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A new explicit double-diode modeling method based on Lambert W-function for photovoltaic arrays

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Abstract

This paper proposes a new explicit double-diode modeling method based on Lambert W-function (EDDMMLW) for solar cells and photovoltaic (PV) arrays with bypass diodes, respectively. There are mainly two kinds of models for solar cells: single-diode model and double-diode model. Since the two kinds of models are implicit and nonlinear, they are usually needed to converted into explicit expressions. Many explicit expressions of *I–V* characteristic for single-diode model have been proposed. However, explicit expressions of *I–V* characteristic for double-diode model are rarely found in the current literature. This paper utilizes Lambert W-function to model explicit expressions of *I–V* characteristic for double-diode model. Firstly, Lambert W-function is used to obtain an explicit double-diode modeling method for solar cells. Secondly, in order to make the method suitable for solar cells with shading, we propose a new explicit double-diode modeling method for PV arrays with bypass diodes. In addition, this paper presents a new explicit expression for series resistance by using EDDMMLW for solar cells in this paper. It is worth mentioning that the seven cell parameters of double-diode model can be obtained by only using four electrical parameters, i.e., voltage and current at maximum power point, open circuit voltage and short circuit current. Finally, we select MSX-60 module, KC200GT module and SM55 PV array to validate the accuracy and simulation speed of the method proposed in this paper. Simulation results show that the method proposed in this paper is accurate and fast for solar cells and PV arrays with bypass diodes at any conditions.

Keywords: Double-diode model; Solar cell; PV array; Lambert W-function; Explicit expression; Series resistance

1. Introduction

In recent years, due to the reduction of fossil fuels, photovoltaic (PV) power generation is becoming more and more concerned by many countries. PV power generation is a kind of sustainable green energy (Lun et al., 2013a,b; Romero et al., 2012). In order to obtain optimal design or optimal operation of PV power generation systems, we need an accurate, fast and simple simulation model of PV power generation systems (Lun et al., 2014). PV arrays, composed of solar cells in series and parallel, are the core parts of PV power generation systems. There are usually two main models for a PV array, i.e., the single-diode model and the double-diode model (Lun et al., 2014). However, these two models are implicit and nonlinear, which must be given an appropriate initial value to calculate by using numerical iterative method. Numerical iterative method is usually sensitive to the initial values and often fail to converge even with good initial guess values. It is very inconvenient to new users, even technical staff also need to debug repeatedly to get the optimal solution.

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Nomenclature

I	output current of solar cells (A)
V	output voltage of solar cells (V)
$I(i)$	output current for each module part (A)
$V(i)$	output voltage for each module part (V)
I_{o1}	saturation current of diode 1 for solar cells (A)
I_{o2}	saturation current of diode 2 for solar cells (A)
I_L	photo current density of solar cells (A)
n_1	ideality factor of diode 1 for solar cells
n_2	ideality factor of diode 2 for solar cells
R_s	series resistance of solar cells (Ω)
R_{sh}	parallel resistance of solar cells (Ω)
T	temperature of solar cells (K)
I_{sc}	short circuit current (A)
V_{oc}	open circuit voltage (V)
V_{mp}	voltage at maximum power point (V)
I_{mp}	current at maximum power point (A)
P_{mp}	power at maximum power point (W)
V_{Load}	load voltage at output of PV array (V)
G_{ref}	irradiance at STC (W/m^2)
$I_{mp,ref}$	current at maximum power point at STC (A)
$V_{oc,ref}$	open circuit voltage at STC (V)
$\beta_{V_{oc}}$	open circuit voltage coefficient (V/°C)
$I_L(i)$	photo current density for each module part (A)
$I_{o1}(i)$	saturation current of diode 1 for each module part (A)
$I_{o2}(i)$	saturation current of diode 2 for each module part (A)
$n_1(i)$	ideality factor of diode 2 for each module part
$n_2(i)$	ideality factor of diode 2 for each module part
$R_{sh}(i)$	parallel resistance for each module part (Ω)
$R_s(i)$	parallel resistance for each module part (Ω)
k	Boltzmann's constant ($1.3806503 \times 10^{-23}$ J/K)
N_s	solar cell number in series
n_{bd}	ideality factor of bypass diode
T_{bd}	bypass diode temperature
$T(i)$	temperature for each module part (K)
V_{Load}	load voltage at output of PV array (V)
I_{sbd}	saturation current of bypass diode (A)
q	electron charge ($1.60217646 \times 10^{-19}$ C)
$V_{mp,G}$	global peak voltage of PV array (V)
$P_{mp,G}$	global peak power of PV array (W)
G	irradiance (W/m^2)
$V_{mp,ref}$	voltage at maximum power point at STC (V)
$I_{sc,ref}$	short circuit current at STC (A)
$\alpha_{I_{sc}}$	short circuit current coefficient (A/°C)
STC	standard testing conditions

Therefore, in order to make the calculation easier, the explicit model of a PV array is needed. Generally, the explicit model is faster than implicit model. The more important thing is that the explicit model does not needed to give the initial value and can directly obtain its solution. That is to say, the explicit model does not exist the problems of selecting initial values and failing to solve.

Currently, for single-diode model of the solar cells, there are mainly two kinds of explicit I–V modeling approaches. One is the approximate explicit methods (Lun et al., 2013a; Fjeldly et al., 1991; Lun et al., 2013b; Das, 2013, 2011; Karmalkar and Saleem, 2011; Das, 2014). The other is exact explicit methods (Romero et al., 2012; Jain and Kapoor, 2004, 2005; Ghani and Duke, 2011; Ghani et al., 2013b,a; Batzelis et al., 2014; Jain et al., 2006; Ghani et al., 2015). Most of the approximate explicit methods usually use some elementary functions, such as Taylor's series expansion (Lun et al., 2013a; Fjeldly et al., 1991), rational function (Lun et al., 2013b; Das, 2013) and power law function (Das, 2011; Karmalkar and Saleem, 2011), to obtain the approximate explicit expressions by some simple operations. Though these approximate explicit methods have certain errors, they usually have simpler explicit expressions and faster computing speed. The exact explicit methods usually use Lambert W-function proposed by Lambert (Jain and Kapoor, 2004) to obtain exact explicit expressions. It is worth mentioning that Lambert W-function is not elementary functions. However, research on explicit modeling method for double-diode model is rarely found in the report. Generally, the double-diode model is more accurate than the single-diode model, especially at low irradiance. Therefore, this paper proposes an accurate explicit double-diode modeling method based on Lambert W-function (EDDMMLW) for solar cells and PV arrays with bypass diodes.

In fact, to really obtain the explicit analytical representation of I–V characteristics, we need to obtain the values of cell parameters: photo current, diode saturation current, series resistance, parallel resistance and ideality factor. In this paper, the cell parameters are determined by using double-diode model and three characteristic points of I–V curve: maximum power point, open circuit voltage and short circuit current. It is worth mentioning that we propose a new accurate explicit expression of series resistance based on Lambert W-function. In resent years, there are some literature, such as Ghani et al. (2013b), Cubas et al. (2014), to obtain series resistance by using Lambert W-function. Ghani et al. (2013b) use Lambert W-function to obtain expressions of series and shunt resistances. However, the expressions are implicit and need using Newton–Raphson method to obtain their solutions. Newton–Raphson method is sensitive to the initial values and often fail to solve. Cubas et al. (2014) use Lambert W-function to establish approximate expressions of cell parameters. Only if the ideality factor is known, are these expressions of cell parameters explicit.

The main contributions of this paper are as follows: (1) The Lambert W-function is used to obtain explicit double-diode models. Since single-diode model only contains an exponential function, a Lambert W-function is needed to establish the explicit expression of single-diode model by using the whole *I–V* characteristic equation. However, double-diode model contains two exponential functions. To make each exponential function turned into a Lambert W-function, the whole *I–V* equation of double-diode model has to be divided into two equations with special form by using the undetermined coefficient method. Thus, each equation of the two equations can use Lambert W-function to generate an explicit current expression about voltage. According to an appropriate linear combination of the two explicit current expressions, we can obtain the explicit *I–V* expression. (2) The explicit double-diode modeling method based on Lambert W-function in this paper is suitable for solar cells and PV arrays with shaded solar cells. Therefore, we propose explicit double-diode models for solar cells and PV arrays with bypass diodes, respectively. (3) This paper proposes a new explicit expression of series resistance based on Lambert W-function. The new explicit expression of series resistance proposed in this paper is completely accurate, which takes into account the influence of temperature and irradiance, and the relationship between series resistance and other parameters. (4) We select MSX-60 module and KC200GT module to validate the EDDMMLW for solar cells in this paper at different irradiation and temperature, respectively. And we select a 20×3 SP (Series–Parallel) SM55 array to validate the EDDMMLW for PV arrays with bypass diodes in this paper under ten different shaded conditions. We use Engineering Equation Solver (EES) to calculate seven cell parameters, and we use Matlab software to obtain the *I–V* curves and record the operating time. Simulation results show that the EDDMMLW in this paper is valid, fast and accurate.

2. The explicit double-diode models based on Lambert W-function for solar cells

The equivalent circuit of a solar cell based on double-diode model consists of two diodes, a current source, a series resistance and a parallel resistance, shown in Fig. 1. Double-diode model is usually more accurate than the single-diode model, especially at low irradiance. Eq. (1) describes the output current of solar cells based on double-diode model:

$$I = I_L - I_{D1} - I_{D2} - \frac{V + IR_s}{R_{sh}} \tag{1}$$

where $I_{D1} = I_{o1}\left(e^{\left(\frac{V+IR_s}{a_1}\right)} - 1\right), I_{D2} = I_{o2}\left(e^{\left(\frac{V+IR_s}{a_2}\right)} - 1\right)$. $I, V, I_{o1}, I_{o2}, n_1, n_2, R_s, R_{sh}$ and I_L are the output current, the output voltage, the reverse saturation current of diode 1 and diode 2, the ideality factor of diode 1 and diode 2, the series resistance, the shunt resistance and the photo current density of a solar cell, respectively. q is the electron charge ($1.60217646 \times 10^{-19}$ C). k is Boltzmann's constant ($1.3806503 \times 10^{-23}$ J/K). N_s is the number of solar cells in series. T is the temperature in Kelvin, $a_1 = n_1 N_s kT/q, a_2 = n_2 N_s kT/q$ (Lun et al., 2014; Hejri et al., 2014; Ishaque et al., 2011b; Karatepe et al., 2007; Ishaque et al., 2011a).

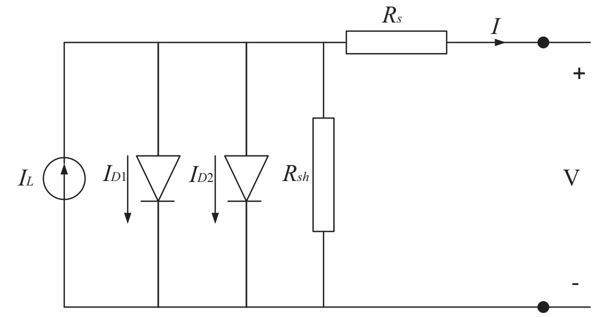

Fig. 1. The double-diode model of solar cells.

Eq. (1) contains two exponential functions. To make each exponential function written into the form of a Lambert W-function, the whole *I–V* equation needs to be divided into two equations with the following form:

$$ax + b = e^{(cx+d)} \tag{2}$$

where a, b, c and d are parameters to be determined.

Both sides of Eq. (2) are multiplied by $-\frac{c}{a}e^{\left(-cx-\frac{cb}{a}\right)}$ at the same time (Ding, 2007). Then, we have

$$\left(-cx - \frac{cb}{a}\right)e^{\left(-cx-\frac{cb}{a}\right)} = -\frac{c}{a}e^{\left(\frac{ad-cb}{a}\right)} \tag{3}$$

If we let $X = -\frac{c}{a}e^{\left(\frac{ad-cb}{a}\right)}$ and $W(X) = -cx - \frac{cb}{a}$, then we can find that Eq. (3) is equivalent to $W(X)e^{W(X)} = X$, which is the definition of Lambert W-function (Jain and Kapoor, 2004). Therefore, Eq. (1) is written as

$$-\frac{R_s + R_{sh}}{R_{sh}}I + \left(I_{o1} + I_{o2} + I_L - \frac{V}{R_{sh}}\right) = I_{o1}e^{\left(\frac{R_s}{a_1}I + \frac{V}{a_1}\right)} + I_{o2}e^{\left(\frac{R_s}{a_2}I + \frac{V}{a_2}\right)} \tag{4}$$

Let $C_1 = \frac{R_s}{a_1}, D_1 = \frac{V}{a_1}, C_2 = \frac{R_s}{a_2}$ and $D_2 = \frac{V}{a_2}$. To take advantage of Lambert W-function, we assume that Eqs. (5) and (6) hold.

$$I_{o1}e^{(C_1 I + D_1)} = I_{o1}(A_1 I + B_1) \tag{5}$$

$$I_{o2}e^{(C_2 I + D_2)} = I_{o2}(A_2 I + B_2) \tag{6}$$

where A_1, A_2, B_1 and B_2 are parameters to be determined by using the undetermined coefficient method.

According to Eqs. (5) and (6), Eq. (4) can be written as

$$I_{o1}(A_1 I + B_1) + I_{o2}(A_2 I + B_2) = -\frac{R_s + R_{sh}}{R_{sh}}I + \left(I_{o1} + I_{o2} + I_L - \frac{V}{R_{sh}}\right) \tag{7}$$

By combining like terms, Eq. (7) becomes

$$(A_1 I_{o1} + A_2 I_{o2})I + (B_1 I_{o1} + B_2 I_{o2}) = -\frac{R_s + R_{sh}}{R_{sh}} I + \left(I_{o1} + I_{o2} + I_L - \frac{V}{R_{sh}}\right) \tag{8}$$

According to the undetermined coefficient method, the corresponding coefficients of like terms on both sides of Eq. (8) should be equal for any I. Then, we can obtain

$$A_1 I_{o1} + A_2 I_{o2} = -\frac{R_s + R_{sh}}{R_{sh}} \tag{9}$$

$$B_1 I_{o1} + B_2 I_{o2} = I_{o1} + I_{o2} + I_L - \frac{V}{R_{sh}} \tag{10}$$

We can see from Eqs. (9) and (10) that the two equations include four variables A_1, A_2, B_1 and B_2. Therefore, the relationship between A_1 and A_2, and the relationship between B_1 and B_2 are linear. In this paper, in order to make the calculation simpler, we suppose $A_1 = A_2 = A$ and $B_1 = B_2 = B$. Substituting A into Eq. (9) and B into Eq. (10), respectively, we can obtain the value of A and B as follows:

$$A = -\frac{R_s + R_{sh}}{R_{sh}(I_{o1} + I_{o2})} \tag{11}$$

$$B = \frac{R_{sh}(I_L + I_{o1} + I_{o2}) - V}{R_{sh}(I_{o1} + I_{o2})} \tag{12}$$

Similar to the deducing process of Eqs. (2) to (3), Eqs. (5) and (6) can both use Lambert W-function to generate an explicit expression of the current I. Substituting A, B, C_1 and D_1 into Eq. (5), we can obtain an explicit expression of current I, denoted by I_1 as Eq. (13).

$$I_1 = \frac{R_{sh}(I_L + I_{o1} + I_{o2}) - V}{R_s + R_{sh}} - \frac{a_1}{R_s} W\left(\frac{R_s R_{sh}(I_{o1} + I_{o2})}{a_1(R_s + R_{sh})} e^{\left(\frac{R_{sh}(R_s I_L + R_s I_{o1} + R_s I_{o2} + V)}{a_1(R_s + R_{sh})}\right)}\right) \tag{13}$$

Substituting A, B, C_2 and D_2 into Eq. (6), we can obtain another explicit expression of current I, denoted by I_2 as Eq. (14).

$$I_2 = \frac{R_{sh}(I_L + I_{o1} + I_{o2}) - V}{R_s + R_{sh}} - \frac{a_2}{R_s} W\left(\frac{R_s R_{sh}(I_{o1} + I_{o2})}{a_2(R_s + R_{sh})} e^{\left(\frac{R_{sh}(R_s I_L + R_s I_{o1} + R_s I_{o2} + V)}{a_2(R_s + R_{sh})}\right)}\right) \tag{14}$$

Therefore, the explicit expression of current I about voltage V can be shown as Eq. (15).

$$\begin{aligned} I &= \frac{1}{2}I_1 + \frac{1}{2}I_2 \\ &= \frac{R_{sh}(I_L + I_{o1} + I_{o2}) - V}{R_s + R_{sh}} \\ &\quad - \frac{a_1}{2R_s} W\left(\frac{R_s R_{sh}(I_{o1} + I_{o2})}{a_1(R_s + R_{sh})} e^{\left(\frac{R_{sh}(R_s I_L + R_s I_{o1} + R_s I_{o2} + V)}{a_1(R_s + R_{sh})}\right)}\right) \\ &\quad - \frac{a_2}{2R_s} W\left(\frac{R_s R_{sh}(I_{o1} + I_{o2})}{a_2(R_s + R_{sh})} e^{\left(\frac{R_{sh}(R_s I_L + R_s I_{o1} + R_s I_{o2} + V)}{a_2(R_s + R_{sh})}\right)}\right) \end{aligned} \tag{15}$$

We call this method explicit double-diode modeling method based on Lambert W-function (EDDMMLW) for solar cells. And we call Eq. (15) explicit expression of current about voltage for solar cells by using EDDMMLW in this paper. It is worth mentioning that Eqs. (13) and (14) are actually equivalent. This is because we supposed $A_1 = A_2 = A$ and $B_1 = B_2 = B$ in Eqs. (9) and (10), respectively. If we change the relationship between A_1 and A_2, and the relationship between B_1 and B_2, Eqs. (13) and (14) will not be equivalent. And we can obtain more explicit expressions of current which are different from Eq. (15).

Similar to the deducing process of explicit current model, we can use Lambert W-function to obtain two explicit expressions of voltage V, denoted by V_1 and V_2, respectively, as follows:

$$V_1 = R_{sh}(I_{o1} + I_{o2} + I_L - I) - IR_s - a_1 W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_1} e^{\left(\frac{R_{sh}(I_{o1} + I_{o2} + I_L - I)}{a_1}\right)}\right) \tag{16}$$

$$V_2 = R_{sh}(I_{o1} + I_{o2} + I_L - I) - IR_s - a_2 W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_2} e^{\left(\frac{R_{sh}(I_{o1} + I_{o2} + I_L - I)}{a_2}\right)}\right) \tag{17}$$

Therefore, the explicit expression of voltage V about current I can be shown as Eq. (18).

$$\begin{aligned} V &= \frac{1}{2}V_1 + \frac{1}{2}V_2 \\ &= R_{sh}(I_{o1} + I_{o2} + I_L - I) - IR_s \\ &\quad - \frac{a_1}{2} W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_1} e^{\left(\frac{R_{sh}(I_{o1} + I_{o2} + I_L - I)}{a_1}\right)}\right) \\ &\quad - \frac{a_2}{2} W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_2} e^{\left(\frac{R_{sh}(I_{o1} + I_{o2} + I_L - I)}{a_2}\right)}\right) \end{aligned} \tag{18}$$

We call Eq. (18) explicit expression of voltage about current for solar cells by using EDDMMLW in this paper. Similar to explicit expressions of current, we can also obtain the more explicit expressions of voltage.

3. The explicit double-diode models based on Lambert W-function for photovoltaic arrays

A PV generation system usually consists of solar PV arrays and electric converters. A PV array is formed by series/parallel combination of PV modules. To describe PV arrays more clearly, we assume that each PV module contains two bypass diodes. Thus, each PV module is divided into two parts: part 1 and part 2, shown in Fig. 2 (Lun et al., 2014; Karatepe et al., 2007). Eq. (19) shows the I–V characteristic of each module part.

$$-I(i)+I_L(i)-I_{o1}(i)\left(e^{\left(\frac{V(i)+I(i)R_s(i)}{pa_1(i)}\right)}-1\right)-I_{o2}(i)\left(e^{\left(\frac{V(i)+I(i)R_s(i)}{pa_2(i)}\right)}-1\right)-\frac{V(i)+I(i)R_s(i)}{R_{sh}(i)}+I_{sbd}\left(e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}-1\right)=0 \text{ for } i=2,4,6,\ldots,2sr \quad (19)$$

where $I(i), V(i), R_s(i), R_{sh}(i), I_{o1}(i), I_{o2}(i), n_1(i), n_2(i), I_L(i)$ and $T(i)$, are the output current, the output voltage, the series resistance, the shunt resistance, the reverse saturation current of diode 1 and diode 2, the ideality factor for diode 1 and diode 2, the photo current density and the temperature in Kelvin for each module part, respectively. $a_1(i)=n_1(i)N_skT(i)/q$ and $a_2(i)=n_2(i)N_skT(i)/q$. s and r are the number of modules in a column and row, respectively. p is the number of solar cells per one bypass diode. I_{sbd}, n_{bd} and T_{bd} are the bypass diode's saturation current, ideality factor and temperature, respectively (Karatepe et al., 2007).

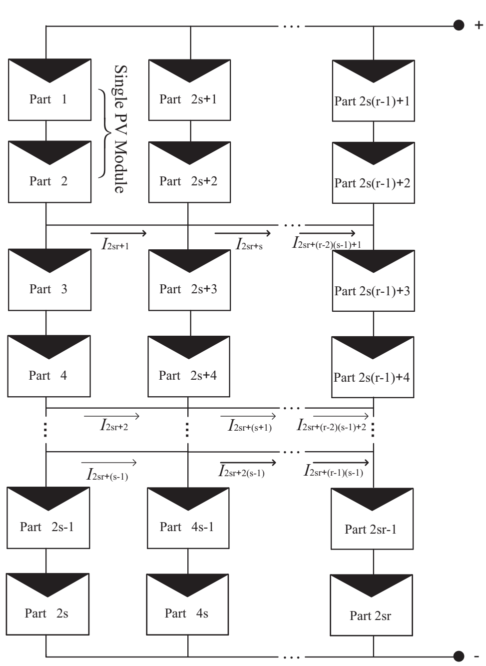

Fig. 2. A general PV array model.

Similar to the deducing process of explicit current model based on Lambert W-function for solar cells, we can use Lambert W-function to obtain an explicit expression of current for PV arrays with bypass diodes. Firstly, Eq. (19) is written as Eq. (20):

$$-\frac{R_{sh}(i)+R_s(i)}{R_{sh}(i)}I(i)+I_L(i)+I_{o1}(i)+I_{o2}(i)-\frac{V(i)}{R_{sh}(i)}+I_{sbd}e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}-I_{sbd}=I_{o1}(i)e^{\left(\frac{R_s(i)}{pa_1(i)}I(i)+\frac{V(i)}{pa_1(i)}\right)}+I_{o2}(i)e^{\left(\frac{R_s(i)}{pa_2(i)}I(i)+\frac{V(i)}{pa_2(i)}\right)} \quad (20)$$

Secondly, according to the undetermined coefficient method, two explicit expressions of current $I(i)$ for a PV array, denoted by $I_1(i)$ and $I_2(i)$ are given in Eqs. (21) and (22), respectively.

$$I_1(i)=\frac{R_{sh}(i)\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I_{sbd}+I_{sbd}e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}\right)-V(i)}{R_{sh}(i)+R_s(i)}-\frac{pa_1(i)}{R_s(i)}W(\varphi_1) \quad (21)$$

$$I_2(i)=\frac{R_{sh}(i)\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I_{sbd}+I_{sbd}e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}\right)-V(i)}{R_{sh}(i)+R_s(i)}-\frac{pa_2(i)}{R_s(i)}W(\varphi_2) \quad (22)$$

where φ_1 and φ_2 are given in Eqs. (23) and (24), respectively.

$$\varphi_1=\frac{R_s(i)R_{sh}(i)(I_{o1}(i)+I_{o2}(i))}{pa_1(i)(R_{sh}(i)+R_s(i))}e^{\left(\frac{R_{sh}(i)\left(V(i)+R_s(i)\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I_{sbd}+I_{sbd}e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}\right)\right)}{pa_1(i)(R_{sh}(i)+R_s(i))}\right)} \quad (23)$$

$$\varphi_2=\frac{R_s(i)R_{sh}(i)(I_{o1}(i)+I_{o2}(i))}{pa_2(i)(R_{sh}(i)+R_s(i))}e^{\left(\frac{R_{sh}(i)\left(V(i)+R_s(i)\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I_{sbd}+I_{sbd}e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}\right)\right)}{pa_2(i)(R_{sh}(i)+R_s(i))}\right)} \quad (24)$$

Therefore, the explicit expression of current about voltage for PV arrays with bypass diodes by using EDDMMLW in this paper can be shown as Eq. (25).

$$I(i)=\frac{1}{2}I_1(i)+\frac{1}{2}I_2(i)=\frac{R_{sh}(i)\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I_{sbd}+I_{sbd}e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}\right)-V(i)}{R_{sh}(i)+R_s(i)}-\frac{pa_1(i)}{2R_s(i)}W(\varphi_1)-\frac{pa_2(i)}{2R_s(i)}W(\varphi_2) \quad (25)$$

When the PV array is not be shaded, i.e., $I_{sbd}(e^{\left(-\frac{qV(i)}{n_{bd}kT_{bd}}\right)}-1)=0$, Eq. (19) can be turned into Eq. (26).

$$-I(i)+I_L(i)-I_{o1}(i)\left(e^{\left(\frac{V(i)+I_iR_s(i)}{pa_1(i)}\right)}-1\right)$$
$$-I_{o2}(i)\left(e^{\left(\frac{V(i)+I(i)R_s(i)}{pa_2(i)}\right)}-1\right)-\frac{V(i)+I(i)R_s(i)}{R_{sh}(i)}=0 \quad (26)$$

And the explicit expression of the current $I(i)$ for PV arrays without shading, denoted by $I_3(i)$ is given in Eq. (27).

$$I_3(i)=\frac{R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i))-V(i)}{R_s(i)+R_{sh}(i)}-\frac{pa_1(i)}{2R_s(i)}W(\theta_1)-\frac{pa_2(i)}{2R_s(i)}W(\theta_2) \quad (27)$$

where θ_1 and θ_2 are defined as

$$\theta_1=\frac{R_s(i)R_{sh}(i)(I_{o1}(i)+I_{o2}(i))}{pa_1(i)(R_s(i)+R_{sh}(i))}e^{\left(\frac{R_{sh}(i)(R_s(i)I_L(i)+R_s(i)I_{o1}(i)+R_s(i)I_{o2}(i)+V(i))}{pa_1(i)(R_s(i)+R_{sh}(i))}\right)} \quad (28)$$

$$\theta_2=\frac{R_s(i)R_{sh}(i)(I_{o1}(i)+I_{o2}(i))}{pa_2(i)(R_s(i)+R_{sh}(i))}e^{\left(\frac{R_{sh}(i)(R_s(i)I_L(i)+R_s(i)I_{o1}(i)+R_s(i)I_{o2}(i)+V(i))}{pa_2(i)(R_s(i)+R_{sh}(i))}\right)} \quad (29)$$

In fact, Eq. (27) is explicit expression of current about voltage for PV array without bypass diodes by using EDDMMLW in this paper.

Similarly, to obtain the explicit expression of voltage for PV array by using Lambert W-function, Eq. (19) is turned into Eq. (30):

$$-\frac{1}{R_{sh}(i)}V(i)+\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-\frac{I(i)R_s(i)}{R_{sh}(i)}-I_{sbd}\right)$$
$$=I_{o1}(i)e^{\left(\frac{1}{pa_1(i)}V(i)+\frac{I(i)R_s(i)}{pa_1(i)}\right)}+I_{o2}(i)e^{\left(\frac{1}{pa_2(i)}V(i)+\frac{I_iR_s(i)}{pa_2(i)}\right)}-I_{sbd}e^{\left(-\frac{q}{n_{bd}kT_{bd}}V(i)\right)} \quad (30)$$

Let $\gamma_1=\frac{1}{pa_1(i)}$, $\delta_1=\frac{I_iR_s(i)}{pa_1(i)}$, $\gamma_2=\frac{1}{pa_2(i)}$, $\delta_2=\frac{I_iR_s(i)}{pa_2(i)}$, $\gamma_3=-\frac{q}{n_{bd}kT_{bd}}$ and $\delta_3=0$, respectively. Similarly to the deducing process of explicit current model based on Lambert W-function for solar cells, we assume that Eqs. (31)–(33) hold.

$$I_{o1}(i)e^{(\gamma_1V(i)+\delta_1)}=I_{o1}(i)(\alpha_1V(i)+\beta_1) \quad (31)$$

$$I_{o2}(i)e^{(\gamma_2V(i)+\delta_2)}=I_{o2}(i)(\alpha_2V(i)+\beta_2) \quad (32)$$

$$-I_{sbd}e^{(\gamma_3V(i)+\delta_3)}=-I_{sbd}(\alpha_3V(i)+\beta_3) \quad (33)$$

where $\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2$ and β_3 are parameters to be determined by using the undetermined coefficient method.

According to Eqs. (31)–(33), Eq. (30) can be written as Eq. (34).

$$I_{o1}(i)(\alpha_1V(i)+\beta_1)+I_{o2}(i)(\alpha_2V(i)+\beta_2)-I_{sbd}(\alpha_3V(i)+\beta_3)$$
$$=-\frac{1}{R_{sh}(i)}V(i)+\left(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-\frac{I(i)R_s(i)}{R_{sh}(i)}-I_{sbd}\right) \quad (34)$$

By combining like terms, the corresponding coefficients of like terms on both sides of Eq. (34) should be equal for any $V(i)$. Then, we can obtain

$$I_{o1}(i)\alpha_1+I_{o2}(i)\alpha_2-I_{sbd}\alpha_3=-\frac{1}{R_{sh}(i)} \quad (35)$$

$$I_{o1}(i)\beta_1+I_{o2}(i)\beta_2-I_{sbd}\beta_3=I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-\frac{I(i)R_s(i)}{R_{sh}(i)}-I_{sbd} \quad (36)$$

From Eqs. (35) and (36), we can find that the two equations include six variables $\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2$ and β_3. Therefore, the relationship between α_1,α_2 and α_3, and the relationship between β_1,β_2 and β_3 are linear. In this paper, in order to make the calculation simpler, we suppose $\alpha_1=\alpha_2=\alpha_3=\alpha$ and $\beta_1=\beta_2=\beta_3=\beta$. Substituting α into Eq. (35) and β into Eq. (36), respectively, we can obtain the value of α and β as follows:

$$\alpha=-\frac{1}{R_{sh}(i)(I_{o1}(i)+I_{o2}(i)-I_{sbd})} \quad (37)$$

$$\beta=\frac{R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})-I(i)R_s(i)}{R_{sh}(i)(I_{o1}(i)+I_{o2}(i)-I_{sbd})} \quad (38)$$

Therefore, Eqs. (31)–(33) can use Lambert W-function to generate an explicit expression of the $V(i)$, respectively. Substituting α,β,γ_1 and δ_1 into Eq. (31), α,β,γ_2 and δ_2 into Eq. (32), α,β,γ_3 and δ_3 into Eq. (33), respectively. We can obtain three explicit expressions of voltage $V(i)$, denoted by $V_1(i),V_2(i)$ and $V_3(i)$ as Eqs. (39)–(41), respectively.

$$V_1(i)=R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})-I(i)R_s(i)-pa_1(i)W(X_1) \quad (39)$$

$$V_2(i)=R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})-I(i)R_s(i)-pa_2(i)W(X_2) \quad (40)$$

$$V_3(i)=R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})-I(i)R_s(i)+\frac{n_{bd}kT_{bd}}{q}W(X_3) \quad (41)$$

where X_1,X_2 and X_3 are defined as

$$X_1=\frac{R_{sh}(i)(I_{o1}(i)+I_{o2}(i)-I_{sbd})}{pa_1(i)}e^{\left(\frac{R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})}{pa_1(i)}\right)} \quad (42)$$

$$X_2 = \frac{R_{sh}(i)(I_{o1}(i) + I_{o2}(i) - I_{sbd})}{pa_2(i)} e^{\left(\frac{R_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})}{pa_2(i)}\right)} \tag{43}$$

$$X_3 = -\frac{qR_{sh}(i)(I_{o1}(i) + I_{o2}(i) - I_{sbd})}{n_{bd}kI_{sbd}} e^{\left(-\frac{qR_{sh}(i)(I_L(i)+I_{o1}(i)+I_{o2}(i)-I(i)-I_{sbd})-I(i)R_s(i)q}{n_{bd}kI_{sbd}}\right)} \tag{44}$$

Then, the explicit expression of voltage $V(i)$ about current $I(i)$ for PV arrays with bypass diodes by using EDDMMLW in this paper can be shown as Eq. (45).

$$\begin{aligned} V(i) &= \frac{1}{3}V_1(i) + \frac{1}{3}V_2(i) + \frac{1}{3}V_3(i) \\ &= R_{sh}(i)(I_L(i) + I_{o1}(i) + I_{o2}(i) - I(i) - I_{sbd}) \\ &\quad - I(i)R_s(i) - \frac{pa_1(i)}{3}W(X_1) - \frac{pa_2(i)}{3}W(X_2) \\ &\quad + \frac{n_{bd}kT_{bd}}{3q}W(X_3) \end{aligned} \tag{45}$$

When the PV array is not be shaded, we can obtain the explicit expression of $V(i)$ from Eq. (26), denoted by $V_4(i)$ as Eq. (46).

$$\begin{aligned} V_4(i) &= R_{sh}(i)(I_{o1}(i) + I_{o2}(i) + I_L(i) - I(i)) - I(i)R_s(i) \\ &\quad - \frac{pa_1(i)}{2}W\left(\frac{R_{sh}(i)(I_{o1}(i) + I_{o2}(i))}{pa_1(i)} e^{\left(\frac{R_{sh}(i)(I_{o1}(i)+I_{o2}(i)+I_L(i)-I(i))}{pa_1(i)}\right)}\right) \\ &\quad - \frac{pa_2(i)}{2}W\left(\frac{R_{sh}(i)(I_{o1}(i) + I_{o2}(i))}{pa_2(i)} e^{\left(\frac{R_{sh}(i)(I_{o1}(i)+I_{o2}(i)+I_L(i)-I(i))}{pa_2(i)}\right)}\right) \end{aligned} \tag{46}$$

In fact, Eq. (46) is the explicit expression of voltage about current for PV arrays without bypass diodes by using EDDMMLW in this paper.

When the double-diode model is used to describe the solar cells of a PV array, we can obtain the following equations according to Kirchhoffs current and voltage laws (Karatepe et al., 2007).

$$\sum_{i=1}^{2s} V_i - V_{\text{Load}} = 0 \tag{47}$$

where V_{Load} is the load voltage at output of PV array.

$$I_{(j-1)2s+(2i-1)} - I_{(j-1)2s+2i} = 0,\ (j = 1, 2, 3, \ldots, r,\ i = 1, 2, 3, \ldots, s) \tag{48}$$

$$\begin{aligned} &I_{(j-1)2s+(2i-2)} + I_{2sr+(j-1)(s-1)+i-1} - I_{(j-1)2s+2i-1} \\ &\quad - I_{2sr+(j-1)(s-1)+i-1-(s-1)} = 0 \quad (j = 1, 2, 3, \ldots, r, \\ &\quad i = 2, 3, 4, \ldots, s-1, \text{ with } I_{2sr+(j-1)(s-1)+i-1-(s-1)} = 0, \\ &\quad \text{if } j = 1, \text{ and } I_{2sr+(j-1)(s-1)+i-1} = 0, \text{ if } j = r) \end{aligned} \tag{49}$$

$$\begin{aligned} &V_{(j-1)2s+2i-1} + V_{(j-1)2s+2i} - V_{j2s+2i-1} - V_{j2s+2i} \\ &\quad + V_{2sr+(j-1)(s-1)+i-V_{2sr+(j-1)(s-1)+i-1}} = 0 \\ &\quad (j = 1, 2, 3, \ldots, r-1, i = 1, 2, 3, \ldots, s, \\ &\quad \text{with } V_{2sr+(j-1)(s-1)+i-1} = 0, \quad \text{if } i = 1, \\ &\quad \text{and } V_{2sr+(j-1)(s-1)+i} = 0, \text{ if } i = s) \end{aligned} \tag{50}$$

4. The selection of parameters

In the above-mentioned explicit double-diode models, the cell parameters $I_{o1}, I_{o2}, a_1, a_2, I_L, R_{sh}$ and R_s need to be determined in advance. In this paper, we give a new method to calculate the seven cell parameters. Especially, explicit and accurate expressions of series resistance based on Lambert W-function are proposed. This makes the cell parameters obtained quickly. We can calculate the seven cell parameters only by using four electrical parameters, namely voltage V_{mp} and current I_{mp} at maximum power point, open circuit voltage V_{oc} and short circuit current I_{sc}, respectively. For a operating photovoltaic power generation systems, generally, we can obtain the four parameters I_{mp}, V_{mp}, I_{sc} and V_{oc} by computer monitoring system. If we can't obtain I_{mp}, V_{mp}, I_{sc} and V_{oc} from photovoltaic power generation systems, we can use Eqs. (51)–(54) to calculate the four parameters (De Soto, 2004; Lun et al., 2013a,b).

$$I_{mp} = I_{mp,ref}\frac{G}{G_{ref}} \tag{51}$$

$$V_{mp} = V_{mp,ref} + \beta_{V_{oc}}(T - T_{ref}) \tag{52}$$

$$I_{sc} = \frac{G}{G_{ref}}(I_{sc,ref} + \alpha_{I_{sc}}(T - T_{ref})) \tag{53}$$

$$V_{oc} = V_{oc,ref} + \beta_{V_{oc}}(T - T_{ref}) \tag{54}$$

where $I_{mp,ref}, V_{mp,ref}, V_{oc,ref}, I_{sc,ref}, G_{ref}$ and T_{ref} are current and voltage at maximum power point, open circuit voltage, short circuit current, irradiance and temperature at Standard Testing Conditions (STC), respectively. STC is solar irradiance of 1000 W/m^2, temperature of 298 K and air mass of 1.5. $G, T, \alpha_{I_{sc}}$, and $\beta_{V_{oc}}$ are irradiance, temperature, short circuit current coefficient and open circuit voltage coefficient, respectively.

The explicit expression of R_s based on Lambert W-function is similar to the deducing process of explicit current model. So we can obtain two explicit expressions of R_s, denoted by R_{s1} and R_{s2} as Eqs. (55) and (56), respectively.

$$\begin{aligned} R_{s1} &= \frac{R_{sh}(I_L + I_{o1} + I_{o2} - I) - V}{I} \\ &\quad - \frac{a_1}{I}W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_1} e^{\left(\frac{R_{sh}(I_L+I_{o1}+I_{o2}-I)}{a_1}\right)}\right) \end{aligned} \tag{55}$$

$$\begin{aligned} R_{s2} &= \frac{R_{sh}(I_L + I_{o1} + I_{o2} - I) - V}{I} \\ &\quad - \frac{a_2}{I}W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_2} e^{\left(\frac{R_{sh}(I_L+I_{o1}+I_{o2}-I)}{a_2}\right)}\right) \end{aligned} \tag{56}$$

In order to make the R_s models describe the I–V characteristics and maximum power point more accurately, we substitute $V = V_{mp}$ and $I = I_{mp}$ in Eqs. (55) and (56), respectively. Then we can obtain two new expressions of R_s, denoted by R_{s3} and R_{s4} as Eqs. (57) and (58), respectively.

Table 1
STC specifications for the three modules used in experiments.

Parameter	MSX60 module	KC200GT module	SM55 module
I_{sc} (A)	3.8	8.21	3.45
V_{oc} (V)	21.1	32.9	21.7
I_{mp} (A)	3.5	7.61	3.15
V_{mp} (V)	17.1	26.3	17.4
$\alpha_{I_{sc}}$ (A/°C)	0.003	0.00318	0.0012
$\beta_{V_{oc}}$ (V/°C)	−0.08	−0.123	−0.077
N_s	36	54	36

$$R_{s3} = \frac{R_{sh}(I_L + I_{o1} + I_{o2} - I_{mp}) - V_{mp}}{I_{mp}} - \frac{a_1}{I_{mp}} W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_1} e^{\left(\frac{R_{sh}(I_L+I_{o1}+I_{o2}-I_{mp})}{a_1}\right)}\right) \tag{57}$$

$$R_{s4} = \frac{R_{sh}(I_L + I_{o1} + I_{o2} - I_{mp}) - V_{mp}}{I_{mp}} - \frac{a_2}{I_{mp}} W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_2} e^{\left(\frac{R_{sh}(I_L+I_{o1}+I_{o2}-I_{mp})}{a_2}\right)}\right) \tag{58}$$

Therefore, the explicit expression of R_s by using EDDMMLW in this paper can be shown as Eq. (59).

$$\begin{aligned} R_s &= \frac{1}{2}R_{s3} + \frac{1}{2}R_{s4} \\ &= \frac{R_{sh}(I_L + I_{o1} + I_{o2} - I_{mp}) - V_{mp}}{I_{mp}} \\ &\quad - \frac{a_1}{2I_{mp}} W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_1} e^{\left(\frac{R_{sh}(I_L+I_{o1}+I_{o2}-I_{mp})}{a_1}\right)}\right) \\ &\quad - \frac{a_2}{2I_{mp}} W\left(\frac{R_{sh}(I_{o1} + I_{o2})}{a_2} e^{\left(\frac{R_{sh}(I_L+I_{o1}+I_{o2}-I_{mp})}{a_2}\right)}\right) \end{aligned} \tag{59}$$

According to the definition of short circuit current, open circuit voltage and maximum power point, we can use Eq. (1) to obtain Eqs. (60)–(63) as follows.

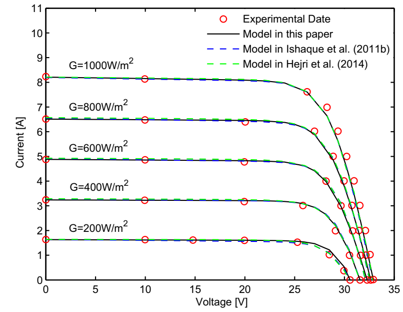

Fig. 3. I–V for KC200GT at $T = 298$ K.

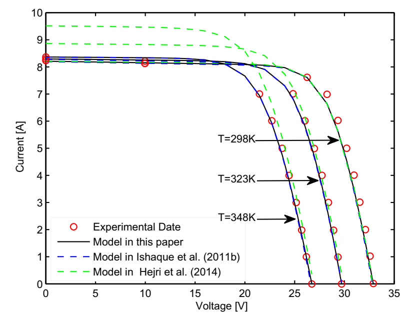

Fig. 4. I–V for KC200GT at $G = 1000$ W/m^2.

$$I_{sc} = I_L - I_{o1}\left(e^{\left(\frac{I_{sc}R_s}{a_1}\right)} - 1\right) - I_{o2}\left(e^{\left(\frac{I_{sc}R_s}{a_2}\right)} - 1\right) - \frac{I_{sc}R_s}{R_{sh}} \tag{60}$$

$$I_L = I_{o1}\left(e^{\left(\frac{V_{oc}}{a_1}\right)} - 1\right) + I_{o2}\left(e^{\left(\frac{V_{oc}}{a_2}\right)} - 1\right) + \frac{V_{oc}}{R_{sh}} \tag{61}$$

$$I_{mp} = I_L - I_{o1}\left(e^{\left(\frac{V_{mp}+I_{mp}R_s}{a_1}\right)} - 1\right) - I_{o2}\left(e^{\left(\frac{V_{mp}+I_{mp}R_s}{a_2}\right)} - 1\right) - \frac{V_{mp} + I_{mp}R_s}{R_{sh}} \tag{62}$$

Table 2
Parameters and operating time of I–V curves for three models at STC.

Item	MSX60 module			KC200GT module		
	Model in Ishaque et al. (2011b)	Model in Hejri et al. (2014)	Model in this paper	Model in Ishaque et al. (2011b)	Model in Hejri et al. (2014)	Model in this paper
I_{o1} (A)	4.704E−10	4.12335E−10	1.8952E−10	4.218E−10	3.795E−10	3.907E−10
I_{o2} (A)	4.704E−10	3.98111E−6	1.8941E−10	4.218E−10	4.433E−6	3.90417E−10
I_L (A)	3.8	3.8046	3.8086	8.21	8.2193	8.22562
R_{sh} (Ω)	176.4	280.20222	166.4854	160.5	278.9255	171.26548
R_s (Ω)	0.35	0.3392	0.37659	0.32	0.3181	0.32579
n_1	–	1	0.99247	–	1	1.02912
n_2	–	2	0.99247	–	2	1.02912
operating time (Unit: s)	0.5671	0.2927	0.2334	1.5811	0.5518	0.4700

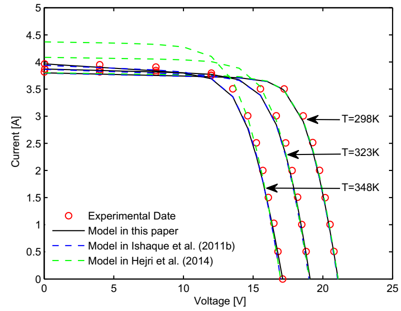

Fig. 5. I–V for MSX-60 at $G = 1000\ \mathrm{W/m^2}$.

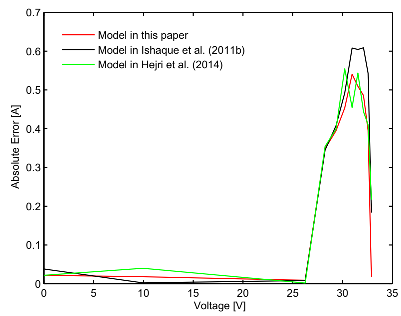

Fig. 6. Absolute errors for KC200GT at $G = 1000\ \mathrm{W/m^2}$, $T = 298$ K.

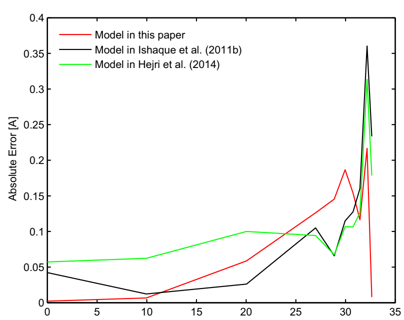

Fig. 7. Absolute errors for KC200GT at $G = 800\ \mathrm{W/m^2}$, $T = 298$ K.

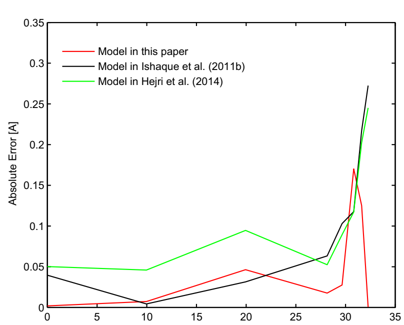

Fig. 8. Absolute errors for KC200GT at $G = 600\ \mathrm{W/m^2}$, $T = 298$ K.

$$0 = I_{mp} + V_{mp}\left(\frac{\frac{-I_{o1}}{a_1}e^{\left(\frac{V_{mp}+I_{mp}R_s}{a_1}\right)} - \frac{I_{o2}}{a_2}e^{\left(\frac{V_{mp}+I_{mp}R_s}{a_2}\right)} - \frac{1}{R_{sh}}}{1 + \frac{I_{o1}R_s}{a_1}e^{\left(\frac{V_{mp}+I_{mp}R_s}{a_1}\right)} + \frac{I_{o2}R_s}{a_2}e^{\left(\frac{V_{mp}+I_{mp}R_s}{a_2}\right)} + \frac{R_s}{R_{sh}}}\right) \tag{63}$$

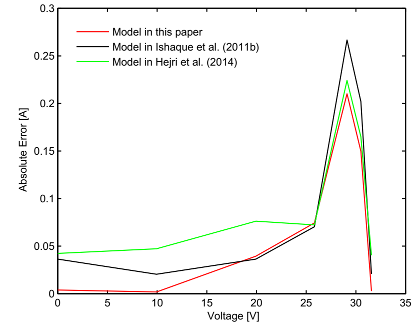

Fig. 9. Absolute errors for KC200GT at $G = 400\ \mathrm{W/m^2}$, $T = 298$ K.

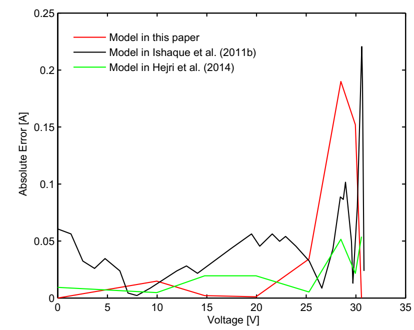

Fig. 10. Absolute errors for KC200GT at $G = 200\ \mathrm{W/m^2}$, $T = 298$ K.

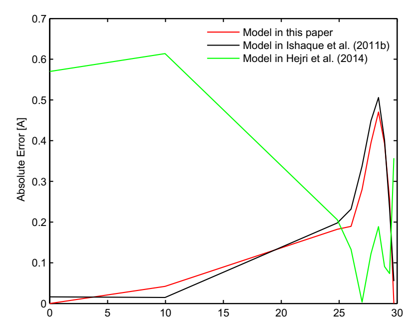

Fig. 11. Absolute errors for KC200GT at $G = 1000\ \mathrm{W/m^2}$, $T = 323$ K.

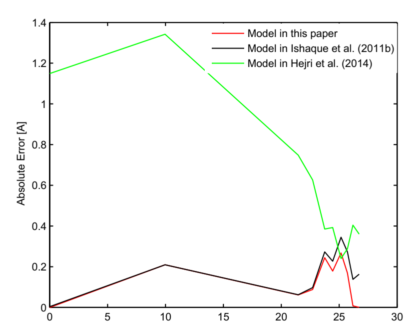

Fig. 12. Absolute errors for KC200GT at $G = 1000\ \mathrm{W/m^2}$, $T = 348$ K.

In fact, Eqs. (57) and (58) imply $a_1 = a_2$. This is due to Eqs. (57) and (58) being equivalent. Thus, we can use six equations, i.e., Eqs. (57), (58) and (60)–(63), to determine the seven cell parameters $I_{o1}, I_{o2}, a_1, a_2, I_L, R_{sh}$ and R_s.

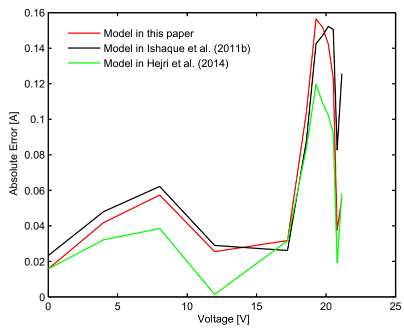

Fig. 13. Absolute errors for MSX-60 at $G = 1000\ \mathrm{W/m^2}$, $T = 298$ K.

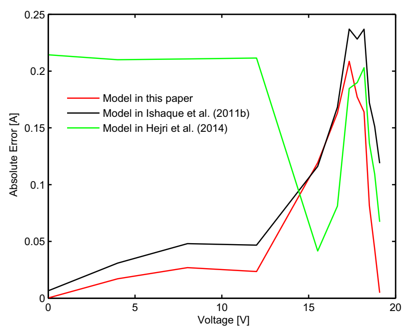

Fig. 14. Absolute errors for MSX-60 at $G = 1000\ \mathrm{W/m^2}$, $T = 323$ K.

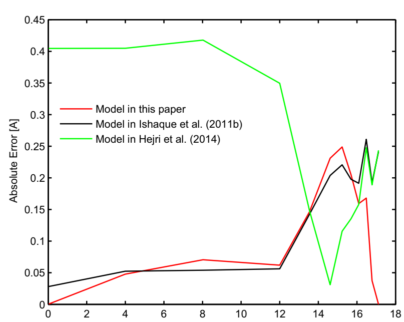

Fig. 15. Absolute errors for MSX-60 at $G = 1000\ \mathrm{W/m^2}$, $T = 348$ K.

5. Model validation and discussion

We select MSX-60 and KC200GT modules to validate the EDDMMLW for solar cells in this paper. And SM55 PV array is selected to validate the EDDMMLW for PV arrays with bypass diodes in this paper. Table 1 shows the specifications of these PV modules. Since the double-diode model of solar cells in Ishaque et al. (2011b) are better than the R_s-model (Walker, 2001) and the R_p-model (Villalva et al., 2009), shown in Ishaque et al. (2011b). So the experimental results in this paper are compared with the model in Ishaque et al. (2011b) and the five-parameter model in Hejri et al. (2014).

Table 4
Parameters for model in Ishaque et al. (2011b) and this paper.

Parameter	SM55 module	
	Model in Ishaque et al. (2011b)	Model in this paper
I_{o1} (A)	2.232E−10	1.5529E−10
I_{o2} (A)	2.232E−10	1.55243E−10
I_L (A)	3.45	3.46188
R_{sh} (Ω)	144.3	144.28908
R_s (Ω)	0.47	0.49696
n_1	–	1.01658
n_2	–	1.01658

For MSX-60 and KC200GT modules, we use Eqs. (57), (58) and (60)–(63) to compute the seven cell parameters of EDDMMLW for solar cells in this paper at different conditions. And we use Eq. (15) to obtain I–V characteristic of EDDMMLW for solar cells in this paper. The seven cell parameters at STC are shown in Table 2. We use Engineering Equation Solver (EES) to calculate seven cell parameters, and we use Matlab software to obtain the I–V curves and record the operating time at any irradiance and temperature. Calculating and plotting a I–V curve takes almost the same time. Here, we only give the operating time of obtaining I–V curve at STC, shown in Table 2. We have recorded 10 times and took the average the operating time of the implicit model in Ishaque et al. (2011b) and Hejri et al. (2014) and the explicit model in this paper. The scope of the voltage satisfies $0 \sim V_{oc}$. The simulation step size is 0.1. From Table 2, we can find that the explicit I–V model proposed in this paper is faster than the two implicit model in Ishaque et al. (2011b) and Hejri et al. (2014).

Figs. 3–5 show the I–V characteristic of the three models for KC200GT and MSX-60 modules at different conditions. Figs. 3 and 4 show the I–V characteristic of KC200GT module when $G = 1000, 800, 600, 400, 200\ \mathrm{W/m^2}$, $T = 298$ K and $G = 1000\ \mathrm{W/m^2}$, $T = 298, 323, 348$ K, respectively. Fig. 5 shows the I–V characteristics of MSX-60 module when $G = 1000\ \mathrm{W/m^2}$, $T = 298, 323, 348$ K, respectively. Figs. 6–15 show the absolute errors of I–V characteristic shown in Figs. 3–5. From Figs. 3–5, the explicit model proposed in this paper can better match with the actual measured data. The absolute

Table 3
RMSEs (Unit: A) for model in Ishaque et al. (2011b), Hejri et al. (2014), and this paper.

	KC200GT module							MSX-60 module		
$G(\mathrm{W/m^2})$	1000	800	600	400	200	1000	1000	1000	1000	1000
T(K)	298	298	298	298	298	323	348	298	323	348
Model in Ishaque et al. (2011b)	0.4225	0.1605	0.138	0.1312	0.1203	0.2972	0.2055	0.1027	0.1523	0.173
Model in Hejri et al. (2014)	0.3704	0.1412	0.1319	0.116	0.0294	0.3087	0.6935	0.0706	0.1668	0.2678
Model in this paper	0.3603	0.1264	0.0774	0.1026	0.087	0.2731	0.1572	0.0936	0.1128	0.1429

Table 5
Shading pattern of the array used in this paper.

Shading pattern (W/m^2)	Case									
	1	2	3	4	5	6	7	8	9	10
A	1000	750	1000	800	900	600	750	1000	1000	1000
B	750	250	500	600	600	500	500	600	1000	500
C	500	250	300	400	300	400	200	300	500	500
D	250	100	100	200	100	300	100	150	250	200

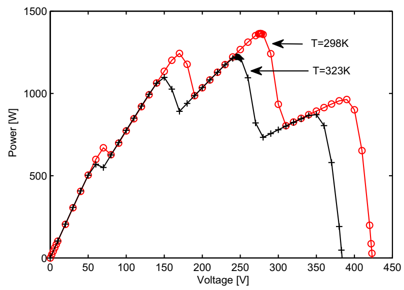

Fig. 16. *P–V* characteristic at Case 1.

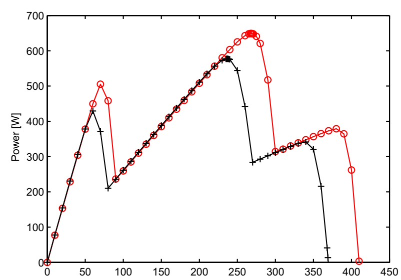

Fig. 17. *P–V* characteristic at Case 2.

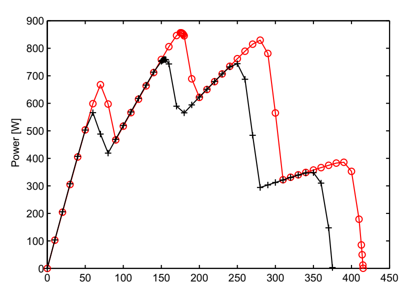

Fig. 18. *P–V* characteristic at Case 3.

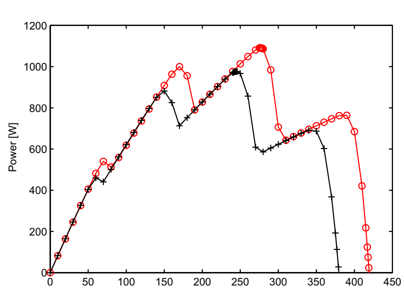

Fig. 19. *P–V* characteristic at Case 4.

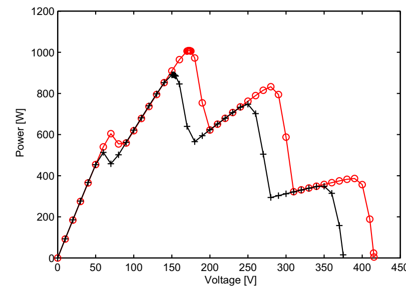

Fig. 20. *P–V* characteristic at Case 5.

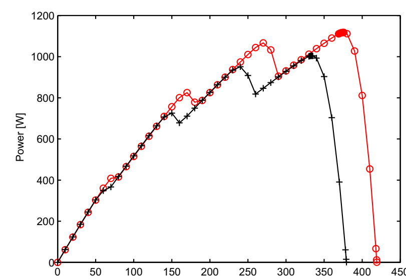

Fig. 21. *P–V* characteristic at Case 6.

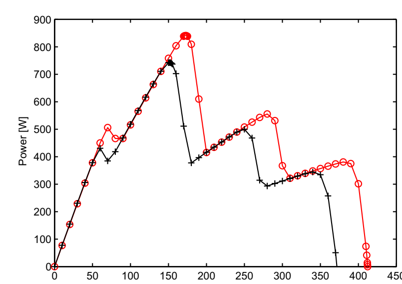

Fig. 22. *P–V* characteristic at Case 7.

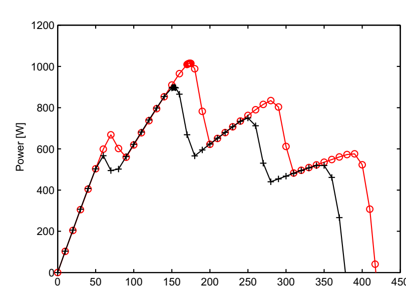

Fig. 23. *P–V* characteristic at Case 8.

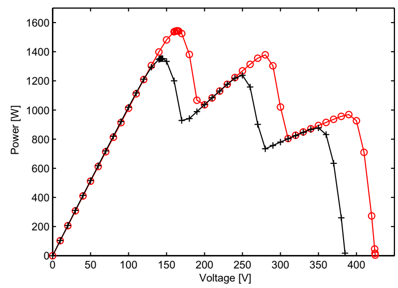

Fig. 24. P–V characteristic at Case 9.

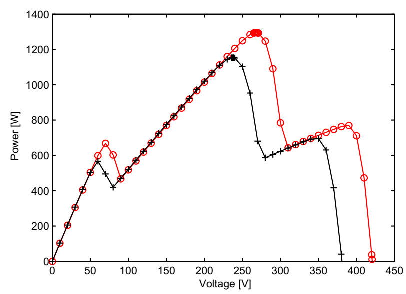

Fig. 25. P–V characteristic at Case 10.

errors of the explicit double-diode model proposed in this paper are close to that of the five-parameter model in Hejri et al. (2014) at 298 K, and are close to that of the model in Ishaque et al. (2011b) at all the conditions. It is worth mentioning that the absolute errors of the explicit double-diode model for solar cells proposed in this paper are far less than that of the five-parameter model in Hejri et al. (2014) at the high temperature conditions.

The Root Mean Square Error (RMSE) (Lun et al., 2013a,b) of the three models for these conditions are shown in Table 3. From Table 3, we can see that RMSEs of the explicit double-diode model proposed in this paper are less than that of the five-parameter model in Hejri et al. (2014) and the model in Ishaque et al. (2011b), expect for the irradiance of 200 W/m^2 and the temperature of 298 K. Therefore, the EDDMMLW for solar cells without bypass diodes in this paper is better than the five-parameter model in Hejri et al. (2014) and the model in Ishaque et al. (2011b).

For a SM55 PV array, the seven cell parameters are shown in Table 4 at STC. We select 20×3 SP (Series–Parallel) configuration shown in Ishaque et al. (2011b) to validate the experimental result. According to Eqs. (25) and (47)–(50) we can calculate the I–V characteristics of PV array and maximum power point (MPP). Here, for the SP array, $I_{2sr+(j-1)(s-1)+i} = 0, i = 1, 2, 3, \ldots, (s-1)$ and $j = 1, 2, 3, \ldots, (r-1)$ (Karatepe et al., 2007). We assume the number of modules in a column is 20 ($s = 20$) and the number of modules in a row is 3 ($r = 3$).

We select 10 shading patterns to verify the precision at 298 K and 323 K, respectively, shown in Table 5. And the simulation data are compared with four other models, namely perturbation and observe (P&O) (Esram and Chapman, 2007), artificial neural network (ANN) (Syafaruddin et al., 2010), single-diode model (Patel and Agarwal, 2008) and model in Ishaque et al. (2011b), respectively. Figs. 16–25 show the P–V curves of SM55 PV array with different shading patterns at 298 K and 323 K, respectively. The corresponding global peak power $P_{mp,G}$ and voltage $V_{mp,G}$ are shown in Tables 6 and 7, respectively. We take ANN model (Syafaruddin et al., 2010) as reference

Table 6
$V_{mp,G}$ (V) and $P_{mp,G}$ (W) outputs of 20×3 PV array under ten shading patterns at 298 K.

Case		P&O Esram and Chapman (2007)	ANN Syafaruddin et al. (2010)	Single-diode model Patel and Agarwal (2008)	Model in Ishaque et al. (2011b)	Model in this paper
1	$V_{mp,G}$	384.72	276.38	271	275	277
	$P_{mp,G}$	971.44	1383.1	1350.8	1359.6	1363.7
2	$V_{mp,G}$	373.67	263.12	247	263	268
	$P_{mp,G}$	377.78	646.13	595.76	620.9	649.1014
3	$V_{mp,G}$	382.51	174.67	178	180	176
	$P_{mp,G}$	387.11	866.76	872.02	883.4	856.1467
4	$V_{mp,G}$	380.3	274.17	265	272	277
	$P_{mp,G}$	770.38	1100	1058.9	1077.9	1091.0
5	$V_{mp,G}$	382.51	172.46	176	178	173
	$P_{mp,G}$	387.7	1020.6	1030.7	1047.7	1008.4
6	$V_{mp,G}$	369.25	369.25	345	360	375
	$P_{mp,G}$	1117.4	1117.4	1046.4	1076.8	1117.9
7	$V_{mp,G}$	375.88	172.46	172	177	173
	$P_{mp,G}$	381.06	847.77	844.17	867.6	840.8682
8	$V_{mp,G}$	382.51	174.67	177	179	174
	$P_{mp,G}$	580.18	1030.4	1044.5	1055.2	1015.3
9	$V_{mp,G}$	386.93	165.83	173	171	164
	$P_{mp,G}$	977.16	1566.3	1621.4	1632.4	1543.3
10	$V_{mp,G}$	382.51	267.54	259	266	268
	$P_{mp,G}$	774.62	1301.6	1247.1	1282.4	1295.4

Table 7
$V_{mp,G}$ (V) and $P_{mp,G}$ (W) outputs of 20 × 3 PV array under ten shading patterns at 323 K.

Case		P&O Esram and Chapman (2007)	ANN Syafaruddin et al. (2010)	Single-diode model Patel and Agarwal (2008)	Model in Ishaque et al. (2011b)	Model in this paper
1	$V_{mp,G}$	338.29	241.01	240	244	246
	$P_{mp,G}$	856.99	1213.4	1206	1220.2	1222.4
2	$V_{mp,G}$	325.03	229.95	217	231	237
	$P_{mp,G}$	329.46	560.97	520.76	553.01	577.3249
3	$V_{mp,G}$	336.08	152.56	158	161	155
	$P_{mp,G}$	339.24	755.81	775.9	790.67	759.0939
4	$V_{mp,G}$	336.08	241.01	236	243	245
	$P_{mp,G}$	677.21	964.26	940.89	965.92	976.3225
5	$V_{mp,G}$	336.08	150.35	156	158	152
	$P_{mp,G}$	339.93	885.98	914.9	935.20	890.8051
6	$V_{mp,G}$	322.81	322.81	307	323	334
	$P_{mp,G}$	979.39	979.39	925.6	964.16	1004.4
7	$V_{mp,G}$	329.45	150.35	153	158	151
	$P_{mp,G}$	333.01	735.27	746.31	774.04	742.6754
8	$V_{mp,G}$	336.08	150.35	159	160	153
	$P_{mp,G}$	509.23	895.62	929.17	943.68	898.8
9	$V_{mp,G}$	340.5	143.72	155	152	144
	$P_{mp,G}$	863.12	1351.1	1435.9	1446.9	1352.8
10	$V_{mp,G}$	336.08	232.16	229	235	237
	$P_{mp,G}$	681.71	1136.5	1101.9	1143.5	1153.8

Table 8
Relative error of P_{mp} (%) at 298 K and 323 K.

T	Method	Case									
		1	2	3	4	5	6	7	8	9	10
298K	Model in Ishaque et al. (2011b)	1.699	3.905	1.92	2.009	2.655	3.633	2.339	2.407	4.22	1.475
	Model in this paper	1.403	4.599	1.224	0.818	1.195	0.0447	0.8141	1.465	1.468	0.4763
323K	Model in Ishaque et al. (2011b)	0.56	1.419	4.612	0.1722	5.555	1.555	5.273	5.366	7.091	0.6159
	Model in this paper	0.7417	2.915	0.4345	1.251	0.545	2.554	1.007	0.3551	0.1258	1.522

values. Since the values of $P_{mp,G}$ in Ishaque et al. (2011b) are in close agreement with ANN (Ishaque et al., 2011b). Therefore, we only compare the model in this paper with the model in Ishaque et al. (2011b). Table 8 shows the relative errors of $P_{mp,G}$ for the model in this paper and the model in Ishaque et al. (2011b), respectively. From Table 8, we can find that the relative errors of the model in this paper are smaller than the model in Ishaque et al. (2011b) at most situations. The RMSE of the model in this paper is 1.6279 W, and the model in Ishaque et al. (2011b) is 3.4750 W. Therefore, for PV array with bypass diodes, the EDDMMLW in this paper is better than the model in Ishaque et al. (2011b).

6. Conclusion

This paper proposes a new explicit double-diode modeling method based on Lambert W-function (EDDMMLW) for solar cells and PV arrays with bypass diodes, respectively. The method in this paper is complete accurate and is suitable for any type of solar cells and any connections of PV arrays with different shading conditions. In addition, this paper proposes a new explicit model based on Lambert W-function for series resistance.

The EDDMMLW for solar cells in this paper is more accurate and faster than the five-parameter model in Hejri et al. (2014) and the model in Ishaque et al. (2011b) at most situations, and it uses fewer electrical parameters than the five-parameter model in Hejri et al. (2014) and the model in Ishaque et al. (2011b). The EDDMMLW for PV arrays with bypass diodes in this paper is more accurate than the model in Ishaque et al. (2011b) at most situations. Therefore, this paper provides a more accurate and effective method to calculate the I–V model of solar cells and PV arrays with bypass diodes.

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