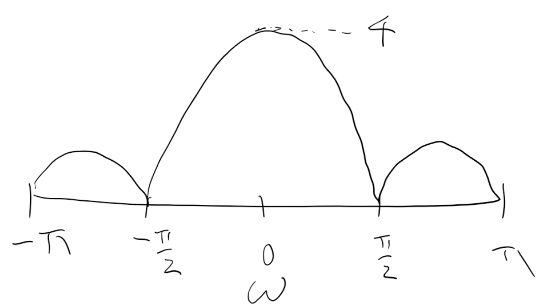
EC516 HW9 Solutions

Problem 9.1

(a)

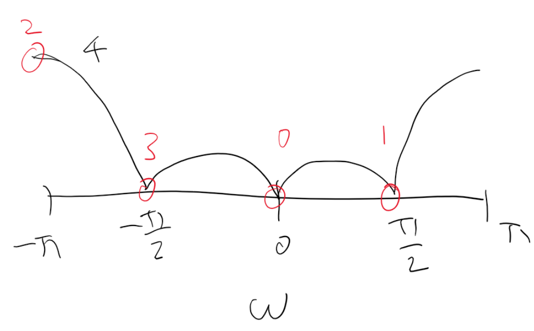
$$X(e^{j\omega}) = e^{-j\frac{3}{2}\omega}\frac{\sin(2\omega)}{\sin(\omega/2)}$$



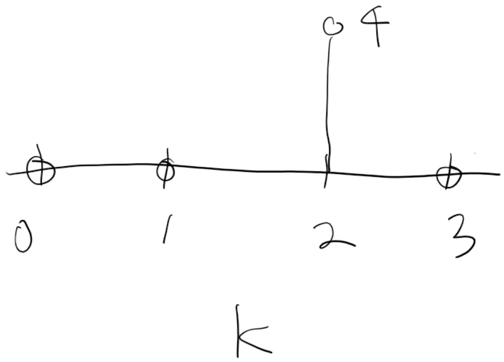
(b) Zero-crossings of $X(e^{j\omega})$ in $-\pi \leq \omega < \pi$ exist at $-\frac{\pi}{2}$, $-\pi$, $\frac{\pi}{2}$. These correspond to $k = 1$, $k = 2$, and $k = 3$.

(c) With the same logic, the zero crossings corresponds to $k = 2$, $k = 4$, and $k = 6$.

(d) $(-1)^n = e^{j\pi n}$. Multiplying $x[n]$ with $e^{j\pi n}$ will result in π frequency shift in the DTFT, as shown below.



$k = 0, 1, 2, 3$ of the 4-point DFT corresponds to DTFT at $\omega = 0, \frac{\pi}{2}, -\pi, -\frac{\pi}{2}$. Hence the 4-point DFT will look like below.



Problem 9.2

A(a) $Q[k]_{256} = Q\left(e^{j\frac{2\pi k}{256}}\right)$. Also $Q\left(e^{-j\frac{\pi}{2}}\right) = Q\left(e^{j\frac{3\pi}{2}}\right)$. Hence, $Q[k_0]_{128}$ becomes $Q\left(e^{-j\frac{\pi}{2}}\right)$ at $k_0 = 192$.

A(b) $R(e^{j\omega}) = e^{j16\omega}Q(e^{j\omega})$. Therefore, $R(e^{j\omega})$ and $Q(e^{j\omega})$ have the same magnitude.

Assuming that $q[n] = 0$ for $n < 0$ and $n \geq 128$, then $r[n] = 0$ for $n < 16$ and $n \geq 144$. Since shifting does not cause the signal to go out of bounds of the DFT ($n < 0$ or $n \geq 256$), both DFT will have the same magnitude.