EC516 HW4 Solutions

Problem 4.1 Since it's a causal filter, $h[n] = 0$ for $n < 0$. If x is an impulse, $x[0] = 1$ and $x[n] = 0$ for $n \neq 0$.

$$h[0] = 0.25 \cdot 0 + 1 = 1$$

$$h[1] = 0.25 \cdot 0 + 0 = 0$$

$$h[2] = 0.25 \cdot 1 + 0 = 0.25$$

$$h[3] = 0.25 \cdot 0 + 0 = 0$$

Notice that

$$h[n] = \begin{cases} 0.25^{n/2} & n \text{even} \\ 0 & n \text{odd} \end{cases}$$

Threfore,

$$h[1001] = 0$$

Problem 4.2

(a) Flowgraph is shown below. This implementation requires 4 retrievals and 4 additions per output sample. No multiplication is needed since all the coefficients are 1's.

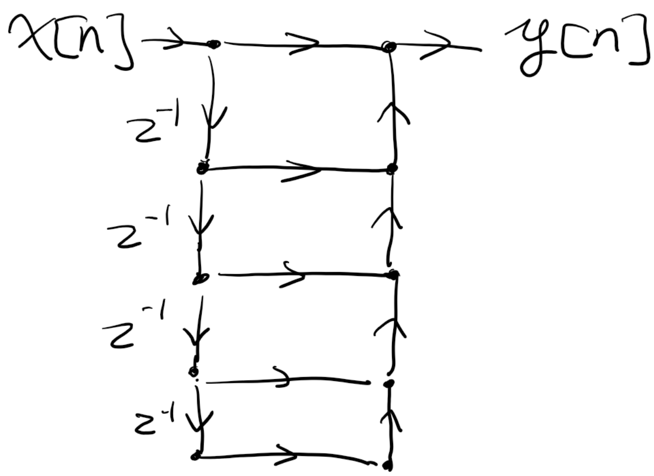

Figure 1: FIR flowgraph

(b)

$$
\begin{aligned}
Y(z) &= X(z) + z^{-1}X(z) + z^{-2}X(z) + z^{-3}X(z) + z^{-4}X(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} \\
&= \sum_{n=0}^{4} z^{-n}
\end{aligned}
$$

(c) We first substitute $z = e^{j\omega}$ to obtain the frequency response.

$$
\begin{aligned}
H(e^{j\omega}) &= \sum_{n=0}^{4} e^{-j\omega n} \\
&= \frac{1 - e^{j5\omega}}{1 - e^{j\omega}} \\
&= \frac{e^{j5\omega/2}\left(e^{-j5\omega/2} - e^{j5\omega/2}\right)}{e^{j\omega/2}\left(e^{-j\omega/2} - e^{j\omega/2}\right)} \\
&= e^{j2\omega}\frac{-2j\sin(5\omega/2)}{-2j\sin(\omega/2)} \\
&= e^{j2\omega}\frac{\sin(5\omega/2)}{\sin(\omega/2)}
\end{aligned}
$$

The magnitude of the frequency response is

$$\begin{aligned} |H(e^{j\omega})| &= \left| e^{j2\omega} \frac{\sin(5\omega/2)}{\sin(\omega/2)} \right| \\ &= \frac{\sin(5\omega/2)}{\sin(\omega/2)} \end{aligned}$$

$|H(e^{j\omega})|$ is drawn in the figure below.

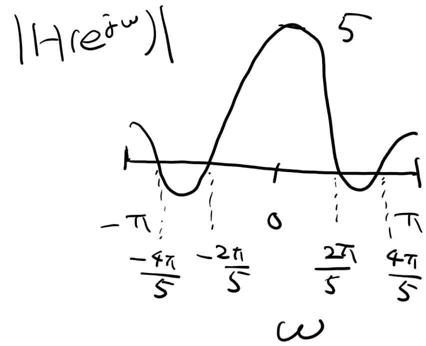

Figure 2: Magnitude of $H(e^{j\omega})$

(d) The filter is a FIR filter with finite coefficients. It is stable.

Problem 4.3

(a) Flowgraph is shown below. This implementation requires 2 retrievals, 2 multiplication, and 2 addition per output sample.

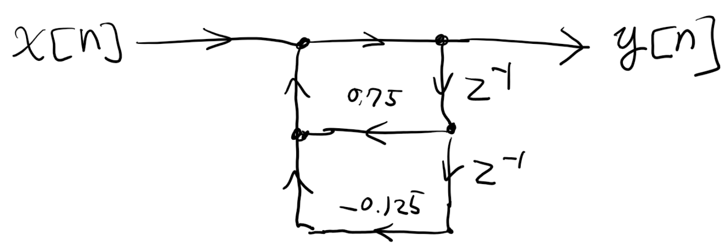

Figure 3: IIR flowgraph

(b)

$$\begin{aligned}
Y(z) &= -0.125z^{-2}Y(z) + 0.75z^{-1}Y(z) + X(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{1}{0.125z^{-2} - 0.75z^{-1} + 1} \\
&= \frac{8z^2}{1 - 6z + 8z^2} \\
&= \frac{8z^2}{(1-2z)(1-4z)}
\end{aligned}$$

(c) The poles of the filter are $z = \frac{1}{2}$ and $z = \frac{1}{4}$. Since all poles are inside the unit circle on the complex plane, this filter is stable.

(d) This filter has 2 zeros at $z = 0$. However since there are not on the unit circle of the complex plane, $|H(e^{j\omega})|$ is non-zero everywhere.

Problem 4.4

(a) The flowgraph is shown below.

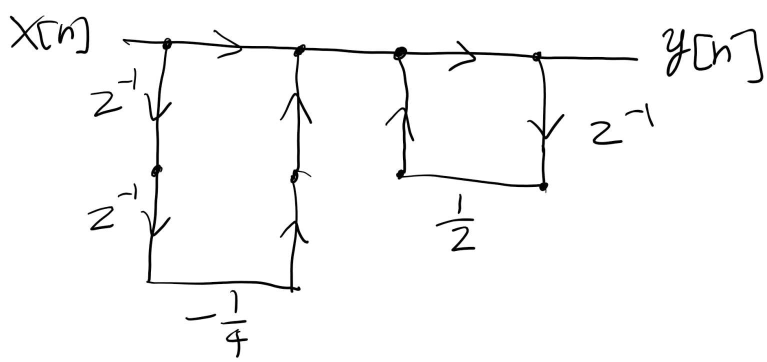

Figure 4: Recursive implementation of FIR

(b)

$$
\begin{aligned}
Y(z) - \frac{1}{2}z^{-1}Y(z) &= X(z) - \frac{1}{4}z^{-2}X(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{1 - \frac{1}{4}z^{-2}}{1 - \frac{1}{2}z^{-1}} \\
&= \frac{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}{1 - \frac{1}{2}z^{-1}} \\
&= 1 + \frac{1}{2}z^{-1} \\
Y(z) &= X(z) + \frac{1}{2}z^{-1}X(z) \\
y[n] &= x[n] + \frac{1}{2}x[n-1]
\end{aligned}
$$

(c) The new flowgraph is shown below.

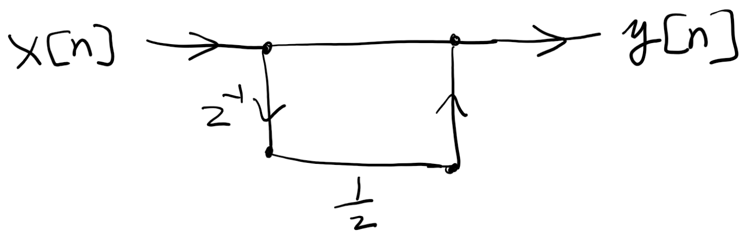

Figure 5: Non-Recursive implementation of FIR

(d) The implementation in (a) requires 2 retrievals, 2 multiplications, and 2 additions.While the implementation in (c) requires 1 retrieval, 1 multiplication, and 1 addition.