EC516 HW 10

10.1 $h[n] = (-1)^n \{u[n] - u[n-4]\}$

a) $H(z) = \sum_{n=0}^{3} h[n] z^{-n}$

$= (z-1)(z+1)$

So z has zero at $z=1$ and $z=\pm j$

b) Since this is a High pass filter (a sinc but with a $\pm\pi$ shift) so at $w=\pi$ it's at max.

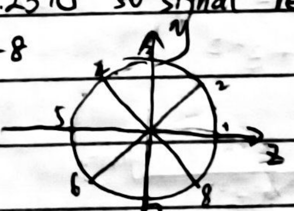
10.2 a) So for $1-z^{-8}$ we could see that except at $z=0$ (origin) he doesn't have pole and it is an FIR (Also limit amount of trace back of x)

b) $x[n] = \cos(0.25\pi n + 0.15\pi n)$

$y[n] = h[n] * x[n] = x[n] - x[n-8]$

w of $x[n]$ is 0.25π so signal repeats every $\frac{2\pi}{0.25\pi} = 8$

the zeros of $1-z^{-8}$



$z^{-8} = 1$

$\frac{2\pi}{8} = \frac{1}{4}\pi$

so any signal input with frequncy: $0, \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi, -\pi, -\frac{1}{4}\pi, -\frac{1}{2}\pi, -\frac{\pi}{4}$ is zero

$\frac{1}{4}\pi = 0.25\pi$ so reaction to $x[n]$ is zero everywhere.

10.3

a) $y[n] = 0.5y[n-1] + x[n] - 0.5^4 \, x[n-4]$

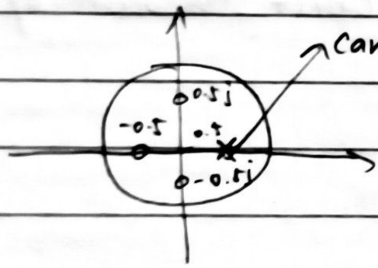
$$Y(z) = 0.5Y(z) \cdot z^{-1} + X(z) - 0.5^4 X(z) \cdot z^{-4}$$

$$(1-0.5z^{-1})Y(z) = (1+0.5^4 z^{-4}) X(z)$$

$$Y(z) = \frac{1-(0.5z^{-1})^4}{1-0.5z^{-1}}$$

$$= \frac{[1+(0.5z^{-1})^2][1-(0.5z^{-1})^2]}{1-0.5z^{-1}}$$

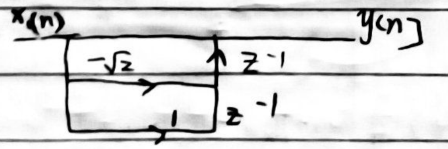
$$= \frac{[1+(0.5z^{-1})^2][1-0.5z^{-1}][1+0.5z^{-1}]}{1-0.5z^{-1}}$$



$1+(0.5z^{-1})^2 = 0$ $\quad\quad$ $1-0.5z^{-1} \rightarrow z=0.5$

b) $H(z) = 1-2\cos(w_0)z^{-1} + z^{-2}$ $\quad\quad$ $w_0 = 0.25\pi$

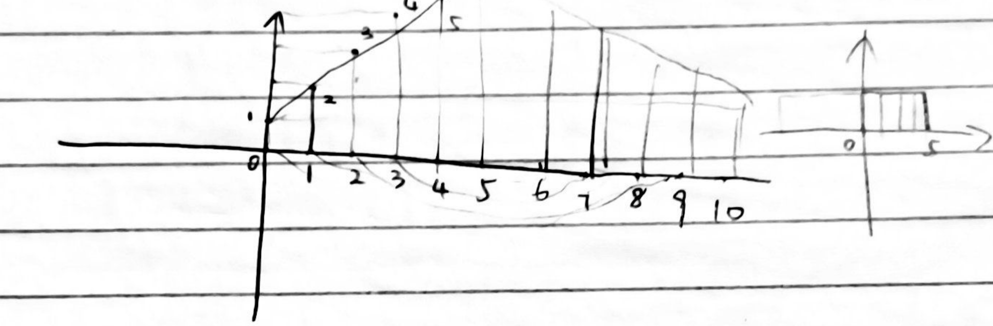
$$= 1-\sqrt{2}z^{-1} + z^{-2}$$



c)

10.4 a) The inverse of this is a convolution of 2 box in time domain.

Convolution of $\{u[n] - u[n-6]\} * \{u[n] - u[n-6]\}$



b) Yes, that is the charactristic of bilinear transformation. it should maintain zero-poles properties.

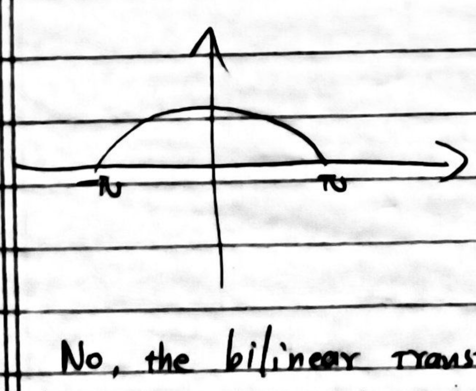
10.5

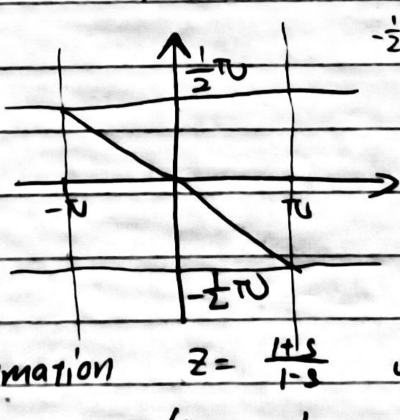
$$G[e^{jw}] = e^{-jw\cdot 0} + e^{-jw\cdot 1}$$
$$= 1 + e^{-jw}$$
$$= e^{-\frac{1}{2}jw}\left(e^{\frac{1}{2}jw} + e^{-\frac{1}{2}jw}\right)$$
$$= e^{-\frac{1}{2}jw}\, 2\cdot\cos\left(\tfrac{1}{2}w\right)$$

So the slope is $-\frac{1}{2}$ while whe cos

$-\frac{1}{2}\cdot w = \pi$ $\quad -\frac{1}{2}\cdot\pi$

$w = .$





10.6 No, the bilinear transformation $z = \frac{1+s}{1-s}$ which could see does not garancee linear phase to linear phase change.