



# Problem Set #1

5 questions

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1.

3-way-Merge Sort : Suppose that instead of dividing in half at each step of Merge Sort, you divide into thirds, sort each third, and finally combine all of them using a three-way merge subroutine. What is the overall asymptotic running time of this algorithm? (Hint: Note that the merge step can still be implemented in  $O(n)$  time.)

- ☐  $n(\log(n))^2$
- ☐  $n$
- ☐  $n \log(n)$
- ☐  $n^2 \log(n)$

1  
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2.

You are given functions  $f$  and  $g$  such that  $f(n) = O(g(n))$ . Is  $f(n) * \log_2(f(n)^c) = O(g(n) * \log_2(g(n)))$  ? (Here  $c$  is some positive constant.) You should assume that  $f$  and  $g$  are nondecreasing and always bigger than 1.

- ☐ False
- ☐ True
- ☐ Sometimes yes, sometimes no, depending on the constant  $c$
- ☐ Sometimes yes, sometimes no, depending on the functions  $f$  and  $g$

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3.

Assume again two (positive) nondecreasing functions  $f$  and  $g$  such that  $f(n) = O(g(n))$ . Is  $2^{f(n)} = O(2^{g(n)})$ ? (Multiple answers may be correct, you should check all of those that apply.)

- ☐ Sometimes yes, sometimes no (depending on  $f$  and  $g$ )
  - ☐ Never
  - ☐ Always
  - ☐ Yes if  $f(n) \leq g(n)$  for all sufficiently large  $n$
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4.

k-way-Merge Sort. Suppose you are given  $k$  sorted arrays, each with  $n$  elements, and you want to combine them into a single array of  $kn$  elements. Consider the following approach. Using the merge subroutine taught in lecture, you merge the first 2 arrays, then merge the 3<sup>rd</sup> given array with this merged version of the first two arrays, then merge the 4<sup>th</sup> given array with the merged version of the first three arrays, and so on until you merge in the final ( $k^{\text{th}}$ ) input array. What is the running time taken by this successive merging algorithm, as a function of  $k$  and  $n$ ? (Optional: can you think of a faster way to do the k-way merge procedure?)

- ☐  $\theta(n \log(k))$
  - ☐  $\theta(n^2 k)$
  - ☐  $\theta(nk^2)$
  - ☐  $\theta(nk)$
- 

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5.

Arrange the following functions in increasing order of growth rate (with  $g(n)$  following  $f(n)$  in your list if and only if  $f(n) = O(g(n))$ ).

a)  $n^2 \log(n)$

b)  $2^n$

c)  $2^{2^n}$

d)  $n^{\log(n)}$

e)  $n^2$

Write your 5-letter answer, i.e., the sequence in lower case letters in the space provided. For example, if you feel that the answer is a->b->c->d->e (from smallest to largest), then type abcde in the space provided without any spaces before / after / in between the string.

You can assume that all logarithms are base 2 (though it actually doesn't matter).

WARNING: this question has multiple versions, you might see different ones on different attempts!

Preview

Enter math expression here

4 questions unanswered

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