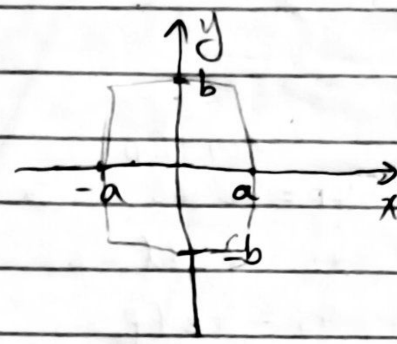
EC 574 PS 2

Problem 1



1) So schrodinger's equation tells us in 2D:

$$\begin{cases}\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\cancel{X}+V(x)\right]X(x)=E_x X(x)\\ \left[\frac{-\hbar^2}{2m}\frac{d^2}{dy^2}+V(y)\right]Y(y)=E_y Y(y)\end{cases}$$

as this is infinite well case so $V(x)$ $V(y)=0$ when within rectangular

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}X=E_x X(x)$$

C should be $\sqrt{\frac{4}{4ab}}=\sqrt{\frac{1}{ab}}$

should we use cosine here?

$$\frac{-\hbar^2}{2m}\frac{d^2}{dy^2}Y=E_y Y$$

Eigenfunction. use sin when x/y is odd and sin whenever

$$X(x)\,Y(y)=C\ \cos/\sin\left(\frac{n_x\pi}{2a}x\right)\cdot\cos/\sin\left(\frac{n_y\pi}{2b}y\right)$$

since we need to apply a shift of $+a$ then normal cos

$$E=\frac{n_x^2\hbar^2}{2m(2a)^2}+\frac{n_y^2\hbar^2}{2m(2b)^2}\rightarrow \text{Energy Eigen value}$$

$$=\frac{\hbar^2}{2m}\left(\frac{n_x^2}{4a^2}+\frac{n_y^2}{4b^2}\right)$$

$$=\frac{\hbar^2}{8m}\left(\frac{n_x^2}{a^2}+\frac{n_y^2}{b^2}\right)$$

the degeneracy could not be exactly determined as we don't know a and b exactly. but # of degeneracy is how many n_x, n_y such that $\frac{n_{x_1}^2}{a^2}+\frac{n_{y_1}^2}{b^2}=\frac{n_{x_2}^2}{a^2}+\frac{n_{y_2}^2}{b^2}$ but it will be less frequent than $a=b$ case.

2) So now Let's do the question

but we have to assume something:

the first 3 states are ψ_1, ψ_2, ψ_3

$$\int_{-b}^{b}\int_{-a}^{a}\left[C(\psi_1+\psi_2+\psi_3)\right]^2 dx\,dy$$

$$= |C|^2(\psi_1^2+\psi_2^2+\psi_3^2) = 1$$

$$C = \frac{1}{\sqrt{3}}$$

So $\psi(x,y) = \frac{1}{\sqrt{3}}(\psi_1+\psi_2+\psi_3)$

$X(x)\ Y(y)$

$\psi_1 = \psi_a \psi_b$

$\psi_2 = \psi_c \psi_d$

$\psi_3 = \psi_e \psi_f$

So in 3D / 2D,

$$\iint \psi_x(x)\,\psi_y(y)\ \psi_{x'}(x)\,\psi_{y'}(y)\,dx\,dy$$

$$= \int \psi_x \psi_{x'}\,dx \int \psi_y \psi_{y'}\,dy$$

and at least one is zero so $\psi_1 \psi_2$ are orthogonal

Problem 2: $k_x = 2k_y$

1) The total potential energy is $V(x,y) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2$

$$\omega_x = \sqrt{\frac{k_x}{m}} \qquad \omega_y = \sqrt{\frac{k_y}{m}}$$

According to general form of the equation

$$\psi_{n_x n_y}(x,y) = \psi_{n_x}(x)\,\psi_{n_y}(y)$$

with $\psi_n(u) = \left(\frac{\alpha}{\sqrt{\pi}\,2^n n!}\right)^{\frac{1}{2}} e^{-\frac{\alpha^2 u^2}{2}} H_n(\alpha u)$

$$\alpha_x = \left(\frac{m\omega_x}{\hbar}\right)^{\frac{1}{2}} \qquad \alpha_y = \left(\frac{m\omega_y}{\hbar}\right)^{\frac{1}{2}}$$

Now Let's come to energy which results in degeneracy.

$$E_{total} = E_x + E_y$$

$$E_x = \hbar\omega_x(n_x + \tfrac{1}{2}) \qquad E_y = \hbar\omega_y(n_y + \tfrac{1}{2})$$

$$\omega_x = \sqrt{\frac{k_x}{m}} \quad \omega_y = \sqrt{\frac{k_y}{m}} \quad \text{so } \omega_x = \sqrt{2}\,\omega_y$$

$$E_{n_x, n_y} = \hbar\omega_y\left(\sqrt{2}(n_x + \tfrac{1}{2}) + (n_y + \tfrac{1}{2})\right)$$

Now when (n_x, n_y) has multiple pairs we have degeneracy

$$\sqrt{2}n_x' + \cancel{\tfrac{1}{2}} + n_y' + \cancel{\tfrac{1}{2}} = \sqrt{2}n_x + \cancel{\tfrac{1}{2}} + n_y + \cancel{\tfrac{1}{2}}$$

$$\sqrt{2}n_x' + n_y' = \sqrt{2}n_x + n_y$$

Now this is very unlikely, as n_x, n_y are all integer so not very possible to $\times\sqrt{2}$ and still get a pair of integer.

2) $\langle x P_y \rangle$ and $\langle P_x y \rangle$

In terms of a^+ and a^-

$$x = \sqrt{\frac{\hbar}{2m\omega_x}}\,(a^+_x + a^-) \qquad P_x = i\sqrt{\frac{\hbar m \omega_x}{2}}\,(a^+_x - a^-_x)$$

and same thing happen to y

$$\text{So } \langle x P_y \rangle = \langle \psi_{nx} \psi_{ny} | (\overset{\text{operator}}{x \; P_y}) | \psi_{nx} \psi_{ny} \rangle$$

$$x \cdot P_y = \sqrt{\frac{\hbar}{2m\omega_x}}\,(a^+_x + a^-) \quad i\sqrt{\frac{\hbar m\omega_y}{2}}\,(a^+_y - a^-_y)$$

$$= i\frac{\hbar}{2}\sqrt{\frac{\omega_y}{\omega_x}}\,((a_x^+ + a_x^-)(a_y^+ + a_y^-))$$

$$= i\frac{\hbar}{2}\sqrt{\frac{\omega_y}{\omega_x}}\,(\underbrace{a_x^+ a_y^+ + a_x^+ a_y^- + a_x^- a_y^+ + a_x^- a_y^-}_{})$$

Note: So a_x and a_y are not the same thing due to different ω.

~~Now $a_x^+ |n_x\rangle = \sqrt{n_x+1}\,|n_x+1\rangle$ $\quad a_x^- |n_x\rangle = \sqrt{n_x}\,|n_x-1\rangle$~~

~~$a_y^+ |n_y\rangle = \sqrt{n_y+1}\,|n_y+1\rangle$ $\quad a_y^- |n_y\rangle = \sqrt{n_y}\,|n_y-1\rangle$~~

~~So $a_x^+ a_y^+ |n_x n_y\rangle =$~~

~~$a_x^+ a_y^- |n_x n_y\rangle =$~~

~~$a_x^- a_y^- |n_x n_y\rangle =$~~

same for otherwise

these are all zero when applying to

$$\langle n_x, n_y | a_x^+ a_y^+ | n_x, n_y \rangle$$

$$= \underset{0}{\langle n_x | a_x^+ | n_x \rangle} \cdot \underset{0}{\langle n_y | a_y^+ | n_y \rangle}$$

so

$$\boxed{\langle x P_y \rangle = 0}$$ and same thing works for $$\boxed{\langle y P_x \rangle = 0}$$

Problem 3

1) $\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = \frac{2mr^2}{\hbar^2}[V(r) - E]R + l(l+1)R$

$u(r) = rR(r)$

$R(r) = \frac{u(r)}{r}, \quad \frac{dR}{dr} = \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}$

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = \frac{d}{dr}\left(r^2\left[\frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right]\right)$$

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = \frac{d}{dr}\left(r\frac{du}{dr} - u\right)$$

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = r\frac{d^2u}{dr^2} + 2\frac{du}{dr} - \frac{du}{dr}$$

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = \frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} \qquad R(r) = \frac{u(r)}{r}$$

$\Downarrow$

$$\frac{d^2u}{dr^2} + \left[\frac{2m}{\hbar^2}[E - V(r)] - \frac{l(l+1)}{r^2}\right]u(r) = 0.$$

2) $l = 0$ then

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}(E - V(r))\,u(r) = 0 \qquad V(r) = \begin{cases} 0 & 0 \leq r \leq a \\ \infty & r > a \end{cases}$$

$$\frac{d^2u}{dr^2} + \frac{2mE}{\hbar}u(r) = 0 \qquad \text{when } 0 \leq r \leq a$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$u(r) = A\sin(kr)$ $\longrightarrow$ cos was removed to satisfy edge case

$u(a) = 0$

So $k = \frac{n\pi}{a}$

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2$$

After normalization, $\int_0^a |u(r)|^2 dr = 1$

$$u_n(r) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi r}{a}\right)$$

At $r=0$, $R_n(r)$ diverges due to $\frac{1}{r}$ term.

but in real $u(r)$ we remains as a finite value

3) $R_n(r) = \frac{u_n(r)}{r} = \sqrt{\frac{2}{a}} \frac{\sin\left(\frac{n\pi r}{a}\right)}{r}$

this is just infinite quantum well but with a radius factor. (particle in a box.

Problem 4

$$\psi_{100}(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} e^{-\frac{r}{a}}$$

1) $$\langle r\rangle = \int_0^\infty r \cdot |\psi_{100}(r)|^2 4\pi r^2 dr$$

$$= \int_0^\infty 4\pi r^3 \cdot \frac{1}{4\pi} \cdot \frac{4}{a^3} e^{-\frac{2r}{a}} dr$$

$$\int_0^\infty x^n e^{-kx} dx = \frac{n!}{k^{n+1}}$$

$$= \frac{16\pi}{a^3} \int_0^\infty r^3 e^{\frac{-2r}{a}} dr = \left(\frac{3!}{\left(\frac{2}{a}\right)^4}\right) \frac{4\pi}{a^3}$$

$$= 3a$$

$$\langle r^2\rangle = \int_0^\infty r^2 |\psi_{100}(r)|^2 4\pi r^2 dr$$

$$= \int_0^\infty 4\pi r^4 \frac{1}{4\pi} \frac{4}{a^3} e^{-\frac{2r}{a}} dr$$

$$= \frac{16\pi}{a^3} \int_0^\infty r^4 a^{\frac{-2r}{a}} dr = \frac{16\pi}{a^3}\left(\frac{24a^5}{\left(\frac{2}{a}\right)^5}\right)$$

$$= 6a^2$$

2) $\langle x\rangle = 0$ the wavefunction is spherically symmetric with respect to origin.

$\langle x^2\rangle = \frac{1}{3}\langle r^2\rangle$ and also due to spherically symmetric property.

$$= \frac{1}{3} 6a^2$$

$$= 2a^2$$

Problem 5

1) So similar to problem 1 except now $a=b$

So $E = \frac{\hbar^2 \pi^2}{8ma^2}(n_x^2 + n_y^2)$

And yes, there will be many degeneracy but different for different energy:

	n_x	n_y	# of degeneracy
E_1 :	1	0	2
	0	1	
E_2 :	1	1	1
E_3 :	1	2	2
	2	1	
E_4 :	2	2	1
⋮			
	3	4	this one has 4.
	4	3	
	0	5	
	5	0	

4+1

2) $X(x) = \begin{cases} \cos\left(\frac{n_x\pi}{2a}\right) & \text{when } n_x \text{ odd} \\ \sin\left(\frac{n_x\pi}{2a}\right) & \text{even} \end{cases}$

same for y

so general expression:

$$\left(\sqrt{\frac{2}{2a}}\right)^2 \begin{matrix}\cos\\ \text{or}\\ \sin\end{matrix}\left(\frac{n_x\pi}{2a}x\right) \begin{matrix}\cos\\ \text{or}\\ \sin\end{matrix}\left(\frac{n_y\pi}{2a}y\right)$$

$$= \frac{1}{a} \begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{n_x\pi}{2a}x\right) \begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{n_y\pi}{2a}y\right)$$

In the book

Problem 6: $L_\pm = L_x \pm i L_y$

1) $L_z = -i\hbar \frac{\partial}{\partial \phi}$ the operator

So $L_z Y(\theta,\phi) = -i\hbar \frac{\partial}{\partial\phi} \frac{\partial Y(\theta,\phi)}{\partial \phi}$

$= -i\hbar \frac{\partial}{\partial\phi} [\Theta(\theta) \cdot \Phi(\phi)]$

$-i\hbar \frac{\partial \Phi(\phi)}{\partial \phi} = \lambda \Phi(\phi) \Rightarrow \frac{\partial \Phi(\phi)}{\partial \phi} = i \frac{\lambda}{\hbar} \Phi(\phi)$

general solution is $Ce^{i\frac{\lambda}{\hbar}\phi}$

and because of periodic: $\Phi(\phi + 2\pi) = \Phi(\phi)$

So $Ce^{i\frac{\lambda}{\hbar}(\phi+2\pi)} = Ce^{i\frac{\lambda}{\hbar}\phi}$

So $e^{i\frac{\lambda}{\hbar}2\pi} = 1$

$\frac{\lambda}{\hbar} 2\pi = 2\pi m$

$\lambda = m\hbar$

2) $[L_z, L_\pm] = L_z \cdot L_\pm - L_\pm \cdot L_z$

$= [L_z, L_x \pm i L_y] = L_z(L_x \pm i L_y) - (L_x \pm i L_y) L_z$

$= [L_z, L_x] \pm i [L_z, L_y]$

From their equation, we have $[L_z, L_x] = i\hbar L_y$

$[L_z, L_y] = -i\hbar L_x$

So $[L_z, L_\pm] = i\hbar L_y \mp i \cdot i\hbar L_x = i\hbar L_y \pm \hbar L_x$

$= \pm\hbar L_\pm$

3) $L_z(L_\pm \phi_m)$

ϕ_m is eigen function of L_z so $L_z \phi_m = m\hbar \phi_m$.

$L_z(L_\pm \phi_m) = L_\pm(L_z \phi_m) + [L_z, L_\pm]\phi_m$

$= L_\pm m\hbar \phi_m + (\pm\hbar L_\pm \phi_m)$

$= (m\pm1)\hbar \phi_m$

Problem 7:

$\langle xy^2 \rangle = ?$

Since this is isotropic harmonic oscillator

$\psi_{nm}(x,y) = \psi_{nx}(x)\,\psi_{ny}(y)$

$\psi_{01}(x,y) = \psi_0(x)\,\psi_1(y)$

$\psi_{10}(x,y) = \psi_1(x)\,\psi_0(y)$.

$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2}$ $\quad\psi_1 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{2}{\sqrt{2}} \sqrt{\frac{m\omega}{\hbar}}\, x\, e^{-\frac{m\omega}{2\hbar}x^2}$

① $\psi_{01}(x,y) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{2}{\sqrt{2}} \sqrt{\frac{m\omega}{\hbar}}\, y\, e^{-\frac{m\omega}{2\hbar}y^2}$

$= \boxed{\frac{m\omega \cdot 2}{\hbar\sqrt{\pi}\sqrt{2}}}\, e^{-\frac{m\omega}{2\hbar}(x^2+y^2)} \cdot y$ all real part

Constant take out

$\langle xy^2 \rangle = \iint xy^2\, dx dy = \iint \Big(\underbrace{e^{-\frac{m\omega}{2\hbar}x^2}}_{\psi_0} \cdot \underbrace{e^{(-\frac{m\omega}{2\hbar})y^2}\, y}_{\psi_1}\Big)^2 \cdot xy^2\, dx dy$

$= \underbrace{\int e^{-\frac{m\omega}{\hbar}x^2} \cdot x\, dx}_{\swarrow \text{ so zero.}} \underbrace{\int e^{-\frac{m\omega}{\hbar}y^2} \cdot y^4\, dy}$

odd

ψ_0 is even

ψ_1 is odd

$= 0$

② $\psi_{10}(x,y)$

$\langle xy^2 \rangle = \iint xy^2\, dx dy = \int e^{-\frac{m\omega}{\hbar}x^2} \cdot x^2 \cdot x\, dx \int e^{-\frac{m\omega}{\hbar}y^2} \cdot y^2\, dy$

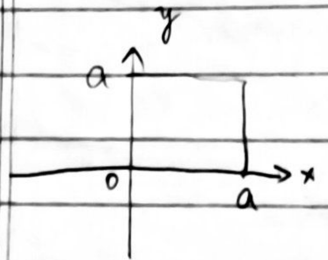
$= \underbrace{\int e^{-\frac{m\omega}{\hbar}x^2} x^3\, dx}_{\swarrow \text{ odd}} \int e^{-\frac{m\omega}{\hbar}y^2} \cdot y^2 dy$

so still zero

$= 0$

Okay.

Problem 8



1) Same as 1 and 5 but now we could apply general sin fomat as we start from 0

$X(x) = \sqrt{\frac{1}{a}} \sin\left(\frac{n_x \pi}{a} x\right)$, $Y(y) = \sqrt{\frac{1}{a}} \sin\left(\frac{n_y \pi}{a} y\right)$

Now X and Y are normalized and so will XY.

So $\psi = \frac{1}{a} \sin\left(\frac{n_x \pi}{a} x\right) \sin\left(\frac{n_y \pi}{a} y\right)$

$E_{n_x, n_y} = \frac{\hbar^2 \pi^2}{2ma^2}(n_x^2 + n_y^2)$ So they do have degeneracy since $n_x^2 + n_y^2$ could $= n_x'^2 + n_y'^2$

2) ground state: $n_x, n_y = (1, 1)$ nice story is we don't have degeneracy in this state.

the unperturbed wavefunction is

$\delta = V_0$ at $\left(\frac{a}{4}, \frac{3}{4}a\right)$ $\quad \psi_{1,1}(x, y) = \frac{2}{a} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{a}\right)$

V_1 at $\left(\frac{3}{4}a, \frac{1}{4}a\right)$ $\Rightarrow \langle n | H' | n \rangle = V_0 \psi_{1,1}\left(\frac{a}{4}, \frac{3}{4}a\right)^2 + V_1 \psi_{1,1}\left(\frac{3}{4}a, \frac{1}{4}a\right)^2$

$= V_0\left(\frac{1}{a}\right)^2 + V_1\left(\frac{1}{a}\right)^2$

$E_{1,1}^{(1)} = \frac{V_0 + V_1}{a^2}$

First E state: $(n_x, n_y) = (2, 1)$ and $(1, 2)$

$W_{ij} = \langle n | H' | n \rangle$

$\begin{pmatrix} ①① & ①② \\ ②① & ②② \end{pmatrix}$

$\psi_{2,1}\left(\frac{a}{4}, \frac{3}{4}a\right) = \frac{\sqrt{2}}{a}$

$\psi_{2,1}\left(\frac{3}{4}a, \frac{1}{4}a\right) = -\frac{\sqrt{2}}{a}$

$\psi_{1,2}\left(\frac{a}{4}, \frac{3}{4}a\right) = \frac{\sqrt{2}}{a}$

$\psi_{1,2}\left(\frac{3}{4}a, \frac{a}{4}\right) = \frac{\sqrt{2}}{a}$

So $W = \frac{2}{a^2}\begin{pmatrix} V_0 + V_1 & V_0 - V_1 \\ V_0 - V_1 & V_0 + V_1 \end{pmatrix}$ $\quad \lambda = \frac{4V_0}{a^2} \,/\, \frac{4V_1}{a^2}$

$E_{1,1}^{(1)} = \frac{4V_0}{a^2}$ and $\frac{4V_1}{a^2}$

3) $\psi_+ = \frac{1}{\sqrt{2}} (\psi_{2,1} + \psi_{1,2})$

$\psi_- = \frac{1}{\sqrt{2}} (\psi_{2,1} - \psi_{1,2})$

Problem 9: 1D oscillator has $E_n = (n+\frac{1}{2})\hbar\omega$ with no degeneracy.

1) $H' = cx$

$E_n^{(1)} = \langle n | H' | n \rangle$

$= \int \psi_n (cx) \cdot \psi_n \, dx$

$= \underbrace{\int \psi_n^2 (cx)}_{\text{odd}}$ Now because ψ_n is either odd or even so ψ_n^2 is always even so

$= 0$

2) $H' = cx^3$

Now same as previous, $\psi_n^2 (cx^3)$ is odd so 0

$E_n^{(1)} = 0$

3) $H' = x^4$

$E_n^{(1)} = \langle n | H' | n \rangle$ $x = \sqrt{\frac{\hbar}{2m\omega}} (a^- + a^+)$

$= \int \psi_n (cx^4) \psi_n \, dx$

$= c \int \psi_n^2 x^4 \, dx$

$= c \cdot \left(\frac{\hbar}{2m\omega}\right)^2 \langle \psi_n | (a^- + a^+)^4 | \psi_n \rangle$

$= c \cdot \left(\frac{\hbar}{2m\omega}\right)^2 \langle \psi_n | \underset{0}{a^{-4}} + 4a^{-3}a^+ + 6a^2(a^+)^2 + 4a(a^+)^3 + \underset{0}{(a^+)^4} | \psi_n \rangle$

$= \frac{3c\hbar^2}{4m^2\omega^2} (2n^2 + 2n + 1)$

Problem 10:

$H' = \frac{p^2}{2m} + \frac{1}{2}k(x^2+y^2)$

1) $\omega = \sqrt{\frac{k}{m}}$

$H' = \frac{1}{2m}(p_x^2+p_y^2) + \frac{1}{2}m\omega^2(x^2+y^2)$

$\psi(x,y) = \psi_x(x)\,\psi_y(y)$

$$\begin{cases} E = E_x+E_y \\ \left[\frac{p^2}{2m}+\frac{1}{2}kx^2\right]\psi_x(x) = E_x\psi_x(x) \\ \left[\frac{p^2}{2m}+\frac{1}{2}kx^2\right]\psi_y(y) = E_y\psi_y(y) \end{cases}$$

$\psi_x(x)$: general HO function to x with n_x

$\psi_y(y)$: general HO function to y with n_y

$E_x = \hbar\omega(n_x+\frac{1}{2})$ $\quad E_y = \hbar\omega(n_y+\frac{1}{2})$

$E = \hbar\omega(n_x+n_y+1)$

Degeneracy: any n_x+n_y who share some value

# of degeneracy: suppose $n_x+n_y = n$ $\Rightarrow$ $n+1$ solution

2) Okay Consider second state we have 2 possible combination

$$\begin{cases} n_x = 0 \\ n_y = 1 \end{cases} \quad \text{and} \quad \begin{cases} n_x = 1 \\ n_y = 0 \end{cases}$$

we end up with 2 ψ_{10} ψ_{01}

$\psi_{1,0}(x,y) = \psi_1(x)\psi_0(y)$ $\qquad H' = cx^4y^4$

$\psi_{0,1}(x,y) = \psi_0(x)\psi_1(y)$

$W_{ab} = \langle\psi_a|H'\psi_b\rangle$ $\qquad$ we compute matrix $\begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix}$

we will get $\begin{pmatrix} & \end{pmatrix}$

Problem 10 continue

$$W_{aa} = c\int |\psi_1(x)|^2 x^4 dx \int |\psi_0(y)|^2 y^4 dy$$

$$= c \cdot \frac{45\hbar^4}{16m^4\omega^4}$$

$$W_{bb} = c\int |\psi_0(x)|^2 x^4 dx \int |\psi_1(y)|^2 y^4 dy$$

$$= c \cdot \frac{45\hbar^4}{16m^4\omega^4}$$

$W_{ab} = W_{ba} = 0$ as ψ_1 odd to ψ_0 even make integrals odd which goes to zero.

So $W = \frac{45\hbar^4}{16m^4\omega^4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad (1-\lambda)^2 = 0 \quad \lambda = 1$

$1-\lambda = 0$

Now the two actually merge into 1 with (only in Energy Term)

$$E^{(1)} = \frac{45c\hbar^4}{16m^4\omega^4}$$

Problem 11 For quantum box $-a<x<a$, $-a<y<a$, $-a<z<a$

then we know

$$\psi_{n_x n_y n_z} = \sqrt{\frac{1}{a^3}} \sin\left(\frac{n_x\pi}{2a}(x+a)\right)\sin\left(\frac{n_y\pi}{2a}(y+a)\right)\sin\left(\frac{n_z\pi}{2a}(z+a)\right)$$

$n_x, n_y, n_z = 1, 2, 3 \cdots$

$$E = \frac{\hbar^2\pi^2}{8ma^2}(n_x^2 + n_y^2 + n_z^2)$$

Now second level is 1, 1, 2 which has some degeneracy are degeneracy.

$(n_x, n_y, n_z) =$ 1, 1, 2

1, 2, 1

2, 1, 1

$W_{ij} = \langle \psi_i | H' | \psi_j \rangle \qquad H' = c x^2 z^4$

$W_{11} = c \, \frac{a^2}{3}\left(1 - \frac{6}{n_x^2 \pi^2}\right) \, 1 \cdot \frac{a^4}{5}\left(1 - \frac{20}{n_z^2 \pi^2} + \frac{120}{n_z^4 \pi^4}\right)$

$W_{22} = c \cdot \frac{a^2}{3}\left(1 - \frac{6}{\pi^2}\right) \cdot \frac{a^4}{5}\left(1 - \frac{20}{\pi^2} + \frac{120}{\pi^4}\right)$

$W_{33} = c \cdot \frac{a^2}{3}\left(1 - \frac{6}{4\pi^2}\right) \; \frac{a^4}{5}\left(1 - \frac{20}{\pi^2} + \frac{120}{\pi^4}\right)$

W_{ij} other would be zero as at least one in x/y would have a odd property that goes to zero

$E^{(1)} = W_{11} / W_{22} / W_{33}$ has 3 possible values

Previous one doubled

Problem 12

1) $H' = -qEz$

So z in polar coordinate is $r\cos\theta$ so $H' = -qEr\cos\theta$

$= -qEr\sqrt{\frac{4\pi}{3}}\, Y_{1,0}(\theta,\phi)$

Now we see there are four

$\psi_1 = \psi_{2,0,0} \qquad \psi_2 = \psi_{2,1,1} \qquad \psi_3 = \psi_{2,1,0} \qquad \psi_4 = \psi_{2,1,-1}$

$W_{ij} = \langle \psi_i | H' | \psi_j \rangle$

Now with different i,j: $2,0,0;\ 2,1,1 : Y_{00} \cdot Y_{11} \cdot Y_{1,0} \rightarrow 0$

$2,0,0;\ 2,1,0 : Y_{00}, Y_{10}, Y_{1,0} \rightarrow \neq 0$

Only nonzero term: $2,0,0;\ 2,1,-1:$

$W_{ii}:$ $2,1,1;\ 2,1,0:$

$2,1,1;\ 2,1,-1:$

$2,1,0;\ 2,1,-1:$

Problem 12 Continue

$$\begin{pmatrix} \psi_{200}\psi_{2,100} & \psi_{200}\psi_{211} & \psi_{200}\psi_{210} & \psi_{200},\psi_{21-1} \\ \psi_{2,1,1}\psi_{2,0,0} & \psi_{211}\psi_{211} & \psi_{211}\psi_{210} & \psi_{211}\psi_{21-1} \\ \psi_{210}\psi_{,200} & \psi_{210}\psi_{211} & \psi_{210}\psi_{210} & \psi_{210}\psi_{21-1} \\ \psi_{21-1}\psi_{200} & \psi_{21-1}\psi_{211} & \psi_{21-1}\psi_{211} & \psi_{21-1}\psi_{21-1} \end{pmatrix}$$

So others are all orthogonal except ψ_{210}

After calculation, we get

$$W = \begin{pmatrix} -210a_0^3 & 0 & -150\sqrt{2}Ea_0^3q & 0 \\ 0 & -210a_0^3 & 0 & 0 \\ 150\sqrt{2}Ea_0^3q & 0 & -210a_0^3 & 0 \\ 0 & 0 & 0 & -210a_0^3 \end{pmatrix}$$

Solving the $\det(W)=0$ when $E = \pm 150\sqrt{2}Ea_0^3q$

$\psi_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $\psi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\psi_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\psi_4 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Problem 13

1) $x' = x - \frac{qE}{m\omega^2}$

$V(x) = \frac{1}{2} m\omega^2 x^2 - qEx$

$= \frac{1}{2} m\omega^2 \left(x' + \frac{qE}{m\omega^2}\right)^2 - qE\left(x' + \frac{qE}{m\omega^2}\right)$

$V(x') = \frac{1}{2} m\omega^2 x'^2 - \frac{(qE)^2}{2m\omega^2}$

$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x'^2 - \underbrace{\frac{(qE)^2}{2m\omega^2}}_{\text{Constant}}$

and so $E_n = (n+\frac{1}{2})\hbar\omega - \frac{(qE)^2}{2m\omega^2}$

2) $E_n^{(1)} = \langle n|H'|n\rangle$

$= -qE\langle n|x|n\rangle$ as we know from HO, $E_n^{(1)} = 0$.

$E_n^{(2)} = \sum_{m\neq n} \frac{|\langle m|H'|n\rangle|^2}{E_n^{(0)} - E_m^{(0)}}$ → ~~non-zero only m=n±1~~

$= \frac{q^2E^2}{2m\omega}\left(\frac{n}{E_n^{(0)} - E_{n-1}^{(0)}} + \frac{n+1}{E_n^{(0)} - E_{n+1}^{(0)}}\right)$

$= \frac{q^2E^2}{2m\omega}\left(\frac{n}{\hbar\omega} - \frac{n+1}{\hbar\omega}\right)$

$= \frac{q^2E^2}{2m\omega^2}$

which is the same as that from 1)