EC516 HW 4

4.1.

$y[n] = 0.25\,y[n-2] + x[n]$ $\qquad$ $x[n] = \delta[0] = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$

$h[n] = \dfrac{y[n]}{x[n]}$ Discrete Function

① $h[0] = y[0]$ $\quad$ $x[0] = 1$ as first input signal $\quad$ $y[0] = 0.25\,y[-2] + x[0] = 1$ $\quad$ $h[0] = 1$

② $h[1] = y[1]$ $\quad$ $x[1] = 0$, $y[-1] = 0$ $\Rightarrow$ $y[1] = 0.25\,y[-1] + x[1] = 0$ $\quad$ $h[1] = 0$

③ $h[2] = y[2]$ $\quad$ $y[0] = 1$, $x[2] = 0$ $\Rightarrow$ $y[2] = 0.25 \cdot 1 + 0 = 0.25$ $\quad$ $h[2] = 0.25$

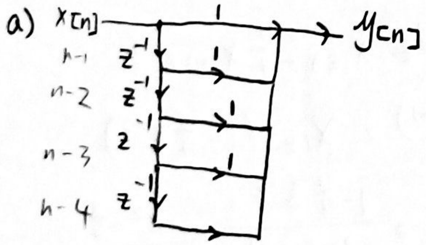
④ $h[3] = y[3]$ $\quad$ $y[1] = 0$, $x[3] = 0$ $\Rightarrow$ $y[3] = 0$ $\quad$ $h[3] = 0$.

⑤ $h[1001] = ?$ $\quad$ From above, we could see that since $x[n] = 0$ after $n=0$, then it is just multiplying 0.25. So $h[n] = \begin{cases} 0 & n \text{ odd} \\ (0.25)^{\frac{n}{2}} \end{cases}$ $\quad$ So $h[1001] = 0$

4.2

$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$

a)



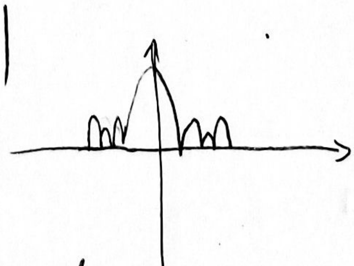
b) $Y(z) = X(z) \cdot (1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$

So $H(z) = \dfrac{Y(z)}{X(z)} = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

* c) $|H(e^{jw})| = \left|1 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w}\right|$

see details

$= \left|\dfrac{1 - e^{-j5w}}{1 - e^{-jw}}\right| = \left|\dfrac{\sin(0.25w)}{\sin(0.5w)}\right|$



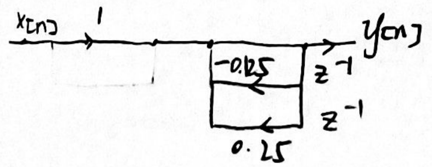
Peak at $w = 0$

zero when $2.5w = k\pi + 2n\pi$

So $w = \dfrac{k + 2n|\pi}{2.5}$ is where zero is at.

d) This system is stable as it doesn't have poles except $z=0$. infact this is how FIR behaves. when with a finite # of coefficients and no poles

4.3 $y[n] = -0.125\,y[n-2] + 0.25\,y[n-1] + x[n]$

a) 

b) $Y(z) = -0.125\,Y(z)z^{-2} + 0.25\,Y(z)z^{-1} + X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

It requires 3 values from memory for $y[n-2]$, $y[n-1]$, $x[n]$ we just need to update them

c) To see if it is stable, we need to find information for poles:

$(1 - 0.75z^{-1} + 0.125z^{-2}) = 0$ $\qquad z = \frac{0.75 \pm 0.25}{2}$

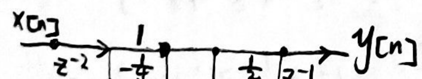
$z^2 - 0.75z + 0.125 = 0$ $\qquad z_1 = 0.5 \quad z_2 = 0.25$

they all lie in 0.5 and 0.25, so they are stable!

d) We could see, the $H(z)$ has 1 on top, there is no value it would be zero.

4.4 $y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-2]$

a) $y[n] = x[n] - \frac{1}{4}x[n-2] + \frac{1}{2}y[n-1]$



b) $Y(z) - \frac{1}{2}Y(z)z^{-1} = X(z) - \frac{1}{4}X(z)z^{-2}$

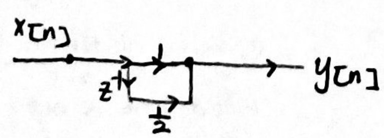
$Y(z)(1 - \frac{1}{2}z^{-1}) = X(z)(1 - \frac{1}{4}z^{-2})$

$$H z = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{4}z^{-2}}{1 - \frac{1}{2}z^{-1}} = \frac{z^2 - \frac{1}{4}}{z^2 - \frac{1}{2}z}$$

$$= \frac{(z - \frac{1}{2})(z + \frac{1}{2})}{z(z - \frac{1}{2})} = \frac{z + \frac{1}{2}}{z}$$

$$\boxed{= 1 + \frac{1}{2}z^{-1}}$$

$y[n] = x[n] + \frac{1}{2}x[n-1]$

c) 

d) In terms of multiplication, the one in a) requires 2 but c) only needs 1.

In terms of addition, the one in a) requires 2, , c) only 1 too

In terms of memory, the one in a) requires 3, a c) only 2

The non-recursive version will accordingly give a simpler and fewer operations filter.