FALL 24 EC516 Problem Set 06

Due: Sunday October 20 (Before 11:59pm)

You must submit your homework attempt on Blackboard Learn. For this purpose, you must convert your homework attempt to a pdf file and upload it at the corresponding homework assignment on Blackboard Learn.

Problem 6.1 (You may use a calculator or a computer to help with calculations)

Let us consider designing an IIR digital lowpass filter with $\delta_p = \delta_s = 0.25$, $\omega_p = \pi/2$, and $\omega_s = 3\pi/4$ by using Bilinear transformation on designed analog Butterworth filter. In this design technique, we first determine $H_a(s)$, the system function of an analog lowpass filter whose magnitude of the frequency response satisfies $\delta_p = \delta_s = 0.25$, $\omega_p = \tan\left(\frac{\pi}{4}\right)$, and $\omega_s = \tan\left(\frac{3\pi}{8}\right)$. Note that determining $H_a(s)$ means that we have determined the numerical values of ω_c and N in $H_a(s)H_a(-s) = 1/\left(1 + (\frac{s}{j\omega_c})^{2N}\right)$. Once we have $H_a(s)$, we can apply Bilinear Transformation to it to obtain $H_d(z) = H_a\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$. From $H_d(z)$ we can finally obtain the difference equation for the desired digital filter specifications. From lecture, we also recall that an analog Butterworth filter has a monotonically decreasing magnitude of the frequency response:

$$|H_a(j\omega)|^{-2} = \left(1 + \left(\frac{\omega}{\omega_c}\right)^{2N}\right).$$

(a) At its passband frequency, $\tan\left(\frac{\pi}{4}\right)$, we can require $|H_a(j\omega)|$ to be equal to $(1 - \delta_p)$. Show that this is equivalent to requiring:

$$\left(\frac{\tan\left(\frac{\pi}{4}\right)}{\omega_c}\right)^{2N} = \frac{1}{(1-\delta_p)^2} - 1$$

(b) At its stopband frequency, $\tan\left(\frac{3\pi}{8}\right)$, we can require $|H_a(j\omega)|$ to be equal to $(1 - \delta_p)$. Show that this is equivalent to requiring:

$$\left(\frac{\tan\left(\frac{3\pi}{8}\right)}{\omega_c}\right)^{2N} = \frac{1}{{\delta_s}^2} - 1$$

(c) Take the ratio of the two equations from the previous two parts and you should find that the ω_c terms cancel each other out in the equation resulting from the ratio. You can now solve the resulting equation for *N*. Round the value of *N* *up* to the nearest integer value.

(d) Use the equation from part (a) with the rounded-up value of *N* from part (c) to solve for ω_c.

(e) Determine the *N* poles of $H_a(s)$ by solving the equation: $1 + \left(\frac{s}{j\omega_c}\right)^{2N} = 0$ and imposing the restriction that only the poles with the negative real parts belong to $H_a(s)$.

(f) Determine the difference equation for the designed digital filter.

<u>Problem 6.2</u> (You may use a calculator or a computer to help with calculations)

Draw a Cascade Form flowgraph for the digital filter you designed in Problem 6.1.

<u>Problem 6.3</u> (You may use a calculator or a computer to help with calculations)

Repeat Problem 6.1 but this time use $\delta_p = \delta_s = 0.1$, $\omega_p = \pi/2$, and $\omega_s = 3\pi/4$ for the specifications on the magnitude of the frequency response of the lowpass digital IIR filter.

<u>Problem 6.4</u>

Consider a FIR filter with impulse response

$$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 3\delta[n-4] + 2\delta[n-5] + \delta[n-6]$$

(a) Draw a flowgraph for this FIR filter that requires only 2 multiplications per output sample.
(b) Draw an approximate sketch of the *magnitude* of the frequency response of this filter (HINT: This impulse response of this filter is equal to the convolution of a box with a box).
(c) Draw a sketch of the *phase* of the frequency response of this filter.