EC516 HW3

3.1

a) $x[n-n_0]$

$x[n] \overset{ZT}{\Longleftrightarrow} \underline{X}(z)$

$\underline{Z}[x[n]] = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ So $\underline{Z}[x[n-n_0]] = \sum_{n=-\infty}^{\infty} x(n-n_0) z^{-n}$

Substituting $(n-n_0)=m$ in above

$\underline{Z}[x[n-n_0]] = \sum_{n=-\infty}^{\infty} x(m) z^{-(m+n_0)}$

$\Rightarrow \underline{Z}[x[n-n_0]] = z^{-n_0} \sum_{m=-\infty}^{\infty} x(m) z^{-m}$

$= z^{-n_0} \underline{X}(z)$

b) $x[-n]$

$\underline{Z}[x[n]] = \underline{X}(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

So, $\underline{Z}[x[-n]] = \sum_{n=-\infty}^{\infty} x[-n] z^{-n}$

Let $-n=m$

$\underline{Z}[x[-n]] = \sum_{m=\infty}^{-\infty} x(m) z^{m}$

$\Rightarrow \underline{Z}[x[-n]] = \sum_{m=-\infty}^{\infty} x(m)(z^{-1})^{m} = \underline{X}(z^{-1})$

c) $x^*[n]$

$\underline{Z}[x[n]] = X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

So $\underline{Z}[x^*[n]] = \sum_{n=-\infty}^{\infty} x^*[n] z^{-n}$

$\Rightarrow \underline{Z}[x^*[n]] = \left[\sum_{n=-\infty}^{\infty} x[n] (z^*)^{-n}\right]^*$

$= [\underline{X}(z^*)]^*$

So $\underline{Z}[x^*[n]] = \underline{X}^*(z^*)$

d) $x[n] * h[n]$

$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h(n-k)$

$\underline{Z}[x[n]] = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

So $\underline{Z}[x[n] * h[n]] = \underline{X}(z) = \sum_{n=-\infty}^{\infty} [x[n] * h[n]] z^{-n}$

$\Rightarrow \underline{X}(z) = \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x[k] h[n-k]\right] z^{-n}$

$\Rightarrow \underline{X}(z) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k] h[n-k] z^{-k} z^{-(n-k)}$

$\Rightarrow \underline{X}(z) = \sum_{k=-\infty}^{\infty} x[k] z^{-k} \sum_{n=-\infty}^{\infty} h[n-k] z^{-(n-k)}$

suppose $(n-k)=m$

$\Rightarrow \underline{X}(z) = \sum_{k=-\infty}^{\infty} x[k] z^{-k} \sum_{m=-\infty}^{\infty} h[m] z^{-m}$

$= X_1(z) H(z)$

So this is true.

3.2

a) $x[n] = \delta[n-3]$

delta function $\delta[n-3]$ is 1 when $n=3$ and 0 elsewhere.

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n-3] z^{-n} = z^{-3}$$

ROC is all z, as the delta function is finite duration and causal.

c) $x[n] = (0.25)^n u[n]$ is right-sided geometric sequence.

$$X(z) = \sum_{n=0}^{\infty} (0.25)^n z^{-n} = \frac{1}{1-0.25z^{-1}} = \frac{z}{z-0.25}$$

ROC is $|z| > 0.25$ so this is because $(0.25z^{-1})^n$ must decay as n increase to be convergent.

d) $x[n] = (0.25)^{n-1} u[n-1]$

$$x[n] = \begin{cases} (0.25)^{n-1} & n \geq 1 \\ 0 & n < 1 \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} (0.25)^{n-1} z^{-n}$$

factor out $(0.25)^0 = 1$

$$X(z) = z^{-1} \sum_{n=0}^{\infty} (0.25)^n z^{-n}$$

$r = 0.25z^{-1}$ geometric series.

$$X(z) = z^{-1} \frac{1}{1-0.25z^{-1}} = \frac{z^{-1}}{1-0.25z^{-1}} = \frac{1}{z-0.25}$$

ROC: $|z| > 0.25$

Same reason for c).

b) $x[n] = u[n] - u[n-5]$

The signal is a finite-length signal is 1 from $n=0$ to $n=4$.

$$x[n] = \begin{cases} 1 & 0 \leq n < 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$X(z) = \sum_{n=0}^{4} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

$$= \frac{z^5 - 1}{z^4(z-1)}$$

ROC is $|z| > 0$ since this is finite duration signal, we exclude $|z|=0$ as it will make z-transform invalid.

e) $x[n] = (0.25)^n u[n-1]$

So like c) we consider as a delay of 1:

$$X(z) = z^{-1} \cdot \frac{z}{z-0.25} = \frac{1}{z-0.25}$$

ROC: $|z| > 0.25$ as similar reason of c.

f) $x[n] = \underbrace{(0.25)^n u[n]}_{①} + \underbrace{(0.5)^n u[n]}_{②}$

$$X(z) = \sum_{n=0}^{\infty} (0.25)^n z^{-n} + \sum_{n=0}^{\infty} (0.5)^n z^{-n}$$

$$X(z) = \frac{1}{1-0.25z^{-1}} + \frac{1}{1-0.5z^{-1}} = \frac{z}{z-0.25} + \frac{z}{z-0.5}$$

$$X(z) = \frac{z(z-0.5) + z(z-0.25)}{(z-0.25)(z-0.5)}$$

$$= \frac{z(2z-0.75)}{(z-0.25)(z-0.5)}$$

$|z| > 0.5$ as ROC must be intersection of ① and ② and their ROC is same idea as in c)

g) $x[n] = (0.25)^n \cos(0.25\pi n)\, u[n]$

$$\bar{X}(z) = \frac{z(z - a\cos(w_0))}{z^2 - 2a\cos(w_0)z + a^2}$$

for $a^n \cos(w_0 n)\, u[n]$

$$\bar{X}(z) = \frac{z(z - 0.25\cos(0.25\pi))}{z^2 - 2\cdot 0.25\cos(0.25\pi)z + 0.25^2}$$

ROC $|z| > 0.25$, since it is right sided signal. for infinite sum to converge.

as $\cos(0.25\pi n)$ is oscillating, not converge but possible to be factored out by other component.

this is for 3.2.

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b) $x[n] = n\{u[n-1] - u[n-5]\}$

$h[n] = 2\delta[n+3]$

$h[n]$ means to shift $x[n]$ by 3 to left and rescale by 2.

So $y[n] = 2x[n+3]$

$y[n] = 2(n+3)(u[n+2) - u[n-2])$

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d) $x[n] = u[n] - u[n-5]$

$h[n] = u[n] - u[n-3]$

$$y[n] = \begin{cases} n+1 & 0 \le n \le 2 \\ 3 & 3 \le n \le 4 \\ 5-n & 5 \le n \le 6 \\ 0 & \text{else} \end{cases}$$

3.3

a) $x[n] = u[n] - u[n-5]$

$h[n] = 0.5\delta[n-3]$

$h[n]$ means to shift $x[n]$ by 3 units to right and rescale it 0.5.

So $y[n] = 0.5x[n-3]$

$= 0.5(u[n-3] - u[n-8])$

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c) $x[n] = u[n] - u[n-5]$

$h[n] = u[n] - u[n-5]$

$x[n]$ and $h[n]$ are both finite length step function.

$$y[n] = \begin{cases} n+1 & 0 \le n \le 4 \\ 10-n & 5 \le n \le 8 \\ 0 & \text{else.} \end{cases}$$

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e) $x[n] = u[n] - u[n-5]$

$h[n] = u[n]$

So same, step function's effect.

$$y[n] = \begin{cases} n+1 & 0 \le n \le 4 \\ 5 & n \ge 5 \\ 0 & n < 0. \end{cases}$$

3.4

A)

a) Things like new knowledge, insight or techniques can be applied to solve problems should be produced.

Example: like discovery of a new noise suppression technique.

b) This usually gives a idea for how the system would be to achieve our desire.

Example: the Design of an echo cancellation system specifically for teleconferencing.

c) This should give path to a real thing. A software / hardware to be put Into use.

Example: a echo cancellation system on embedded DSP chip. It's the thing that is made already not a thought on how.

d) Then they need to go back to implemetation to update.

B)

a) Digital Filter

b) Spectral Analyzer

c) Spetrogram Analyzer

d) Filterback

e) Parametric Signal Modeling

f) Capstral Analyzer.

C)

a)

① Usually in terms of # of arithmetic operations like multiplications and additions.

② Memory usage: this cost space and money to produce.

③ Executio time: this could be effected by ① but still real life might also take this into account

④ Power consumption.

b)

Audio equalization in hearing aids. Here the IIR would be better as it could achieve a given frequency response with fewer filter coefficients. It would reduce size and power usage of the small system.

c) Data Trasmission system.

In these, linear phase response is critical to prevent signal distortion. FIR provides linear phase response ensuring all frequency are delayed equally. That makes FIR more stable.