EC516 HW1 Solutions

Problem 1.1 (a) magnitude:

$$\sqrt{1^2+1^2}=\sqrt{2}$$

phase:

$$\arctan\left(\frac{1}{1}\right)=\frac{\pi}{4}$$

(b) magnitude: (same as (a))

$$\sqrt{2}$$

phase: (conjugation results in opposite phase)

$$-\frac{\pi}{4}$$

(c) magnitude:

$$\sqrt{0.5^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{1/4+3/4}=1$$

phase:

$$\arctan\left(\sqrt{3}\right)=\frac{\pi}{3}$$

(d) magnitude: (same as (c))

$$1$$

phase: (conjugation results in opposite phase)

$$-\frac{\pi}{3}$$

(e) magnitude:

$$\sqrt{2^2+0^2}=2$$

phase: (since it's a negative number)

$$\pi$$

Problem 1.2 A)

$$\begin{aligned}
0.5e^{j\omega t}+0.5e^{-j\omega t} &= 0.5\left(\cos(\omega t)+j\sin(\omega t)\right)+0.5\left(\cos(-\omega t)+j\sin(-\omega t)\right)\\
&= 0.5\left(\cos(\omega t)+j\sin(\omega t)\right)+0.5\left(\cos(\omega t)-j\sin(\omega t)\right)\\
&= \cos(\omega t)
\end{aligned}$$

B)

$$
\begin{aligned}
x(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\pi\delta(\omega-400\pi)+\pi\delta(\omega+400\pi)\right)e^{j\omega t}d\omega \\
&= \frac{1}{2}e^{j400\pi t}+\frac{1}{2}e^{-j400\pi t} \\
&= \cos(400\pi t)
\end{aligned}
$$

In order to make $x(t)$ odd, we must shift by $\pi/2$ radians or $1/800$ in t domain.

$$\sin(400\pi t) = \cos(400\pi t - \pi/2) = \cos(400\pi(t - 1/800))$$

C) a) Yes. The function is a linear combination of sinusoids.
b) Check Figure 1.
c) Check Figure 1.
d) Check Figure 1.

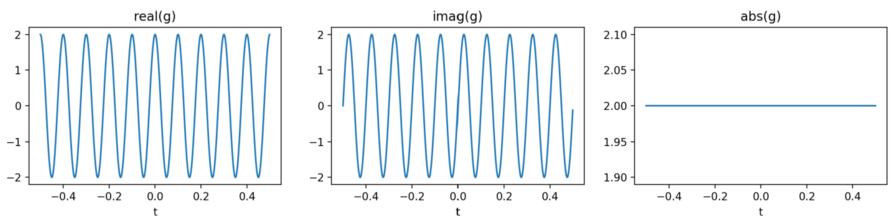

Figure 1: b)-d) real, imaginary, and absolute value of $g(t)$

Problem 1.3 a) From finite sum formula for geometric series give us

$$
\begin{aligned}
T_n &= \sum_{n=0}^{N-1} a^n \\
T_n - aT_n &= \sum_{n=0}^{N-1} a^n - \sum_{n=0}^{N-1} a^{n+1} \\
(1-a)T_n &= 1 - a^N \\
T_n &= \frac{1-a^N}{1-a}
\end{aligned}
$$

The proof is valid as long as $1 - a \neq 0$.

b) Infinite sum formula is derived by assuming that $\lim_{N\to\infty} a^N = 0$.

$$\lim_{N\to\infty}\frac{1-a^N}{1-a} = \frac{1}{1-a}$$

This is true when $|a| < 1$.

Problem 1.4

a) Same hardware can be used to implement various filters and signal processing algorithms. General purpose hardware can be manufactured in mass since hardware design does not need to be application specific thanks to digital signal processing. More flexibility is given to the users.

b) The overhead of A/D and D/A may be too much of a price to pay in some applications to replace a simple analog filter by a simple digital filter. In some other applications, the sampling rate needed for the A/D to avoid aliasing may be so high that it is either impractical or too expensive to build the A/D.