Problem Set III
EC/MS574, Fall 2024
Assigned November 19, 2024

– Please complete and return the solution of all problems

- Problem 1
  The direct lattice primitive vectors of an hexagonal crystal are given by:

$$\vec{a}_1 = a\,\widehat{x}$$
$$\vec{a}_2 = -\frac{1}{2}\,a\,\widehat{x} + \frac{\sqrt{3}}{2}\,a\,\widehat{y}$$
$$\vec{a}_3 = c\,\widehat{z}$$

  Where a and c are the lattice constants. Compute:

  - the volume of the primitive cell.
  - The basis vector of the reciprocal lattice.
  - The volume of the primitive cell of the reciprocal lattice.
  - The product of the two volumes.

- Problem 2

  Calculate the electronic dispersion relation $E_n(k)$ for a face-centered, body centered, and simple cubic crystal using the tight-binding model. Use the result to derive an expression for the effective mass m^*.

- Problem 3

  Calculate the electronic dispersion relation $E_n(k)$ for a two-dimensional exagonal lattice of lattice constant a using the tight-binding model. Use the result to derive an expression for the effective mass m^*.

- Problem 4
  Consider an hypothetical semiconductor held at a temperature of 300K. The conduction band energy as a function of the wavevector is given by:

$$\mathrm{E}_c(k) = \frac{40\,\hbar^2}{a^2\,m_o}\left[1 - cos\left(\frac{k\,a}{2}\right)\right]$$

  where a is the lattice constant, m_o the electron rest mass and $-\frac{2\,\pi}{a} \leq \mathrm{k} \leq \frac{2\,\pi}{a}$.

  - Compute the value of the electron effective mass in unit of m_o at the center of the Brillouin zone.
  - Compute the carrier group velocity at the zone edge.

- Problem 5
  Consider an hypothetical semiconductor where the conduction band is anysotrpic with masses m_x, m_y, m_x along the principal axes x, y, and z.
  - Compute and expression for the density of states for this band.