EC516 HW3 Solutions

Problem 3.1

(a)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x[n-n_0]z^{-n} &= \sum_{n+n_0=-\infty}^{\infty} x[n]z^{-(n+n_0)} \\
&= \sum_{n=-\infty}^{\infty} x[n]z^{-(n+n_0)} \\
&= z^{-n_0} \sum_{n=-\infty}^{\infty} x[n]z^{-n} \\
&= z^{-n_0} X(z)
\end{aligned}$$

(b)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x[-n]z^{-n} &= \sum_{-n=-\infty}^{\infty} x[n]z^{n} \\
&= \sum_{n=-\infty}^{\infty} x[n]z^{n} \\
&= \sum_{n=-\infty}^{\infty} x[n](z^{-1})^{-n} \\
&= X(z^{-1})
\end{aligned}$$

(c)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x^*[n]z^{-n} &= \sum_{n=-\infty}^{\infty} \big(x[n](z^{-n})^*\big)^* \\
&= \left(\sum_{n=-\infty}^{\infty} x[n](z^{-n})^*\right)^* \\
&= \left(\sum_{n=-\infty}^{\infty} x[n](z^*)^{-n}\right)^* \\
&= (X(z^*))^*
\end{aligned}$$

(d)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} x[n-m]h[m]z^{-n} &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} x[n-m]h[m]z^{-(n-m)}z^{-m} \\
&= \sum_{m=-\infty}^{\infty}\sum_{\ell=-\infty-m}^{\infty-m} x[\ell]h[m]z^{-\ell}z^{-m} \\
&= \sum_{m=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty} x[\ell]h[m]z^{-\ell}z^{-m} \\
&= \left(\sum_{\ell=-\infty}^{\infty} x[\ell]z^{-\ell}\right)\cdot\left(\sum_{m=-\infty}^{\infty} h[m]z^{-m}\right) \\
&= X(z)H(z)
\end{aligned}$$

Problem 3.2

(a)

$$\sum_{n=-\infty}^{\infty} \delta[n-3]z^{-n} = z^{-3} \qquad \text{for } |z| > 0$$

(b)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} \left(u[n]-u[n-5]\right)z^{-n} &= \sum_{n=0}^{4} z^{-n} \\
&= \begin{cases} \frac{1-z^{-5}}{1-z^{-1}} & \text{for } z \neq 1 \\ 5 & \text{for } z = 1 \end{cases}
\end{aligned}$$

(c)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} (0.25)^n u[n]z^{-n} &= \sum_{n=0}^{\infty} (0.25z^{-1})^n \\
&= \frac{1}{1-0.25z^{-1}} \qquad \text{for } |z| > 0.25
\end{aligned}$$

(d)

$$\sum_{n=-\infty}^{\infty} (0.25)^{n-1} u[n-1]z^{-n} = \frac{z^{-1}}{1-0.25z^{-1}} \qquad \text{for } |z| > 0.25$$

(e)

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} (0.25)^n u[n-1]z^{-n} &= 0.25\cdot\sum_{n=-\infty}^{\infty} (0.25)^{n-1} u[n-1]z^{-n} \\
&= \frac{0.25z^{-1}}{1-0.25z^{-1}} \qquad \text{for } |z| > 0.25
\end{aligned}$$

(f)

$$\sum_{n=-\infty}^{\infty} \left((0.25)^n u[n] + (0.5)^n u[n]\right) z^{-n} = \frac{1}{1-0.25z^{-1}} + \frac{1}{1-0.5z^{-1}} \qquad \text{for } |z| > 0.5$$

(g)

$$\begin{aligned}\sum_{n=-\infty}^{\infty} (0.25)^n \cos(0.25\pi n) u[n] z^{-n} &= \sum_{n=0}^{\infty} (0.25)^n \frac{1}{2}\left(e^{j0.25\pi n} + e^{-j0.25\pi n}\right) z^{-n} \\ &= \frac{1}{2}\sum_{n=0}^{\infty} (0.25)^n e^{j0.25\pi n} z^{-n} + \frac{1}{2}\sum_{n=0}^{\infty} (0.25)^n e^{-j0.25\pi n} z^{-n} \\ &= \frac{1}{2\cdot(1-0.25e^{j0.25\pi}z^{-1})} + \frac{1}{2\cdot(1-0.25e^{-j0.25\pi}z^{-1})} \\ &\qquad\qquad\qquad \text{for } |z| > 0.25\end{aligned}$$

Problem 3.3

(a)

$$(u[n] - u[n-5]) * 0.5\delta[n-3] = 0.5(u[n-3] - u[n-8])$$

(b)

$$n(u[n-1] - u[n-5]) * 2\delta[n+3] = 2(n+3)(u[n+2] - u[n-2])$$

(c)

$$\begin{aligned}(u[n] - u[n-5]) * (u[n] - u[n-5]) &= \sum_{m=-\infty}^{\infty} (u[m] - u[m-5])(u[n-m] - u[n-m-5]) \\ &= \sum_{m=0}^{4} (u[n-m] - u[n-m-5]) \\ &= \begin{cases} 1 & n = 0 \\ 2 & n = 1 \\ \vdots & \\ 5 & n = 4 \\ \vdots & \\ 2 & n = 7 \\ 1 & n = 8 \\ 0 & \text{else} \end{cases}\end{aligned}$$

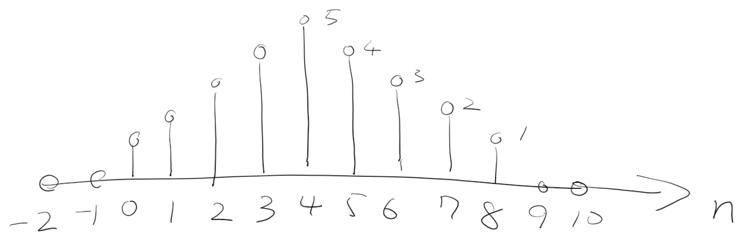

Figure 1: Problem 3.3 (c)

(d)

$$
\begin{aligned}
(u[n]-u[n-5])*(u[n]-u[n-3]) &= \sum_{m=-\infty}^{\infty}(u[m]-u[m-5])(u[n-m]-u[n-m-3]) \\
&= \sum_{m=0}^{4}(u[n-m]-u[n-m-3]) \\
&= \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 3 & n=3 \\ 3 & n=4 \\ 2 & n=5 \\ 1 & n=6 \\ 0 & \text{else} \end{cases}
\end{aligned}
$$

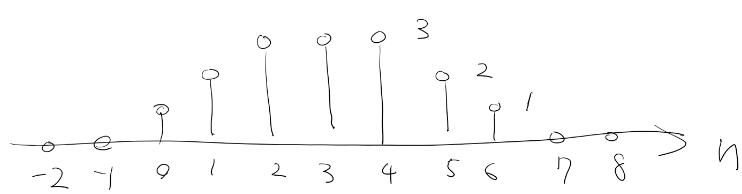

Figure 2: Problem 3.3 (d)

(e)

$$\begin{aligned}
(u[n] - u[n-5]) * u[n] &= \sum_{m=-\infty}^{\infty} (u[m] - u[m-5])u[n-m] \\
&= \sum_{m=0}^{4} u[n-m] \\
&= \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ 2 & n = 1 \\ 3 & n = 2 \\ 4 & n = 3 \\ 5 & n = 4 \\ 5 & n = 5 \\ 5 & n = 6 \\ \vdots & \end{cases}
\end{aligned}$$

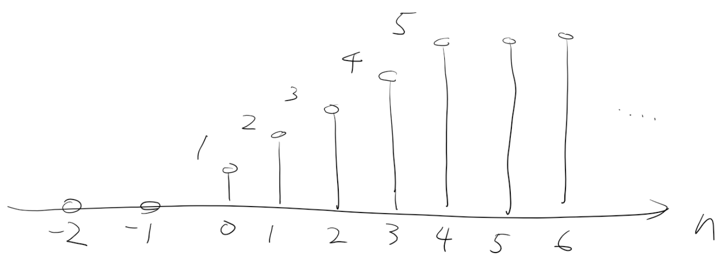

Figure 3: Problem 3.3 (e)

Problem 3.4

Part(A) (a) *Discovery* step is for determining what filter types and specifications are needed (low pass filter, high pass filter? What cutoff frequency?). For example, we have a narrow bandwidth signal but it is corrupted by a wide bandwidth noise. In order to get rid of some of the noise, we decide that we need a low-pass filter to get rid of high frequency components where the noise is dominant.

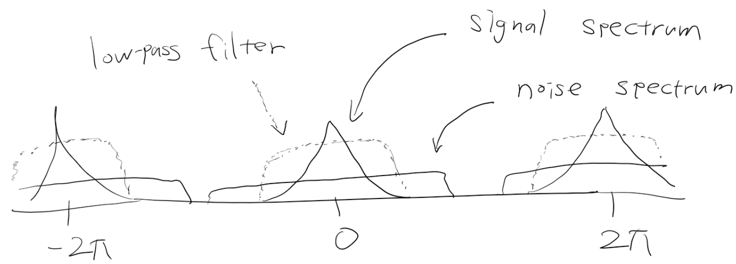

Figure 4: We have a signal corrupted with noise, we would like to get rid of some of it to recover a cleaner signal. We can apply low-pass filter to suppress the high frequency components.

(b) *Design* step is where we can determine filter coefficients. As an example, we can design a IIR filter with some values $\beta_0, \beta_1, \alpha_1$.

$$y[n] = \beta_0 x[n] + \beta_1 x[n-1] + \alpha_1 y[n-1]$$

(c) *Implementation* step is where we come up with detailed instructions on how to perform computation (i.e. how to store and retrieve data, when to multiply and add). Flowgraph can be a helpful tool to clearly notate these instructions. We can use the similar example from the class.

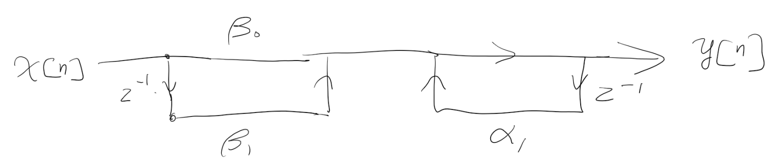

Figure 5: Let $n = n_0$. 1) We retrieve $x[n_0 - 1]$ and store $x[n_0]$. 2) we compute $\beta_0 x[n_0] + \beta_1 x[n_0 - 1]$. 3) we retrieve $y[n_0 - 1]$, compute $\alpha_1 y[n_0 - 1]$, and add to the previous value. 4) We store $y[n_0]$.

(d) If evaluation step concludes that our filter does not meet the criteria, it is time to come up with another implementation of our filter. For example, previous implementation will require storing $y[n_0]$ and $x[n_0]$. If we have limited memory, we can change our implementation to store only one value.

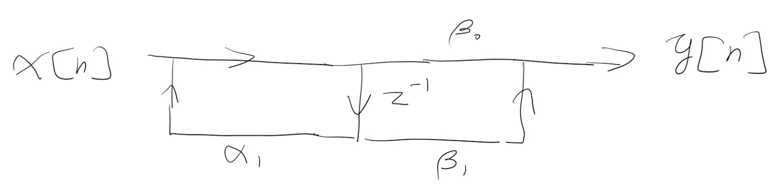

Figure 6: In this new flowgraph, we only need to store one value instead of two values, thus saving memory.

Part(B) (a) **D**igital **F**ilter

(b) **S**pectral **A**nalyzer

(c) **S**pectrogram **A**nalysis

(d) **F**ilterbank

(e) **P**arametric **S**ignal **M**odeling

(f) **C**epstral **A**nalyzer

Part(C) (a) number of multiply and accumulate (MAC) operations

(b) IIR filter is generally less computationally expensive than FIR, however comes with nonlinear phase response. Audio processing for phone calls is an example where fast processing is desired and nonlinear phase response does not have significant impact on the audio quality.

(c) Images are sensitive to nonlinear phase response of the filter. Image processing is an example where linear phase response is desired, hence FIR is a preferred choice.