FALL 24 EC516 Problem Set 04

Due: Sunday October 6 (Before 11:59pm)

You must submit your homework attempt on Blackboard Learn. For this purpose, you must convert your homework attempt to a pdf file and upload it at the corresponding homework assignment on Blackboard Learn.

Problem 4.1 (IIR Filter – Impulse Response from Difference Equation)

Consider a digital filter with impulse response $h[n]$ which is zero for n<0 and whose input $x[n]$ and output $y[n]$ are related through the following difference equation:

$$y[n] = 0.25y[n-2] + x[n]$$

By noting that the impulse response $h[n]$ is the output of the digital filter when the input is $\delta[n]$, use the above difference equation to determine the numerical values of $h[0], h[1], h[2], h[3]$ & $h[1001]$. Justify your answers

Problem 4.2 (Fundamentals of FIR Filter Implementation)
Consider a digital FIR filter whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$$

(a) Draw a flowgraph for this filter. How many multiplications per output sample does this filter implementation (as represented by your flowgraph) require? What amount of memory retrieval is needed for this filter implementation. Justify your answers.
(b) Write the algebraic expression for the system function $H(z)$ for this filter.
(c) Draw the magnitude of the frequency response of this filter.
(d) Is this filter stable? Justify your answer.

Problem 4.3 (Fundamentals of IIR Filter Implementation
Consider a digital IIR filter whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] = -0.125y[n-2] + 0.75y[n-1] + x[n]$$

(a) Draw a flowgraph for this filter. How many multiplications per output sample does this filter implementation (as represented by your flowgraph) require? What amount of memory retrieval is needed for this filter implementation. Justify your answers.
(b) Write the algebraic expression for the system function $H(z)$ for this filter.
(c) Give your reasoning for why this filter is stable.
(d) Determine all the values of ω for which it is guaranteed that $H(e^{j\omega}) = 0$.

Problem 4.4 (Recursive Implementations of FIR filters)

Consider a *causal* digital filter whose input $x[n]$and output $y[n]$ are related through the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-2]$$

(a) Draw the flowgraph corresponding to the above difference equation.
(b) Show that the same filter can be implemented using a non-recursive difference equation (HINT: Figure out the System Function $H(z)$ for the filter, simplify it by cancelling any common factors between the numerator and denominator, and go back to the difference equation in the time domain).
(c) Draw a flowgraph corresponding to the non-recursive difference equation.
(d) How does the computational complexity of the flowgraph you obtained in part (a) of this problem compare to the computational complexity of the flowgraph you obtained in part (c) of this problem?