EC 516 HW 1

1.1

a) $1+j$

Magnitude

$$|1+j| = \sqrt{1^2+1^2} = \sqrt{2}$$

Angle : $\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

b) $(1+j)^*$

$= 1-j$

Magnitude : $\sqrt{1^2+(-1)^2} = \sqrt{2}$

Angle : $\theta = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

c) $0.5 + \frac{\sqrt{3}}{2} j$

Magnitude : $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= 1$

Angle : $\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$

$= \frac{\pi}{3}$

d) $0.5 - \frac{\sqrt{3}}{2}j$

Magnitude : $\sqrt{(\frac{1}{2})^2 + (-\frac{\sqrt{3}}{2})^2}$

$= 1$

Angle : $\tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$

$= \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

e) -2.0

Magnitude : $\sqrt{(-2)^2} = 2$

Angle : $\theta = \tan^{-1}\left(\frac{0}{-2}\right) = \tan^{-1}(0)$

$= \pi$

While these are first hand answer, they holds true by $+2n\pi$ (n is integer).

1.2

A)

Since $e^{jwt} = \cos(wt) + j\sin(wt)$

we would know:

$$e^{jwt} = \cos(wt) + j\sin(wt)$$

$$e^{-jwt} = \cos(-wt) + j\sin(-wt)$$

$$= \cos(wt) - j\sin(wt)$$

$$e^{jwt} + e^{-jwt}$$

$$= (\cos(wt) + j\sin(wt)) + (\cos(wt) - j\sin(wt))$$

$$= 2\cos(wt) \quad \text{— ①}$$

So, divide by 2 on both sides in ①

we get

$$\cos(wt) = 0.5(e^{-jwt} + e^{jwt})$$

$$= 0.5e^{jwt} + 0.5e^{-jwt}.$$

B) Given that $X(j\omega) = \pi\delta(\omega - 400\pi) + \pi\delta(\omega + 400\pi)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[(\pi\delta(\omega - 400\pi) + \pi\delta(\omega + 400\pi))\right] e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left(\pi e^{j400\pi t} + \pi e^{-j400\pi t}\right)$$

$$= \cos(400\pi t)$$

Thus, $x(t) = \cos(400\pi t)$

Now, to make a cos function ODD, we just have to shift $\frac{1}{2}$ cycle, which is

$$\frac{\pi}{2\omega} = \frac{\pi}{2 \cdot 400\pi} = \frac{1}{800}\ s$$

c) $g(t) = 2e^{j20\pi t}$

a)

Yes, It is periodic.

$g(t+T) = g(t)$ if One is periodic,

$g(t+T) = 2e^{j(4\pi t + 4\pi T + \frac{\pi}{4})}$

for a $e^{j\omega t}$, usually adding 2π will make it periodic (same)

So Let's try $T = \frac{n}{2}$ (n is integer)

where we found it does work!

So this is a periodic function with $T = \frac{1}{2}$.

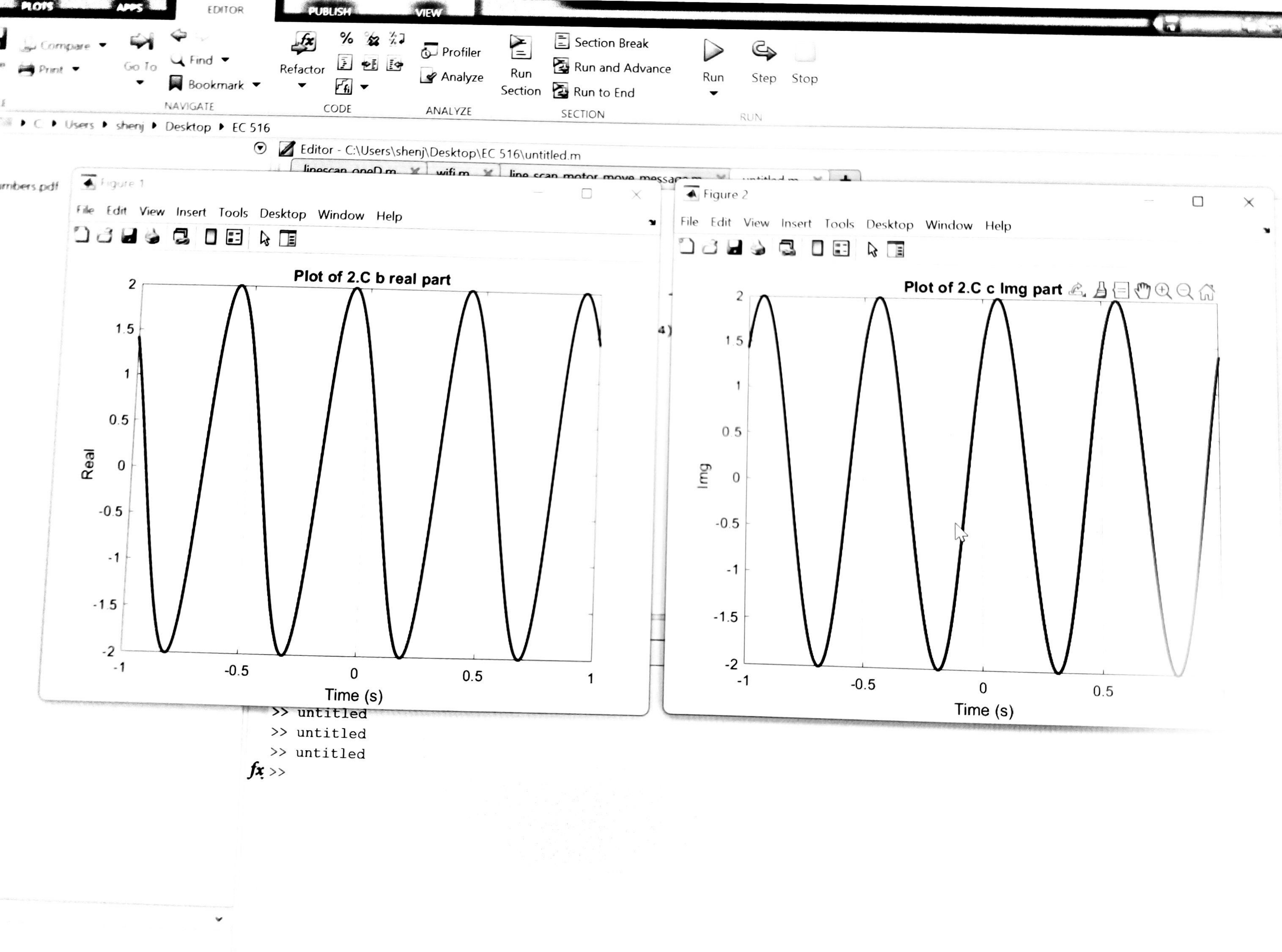
b)

$\mathrm{Re}\{g(t)\} = 2\cos(4\pi t + \frac{\pi}{4})$

See Matlab plot

c) $\mathrm{Im}\{g(t)\} = 2 \cdot \sin(4\pi t + \frac{\pi}{4})$

See Matlab plot



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SECTION
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Step
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Editor - C:\Users\shenj\Desktop\EC 516\untitled.m
Figure 1
File Edit View Insert Tools Desktop Window Help
Plot of 2.C b real part
Real
Time (s)
Figure 2
File Edit View Insert Tools Desktop Window Help
Plot of 2.C c Img part
Img
Time (s)
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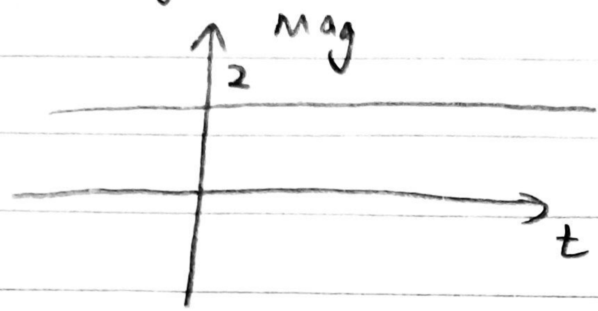
d) $|g(t)| = 2$



1.3

a) $\sum_{n=0}^{N-1} \alpha^n = S_N$

So sum of finite geometric series:

$S_N = 1 + \alpha + \alpha^2 + \cdots + \alpha^{N-1}$ by Σ definition

Multiply both side by $1-\alpha$

$$(1-\alpha) S_N = (1-\alpha)(1+\alpha+\alpha^2+\cdots+\alpha^{N-1})$$

$$\begin{aligned}(1-\alpha) S_N &= (1+\alpha+\alpha^2+\cdots+\alpha^{N-1}) \\ &\quad - (\alpha+\alpha^2+\cdots+\alpha^N) \\ &= 1-\alpha^N\end{aligned}$$

thus $S_N = \dfrac{1-\alpha^N}{1-\alpha}$

b) $S_\infty = \sum_{n=0}^{\infty} \alpha^n$

In order for it to converge, we first would exclude $|\alpha| \geq 1$, these will just go larger and larger and just be ∞ Now for $=1$, even -1, we could not determine value, so we exclude this

Now Let's see if for all $|\alpha| < 1$

$$S_\infty = 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \cdots$$

$$S_\infty \cdot (1-\alpha)$$

$$= (1 + \alpha + \alpha^2 + \alpha^3 + \cdots)$$

$$- (\alpha + \alpha^2 + \alpha^3 + \cdots)$$

Since below are infinite, getting smaller smaller, we just get 1 (Last term more is infinitely closer to 0)

$$S_\infty (1-\alpha) = 1$$

thus $S_\infty = \frac{1}{1-\alpha}$

1.4.

a) first , according to what we are taught in class, DSP is cheaper It can be massively produced and applied to multiple devices. It also allows devices to pass signals to each other as a second language for them. More over, it allows the data to be saved in memory and software refine.

b) While DS is better than AS in price and usage, it gives up precision. Also, when you convert real world AS to DS, you will likely has latency in converting it.