EC516 HW6

6.1 $w_p = \frac{\pi}{2}$, $W_s = \frac{3}{4}\pi$, $\delta p = \delta s = 0.25$

$$\cancel{W_c = \frac{\tan\left(\frac{w_p}{2}\right)}{\tan\left(\frac{w_s}{2}\right)} = \frac{\tan\left(\frac{\pi}{4}\right)}{\tan\left(\frac{3}{8}\pi\right)}}$$

a) $$|Ha(jW)|^2 = \frac{1}{1+\left(\frac{W}{Wc}\right)^{2N}}$$

$|Ha(j\tan(\frac{\pi}{4}))| = 1 - \delta p$ as we set,

then

with equation we know:

$$||Ha(j\tan(\frac{\pi}{4}))|^{-2} = 1 + \left(\frac{W}{Wc}\right)^{2N}$$

$W = \tan\frac{\pi}{4}$ $$|Ha(j\tan(\frac{\pi}{4}))| = \frac{1}{\sqrt{1+\left(\frac{W}{Wc}\right)^{2N}}} = 1 - \delta p$$

$$1 + \left(\frac{W}{Wc}\right)^{2N} = \frac{1}{(1-\delta p)^2}$$

$$\left(\frac{W}{Wc}\right)^{2N} = \frac{1}{(1-\delta p)^2} - 1$$ So this is it.

b) We set $$|Ha(j\tan(\frac{3}{8}\pi))| = \delta s = \frac{1}{\sqrt{1+\left(\frac{\tan(\frac{3}{8}\pi)}{Wc}\right)^{2N}}}$$

$$\frac{1}{\delta s^2} = 1 + \left(\frac{\tan(\frac{3}{8}\pi)}{We}\right)^{2N}$$

So $$\left(\frac{\tan(\frac{3}{8}\pi)}{Wc}\right)^{2N} = \frac{1}{\delta s^2} - 1$$

c) $$\frac{\left(\frac{\tan(\frac{3}{8}\pi)}{Wc}\right)^{2N}}{\left(\frac{\tan(\frac{\pi}{4})}{Wc}\right)^{2N}} = \frac{\frac{1}{\delta s^2} - 1}{\frac{1}{(1-\delta p)^2} - 1}$$

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$$\left(\frac{\tan\left(\frac{3}{8}\pi\right)}{\tan\left(\frac{\pi}{4}\right)}\right)^{2N} = \frac{\frac{1}{\delta_s^2}-1}{\frac{1}{(1-\delta_p)^2}-1}$$ solved $N \approx 1.68$

and we take $N=2$.

d) $$\left(\frac{\tan\left(\frac{\pi}{4}\right)}{w_c}\right)^4 = \frac{1}{(1-0.25)^2}-1$$

$$w_c \approx 1.065$$

e) $$1+\left(\frac{s}{jw_c}\right)^{2N} = 0$$

$$\left(\frac{s}{(jw_c)}\right)^{2N} = -1$$

$$\frac{s}{jw_c} = e^{j\frac{(2k+1)\pi}{2N}} \qquad k = 0, 1, 2, \cdots, 2N-1$$

$$s = jw_c e^{j\frac{(2k+1)\pi}{2N}} \qquad N=2$$

$$s_1 = -0.753 + 0.753j$$

$$s_2 = -0.753 - 0.753j$$

f) $$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \qquad H_d(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1+a_1 z^{-1} + a_2 z^{-2}}$$

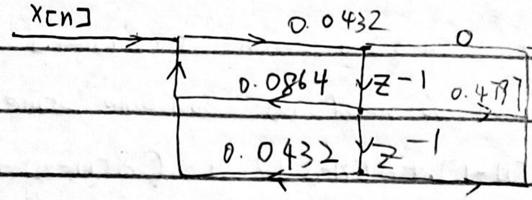
Plug in we get $b_0 = 0.0432$ $a_1 = -1.2839$

$b_1 = 0.0864$ $a_2 = 0.4797$

$b_2 = 0.0432$

$$y[n] = 0.0432 \cdot x[n] + 0.0864\,x[n-1] + 0.0432\,x[n-2] - 1.2839\,y[n-1] + 0.4797 \cdot y[n-2]$$

6.2



6.3

a) $\left|H_a(j\tan(\frac{\pi}{2}))\right| = 1-\delta_p = \frac{1}{\sqrt{1+\left(\frac{\tan(\frac{\pi}{2})}{w_c}\right)^{2N}}}$

$$1+\left(\frac{\tan(\frac{\pi}{2})}{w_c}\right)^{2N} = \frac{1}{(1-\delta_p)^2}$$

$$\left(\frac{\tan(\frac{\pi}{2})}{w_c}\right)^{2N} = \frac{1}{(1-\delta_p)^2} - 1$$

b) this is exactly the same as 6.1 as it only involves only w_c's value to prove

c) $\left(\frac{\tan(\frac{3}{8}\pi)}{\tan(\frac{3}{4}\pi)}\right)^{2N} = \frac{\frac{1}{\delta_s^2}-1}{\frac{1}{(1-\delta_p)^2}-1}$ $N \approx 4$

d) $1+\left(\frac{s}{jw_c}\right)^{2N} = -1$ $s = jw_c e^{j\frac{(2k+1)\pi}{2N}}$

$\Downarrow$

$s_1 = -0.408 + 0.984j$

$s_2 = -0.984 + 0.408j$

$s_3 = -0.984 - 0.408j$

$s_4 = -0.408 - 0.984j$

f) $H_d(z) = \frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_Nz^{-N}}{1+a_1z^{-1}+a_2z^{-2}+\cdots+a_Nz^{-N}}$ ⇐ plug in bilinear equation to $|H_a|$ to get.

we have four coefficient.

$b_0 = 0.00196$ $b_1 = 0.00784$ $b_2 = 0.01177$ $b_3 = 0.00784$ $b_4 = 0.00196$

$a_1 = -2.6468$ $a_2 = 2.7840$ $a_3 = -1.3488$ $a_4 = 0.2520$

$y[n] = 0.00196\,x[n] + 0.00784\,x[n-1] + 0.01177\,x[n-2] + 0.00784\,x[n-3]$
$+ 0.00196\,x[n-4] - 2.6468\,y[n-1] + 2.7840\,y[n-2] - 1.3488\,y[n-3] + 0.2520\,y[n-4]$

6.4 $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 2\delta[n-5] + \delta[n-6] + 3\delta[n-4]$

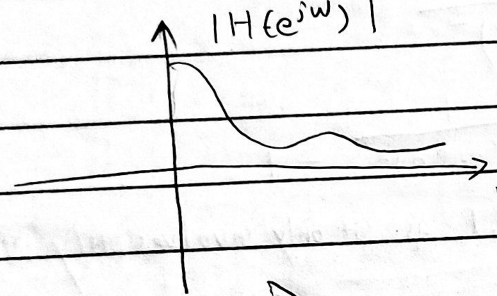
a) we want only 2 multiplication, then we need to combine some.

$$h[n] = \delta[n] + \delta[n-6] + 2(\delta[n-1]+\delta[n-5]) + 3(\delta[n-2]+\delta[n-4]) + 4\delta[n-3]$$

$$= \delta[n] + \delta[n-6] + 2(\delta[n-1]+\delta[n-5]) + 3(\delta[n-2]+\delta[n-4]) + 4\delta[n-3]$$

3 multiplication.

b)



c)