EC516 HW7 Solutions

Problem 7.1

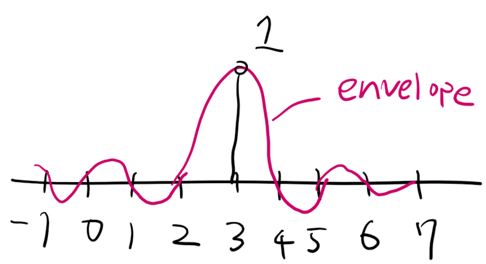
(a) Taking the DTFT of $\delta[n-3]$ will give us $X(e^{j\omega}) = e^{-j\omega}$. To find the envelope, we find the inverse CTFT of

$$E_x(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \pi \\ 0 & \text{else} \end{cases}$$

Taking inverse CTFT gives us

$$\frac{\sin(\pi(t-3))}{\pi(t-3)}$$

Note that the envelope is zero at any integer values except for $t = 3$.

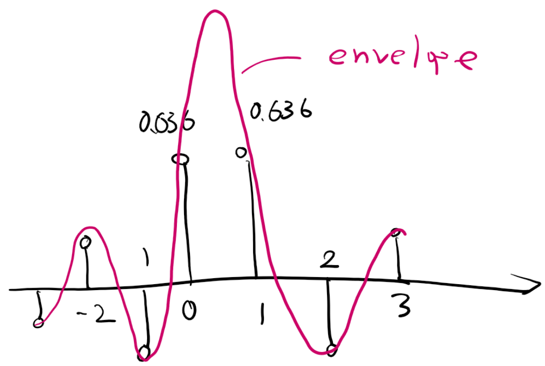


(b) We take the CTFT of $\frac{\sin(\pi(t-0.5))}{\pi(t-0.5)}$, which is

$$E_x(j\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & \text{else} \end{cases}$$

Taking the inverse DTFT gives us

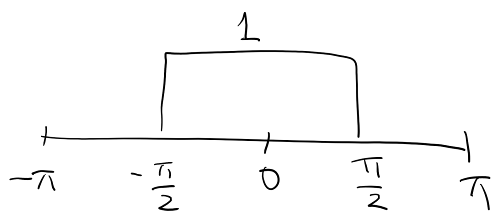
$$x[n] = \frac{\sin(\pi(n-0.5))}{\pi(n-0.5)}$$



Problem 7.2

(a)

$$H_{id}(e^{j\omega}) = \text{rect}\left(\frac{\omega}{\pi}\right) = \begin{cases} 1 & |\omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq |\omega| \leq \pi \end{cases}$$



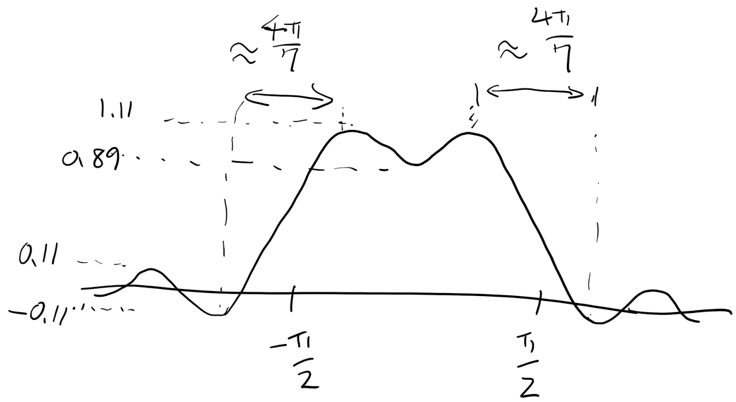
(b) DTFT of $w[n]$ is

$$W(e^{j\omega}) = e^{-j3\omega}\frac{\sin(7\omega/2)}{\sin(\omega/2)}$$

Multiplications in time domain corresponds to convolution in frequency domain. Recall that DTFT$\{h_{id}[n-3]\} = e^{-j3\omega}H_{id}(e^{j\omega})$.

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j3\theta}e^{-j3(\omega-\theta)}\frac{\sin(7(\omega-\theta)/2)}{\sin((\omega-\theta)/2)}d\theta \\ &= e^{-j3\omega}\cdot\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}\frac{\sin(7(\omega-\theta)/2)}{\sin((\omega-\theta)/2)}d\theta \\ &= R(\omega)e^{-j3\omega} \end{aligned}$$

Transition bandwidth of $H(e^{j\omega})$ will roughly equal to the width of mainlobe of $W(e^{j\omega})$.



Problem 7.3

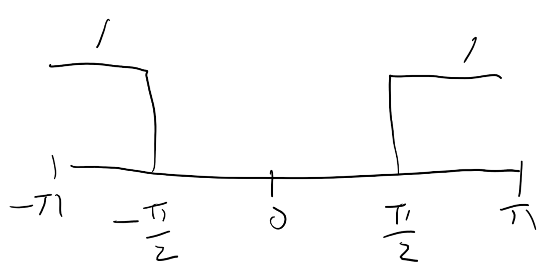
(a) Notice that $(-1)^n = e^{j\pi n}$. DTFT of $e^{j\pi n}$ is

$$\text{DTFT}\{e^{j\pi n}\} = \delta(\omega - \pi) \quad \text{for } -\pi < \omega \leq \pi$$

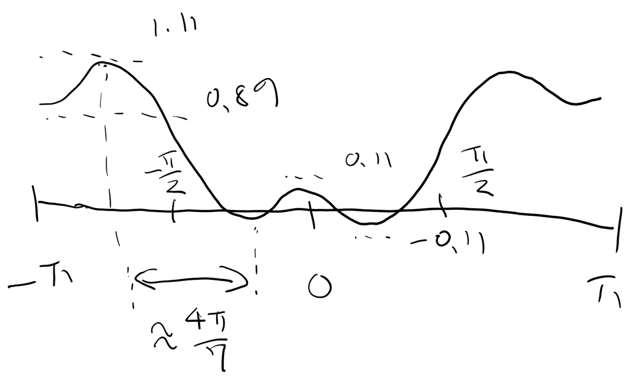
Since $(-1)^n$ is being multiplied in time domain, we perform convolution in frequency domain.

$$\begin{aligned}\text{DTFT}\left\{(-1)^n \cdot \frac{\sin(0.5\pi n)}{\pi n}\right\} &= \delta(\omega - \pi) * \text{rect}\left(\frac{\omega}{\pi}\right) \\ &= \begin{cases} 1 & \frac{\pi}{2} < |\omega| \leq \pi \\ 0 & |\omega| \leq \frac{\pi}{2} \end{cases}\end{aligned}$$

Contrary to previous problem, this is a high pass filter.



(b) $H(e^{j\omega})$ can still be written in the form $R(\omega)e^{j3\omega}$.



Problem 7.4

(a) $H_{id}(e^{j\omega})$ is the same.

(b) $w[n]$ is a triangular function (convolved with rectangle with rectangle). Its DTFT is simply the square of sinc function.

$$W(e^{j\omega}) = \left(e^{-j3\omega/2}\frac{\sin(4\omega/2)}{\sin(\omega/2)}\right)^2$$
$$= e^{-j3\omega}\frac{\sin^2(4\omega/2)}{\sin^2(\omega/2)}$$

The phase shift would be the same as before.

The width of the mainlobe in $W(e^{j\omega})$ is π hence the transition width will appeoximately be π. Since $W(e^{j\omega})$ will have smaller sidelobes, $H(e^{j\omega})$ will have smaller ripples.