All Matlab graph is in the end and named accordingly

EC516 HW 2

2.1 Part (A)

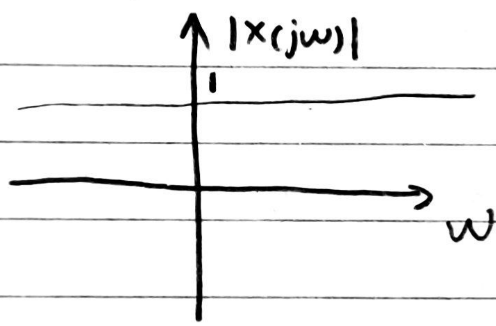
a) $x(t) = \delta(t - t_0)$

CTFT:

$$X(jw) = \int_{-\infty}^{\infty} \delta(t-t_0)\, e^{-jwt}\, dt$$

sifting property → $$= e^{-jw t_0}$$

$$|X(jw)| = |e^{-jw t_0}| = 1$$



b) $x(t) = u(t+T) - u(t-T)$

CTFT:

$$X(jw) = \int_{-\infty}^{\infty} [u(t+T) - u(t-T)]\, e^{-jwt}\, dt$$

$$= \int_{-T}^{T} e^{-jwt}\, dt$$

$$= \left. \frac{e^{-jwt}}{-jw} \right|_{-T}^{T}$$

$$= \frac{e^{-jwT} - e^{jwT}}{-jw} = \frac{2\sin(wT)}{w}$$

$$|X(j\omega)| = \left| \frac{2\sin(\omega T)}{\omega} \right|$$

This is a Sinc function

See Matlab graph

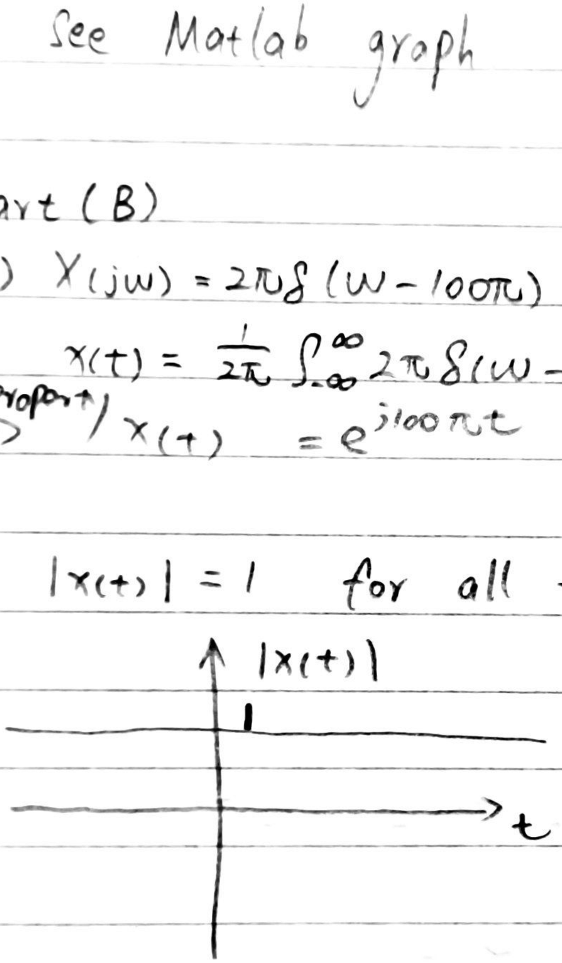
Part (B)

a) $X(j\omega) = 2\pi\delta(\omega - 100\pi)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\,\delta(\omega - 100\pi)\, e^{j\omega t}\, d\omega$$

sifting property $\rightarrow$ $x(t) = e^{j100\pi t}$

$|x(t)| = 1$ for all t



b) $X(j\omega) = u(\omega + 2\pi) - u(\omega - 2\pi)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [u(\omega+2\pi) - u(\omega-2\pi)]\, e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-2\pi}^{2\pi}$$

$$= \frac{\sin(2\pi t)}{\pi t}$$

$|x(t)| = \left|\frac{\sin(2\pi t)}{\pi t}\right|$

See Matlab Plot

Part (c)

$X(j1000\pi) = 1+j$

$\overline{X(j1000\pi)} = 1-j$

$X(-j\omega) = \overline{X(j\omega)}$ So $X(-j1000\pi) = (1+j)^* = 1-j$)

Prove:

$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$$\overline{X(j\omega)} = \overline{\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt}$$

$$= \int_{-\infty}^{\infty} \overline{x(t)}\, \overline{e^{-j\omega t}}\, dt$$

Since $x(t)$ is real

$$= \int_{-\infty}^{\infty} x(t)\, \overline{e^{-j\omega t}}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{j\omega t} dt$$

$$= X(-j\omega)$$

2.2

Part (A)

$x[n] = \delta[n-3]$

$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$

$= \sum_{-\infty}^{\infty} \delta[n-3] e^{-jwn}$ since δ is zero else where

$= e^{-jw3}$

Part (B)

a) $x[n] = u[n+2] - u[n-3]$

$x[n] = \begin{cases} 1 & n = -2, -1, 0, 1, 2 \\ 0 & \text{else where} \end{cases}$

$X(e^{jw}) = \sum_{n=-2}^{2} e^{-jwn}$

$= e^{jw2} + e^{jw} + 1 + e^{-jw} + e^{-j2w}$

$|X(e^{jw})|$ see matlab plot

b) $x[n] = u[n] - u[n-5]$

$$x[n] \begin{cases} 1 & n = 0,1,2,3,4 \\ 0 & \text{else where} \end{cases}$$

$$X(e^{jw}) = \sum_{n=0}^{4} e^{-jwn} = e^0 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w}$$

See plot in Matlab

c) $x[n] = u[n] - u[n-4]$

$$x[n] \begin{cases} 1 & n = 0,1,2,3 \\ 0 & \text{elsewhere} \end{cases}$$

$$X(e^{jw}) = \sum_{n=0}^{3} e^{-jwn} = e^0 + e^{-jw} + e^{-j2w} + e^{-j3w}$$

See plot in Matlab

Part (C)

a) $X(e^{jw}) = \sum 2\pi \delta(w - 0.5\pi - 2\pi k)$

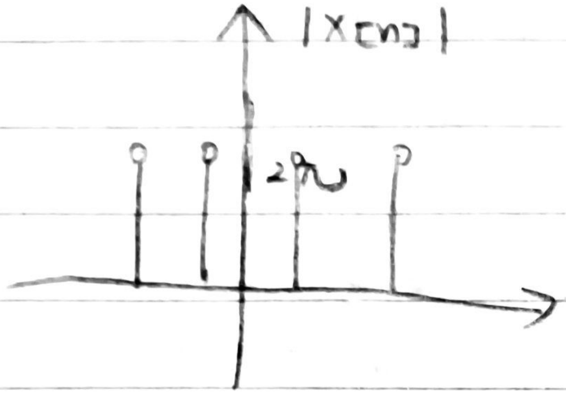
$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\sum 2\pi \delta(w - 0.5\pi - 2\pi k) \right) e^{-jwn} dw$$

sifting property $= \sum 2\pi e^{j(0.5\pi + 2\pi k)n}$

$$= 2\pi e^{j0.5\pi n} \sum_{k=-\infty}^{\infty} \delta[n]$$

$$= 2\pi e^{j\frac{1}{2}\pi n}$$

$|x[n]| = 2\pi$



b) $X(e^{jw}) = \sum_{k=-\infty}^{\infty} \{ \pi \delta(w + 0.5\pi - 2\pi k) +$

$\pi \delta(w - 0.5\pi - 2\pi k) \}$

$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} (\Sigma \cdots\cdots) e^{jwn} dw$

sifting property $\rightarrow$ $= \pi \sum_{k=-\infty}^{\infty} (e^{-j\frac{1}{2}\pi n} + e^{j\frac{1}{2}\pi n})$

$x[n] = 2\pi \cos(\frac{1}{2}\pi n)$

See Matlab Plot.

2.3

a) $y[n] = 0.5y[n-1] + x[n]$

$Y(e^{jw}) = 0.5Y(e^{jw})e^{-jw} + X(e^{jw})$

(DTFT on both sides)

$Y(e^{jw}) - 0.5Y(e^{jw})e^{-jw} = X(e^{jw})$

$Y(e^{jw}) = \dfrac{X(e^{jw})}{1-\frac{1}{2}e^{-jw}}$

b) $X(e^{j0.25\pi}) = 1+j$

As for property $X(e^{-jw}) = \overline{X(e^{jw})}$

So $X(e^{-j0.25\pi}) = 1-j$

Prove:

$$DTFT(x) = \sum_{n=-\infty}^{\infty} x(n)e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} x(n)[\cos(wn) + j\sin(wn)]$$

$$= \sum_{n=-\infty}^{\infty} x(n)\cos(wn) + j\underbrace{\sum_{n=-\infty}^{\infty} \sin(wn)}_{\text{goes to zero}}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\cos(wn)$$

So real and even

2.4

$$X(e^{jw}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\left(\frac{w+2\pi k}{T}\right)\right)$$

with Nyquist-Shannon Sampling theorem,

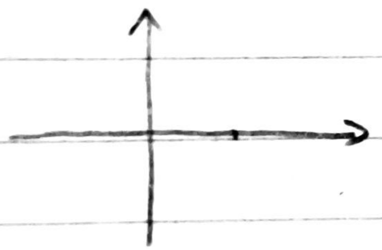
$f_s > 2 f_{max} = \frac{1}{T} > 2 \times 5000\pi$ rad/s

$$f_{Nyquist} = \frac{10000\pi}{2\pi} = 5000\,Hz$$

$f_s > 2 f_{Nyquist} = 10{,}000\,Hz$

a) $T = 0.0001\,s$

$$\frac{\pi}{T} = \frac{\pi}{0.0001s} = 10000\pi$$



Here we only talk about value in $[-\pi, \pi]$ the other part of DTFT will behave periodically and so does zero values

guarantee to be zero : $-\pi$ and π

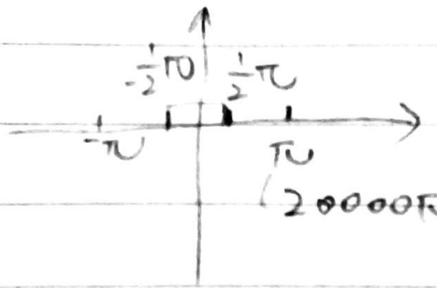
As $[-\pi, \pi]$ corresponding to $[-10000\pi, 10000\pi]$

So 10000 Hz will have value zero as

$|w| \geq 10{,}000\pi$ is zero.

b) $T = 0.00005\ s$

$$\frac{\pi}{T} = \frac{\pi}{0.00005\ s} = 20000\pi\ rad/s$$



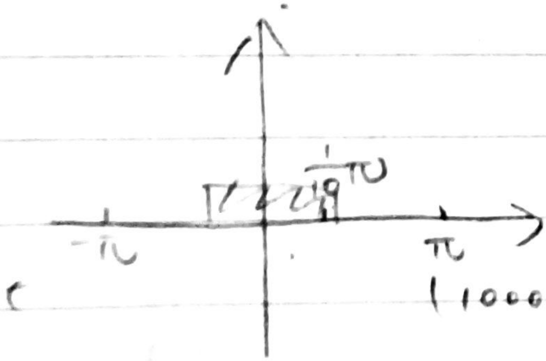
guarantee to be zero from
① $[-\pi, -\frac{1}{2}\pi]$ and $[\frac{1}{2}\pi, \pi]$

So same as before Now the $[-\pi, \pi]$ represents in CTFT $[-20000\pi, 20000\pi]$ and $|\omega| \geq 100000\pi$ is guarantee to be zero, so correspondimly ① is right

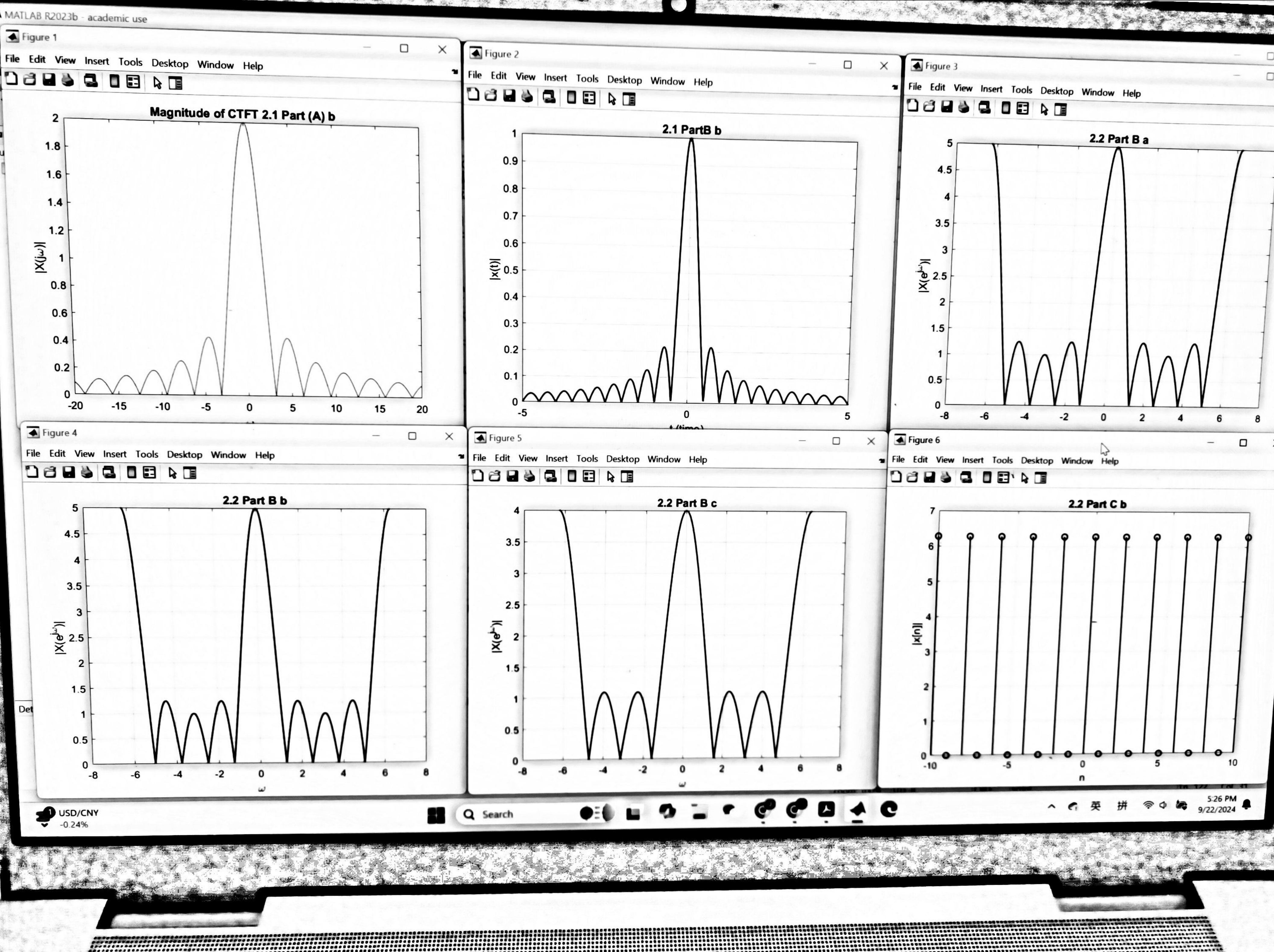
c) $T = 0.00001\ s$

$$\frac{\pi}{T} = \frac{\pi}{0.00001\ s} = 100000\pi\ rads$$

$[-\pi, -\frac{1}{10}\pi]$ and
$[\frac{1}{10}\pi, \pi]$ ②



Same as before Now $[-\pi, \pi]$ represents CTFT $[-100000\pi, 100000\pi]$, So $|\omega| \geq 10000$ is corresponding to ②



MATLAB R2023b - academic use
Figure 1
File Edit View Insert Tools Desktop Window Help
Magnitude of CTFT 2.1 Part (A) b
|X(jω)|
Figure 2
File Edit View Insert Tools Desktop Window Help
2.1 PartB b
|x(t)|
Figure 3
File Edit View Insert Tools Desktop Window Help
2.2 Part B a
Figure 4
File Edit View Insert Tools Desktop Window Help
2.2 Part B b
ω
Figure 5
File Edit View Insert Tools Desktop Window Help
2.2 Part B c
ω
Figure 6
File Edit View Insert Tools Desktop Window Help
2.2 Part C b
|x[n]|
n
USD/CNY
-0.24%
Search
5:26 PM
9/22/2024