FALL 24 EC516 Problem Set 11

Due: Sunday November 24 (Before 11:59pm)

You must submit your homework attempt on Blackboard Learn. For this purpose, you must convert your homework attempt to a pdf file and upload it at the corresponding homework assignment on Blackboard Learn.

Problem 11.1

In this problem, let $R_N[k] = u[n] - u[n-N]$. Consider the signal $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$

(a) Sketch the 5-point signal $x[(n-1)_5]R_5[n]$
(b) Sketch the 10-point signal $x[(n-1)_{10}]R_{10}[n]$
(c) Sketch the 5-point signal $x[(-n)_5]R_5[n]$
(d) Sketch the 10-point signal $x[(-n)_{10}]R_{10}[n]$
(e) Sketch the 5-point signal $x[(-n-2)_5]R_5[n]$
(f) Sketch the 10-point signal $x[(-n-2)_{10}]R_{10}[n]$

Problem 11.2

Let $x[n] = 2\{u[n] - u[n-8]\}$ with 8-point DFT $X[k]_8$ and let $g[n] = 3\{u[n] - u[n-8]\}$ with 8-point DFT $G[k]_8$.

a) Determine an expression for the DTFT $X(e^{j\omega})$ and sketch $|X(e^{j\omega})|$.
b) Use the fact that $X[k]_8$ is samples of the first period of $X(e^{j\omega})$ to determine and sketch $X[k]_8$.
c) Determine an expression for the DTFT $G(e^{j\omega})$ and sketch $|G(e^{j\omega})|$.
d) Use the fact that $G[k]_8$ is samples of the first period of $X(e^{j\omega})$ to determine and sketch $G[k]_8$.
e) Let $Y(e^{j\omega}) = X(e^{j\omega})G(e^{j\omega})$. Sketch $y[n]$, the inverse DTFT of $Y(e^{j\omega})$.
f) Let $Q[k]_8 = X[k]_8 G[k]_8$. Sketch $Q[k]_8$.
g) Show that $Q[k]_8$ is the 8-point DFT of $q[n] = 6\{u[n] - u[n-8]\}$
h) Is $q[n]$ from part (g) the same signal as $y[n]$ from part (e)? Why or why not?
i) According to the circular convolution property of the DFT discussed in lecture 20, $q[n] = \sum_{k=0}^{N-1} x[k]g[(n-k)_8]\{u[k] - u[k-N]\}$ for $0 \le n < N$. Evaluate the right side of this equation to show that you obtain the same $q[n]$ as given in part (g).

Problem 11.3

Throughout this problem, let $x[n]$ be and $g[n]$ both be arbitrary 4-point signals. The circular *N*-point convolution of $x[n]$ and $g[n]$ is given as

$$q[n] = \sum_{k=0}^{N-1} x[k]g[(n-k)_8]\{u[k] - u[n-N)\} \text{ for } 0 \le n < N.$$

Furthermore, the linear convolution of $x[n]$ and $g[n]$ is given as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-k]$$

a) Show $y[n]$ is guaranteed to be 7-points long.
b) Show that if $N \ge 7$, $q[n] = y[n]$.

Problem 11.4

Let $p[n]$ be an arbitrary N-point (N>2) signal with DTFT $P(e^{j\omega})$.

(a) Let the signal $q[n]$ have a DTFT that is obtained by sampling $P(e^{j\omega})$ every $2\pi/N$. Use sampling concepts to argue that $q[n] = \sum_{m=-\infty}^{\infty} p[n-mN]$. What is the smallest guaranteed period of $q[n]$?
(b) If we form a signal $r[n]$ by extracting the *first period* of $q[n]$ from the previous part, show that signal $r[n]$ is the same as $p[n]$
(c) Let the signal $q[n]$ have a DTFT that is obtained by sampling $P(e^{j\omega})$ every $2\pi/(N\text{-}1)$. Use sampling concepts to argue that $q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-1)]$. What is the smallest guaranteed period of $q[n]$?
(d) If we form a signal $r[n]$ by extracting the *first period* of $q[n]$ from the previous part, show that signal $r[n]$ is the same as the first $N-1$ points of $p[n]$ except that the first sample in $r[n]$ is contaminated as follows: $r[0] = p[0] + p[N-1]$. Informally, we say that the last sample of $p[n]$ has wrapped around to "contaminate" the first sample of $r[n]$.
(e) Now, let the signal $q[n]$ have a DTFT that is obtained by sampling $P(e^{j\omega})$ every $2\pi/(N\text{-}2)$. Use sampling concepts to argue that $q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-2)]$. What is the smallest guaranteed period of $q[n]$?
(f) If we form signal $r[n]$ by extracting the *first period* of the signal $q[n]$ from the previous part, show that the signal $r[n]$ is the same as the first $N-2$ points of $p[n]$ except that the first 2 samples in $r[n]$ are contaminated as follows: $r[0] = p[0] + p[N-2]$ and $r[1] = p[1] + p[N-1]$. Informally, we say that the last two samples of $p[n]$ have wrapped around to "contaminate" the first two samples of $r[n]$.
(g) Generalize the result of the previous parts to argue that if the DTFT of $p[n]$ is sampled every $2\pi/M$ (where $M < N$), then there exists an M-point signal $r[n]$ whose M-point DFT is equal to the *first period* of those samples of $P(e^{j\omega})$. Furthermore, explain why $r[n]$ is the same as first M samples of $p[n]$ except that the first $(N-M)_M$ samples of $r[n]$ have been "contaminated."
(h) Use the result from the previous part to argue that if two signals have a linear convolution that is N points long, then the circular M-point convolution of those two signals is equal to their linear convolution when $M \ge N$. Furthermore, explain that if $M < N$, then the circular M-point convolution of the two signals is equal to the first M points of the linear convolution, except that the first $(N-M)_M$ samples of the circular convolution have been "contaminated."