

Problem Set 0  
EC(MS)574A1, Fall 2024  
Assigned September 3, 2024  
Due October 17, 2024

Review of some relevant mathematical techniques.

The solution of the assigned problem set is **mandatory** and it is your responsibility. If you do not work on the homework you will not be able to solve the exam problems. It is strongly suggested that you start solving the homework sets immediately without waiting the day before the exam. You are required to turn in the solution of Problem Set I the day of the exam.

- Problem 1

The Gaussian integral and its variations are going to be useful going forward.

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

Show that the result above is correct using two of the four following approaches: Feynman's technique, integration using polar coordinates, Laplace method, **complex integration**.

- Problem 2

Determine the solution of the following indefinite integrals:

$$\begin{aligned} \int_{-\pi}^{\pi} \cos(nx)\cos(mx)dx \\ \int_{-\pi}^{\pi} \cos(nx)\sin(mx)dx \\ \int_{-\pi}^{\pi} \sin(nx)\sin(mx)dx \end{aligned}$$

when  $n$  and  $m$  are positive integers.

- Problem 3

Consider following first order ODE with constant coefficients:

$$\frac{dy}{dt} = ay + b$$

1 – Compute the general solution of this ODE.

2 – Determine the solution of the same ODE assuming that  $a = 1$ ,  $b = 4$ , with the condition  $y(0) = 1$ .

- Problem 4

Consider following second order homogeneous ODE with constant coefficients:

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

- 1 – Compute the general solution of this ODE assuming first that  $a = 1$ ,  $b = 4$ ,  $c = 3$ .
- 2 – Determine again the general solution of the same ODE assuming that  $a = 1$ ,  $b = 4$ ,  $c = 5$ .
- 3 – Finally, given the boundary conditions  $y(0) = 1$  and  $y(1) = 3$  compute the solutions of both cases.