An Explicit Multiexponential Model as an Alternative to Traditional Solar Cell Models With Series and Shunt Resistances

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***Abstract*—Classical analyses of various conventional solar cell models are examined. They are unified through the separation of their linear and nonlinear components and the application of Thevenin's theorem to the linear terms. An explicit multiexponential model with series and shunt resistances is proposed as an alternative to conventional implicit multiexponential models commonly used to describe significant parallel conduction mechanisms in real solar cells. The proposed model is better suited than conventional models for repetitive simulation applications because of its inherently higher computational efficiency. Its explicit nature is a very useful feature for direct analytic differentiation and integration. The model's applicability has been assessed by parameter extraction and subsequent playback using synthetic *I–V* characteristics of a hypothetical solar cell at various illumination levels chosen purely for illustrative purposes.**

***Index Terms*—Explicit junction model, Lambert function, multiexponential diode model, solar cell model, Thevenin equivalent circuit.**

I. Introduction

The simplest approach to describe the electrical behavior of illuminated solar cells consists of using a parallel combination of a photogenerated current source and an ideal diode described by the single-exponential Shockley equation [1]. This well-known equation becomes implicit when modified to include the effects of possible series and parallel parasitic loss mechanisms. However, instead of solving it either numerically or approximately [2], [3], the current and voltage of this modified single-exponential equation may be explicitly solved in terms of the Lambert functions [4]. The Lambert *W* function, which is defined as the solution to the equation $W(x)\exp[W(x)] = x$ [5], is being increasingly used in electronics problems, in particular, for diode [6]–[10], solar cell [11]–[18], bipolar transistor [19], and metal-oxide-semiconductor field-effect transistor (MOSFET) [20]–[22] modeling purposes.

However, a single-exponential equation is frequently not enough to model many types of real solar cells, because it cannot adequately represent the several conduction phenomena that significantly contribute to the total current of the junction [1]. Fig. 1 presents the equivalent circuit of a solar cell that consists of various exponential-type ideal diodes, a photogenerated current source, a series parasitic resistance R_s, and two parallel parasitic conductances G_{p1} and G_{p2}. Its mathematical description is given by the implicit equation

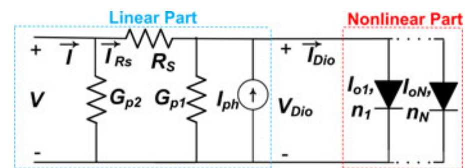

Fig. 1. Generic solar cell equivalent circuit including multiple diodes, parasitic series resistance, and two parallel conductances. For convenience, the linear and the nonlinear parts are separated by dashed boxes.

$$I = \left\{ \sum_{k=1}^{N} I_{\mathrm{ok}} \left[\exp\left(\frac{V - R_S\left(I - G_{p2}V\right)}{n_k v_{\mathrm{th}}} \right) - 1 \right] \right\} - I_{\mathrm{ph}} + G_{p2}V\left(1 + G_{p1}R_S\right) + G_{p1}\left(V - R_S I\right) \quad (1)$$

where I is the total current, V is the terminal voltage, I_{ph} is the photogenerated current, N is the number of different conduction mechanisms to be modeled, I_{ok} is the reverse current coefficient corresponding to each kth mechanism, n_k is the respective junction "ideality" factor, R_S is the parasitic series resistance, G_{p1} is the parallel parasitic conductance at the junction, G_{p2} is the parallel parasitic conductance at the periphery, and $v_{\mathrm{th}} = kT/q$ is the thermal voltage. The summation term inside the brackets in (1) is represented in Fig. 1 by I_{Dio}. Parallel conductances G_{p1} and G_{p2} are used instead of parallel resistances $R_{p1} = 1/G_{p1}$ and $R_{p2} = 1/G_{p2}$ for mathematical convenience, since conductance vanishes from the equations when it is set equal to zero. Using parallel resistances instead would mean having to find limits of equations as resistance goes to infinity. This notation is, therefore, more convenient to easily translate the general model into any particular case.

An equation such as (1), with a suitable number N of conduction mechanisms included, usually provides a sufficiently faithful representation of the *I–V* characteristics for most practical lumped-parameter modeling applications.

Unfortunately, (1) has the serious shortcoming of not being explicitly solvable, in general, for either the terminal current or voltage. There are two very specific exceptions to this

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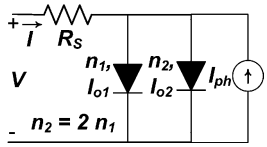


Fig. 2. Double-exponential ($N = 2$) solar cell model with only series resistance and ideality factors such that one is equal to twice the other ($n_2 = 2\, n_1$).

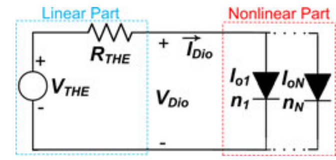


Fig. 3. Thevenin equivalent circuit used to obtain the voltage and the total current in the nonlinear part.

insolvability: 1) the case of the single-exponential ($N = 1$) model already mentioned earlier, whose explicit solutions are well known [11], [12]; and 2) the case of a double-exponential ($N = 2$) model with series resistance in which the ideality factors are known fixed quantities, and one is equal to twice the other ($n_2 = 2n_1$), as shown in Fig. 2. In the second case, although there is no explicit solution for the terminal current, an explicit solution for the terminal voltage is possible

$$V = R_S\, I + n_2\, v_{\mathrm{th}} \ln\left[\sqrt{\left(\frac{I_{02}}{2\, I_{01}} + 1\right)^2 + \frac{I + I_{\mathrm{ph}}}{I_{01}}} - \frac{I_{02}}{2\, I_{01}}\right]. \tag{2}$$

This result is analogous, under illuminated conditions, to a recently published equation used to model diodes under dark conditions [23].

The availability of analytically explicit device model equations constitutes a very desirable feature for simulation applications, especially when the model is to be used repeatedly. Simulation times can be significantly reduced by the computational efficiency improvement afforded by avoiding the numerical iterations required by implicit equation models.

An additional motivation for an explicit model is that it allows easier derivation of expressions to describe other variables, such as output power and maximum power point. Another advantage of models consisting of analytic explicit equations is that they may be directly differentiated and integrated. This is certainly a desirable attribute to develop other model-derived functions, such as the dynamic resistance as a function of applied voltage. Not less important is the fact that, in general, any procedure to be used for model parameter extraction is easier to conceive and more efficiently implemented if it is based on explicit model equations.

It is important to emphasize that rigorous 2-D physical models of solar cells [24], [25] do require a lot of computing power. Therefore, these rigorous models are not practical for circuit simulations and there is a need for more efficient ways of modeling solar cells in order to reduce computing time. The presently proposed lumped-parameter model does not account for second-order effects, such as distributed series resistance, carrier density gradients, or contact or interfacial barriers [24].

In Section II, we present a rigorous general analysis, based on Thevenin's theorem, which requires, in general, a numerical solution. In Section II-A, we show how such general analysis yields the analytical solutions for the case of a single-exponential model. In Section III, we propose the use of an alternative explicit multiexponential model to circumvent the implicit insolvability of (1) and, thus, be able to benefit from the aforementioned advantages of an explicit model.

The proposed novel model will be useful to model solar cells whose *I–V* characteristics show significant signs of the presence of more than one type of conduction mechanism, along with both series and shunt resistive-like losses.

II. Rigorous General Analysis

Approximated simplified nonlinear Thevenin's models have already been used to analyze solar panels [26]–[28]. In order to use Thevenin's theorem for solar cells in a rigorous way, we proceed first to separate the linear and nonlinear terms, as indicated in Fig. 1. The Thevenin equivalent circuit of the linear part defined in Fig. 3 is given by the Thevenin equivalent voltage V_{THE}, which includes the photogenerated current I_{ph}, and the Thevenin equivalent resistance R_{THE}, which only includes R_S and G_{p1}

$$V_{\mathrm{THE}} = \left(\frac{V}{R_S} + I_{\mathrm{ph}}\right) R_{\mathrm{THE}} \tag{3}$$

$$R_{\mathrm{THE}} = \frac{R_S}{1 + G_{p1} R_S}. \tag{4}$$

Note that the absence of G_{p2} from the Thevenin equivalent resistance is correct, since it is perfectly congruent with the fact that I_{Dio} does not depend on G_{p2}. The effect of G_{p2} on the total current will be accounted for once I_{Dio} is calculated.

The total current I_{Dio} through the nonlinear part defined in Fig 3 would be numerically obtained for the general case of N diodes in parallel. Such procedure is the equivalent of numerically solving (1). The importance of the Thevenin equivalent circuit shown in Fig. 3 will be discussed in Sections II-A and C.

There are variables that are apparently concealed in the Thevenin equivalent circuit. Nonetheless, they may be evaluated. The voltage across the nonlinear part V_{Dio} can be expressed in terms of the known I_{Dio} as

$$V_{\mathrm{Dio}} = V_{\mathrm{THE}} - I_{\mathrm{Dio}}\, R_{\mathrm{THE}} \tag{5}$$

and according to the circuit in Fig. 1, the total current can be expressed in terms of I_{Dio} and V_{Dio} as

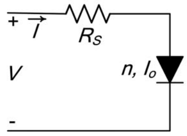
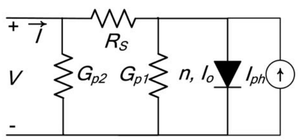
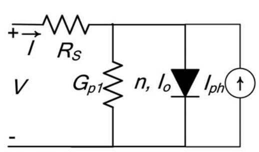
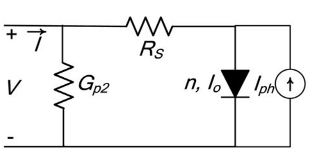
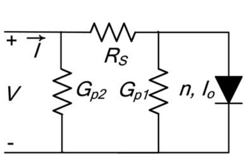
$$I = G_{p2}\, V + G_{p1}\, V_{\mathrm{Dio}} - I_{\mathrm{ph}} + I_{\mathrm{Dio}}. \tag{6}$$

A particular case in which I_{Dio} can be analytically solved will be presented in Section II-A.

A. Single-Diode Model

In the particular case when the junction conduction process is sufficiently well modeled by a single diode (the nonlinear part

TABLE I
EXACT EXPLICIT SOLUTIONS FOR SEVERAL CASES USING A SINGLE-DIODE MODEL

Case	**Explicit solutions**
	$I = \frac{n v_{th}}{R_S} W_0\left\{\frac{I_0 R_S}{n v_{th}}\exp\left[\frac{(V+I_0R_S)}{n v_{th}}\right]\right\} - I_0$ $V = I R_S + n v_{th}\ln\left(\frac{I+I_0}{I_0}\right)$
	$I = \frac{n v_{th}}{R_S} W_0\left\{\frac{I_0R_S}{n v_{th}(1+R_S G_{P1})}\exp\left[\frac{V+R_S(I_0+I_{ph})}{n v_{th}(1+R_S G_{P1})}\right]\right\} + \frac{V(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S)-(I_0+I_{ph})}{1+R_S G_{P1}}$ $V = -\frac{n v_{th}}{1+R_S G_{P2}} W_0\left[\frac{I_0(1+R_S G_{P2})}{n v_{th}(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S)} \exp\left(\frac{I+(I_0+I_{ph})(1+R_S G_{P2})}{n v_{th}(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S)}\right)\right] + \frac{I(1+R_S G_{P1})+I_0+I_{ph}}{(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S)}$
	$I = \frac{n v_{th}}{R_S}W_0\left\{\frac{I_0R_S}{n v_{th}(1+R_S G_{P1})}\exp\left[\frac{V+R_S(I_0+I_{ph})}{n v_{th}(1+R_S G_{P1})}\right]\right\} + \frac{VG_{P1}-(I_0+I_{ph})}{1+R_S G_{P1}}$ $V = -n v_{th} W_0\left[\frac{I_0}{n v_{th}G_{P1}}\exp\left(\frac{I+I_0+I_{ph}}{n v_{th} G_{P1}}\right)\right] + I\left(R_S+\frac{1}{G_{P1}}\right) + \frac{I_0+I_{ph}}{G_{P1}}$
	$I = \frac{n v_{th}}{R_S}W_0\left\{\frac{I_0R_S}{n v_{th}}\exp\left[\frac{V+R_S(I_0+I_{ph})}{n v_{th}}\right]\right\} + G_{P2}V-(I_0+I_{ph})$ $V = -\frac{n v_{th}}{1+R_SG_{P2}}W_0\left[\frac{I_0(1+R_SG_{P2})}{n v_{th}G_{P2}}\exp\left(\frac{I+(I_0+I_{ph})(1+R_SG_{P2})}{n v_{th}G_{P2}}\right)\right] + \frac{I+I_0+I_{ph}}{G_{P2}}$
	$I = \frac{n v_{th}}{R_S}W_0\left\{\frac{I_0R_S}{n v_{th}(1+R_SG_{P1})}\exp\left[\frac{(V+I_0R_S)}{n v_{th}(1+R_SG_{P1})}\right]\right\} + \frac{(V G_{P1}-I_0)}{(1+R_SG_{P1})} + V G_{P2}$ $V = -\frac{n v_{th}}{(1+R_S G_{P2})}W_0\left\{\frac{I_0(1+R_S G_{P2})}{n v_{th}(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S)}\exp\left[\frac{(I+I_0(1+R_S G_{P2}))}{n v_{th}(G_{P1}+G_{P2}+G_{P1}G_{P2} R_S)}\right]\right\} + \frac{I(1+R_S G_{P1})+I_0}{(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S)}$

of the circuit in Fig. 3 contains only one diode), the Thevenin equivalent circuit yields the following analytic solution:

$$I_{\rm Dio} = \frac{n\,v_{\rm th}}{R_{\rm THE}} W_0 \left\{ \frac{I_0\,R_{\rm THE}}{n\,v_{\rm th}} \exp\left[\frac{(V_{\rm THE}+I_0\,R_{\rm THE})}{n\,v_{\rm th}} \right] \right\} - I_0 \tag{7}$$

where W_0 is the Lambert *W* function's principal branch [5].

Therefore, the total current can be readily obtained as an explicit analytic function of the terminal voltage by substituting (7) into (6) and using (3)–(5)

$$I = \frac{n\,v_{\rm th}}{R_S} \times W_0 \left\{ \frac{I_0 R_S}{n\,v_{\rm th}\,(1+R_S\,G_{P1})} \exp\left[\frac{V + R_S\,(I_0 + I_{\rm ph})}{n\,v_{\rm th}\,(1+R_S\,G_{P1})} \right] \right\} + \frac{V\,(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S) - (I_0 + I_{\rm ph})}{1+R_S G_{P1}}. \tag{8}$$

The aforementioned equation embodies the general description of a solar cell by a single-exponential lumped-parameter model with series resistance and two parallel resistive-like losses (across junction and periphery). It readily yields explicit analytic solutions for the current and terminal voltage of several dark and illuminated cases, five of which are illustrated in Table I.

It is worth mentioning that if we let $I_{\rm ph} \to 0$, the general solution (8) turns into the dark diode case solution already described by Ortiz-Conde and García-Sánchez [7]. Likewise, letting $G_{p2} \to 0$ reduces (8) to the case published by Jain and Kapoor [11].

III. Proposed Alternative Model

We propose an explicit model as an alternative to the previously discussed conventional implicit model depicted by the circuit shown in Fig. 1 and mathematically described by (1). The idea for this model ensues from recently published work on explicit models for PIN diodes [29]. This alternative explicit model's equivalent circuit is shown in Fig. 4.

The model's *I–V* characteristics may be found by solving each branch separately and then adding their solutions together. Thus, the following explicit equation may be written for the total current into the diodes:

$$I_{\rm Dioa} = \sum_{k=1}^{N} \left\{ \frac{n_{\rm ka}\,v_{\rm th}}{a_k\,R_{\rm THE}} W_0 \left[\frac{a_k\,R_{\rm THE}\,I_{0k}}{n_{\rm ka}\,V_{\rm th}} \times \exp\left(\frac{V_{\rm THE} + a_k\,R_{\rm THE}\,I_{0k}}{n_{\rm ka}\,v_{\rm th}} \right) \right] - I_{0k} \right\} \tag{9}$$

where the single global series resistance $R_{\rm THE}$ of the conventional model has been replaced in this alternative model by individual resistances $a_k\ R_{\rm THE}$ placed in series with each of the *N* parallel diodes that represent the *N* significant conduction mechanisms.

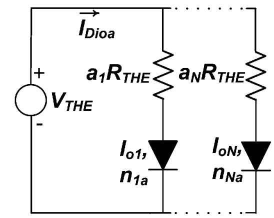


Fig. 4. Presently proposed alternative model's equivalent circuit with multiple diodes and the Thevenin equivalent resistance in series with each diode.

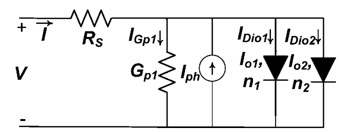


Fig. 5. Equivalent circuit of a ZnO/CdS/CuGaSe$_2$ solar cell studied by Saad and Kassis [34] (same as the circuit in Fig. 1 with $G_{p2} = 0$ and $N = 2$).

Finally, the total current is obtained by substituting (9) into (6) in combination with (3)–(5)

$$I = \sum_{k=1}^{N} \left\{ \frac{n_{\rm k\,a}\,v_{\rm th}}{a_k\,R_S} W_0 \left[\frac{a_k\,R_S\,I_{0k}}{n_{\rm ka}\,v_{\rm th}\,(1+R_S\ G_{P1})} \times \exp\left(\frac{V + I_{\rm ph}R_S + a_k\,R_S\,I_{0k}}{n_{\rm ka}\,v_{\rm th}\,(1+R_S\ G_{P1})} \right) \right] - \frac{I_{0k}}{(1+R_S\ G_{P1})} \right\} + \frac{V\,(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S) - I_{\rm ph}}{1+R_S\,G_{P1}}. \tag{10}$$

In this alternative model, described by the equivalent circuit of Fig. 4, the terminal current turns out to be an explicit analytic function of the terminal voltage, as indicated by (10). It should be noted that this alternative model is an approximation to the conventional model of Fig. 1, since they are not exactly analogous for all possible arbitrary sets of parameters. However, there is a very broad range of practical situations, or model parameter sets, for which the correspondence between both models happens to be excellent. Most useful solar cells fall into this category.

Models somehow comparable with this alternative model have been proposed to individually describe the various current conduction mechanisms present in poly- and multicrystalline solar cells [30]–[33].

IV. Example of Model Correspondence

The proposed alternative model was validated on synthetic *I–V* characteristics generated with the conventional two-exponential model, using the parameter values recently published for a ZnO/CdS/CuGaSe$_2$ solar cell [34]. These synthetic data are used here purely for illustrative purposes. No other inference is implied or intended about the physical meaning of the model or its parameter values regarding this particular cell.

The model used in [34] and shown in Fig. 5 corresponds to the circuit presented in Fig. 1 with $N = 2$ and $G_{p2} = 0$. The synthetic *I–V* characteristics are defined by the following

TABLE II
COMPARISON OF CONVENTIONAL AND ALTERNATIVE MODELS' EXTRACTED PARAMETERS FOR THE SOLAR CELL OF [32] AT VARIOUS ILLUMINATION LEVELS

Extracted Model Parameters	**Illumination level (in terms of I_{ph} [mA cm^{-2}])**						
	15.7	**9.3**	**6.4**	**4.3**	**2.5**	**1.0**	**0.03**
Alternative							
Conventional							
I_{01a} [A.cm^{-2}]	1.48×10^{-5}	9.77×10^{-6}	8.13×10^{-6}	7.08×10^{-6}	7.18×10^{-6}	6.41×10^{-6}	4.90×10^{-6}
I_{01} [A cm^{-2}]	1.91×10^{-5}	12.50×10^{-6}	9.5×10^{-6}	7.90×10^{-6}	7.20×10^{-6}	6.40×10^{-6}	4.90×10^{-6}
I_{02a} [A.cm^{-2}]	1.55×10^{-14}	9.25×10^{-16}	1.11×10^{-15}	6.74×10^{-16}	9.40×10^{-16}	7.77×10^{-16}	4.69×10^{-17}
I_{02} [A cm^{-2}]	3.60×10^{-15}	1.40×10^{-16}	1.20×10^{-16}	1.10×10^{-16}	1.00×10^{-16}	9.40×10^{-17}	2.80×10^{-18}
n_{1a}	6.54	6.44	6.24	6.42	6.35	6.39	6.08
n_1	7.16	7.00	6.55	6.66	6.35	6.39	6.08
n_{2a}	1.37	1.27	1.28	1.24	1.27	1.23	1.13
n_2	1.27	1.16	1.15	1.14	1.14	1.11	1.00
a_1R_{TH} [Ωcm^2]	25.68	33.94	25.55	27.58	16.03	18.27	20.98
R_{TH} [Ω cm^2]	7.00	12.17	13.27	15.07	16.36	18.16	20.67
a_2R_{TH} [Ωcm^2]	8.08	14.74	17.11	18.72	22.83	23.63	27.86
G_{p1} [mS cm^{-2}]	0.200	0.204	0.189	0.152	0.137	0.112	0.077
R_s [Ω cm^2]	7.01	12.20	13.30	15.10	16.40	18.20	20.70

conventional implicit double-exponential equation [35]–[37]:

$$I = I_{01}\left[\exp\left(\frac{V - R_S I}{n_1 v_{\rm th}}\right) - 1\right] + I_{02}\left[\exp\left(\frac{V - R_S I}{n_2 v_{\rm th}}\right) - 1\right] - I_{\rm ph} + G_{P1}\left(V - R_S I\right). \tag{11}$$

The alternative model's explicit current equation for this particular case (using (10) with $N = 2$ and $G_{p2} = 0$) is

$$\begin{aligned} I_A = &\left\{\frac{n_{1a} v_{\rm th}}{a_1 R_S} W_0\left[\frac{a_1 R_S I_{01}}{n_{1a} v_{\rm th}\left(1 + R_S\, G_{P1}\right)}\right.\right. \\ &\left.\left.\times\exp\left(\frac{V + I_{\rm ph} R_S + a_1 R_S I_{01}}{n_{1a} v_{\rm th}\left(1 + R_S\, G_{P1}\right)}\right)\right] - \frac{I_{01}}{\left(1 + R_S\, G_{P1}\right)}\right\} \\ &+ \left\{\frac{n_{2a} v_{\rm th}}{a_2 R_S} W_0\left[\frac{a_2 R_S I_{02}}{n_{2a} v_{\rm th}\left(1 + R_S\, G_{P1}\right)}\right.\right. \\ &\left.\times\exp\left(\frac{V + I_{\rm ph} R_S + a_2 R_S I_{02}}{n_{2a} v_{\rm th}\left(1 + R_S\, G_{P1}\right)}\right)\right] \\ &\left.- \frac{I_{02}}{\left(1 + R_S\, G_{P1}\right)}\right\} + \frac{V G_{P1} - I_{\rm ph}}{1 + R_S\, G_{P1}}. \end{aligned} \tag{12}$$

The two terms inside square brackets in the aforementioned equation correspond to the currents through the diodes divided by $(1{+}R_S\ G_{p1})$.

The alternative model parameters were extracted by directly fitting (12) to the original synthetic *I–V* characteristics generated with (11). Relative error $(I_A/I{-}1)$ was minimized to get the best fit for both linear and logarithmic scales.

Table II presents the parameter values at various illumination levels extracted from fitting the presently proposed alternative model (12) to the synthetically generated solar cell data. They are denoted as "alternative."

The parameter values used here to calculate the synthetic data with (11) are those originally published in [32], and are denoted as "conventional" in Table II. A value of 0.024 V was used for $v_{\rm th}$. The table indicates that, as expected, $I_{01a} \approx I_{01}$, $I_{02a} \approx I_{02}$, $n_{1a} \approx n_1$, $n_{2a} \approx n_2$, and $a_1 \approx a_2 \approx 1$.

It is worth pointing out that the very large values of n_1 reported for this particular solar cell [34] and indicated in Table II, which apparently are incompatible with classical diode theory [38], might be a nonphysical artifact. On the other hand, a high illumination-enhanced interface state density, due to buffer-absorber heterojunction lattice mismatch, has been alleged as a possible justification for such large ideality factor values [39], [40]. A discussion of this controversial issue falls beyond the scope of the present work.

The decreasing series resistance and increasing shunt conductance as the illumination intensity increases is consistent with a photoconductivity modulation effect. Photogenerated current illumination dependence, which is typical of chalcopyrite cells [41], prevents modeling this type of cell by applying the "superposition principle" or "*I–V* translation approximation."

It is interesting to mention that (12) resembles the defining equation of a model proposed by Miranda *et al.* to describe the leakage postbreakdown *I–V* characteristics of HfO_2/TaN/TiN gate oxide stacks used in MOSFETS [42]. In that unrelated model, the most relevant mechanisms are described by a parallel combination of two oppositely connected diodes with individual series resistances and a shunt leakage path [42].

Fig. 6 presents the original synthetic data generated by the conventional implicit model (11), together with the alternative model's playback of the explicit double-exponential (12) using the extracted parameters shown in Table II.

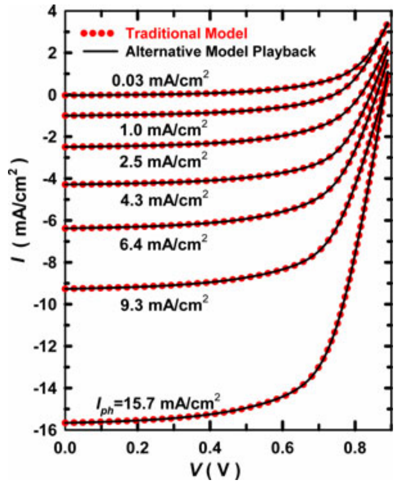


Fig. 6. Solar cell synthetic *I–V* characteristics (red symbols) from [34] and the presently proposed alternative model's playback (black solid lines).

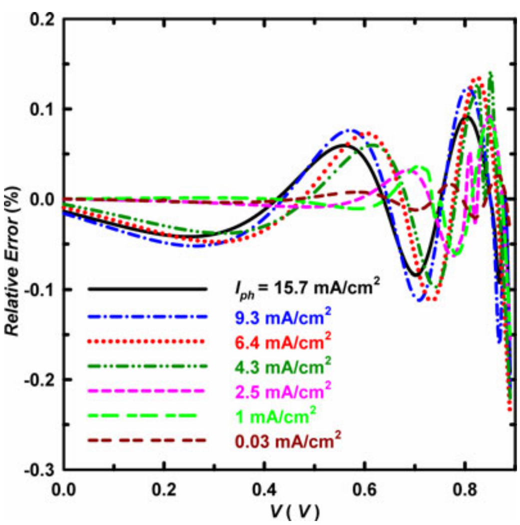


Fig. 7. Relative error produced by the presently proposed alternative model with respect to the conventional model for the solar cell of [34].

The excellent correspondence between both models, with a calculated maximum relative error <0.25%, as shown in Fig. 7, illustrates the suitability of the presently proposed alternative explicit equation to faithfully reproduce the *I–V* characteristics of solar cells.

The rigorous solution of the total current and all of its components are presented in Fig. 8 for the highest illumination case of $I_{\rm ph} = 15.70\,{\rm mA/cm}^2$ (short-circuit current $I_{\rm sc} = 15.66\,{\rm mA/cm}^2$) to visualize the relative importance of each component. In this particular case with $N = 2$ and $G_{p2} = 0$, the total current is

$$I = I_{\rm Gp1} + I_{\rm Dio1} + I_{\rm Dio2} - I_{\rm ph}. \qquad (13)$$

The figure presents $I + I_{\rm ph}$, $I_{\rm Gp1}$, $I_{\rm Dio1}$, and $I_{\rm Dio2}$ in linear and logarithmic scales for the case with the highest illumination level ($I_{\rm ph} = 15.7\,{\rm mA/cm}^2$). As can be observed from the figure, none of the current components in (13) can be neglected for all bias conditions in this particular solar cell.

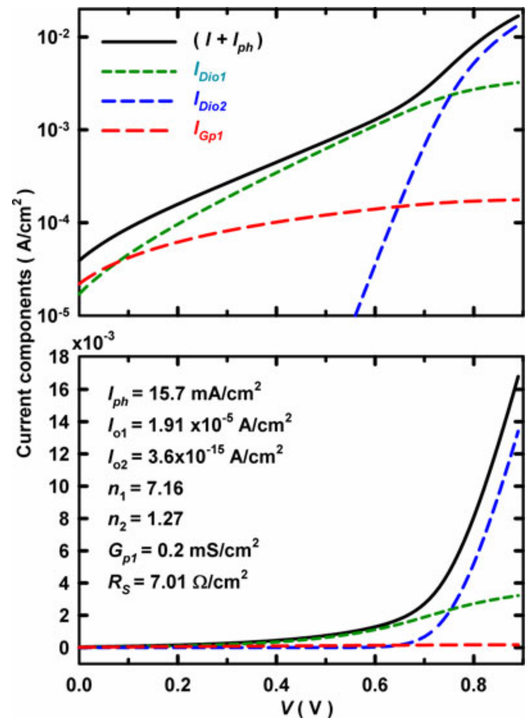


Fig. 8. Current components of the synthetic *I–V* characteristics of the solar cell reported in [34], for the highest illumination case. (Top) Linear scale. (Bottom) Logarithmic scale.

As already mentioned, the use of the presently proposed alternative model inherently offers an obvious computational advantage, because its explicit nature circumvents the need for numerical iteration, which would be unavoidably required if the conventional implicit model were used.

A quick calculation test indicates that the computation time of thousand points can be reduced typically by a factor of 10 using the alternative model as compared with using the conventional model. This higher efficiency illustrates the proposed model's appeal for repetitive simulation purposes and for expediting parameter extraction procedures.

V. Conclusion

We have reviewed and unified the analysis of the various conventional lumped circuits used to model solar cells. This unification is made possible by separating their nonlinear components and applying Thevenin's theorem to their remaining linear components. The application of Thevenin's theorem is rigorous in this case, in contrast with the previous usage of the approximated simplified nonlinear Thevenin's model [26]–[28].

Our recent proposal for PIN diode modeling [27] has been generalized to include the various current transport mechanisms that could coexist in real solar cells with series and shunt losses. The distinctive and most valuable feature of the presently proposed alternative approach is that the current may be expressed as an explicit analytic function of the applied terminal voltage. On the contrary, the current–voltage equation of the conventional multidiode model must be solved by numerical iteration since it is, in general, unavoidably implicit [43], [44].

The explicit nature of the alternative model allows a significantly higher numeric computational efficiency in simulation applications, facilitates curve-fitting procedures for model parameter extraction [45], and allows straightforward analytic differentiation and integration to derive analytic expressions of useful quantities such as temperature and parameter variation sensitivity.

The validity of the proposed alternative model was studied by comparing the extracted parameter values of the presently proposed explicit model to those of the conventional implicit two-exponential model's parameters, which were originally used to generate the illustrative synthetic *I–V* characteristics. The close match observed between the original and the extracted parameters corroborates the suitability of the presently proposed model as an alternative to explicitly describe solar cell multiexponential models. Although a particular example was presented here purely for illustrative purposes, the proposed alternative explicit multiexponential model's applicability extends in general to any solar cell that is presently described by a conventional implicit multiexponential model.

Future studies to test the validity range, robustness, and sensitivity of the proposed alternative model would be desirable and could be done by introducing artificial series and shunt resistances into a well-functioning device.

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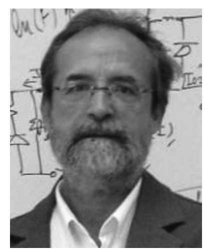

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