EC516 HW10 Solutions

Problem 10.1

(a)

$$
\begin{aligned}
H(z) &= 1 - z^{-1} + z^{-2} + z^{-3} \\
&= (1 - z^{-1})(1 + z^{-2}) \\
&= (1 - z^{-1})(1 + jz^{-1})(1 - jz^{-1})
\end{aligned}
$$

The zeros are at $z = 1, j, -j$.

(b) Note the DTFT is

$$
\begin{aligned}
H(e^{j\omega}) &= e^{-j3\omega/2} \frac{\sin(2\omega)}{\sin(\omega/2)} * \delta(\omega - \pi) \\
&= e^{-j3(\omega-\pi)/2} \frac{\sin(2(\omega - \pi))}{\sin((\omega - \pi)/2)}
\end{aligned}
$$

The peak is located at $\omega = \pi$ or $(-\pi)$ and its peak value is 4.

Problem 10.2

(a) Since all poles are at $z = 0$, it is a FIR filter.

(b) From the impluse response, we know that

$$
y[n] = x[n] - x[n-8]
$$

Therefore,

$$
\begin{aligned}
y[n] &= \cos(0.25\pi n + 0.15\pi) - \cos(0.25\pi((n-8) + 0.15\pi) \\
&= \cos(0.25\pi n + 0.15\pi) - \cos(0.25\pi n - 2\pi + 0.15\pi) \\
&= \cos(0.25\pi n + 0.15\pi) - \cos(0.25\pi n + 0.15\pi) \\
&= 0
\end{aligned}
$$

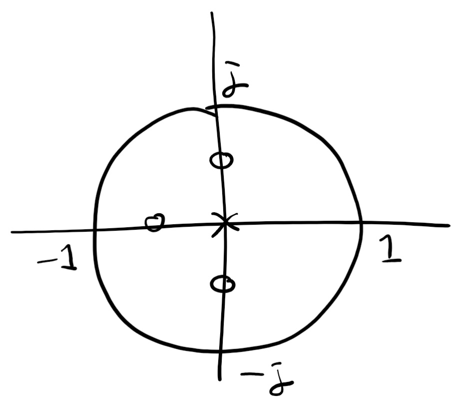
$y[n]$ is 0 at all n.

Problem 10.3

(a)

$$
\begin{aligned}
H(z) &= \frac{1-(0.5)^4 z^{-4}}{1-0.5z^{-1}} \\
&= \frac{(1-(0.5)^2 z^{-2})(1+(0.5)^2 z^{-2})}{1-0.5z^{-1}} \\
&= \frac{(1-0.5z^{-1})(1+0.5z^{-1})(1-j0.5z^{-1})(1+j0.5z^{-1})}{1-0.5z^{-1}} \\
&= (1+0.5z^{-1})(1-j0.5z^{-1})(1+j0.5z^{-1})
\end{aligned}
$$

The three zeros are at $z = -0.5, j0.5, -j0.5$ and three poles at $z = 0$.



(b) DTFT of $\cos(0.25\pi n)$ is

$$\frac{1}{2}\delta(\omega - \pi/4) + \frac{1}{2}\delta(\omega + \pi/4)$$

We can create a filter with zero response at $e^{j\pi/4}$ and $e^{j\pi/4}$, and pole at arbitrary place inside the unit circle (to make the filter stable).

$$H(z) = \frac{(1-e^{j\pi/4}z^{-1})(1-e^{-j\pi/4}z^{-1})}{1-\alpha z^{-1}}, \quad |\alpha| < 1 \tag{1}$$

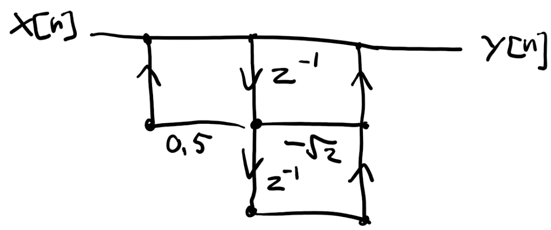
If we let $\alpha = 0.5$, we get

$$H(z) = \frac{1-2\cos(\pi/4)z^{-1}+z^{-2}}{1-0.5z^{-1}}$$

In time domain this filter is

$$y[n] = 0.5y[n-1] + x[n] - \sqrt{2}x[n-1] + x[n-2]$$

The flowgraph will look like below.



Problem 10.4

(a) Inverse DTFT of

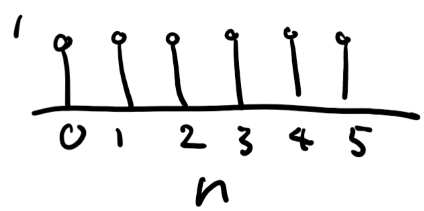
$$e^{-(N-1)\omega/2}\frac{\sin(N\omega/2)}{\sin(\omega/2)}$$

is

$$u[n] - u[n-N]$$

Therefore inverse DTFT of $X_a(e^{j\omega})$ is

$$x[n] = u[n] - u[n-6]$$

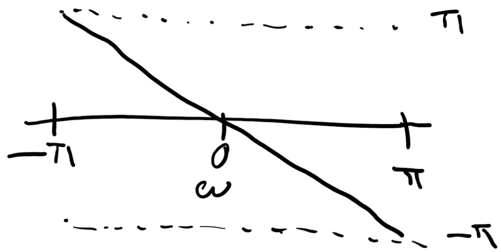


(b) Bilinear transformation is a one-to-one mapping of left-half of complex plane ($\mathrm{Re}(s) \le 0$) to unit-circle ($|z| \le 1$). $s = j\omega_0$ is mapped to $z = e^{j2\tan^{-1}(\omega_0)}$. The answer is yes, there will be at least one zero on the unit-circle.

Problem 10.5 $x[n]$ is a convolution of 2-point boxes. Its DTFT will be the square of the DTFT of a 2-point box.

$$X(e^{j\omega}) = \left(e^{-j\omega/2}\frac{\sin(\omega)}{\sin(\omega/2)}\right)^2 = e^{-j\omega}\frac{\sin^2(\omega)}{\sin^2(\omega/2)}$$

Since $\frac{\sin^2(\omega)}{\sin^2(\omega/2)}$ doesn't changes its sign, the phase is simply $-\omega$.



<u>Problem 10.6</u> Bilinear transformation is a nonlinear transformation. Applying the bilinear transformation to an analog filter with linear phase will not result in a digital filter with linear phase. Bilinear transformation is not the best tool to use for creating filters with linear phase.