FALL 24 EC516 Problem Set 02

Due: Sunday September 22 (Before 11:59pm)

You must submit your homework attempt on Blackboard Learn. For this purpose, you must convert your homework attempt to a pdf file and upload it at the corresponding homework assignment on Blackboard Learn.

Problem 2.1 (CTFT Basics Review)

Part (A):
Calculate the CTFT of each of the following signals using $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ and plot $|X(j\omega)|$ as a function of ω.

a) $x(t) = \delta(t - t_0)$, where $\delta(t)$ is the continuous-time unit impulse.
b) $x(t) = u(t + T) - u(t - T)$, where $u(t)$ is the continuous-time unit step.

Part(B):
For each CTFT given below, compute the corresponding signal using $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$ and plot $|x(t)|$ as a function of *t*.

(a) $X(j\omega) = 2\pi\delta(\omega - 100\pi)$
(b) $X(j\omega) = u(\omega + 2\pi) - u(\omega - 2\pi)$

Part (C):
If $x(t)$ is a real-valued signal with $X(j1000\pi) = 1 + j$, use $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ to determine the value of $X(-j1000\pi)$. This result shows that any algorithm for determining the CTFT of a real-valued signal need not directly calculate the CTFT for negative frequencies.

Problem 2.2 (DTFT Basics Review)

Part (A):
Determine the DTFT $X(e^{j\omega})$ of $x[n] = \delta[n - 3]$ by using $X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ where $\delta[n]$ is the discrete-time unit impulse.

Part (B):
Calculate the DTFT of each of the following signals using $X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ and plot $|X(e^{j\omega})|$ as a function of ω.

a) $x[n] = u[n + 2] - u[n - 3]$, where $u[n]$ is the discrete-time unit step.
b) $x[n] = u[n] - u[n - 5]$, where $u[n]$ is the discrete-time unit step.
c) $x[n] = u[n] - u[n - 4]$, where $u[n]$ is the discrete-time unit step.

Part (C):
For each DTFT given below, compute the corresponding signal using $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$ and plot $|x[n]|$ as a function of *n*.

a) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - 0.5\pi - 2\pi k)$
b) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}\{\pi\delta(\omega + 0.5\pi - 2\pi k) + \pi\delta(\omega - 0.5\pi - 2\pi k)\}$

<u>Problem 2.3</u> (DTFT Properties Review)

a) Suppose that the input $x[n]$ to a digital circuit is related to the output $y[n]$ of that digital circuit through the following difference equation:

$$y[n] = 0.5y[n-1] + x[n]$$

Determine the mathematical relationship between $Y(e^{j\omega})$ and $X(e^{j\omega})$.

b) If $x[n]$ is a real-valued signal with $X\left(e^{j0.25\pi}\right) = 1 + j$, use $X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ to determine the value of $X(e^{-j0.25\pi})$. This result shows that any algorithm for determining the DTFT of a real-valued signal need not directly calculate the DTFT for negative frequencies.

<u>Problem 2.4</u> (A/D Conversion)

Suppose $x(t)$ is a *real-valued* speech-signal whose CTFT is $X(j\omega)$ and it is known that $|X(j\omega)| = 0$ for $|\omega| \geq 10{,}000\pi$. Let $x[n] = x(nT)$ be the output of an A/D converter where T represents the sampling interval. Answer the following questions about $X(e^{j\omega})$, the DTFT of $x[n]$, for the specified values of *T*.

(a) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.0001$ secs. *Justify your answer.*

(b) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.00005$ secs. *Justify your answer.*

(c) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.00001$ secs. *Justify your answer.*