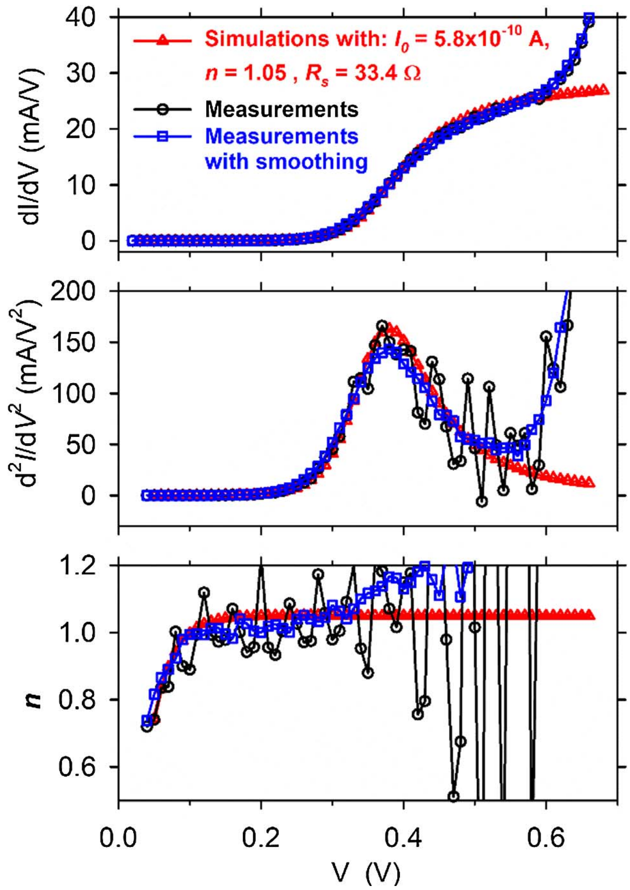

Fig. 7. First and second derivatives and corresponding extracted parameter n as a function of the voltage for previous simulated and measured I-V characteristics.

$$\frac{dV}{dI}=\left(\frac{dI}{dV}\right)^{-1} \tag{15}$$

and

$$\frac{d^2V}{dI^2}=-\frac{\frac{d^2I}{dV^2}}{\left(\frac{dI}{dV}\right)^3}. \tag{16}$$

Substituting (16) into (14) and solving for the parameter n:

$$n=-\frac{(I+I_0)^2\frac{d^2I}{dV^2}}{v_{th}\left(\frac{dI}{dV}\right)^3}. \tag{17}$$

Then, assuming $I >> I_0$, (17) yields:

$$n\approx-\frac{I^2\frac{d^2I}{dV^2}}{v_{th}\left(\frac{dI}{dV}\right)^3} \tag{18}$$

which allows to extract the parameter n without the influence of the other two parameters. Then, using (13) and assuming $I >> I_0$, yields to the value of parameter R:

$$R=\frac{dV}{dI}-\frac{n\,v_{th}}{I}. \tag{19}$$

Finally, knowing n and R, parameter I_0 is obtained from (1):

$$I_0=\frac{I}{\left[\exp\left(\frac{V-RI}{n\,v_{th}}\right)-1\right]}\approx I\exp\left(\frac{RI-V}{n\,v_{th}}\right) \tag{20}$$

The top and medium part of Fig. 7 illustrates the first and second derivatives of I with respect to V as a functions of the voltage for previous measured and simulated (n = 1.05, and I_o = 0.58 nA and R = 33.4 Ω) I-V characteristics. This figure also presents the results of the measured data after using smoothing technique to decrease the effects of the experimental errors. We observe that for $V > 0.6$ V, the first and second derivatives of the measured data are higher than those of the simulated data because of high injections effects. It is important to note how the level of noise is higher in the second derivative because the derivative acts a high pass filter. The bottom part of Fig. 7 shows the parameter n extracted with Eq. (18). In this figure, the value n for the simulations and $V > 0.2$ V is a constant of 1.05, which implies that this method is successful to extract the parameter n. On the other hand, for raw measured data, the effects of the experimental noise are so large that this method fails for parameter extraction.

5. Conclusions

A novel simple procedure, based on integrations, has been proposed to extract the model parameter values of a single-exponential diode with series resistance. A simple procedure, based on differentiations, has also been proposed to extract the model parameter values. Both procedures are based on using functions that isolate the effects of each of the parameters. The validity of both proposed procedures were confirmed by parameter extraction from experimental and simulated I–V characteristics of diodes. The differentiation method is simpler and can be used when the noise in the measured data is very small. On the other hand, the integration method is more sophisticated and more immune to the noise in the measured data. Analogous procedures could be attempted to extract the parameters of other semiconductor devices.

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