Problem Set I
EC574A1, Fall 2024
Assigned September 9, 2024
Due October 17, 2024

The solution of the assigned problem set is **mandatory** and it is your responsibility. If you do not work on the homework you will not be able to solve the exam problems. It is strongly suggested that you start solving the homework sets immediately without waiting the day before the exam. You are required to turn in the solution of Problem Set I the day of the exam.

- Problem 1
  Consider the classical harmonic oscillator discussed in class. Derive the expression for the classical probability density:

$$P_{cl}(x) = \frac{\sqrt{m\omega_0^2}}{\pi} \frac{1}{\sqrt{2E - m\omega_0^2 x^2}}$$

- Problem 2
  The ionization energy of the Hydrogen atom ground level is E=13.6eV. Compute the frequency, wavelength, and wavenumber of the electromagnetic radiation that will ionize the atom.

- Problem 3
  Assume an electron is confines in a 1D box of dimension Δx = 10nm, estimate the uncertainty to which the x component of the momentum can be measured. What is the unit of measurement for the momentum?

- Problem 4
  Consider a set of CON wavefunctions $\{\psi_n\}$:

$$\psi_n(x) = \begin{cases} \sqrt{\frac{1}{a}}\, sin(\frac{n\pi x}{a}) & |x| \leq a \\ 0 & |x| \geq a \end{cases}$$

  Show that are normalized and they are orthogonal.

- Problem 5
  Two wavefunctions ψ_1 and ψ_2 are normalized eigenfunctions corresponding to the same eigenvalue. We know that

$$\int \psi_1^* \psi_2 \, dx = d$$

why you could find things like this?

  where d is a real number. Find normalized linear combinations of ψ_1 and ψ_2, ϕ=$c_1\psi_1$+$c_2\psi_2$ (assume c_1 and c_2 real number) that are orthogonal to a) ψ_1 and b) ψ_1+ψ_2

- Problem 6
We have seen that for a quantum mechanical system the probability current density is given by:
$$\vec{\mathrm{J}} = \frac{\hbar}{2\,m\,\imath}\,[\,\Psi^*(\vec{r},t)\,\nabla\,\Psi(\vec{r},t)\;-\;\Psi(\vec{r},t)\,\nabla\,\Psi^*(\vec{r},t)\,]$$
Consider a free particle (we computed the wavefunction in class) and show that we can write the probability current density as:
$$\vec{\mathrm{J}} = velocity * particle\,density$$

- Problem 7
The following momentum space wavefunction is given:
$$\phi(p_x) = C\,e^{\left[-\frac{(p_x-p_0)^2}{2(\Delta p_x)^2}\right]}$$
where p_0 and Δp_x are constants.

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- Compute the normalization constant C.

- Compute the conjugate real space wavefunction $\psi(x)$.

- Evaluate the product of the indetermination of the position and momentum coordinates.

- Problem 8
Solve Problem 2.1 page 76 of the textbook.

- Problem 9
Consider a 10Å wide and 2eV high potential barrier. An electron with energy 1.5eV impinges on the barrier. What are the transmission and reflection probabilities?

- Problem 10
Consider a triangular quantum well where $V(x) = \infty$ for $x \leq 0$ and $V(x) = qEx$ for $x > 0$. Find the solution of the Schroedinger equation by determining the eigenvalues and eigenfunctions.

- Problem 11

Consider a system characterized by a time independent potential $U(x)$. We know the solution of the Schroedinger equation for this system and $\psi_m(x,t)$ and E_n are known. It is also know that the system is in a state in which the first two eigenfunctions have the same probability.

– Write the expression for the time dependent state function $\Psi(x,t)$ when the first two eigenfunction have the same probability

– Compute the average energy $< E >$ corresponding to $\Psi(x,t)$;

– Compute the energy uncertainty $< \Delta E >$ corresponding to $\Psi(x,t)$;

– Using the expression of $\Psi(x,t)$, determine the average position $< x(t) >$ of a particle. Assume that:

$$x_0 = \frac{1}{2}\int_{-\infty}^{+\infty} x[\psi_1^2(x) + \psi_2^2(x)]dx$$

and

$$a = \int_{-\infty}^{+\infty} x\psi_1(x)\psi_2(x)dx$$

- Problem 12

Consider a particle of mass m in a state described by the following state function:

$$\Psi(x,t) = A\, e^{-a[\frac{mx^2}{\hbar} + it]} \tag{1}$$

where A and a are real constant.

$-1-$ Compute A.

$-2-$ Determine for what potential $V(x)$ the state function $\Psi(x,t)$ satisfies the Schroedinger equation.

$-3-$ Calculate the expectation values $< x >$ and $< x^2 >$.

$-4-$ Calculate the expectation values of $< p >$ and $< p^2 >$.

- Problem 13

Consider a particle of mass m that at time $t = 0$ is represented by the following wavefunction:

$$\psi(x,0) = \begin{cases} \psi(x,0) = A\,(a^2 - x^2) & -a \leq x \leq a \\ \psi(x,0) = 0 \quad |x| > a & \end{cases}$$

where A and a are real constant.

$-1-$ Compute A.

$-2-$ Calculate the expectation values $< x >$ and $< x^2 >$.

$-3-$ Calculate the expectation values of $< p >$ and $< p^2 >$.

$-4-$ Calculate the uncertainty for x (σ_x) and p (σ_p) and check the indetermination principle.

- Problem 14

You have a harmonic oscillator with $\alpha = (mk/\hbar^2)^{1/4}$, where k is the oscillator spring constant and $\omega = (k/m)^{1/2}$ the corresponding frequency. The eigenfunctions solution of the Schroedinger equation are given by:

$$\psi_n(x) = \left(\frac{\alpha}{\sqrt{\pi}\, 2^n\, n!} \right)^{1/2} e^{-\dfrac{\alpha^2 x^2}{2}} H_n(\alpha\, x)$$

and the corresponding generating function of the Hermite polynomial is:

$$H_n(\xi) = (-1)^n\, e^{\xi^2} \frac{d^n\, e^{-\xi^2}}{d\xi^n}$$

– Determine the eigenfunction for the ground state.

– Compte Δx and Δp_x for the ground state.

– Compute the product $\Delta x \Delta p$ for the ground state. What is the meaning of the result?

- Problem 15

Consider again the expression for the equation of motion of an operator $\hat{Q}$ that represents the physical observable Q.

$$\frac{d}{dt} < Q >= \frac{i}{\hbar} < [H, \hat{Q}] > + \left\langle \frac{\partial}{\partial t} \hat{Q} \right\rangle$$

– For each of the following cases: (1) $\hat{Q} = 1$, (2) $\hat{Q} = H$, (3) $\hat{Q} = x$ and (4) $\hat{Q} = p$ determine the values of $d/dt\ < Q >$ and provide a physical interpretation.

– What is the condition H, x, and p have to satisfy?

- Problem 16

Consider the harmonic oscillator and the raising a_+ and lowering a_- operators.

– Using a_+ and a_- compute the expectation values of the potential energy $< V(x) >$ for a state of index n.

– Using a_+ and a_- compute the expectation values of the kinetic energy $< T(x) >$ for a state of index n.

– What does the sum $< T(x) > + < V(x) >$ correspond to and Why?.

- Problem 17

Consider the harmonic oscillator,

– Compute the expectation values $< x >$, $< p_x >$, $< x^2 >$, and $< p_x^2 >$ using explicit integration for the states $\psi_0(x)$ and $\psi_1(x)$.

– Compute the expectation values of the kinetic $< T(x) >$ and potential $< V(x) >$ energy for $\psi_0(x)$ and $\psi_1(x)$.

– Check that the sum $< T(x) > + < V(x) >$ provide the ssame results of Problem 16.

- Problem 18

  Consider the harmonic oscillator and the raising a_+ and lowering a_- operators.

  – Starting from the ground state $\psi_0(x)$ construct $\psi_1(x)$ and $\psi_2(x)$ using a_+ and a_-.

  – Check the orthogonality of $\psi_0(x)$, $\psi_1(x)$, and $\psi_2(x)$ using the explicit integration.

- Problem 19

  Consider the harmonic oscillator and the raising a_+ and lowering a_- operators.

  – Using a_+ and a_- compute the expectation values $< x >$, $< p_x >$, $< x^2 >$, and $< p_x^2 >$ for the n-th state.

  – Check the uncertainty pricinple for these states.