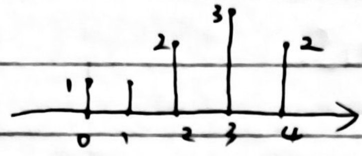
EC 516 HW 11

11.1 $R_N[k] = u[n] - u[n-N]$

a)



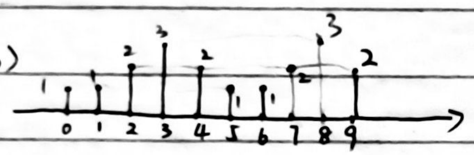
$R_5[n] = u[n] - u[n-5]$

$R_5[0] = x[4]R[0] = 1$ $R[1] = x[0]R[1] = 1$

$R_5[2] = x[1]R[2] = 2$ $R[3] = x[2]R[3] = 3$

$R_5[4] = x[3]R[4] = 2$

b)

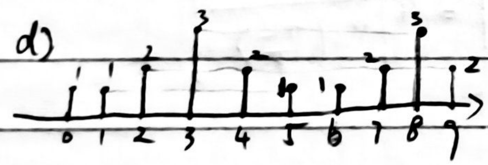


This is actually just rotations of $x[n]$ by 1 shift which follows 1, 1, 2, 3, 2

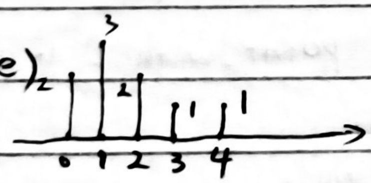
c)

Still 1, 1, 2, 3, 2 Because we start from $\delta[n]$ and rotate backward.

d)



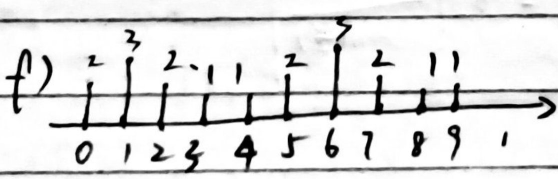
e)



$n[0] = x[-2]$

2, 3, 2, 1, 1

f)



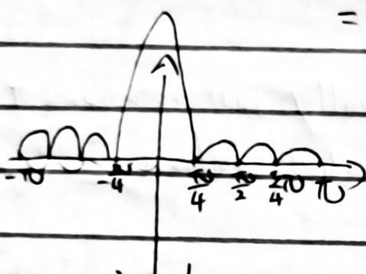
11.2

$x[n] = 2\{u[n] - u[n-8]\}$ $\qquad$ $g[n] = 3\{u[n] - u[n-8]\}$

a) $X(e^{jwn}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$

$= \sum_{n=0}^{7} 2e^{-jwn} = 2\,\frac{1-e^{-j8w}}{1-e^{-jw}}$

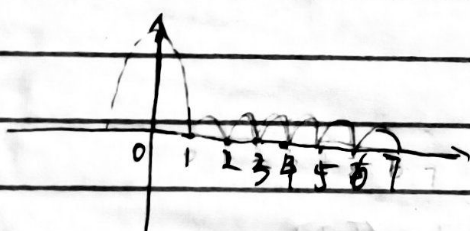
$= 4\left|\frac{\sin(4w)}{\sin(\frac{w}{2})}\right|$



b) $x[k] = X(e^{jw})\big|_{w=\frac{2\pi k}{N}}$ $\qquad k = 0, 1, \ldots, N-1$

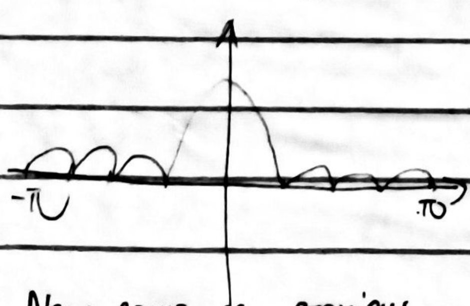
$N = 8$ so $w = \frac{2\pi k}{8} = \frac{\pi k}{4}$

$|X[k]| = 4\left|\frac{\sin(4 \cdot \frac{\pi k}{4})}{\sin(\frac{\pi k}{8})}\right| = 4\left|\frac{\sin(\pi k)}{\sin(\frac{\pi k}{8})}\right|$

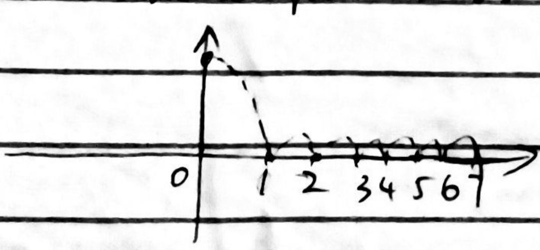


All zero point with 0 at a max value

c) $G(e^{jw}) = 3 \cdot \frac{1-e^{-j8w}}{1-e^{-jw}} \Rightarrow |G(e^{jw})| = 6\left|\frac{\sin(4w)}{\sin(\frac{w}{2})}\right|$



d) Now same as previous, d) is just resale of b



e) $Y(e^{jw}) = X(e^{jw})\, G(e^{jw})$

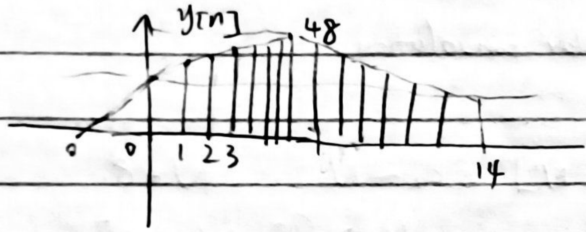
$$X(e^{jw}) = 4\frac{\sin(4w)}{\sin(\frac{w}{2})} e^{-j\frac{7}{2}w}$$

$$G(e^{jw}) = 6\frac{\sin(4w)}{\sin(\frac{w}{2})} e^{-j\frac{7}{2}w}$$

$$Y(e^{jw}) = 24\frac{\sin^2(4w)}{\sin^2(\frac{w}{2})} e^{-j7w}$$

$y[n] = x[n] * g[n]$

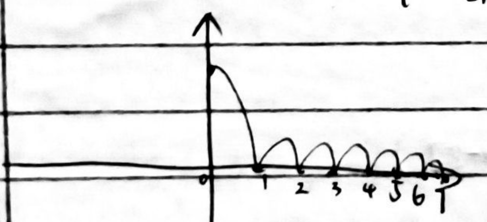
$2(u[n]-u[n-8])$ $\quad$ $g[n]=3(u[n]-u[n-8])$




f) $Q[k] = X[k]\,G[k] \quad k=0,1,\dots,7$

$$|Q[k]| = 24\left|\frac{\sin^2(4w_k)}{\sin^2(\frac{w_k}{2})}\right| \qquad w_k = \frac{\pi k}{4}$$

$$= 24\left|\frac{\sin^2(\pi k)}{\sin^2(\frac{\pi k}{8})}\right|$$




g) $g[n] = 6(u[n]-u[n-8])$ is a rectangular pulse with $A=6$ width 8.

DFT of $g[n] = \sum_{n=0}^{7} g[n] e^{-j\frac{2\pi kn}{8}}$ $\quad n=0,1,2,\dots,7$

$$\sum_{n=0}^{N-1} e^{-j\frac{2\pi kn}{N}} = \begin{cases} N & k=0 \\ 0 & k\neq 0 \end{cases}$$ which we see is the same as above.

$Q[0] = 6\cdot 8 = 48$

h) $Y(e^{jw}) = X(e^{jw})\, G(e^{jw})$

$y[n]$ is a triangular pulse with:

$n=0$ to $n=14$

Peak at 7

$g[n] = 6(u[n] - u[n-8])$ is a rectangular pulse

with amplitude 6 width: 8

so they are not exactly the same.

$y[n]$ is from linear convolution

while $g[n]$ is from circular convolution.

i) $g[n] = \sum_{k=0}^{N-1} x[k]\, g[(n-k) \bmod N]$ $0 \le n < N$ $N=8$

$x[n] = \begin{cases} 2 & 0 \le n < 7 \\ 0 & \text{otherwise} \end{cases}$ $g[n] = \begin{cases} 3 & 0 \le n < 7 \\ 0 & \text{otherwise} \end{cases}$

$g[n] = \sum_{k=0}^{7} x[k]\, g[(n-k) \bmod 8]$

$= \sum_{k=0}^{7} (2)(3) = 6 \cdot 8 = 48$ $0 \le n \le 7$

$g[n] = \begin{cases} 6 & 0 \le n < 7 \\ 0 & \text{otherwise} \end{cases}$

this is same as $6(u[n] - u[n-8])$

11.3 a) Length of $x[n] = 4$

Length of $g = 4$

$$L_y = L_x + L_g - 1 = 7$$

b) So from previous, we know $L_y = 7$

$$g[n] = \sum_{k=0}^{N-1} x[k]\, g[(n-k) \bmod N] \quad 0 \leq n \leq N$$

$N \geq 7$ the periodic nature of the circular convolution will not truncate or overlap contributions of $x[n]$ and $g[n]$ because $y[n]$ already fits entirely within $N=7$.

$N=7$ the length matches exactly so each index of the circular convolution matches the corresponding index of the linear convolution

So $N \geq 7$, then $x[n]\, g[n]$ is exactly equal to their linear convolution

11.4. a) $g[n] = \sum_{m=-\infty}^{\infty} P[n-mN]$ = IDTFT of sampled $P(e^{j\omega})$

Inverse DTFT, the DTFT is at intervals $\frac{2\pi}{N}$ so

$$g[n] = \sum_{m=-\infty}^{\infty} P[n-mN]$$

The signal $g[n]$ repeats every $N$ samples because sampling $P(e^{j\omega})$ at $\frac{2\pi}{N}$ creates a frequency spectrum that repeats every $2\pi$.

The periodic in the frequency domain translates to periodicity in the time domain with a period of $N$.

Thus the smallest guaranteed period of $g[n]$ is $N$.

b) $q[n] = \sum_{m=-\infty}^{\infty} p[n-mN]$

$r[n] = q[n] = \quad 0 \le n < N$

$r[n] = \sum_{m=-\infty}^{\infty} p[n-mN] \quad 0 \le n < N$

Now for $0 \le n < N$ only $m=0$ contributes to $r[n]$ because $m \ne 0$, $n-mN$ lies outside the support of $p[n]$. Assuming $p[n]$ is defined for $0 \le n < N$

Thus for $0 \le n < N$

$$r[n] = p[n]$$

c) when the DTFT $P(e^{j\omega})$ is sampled at intervals of $\frac{2\pi}{N-1}$, the resulting $q[n]$ becomes the inverse DTFT of these samples. The periodicity introduced by this sample causes $q[n]$ to repeat with a period of $N-1$

$$q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-1)]$$

Sampling DTFT at $\frac{2\pi}{N-1}$ creates periodicity in the frequency domain with a period of $2\pi$. In time-domain this means $N-1$.

So $q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-1)]$

Smallest Guaranteed Period $= N-1$

d) $r[0] = p[0] + p[N-1]$

$r[n] = q[n] \quad 0 \le n < N-1$

$r[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-1)]$

$r[0] = \sum_{m=-\infty}^{\infty} p[-m(N-1)] \qquad r[0] = p[0] + p[N-1]$

for $1 \le n < N-1$

$r[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-1)]$ only $m=0$ contributes, since $p[n]$ is nonzero only for $0 \le n < N$

$$r[n] = p[n] \quad 1 \le n < N-1$$

$r[n]$ matches $p[n]$ for $1 \le n < N-1$

the first sample of $r[n]$ is contaminated as $r[0] = p[0] + p[N-1]$

e) when DTFT $P(e^{j\omega})$ is sampled at intervals of $\frac{2\pi}{N-2}$, the resulting $q[n]$ becomes the inverse DTFT of these samples

$$q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-2)]$$

Sampling the DTFT at $\frac{2\pi}{N-2}$ creates periodicity in the frequency domain with a period of $2\pi$.

This periodicity in the frequency domain translates to a time domain period of $N-2$. So smallest guaranteed period of $q[n]$ is $N-2$.

f) $r[0] = p[0] + p[N-2]$ $\quad r[1] = p[1] + p[N-1]$

$$r[n] = q[n] \qquad 0 \le n < N-2$$

$$r[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-2)]$$

$n=0$ : $r[0] = p[0] + p[N-2]$

$n=1$ : $r[1] = \sum_{m=-\infty}^{\infty} p[1-m(N-2)]$

but only $m=0$ and $m=1$ contribute

So $r[1] = p[1] + p[N-1]$

$2 \le n < N-2$ only $m=0$ contributes as $p[n]$ is nonzero for $0 \le n < N$

$$r[n] = p[n] \qquad 2 \le n < N-2$$

Thus $r[n]$ is the same as the first $N-2$ points of $p[n]$ except that the first two samples are contaminated.

g) when DTFT $P(e^{jw})$ is sampled at intervals of $\frac{2\pi}{M}$, the resulting time-domain signal $q[n]$ is

$$q[n] = \sum_{m=-\infty}^{\infty} p[n-mM]$$

Let $r[n] = q[n] \quad 0 \leq n < M$

$$r[n] = \sum_{m=-\infty}^{\infty} p[n-mM] \qquad 0 \leq n < M$$

For $0 \leq n < N-M$: Contamination occurs because $n-mM$ can access indices of $p[n]$ outside its original range $0 \leq n < N$.

$$r[n] = p[n] + \sum_{m=1}^{\frac{N}{M}} p[n+mM]$$

$N-M \leq n < M$: There is no contamination because $n-mM$ stays within the range $0 \leq n < N$

Contamination rule: The first $N-M$ samples of $r[n]$ are contaminated by contributions from wrapped-around samples of $p[n]$.

DFT properties: The $M$-point DFT of $r[n]$ equals the first period of the sampled DTFT $P(e^{jw})$

h) The linear convolution of two signals $x[n]$ $g[n]$ has length

$$N \sim L_y = L_x + L_g - 1$$

$$q[n] = \sum_{k=0}^{N-1} x[k] [g(n-k) \bmod M]$$

$q[n]$ is periodic with a period $M$. Overlaps occur when $M < N$

① $M > N$ $\quad q[n] = y[n] \quad 0 \leq n < N \quad$ $y[n]$ is the linear convolution

② $M < N$ $\quad q[n]$ = first $M$ points of $y[n]$

① is because $M \geq N$, the the circular convolution includes all $N$ points of the linear convolution without any overlap. $q[n]$ wraps around due to the periodicity of the circular convolution, but this does not affect the linear convolution since $N$ points fit entirely within $M$.

② If $M<N$, the circular convolution result $q[n]$ contains only $M$ points of the linear convolution. However, due to the periodic nature of circular convolution,

The contributions indices beyond $M$ wrap around and contaminate the first $M$ points of $q[n]$

The first $N-M$ points of $q[n]$ are contaminated by contributions from the wrapped-around portion of $y[n]$