FALL 24 EC516 Problem Set 12

Due: Sunday December 8 (Before 11:59pm)

You must submit your homework attempt on Blackboard Learn. For this purpose, you must convert your homework attempt to a pdf file and upload it at the corresponding homework assignment on Blackboard Learn. There are 6 problems on this problem set. There will be no quiz on this problem set, but you will be tested on it in the Final Exam on December 16, 9am.

Problem 12.1 (FFT)

In this problem, you are given a 55-point real-valued signal $x[n]$. Compare the computational cost (in terms of multiplications of real numbers with real numbers) of the direct computation of the 55-point DFT of $x[n]$ with that of the 64-point DFT of $x[n]$ computed using the radix-2 decimation-in-time FFT Show your work.

Problem 12.2 (Spectrogram)

(A) Consider the 5-point signal given as $x[n] = u[n] - u[n-5]$, where $u[n]$ is the unit step. Let $X_w[n, \omega)$ be the TDFT of $x[n]$ with respect to analysis window $w[n] = \delta[n]$. Also let $f_n[m] = w[m]x[n+m]$ denote the short-time section at each time *n* that is a signal as a function of *m*.

(a) Sketch magnitude and phase of $X(e^{j\omega})$.
(b) Sketch the short-time sections $f_{-1}[n], f_0[n]$, and $f_{10}[n]$. Justify your answers.
(c) Sketch $X_w[n, \frac{\pi}{2})$ as a function of *n*. Justify your answer
(d) Sketch $X_w[0, \omega)$ as a function of ω. Justify your answer.

(B) Consider the 5-point signal given as $x[n] = u[n] - u[n-5]$, where $u[n]$ is the unit step. Let $X_w[n, \omega)$ be the TDFT of $x[n]$ with respect to analysis window $w[n] = 1$ for all *n*.
(a) Sketch the short-time sections $f_{-1}[n], f_0[n]$, and $f_{10}[n]$. Justify your answers.
(b) Sketch $X_w[n, \frac{2\pi}{5})$ as a function of *n*. Justify your answer
(c) Sketch $X_w[n, \frac{3\pi}{5})$ as a function of *n*. Justify your answer
(d) Sketch $X_w[0, \omega)$ as a function of ω. Justify your answer.

Problem 12.3 (Spectrogram)

a) Let $X_w[n, \omega)$ denote the TDFT of a signal $x[n]$ with respect to an analysis window $w[n]$. Use the filtering view of the TDFT to argue that the TDFT of $x[n-n_0]$ is $X_w[n-n_0, \omega)$. HINT: A filter is a time-invariant system.

b) Let $X_w[n,\omega)$ denote the TDFT of a real-valued signal $x[n]$ with respect to a real-valued analysis window $w[n]$. Use the Fourier transform view of the TDFT to argue that $X_w[n,\omega) = X_w^*[n,-\omega)$

c) Using the filtering view of the TDFT, construct a *counterexample* to show that the TDFT of the convolution of two signals is not necessarily equal to the product of their individual TDFTs.

Problem 12.4 (Spectrogram in MATLAB)

(A) Use MATLAB to record your voice while you say the word "sad" using a sampling rate of 16 KHz. Splice off the beginning portion from the recorded signal so that the resulting signal $x[n]$ has no silence preceding the sound of the "s" in "sad." Use MATLAB to plot the signal $x[n]$.

(B) Let $w[n] = u[n] - u[n-256]$ and let $f_n[m] = w[m]x[n+m]$.

(a) Use MATLAB to calculate and plot $f_0[m]$ for $0 \le m < 256$.

(b) Determine an integer value n_0 such that $f_{n_0}[m]$ falls in a portion of $x[n_0+m]$ during which the "a" sound is being spoken during "sad." Use MATLAB to calculate and plot $f_{n_0}[m]$ for $0 \le m < 256$. You should observe that this signal has a somewhat periodic structure.

(c) Plot (using MATLAB) the magnitude of the 512-point DFT of $f_{n_0}[m]$ you observed in the previous part. How is this plot consistent with the fact that the corresponding short-time section has a somewhat periodic structure.

Problem 12.5 (Parametric Signal Modeling and FFT)

Let $g[n]$ be a 128-point signal whose Parametric Signal Model is given as $H(z) = \frac{G}{1+\sum_{k=1}^{10} a_k z^{-k}}$. Show that the values of $H\left(e^{j\frac{2\pi k}{256}}\right)$ for $k = 0,1,\cdots,255$ can be calculated by taking the DFT of some signal (using, say, an FFT algorithm) and taking the reciprocal of those DFT values.

Problem 12.6 (Cepstral Analysis)

What shift and what amplitude scaling would you perform on the signal $x_c[n] = -2\delta[n+2] + 4\delta[n+1]$ such that the resulting signal has a z-transform that satisfies the restricted model for computing the complex cepstrum. *Justify your answer*.