FALL 2021 EC516 PRACTICE TEST 1

Closed Book; Total Time=60 minutes; No collaboration with Anyone; No Electronics (such as cell phones, calculators etc.

Problem 1 (20 Points)

Consider a digital filter ***F*** with impulse response $h[n] = \sum_{k=0}^{9} 2\delta[n-k]$

(a) What are the locations of the <u>poles</u> of the system function, $H(z)$, of the filter ***F***? Justify your answer.

(b) For what frequencies in the range $0 \le \omega \le \pi$, is it guaranteed that the frequency response of filter ***F*** is zero? Justify your answer,

Problem 2 (20 Points)

Consider a filter ***G*** with corresponding frequency response $H(e^{j\omega}) = e^{-j4\omega}\left(\frac{\sin\left(\frac{5\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right)^2$.

(a) If the filter ***G*** is provided the input $x[n] = u[n+3]$, for what values of n is it *guaranteed* that the output signal $y[n]$ is equal to zero? Justify your answer.

(b) Draw a flowgraph for filter G as a cascade of <u>second order sections</u>. Show your work.

Problem 3 (20 Points)

(a) Determine a <u>non-recursive</u> difference equation for a digital FIR filter ***H*** whose magnitude of the frequency response is zero at frequencies $\omega = 0$ and $\omega = \pi$ and is non-zero in the frequency range $0 < \omega < \pi$. Justify your answer.

(b) Determine a <u>recursive</u> difference equation for the digital FIR filter ***H*** in the previous part of this problem. Justify your answer.

Problem 4 (20 Points)

(a) Draw the <u>Pole-Zero plot</u> for an IIR filter ***S*** whose input and output are related by the following difference equation:

$$y[n] = 0.25y[n-4] + x[n] + x[n-2]$$

(b) Would you describe a digital filter ***T*** obtained by applying <u>Bilinear transformation</u> to an analog filter with frequency response $H(j\omega) = 1/(j\omega + 5000)$ as a lowpass filter? Justify your answer.

Problem 5 (10 Points)

Sketch the <u>phase</u> of the DTFT of the signal specified as $g[n] = \delta[n] + \delta[n-4]$ where $\delta[n]$ is the unit impulse. *Justify your answer*

Problem 6 (10 points)

During the <u>discovery process</u> for a digital filtering application, it is reported that the filter is required to have <u>linear phase</u>. What does that imply for the filter <u>design process</u>? Explain your answer.

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Unit Step

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases}$$

Unit Impulse

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Complex Exponentials and Sinusoids

$$e^{j\omega n} = \cos(\omega n) + j\sin(\omega n) \quad \cos(\omega n) = (1/2)(e^{j\omega n} + e^{-j\omega n}) \quad \sin(\omega n) = (1/2j)(e^{j\omega n} - e^{-j\omega n})$$

DT Convolution : $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ **CT Convolution** : $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$

FSF : $\sum_{k=0}^{N-1} \alpha^k = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha} & ;\alpha \neq 1 \\ N & ;\alpha = 1 \end{cases}$ **ISF** : $\sum_{k=0}^{\infty} \alpha^k = \dfrac{1}{1-\alpha} \quad ;|\alpha| < 1$

DTFT: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n};$ **Inverse DTFT**: $x[n] = \dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$

Basic DTFT Properties:

$$x[n-n_0] \Leftrightarrow e^{-j\omega n_0}X(e^{j\omega}) \qquad e^{j\omega_0 n}x[n] \Leftrightarrow X(e^{j(\omega-\omega_0)}) \qquad x^*[n] \Leftrightarrow X^*(e^{-j\omega})$$

$$x[-n] \Leftrightarrow X(e^{-j\omega}) \qquad x[n] * h[n] \Leftrightarrow X(e^{j\omega})H(e^{j\omega})$$

Common DTFT Pairs

$$e^{j\omega_0 n} \Leftrightarrow 2\pi \textstyle\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

$$\delta[n-n_0] \Leftrightarrow e^{-j\omega n_0} \qquad u[n] - u[n-N] \Leftrightarrow \frac{\sin(\omega N/2)}{\sin(\omega/2)}e^{-j\omega(N-1)/2}$$

$$\frac{\sin\omega_0 n}{\pi n} \Leftrightarrow \begin{cases} 1 & 0 \leq |\omega| \leq \omega_0 \\ 0 & \omega_0 < |\omega| \leq \pi \end{cases}$$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Properties of z-transform:

$x[n-n_0] \Leftrightarrow z^{-n_0}X(z)$ $\quad$ $x[-n] \Leftrightarrow X(z^{-1})$ $\quad$ $x^*[n] \Leftrightarrow X^*(z^*)$

$x[n]*h[n] \Leftrightarrow X(z)\times H(z)$

Basic z-transform pair: Decaying Exponential $a^n u[n] \Leftrightarrow \dfrac{1}{1-az^{-1}}$ ROC:$|z| > |a|$

Recursive Difference Equation

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{m=0}^{M} b_m x[n-m]$$

Non-Recursive Difference Equation

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FIR filters

A FIR filter has an impulse response of length *N* such that $h[n] = 0$ for *n<0* and for *n>N-1*

IIR filters

Impulse response $h[n] = 0$ for *n<0* and system function $H(z)$ is rational and has at least one pole not at the origin.

Linear Phase FIR Filters:

Type I: Odd Length, Symmetric
Type II: Even Length, Symmetric
Type III: Odd Length, Anti-symmetric
Type IV: Even Length, Anti-symmetric

N-Point Signal

$h[n] = 0$ for *n<0* and for *n>N-1*

Bilinear Transformation

$$H_d(z) = H_a\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$