EC516 HW8 Solutions

Problem 8.1

a) $\Delta\omega = 0.25\pi$, $A = -20\log_{10}(0.15) \approx 16.478$. Therefore, $\beta = 0$, and

$$N = \frac{16.478 - 8}{2.285 \cdot 0.25\pi} \approx 4.724$$

Picking the smallest integer larger than the above, the length will be 5.

b) $\Delta\omega = 0.15\pi$, $A = -20\log_{10}(0.15) \approx 16.478$. Therefore, $\beta = 0$, and

$$N = \frac{16.478 - 8}{2.285 \cdot 0.15\pi} \approx 7.873$$

The length is 8.

c) $\Delta\omega = 0.25\pi$, $A = -20\log_{10}(0.09) \approx 20.915$. Therefore, $\beta = 0$, and

$$N = \frac{20.915 - 8}{2.285 \cdot 0.25\pi} \approx 7.196$$

The length is 8.

d) $\Delta\omega = 0.15\pi$, $A = -20\log_{10}(0.09) \approx 20.915$. Therefore, $\beta = 0$, and

$$N = \frac{20.915 - 8}{2.285 \cdot 0.25\pi} \approx 11.994$$

The length is 12.

Problem 8.2

a) Window Design will produce equiripple frequency response (same error in passband and stopband). Optimal FIR Filter is more flexible when you want to allow varying error in the passband and stopband.

b)

$$\frac{-10\log_{10}(0.10 \cdot 0.05) - 13}{14.6(0.75\pi - 0.5\pi)/(2\pi)} \approx 5.485$$

Smallest odd integer that is greater than the above value is 7 (or $L = 3$).

c) $\Delta\omega = 0.25\pi,\ A = -20\log_{10}(0.05) \approx 26.021$. Therefore,

$$N = \frac{26.021 - 8}{2.285 \cdot 0.25\pi} \approx 12.017$$

Smallest integer greater than the above value is 13. Kaiser window requires filter with longer length to achieve error within $\delta_s = 0.05$ in the stopband.

d) Note that by trig identity we get

$$\cos^k(\omega) = \sum_{m=0}^{k} a_m \cos(m\omega)$$

In order to have k-th power of $\cos(\omega)$, the frequency response must include $\cos(k\omega)$. That means the frequency response found by optimal filter design $H(e^{j\omega})$ can be written in the following form.

$$H(e^{j\omega}) = e^{j\omega n_0}\left(\alpha \cdot \cos(k\omega) + R(\omega)\right)$$

n_0, α are some arbitrary numbers and $R(\omega)$ is a real-valued function. Inverse DTFT of $\cos(k\omega)$ is

$$\begin{aligned}\mathrm{DTFT}^{-1}\left\{e^{j\omega n_0}\cos(k\omega)\right\} &= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega n_0}\frac{1}{2}\left(e^{jk\omega} + e^{-jk\omega}\right)e^{-j\omega n}d\omega \\ &= \frac{1}{2}\delta[n - n_0 - k] + \frac{1}{2}\delta[n - n_0 + k]\end{aligned}$$

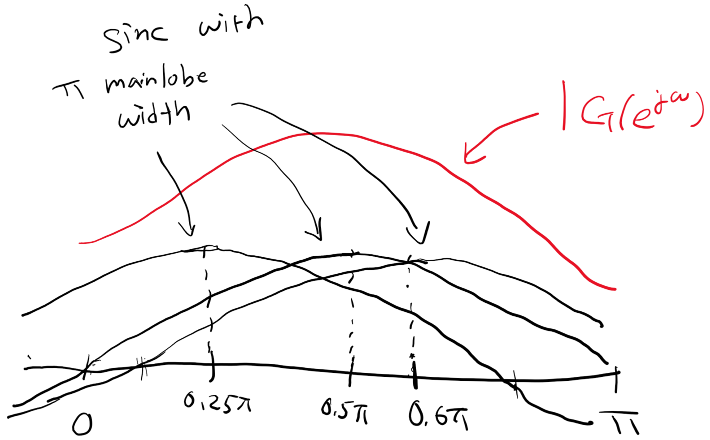
This means that filter length must be at least $2k+1$ to have k-th power of $\cos(\omega)$. From the number we found from part (b), the highest order of $\cos(\omega)$ is 3.

Problem 8.3

a) 4-point rectangular window has the DTFT

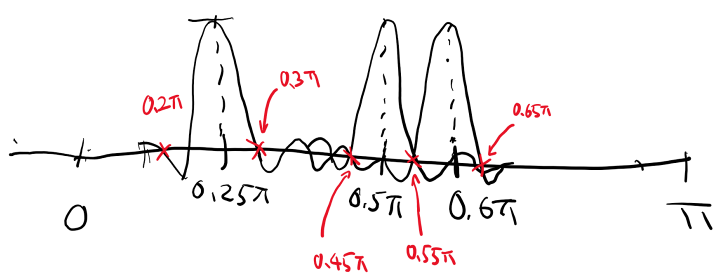
$$W(e^{j\omega}) = e^{-j3\omega/2}\frac{\sin(2\omega)}{\sin(\omega/2)}$$

The mainlobe width is π. $|G(e^{j\omega})|$ will be a combination of this sinc function as shown below.



Because the mainlobe width is much larger than the distance between the invidual frequencies, the resulting frequency response $|G(e^{j\omega})|$ will smoothed out and fail to separate the frequencies.

b) Smallest distance between separate frequencies is 0.1π (between 0.6π and 0.5π). We need a sinc function with mainlobe width that is at most 0.1π to have a complete separation of the frequencies, as shown below.



In order to achieve this, N must be 40. We can confirm from its DTFT,

$$W(e^{j\omega}) = e^{-j39\omega/2}\frac{\sin(20\omega)}{\sin(\omega/2)}$$

The zero crossings are $\pi k/20$ for $k = \pm 1, \pm 2, \ldots$. We can see that its mainlobe width is $\pi/10$.

c) As $N \to \infty$, $|W(e^{j\omega}|$ becomes closer to $\delta(\omega)$. Hence $|G(e^{j\omega})|$ becomes closer to $|X(e^{j\omega})|$.

Problem 8.4

(a) **F**requency **R**esolution, **D**iscovery

(a) **L**ength, **S**hape, **D**esign