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Problem Set #3

5 questions

1 point

1.

Let $0 < \alpha < .5$ be some constant (independent of the input array length n). Recall the Partition subroutine employed by the QuickSort algorithm, as explained in lecture. What is the probability that, with a randomly chosen pivot element, the Partition subroutine produces a split in which the size of the smaller of the two subarrays is $\geq \alpha$ times the size of the original array?

- $1 - 2 * \alpha$
- α
- $1 - \alpha$
- $2 - 2 * \alpha$

1 point

2.

Now assume that you achieve the approximately balanced splits above in every recursive call --- that is, assume that whenever a recursive call is given an array of length k, then each of its two recursive calls is passed a subarray with length between αk and $(1 - \alpha)k$ (where α is a fixed constant strictly between 0 and $.5$). How many recursive calls can occur before you hit the base case? Equivalently, which levels of the recursion tree can contain leaves? Express your answer as a range of possible numbers d, from the minimum to the maximum number of recursive calls that might be needed.

- $-\frac{\log(n)}{\log(\alpha)} \leq d \leq -\frac{\log(n)}{\log(1-\alpha)}$

- $0 \leq d \leq -\frac{\log(n)}{\log(\alpha)}$
- $-\frac{\log(n)}{\log(1-\alpha)} \leq d \leq -\frac{\log(n)}{\log(\alpha)}$
- $-\frac{\log(n)}{\log(1-2*\alpha)} \leq d \leq -\frac{\log(n)}{\log(1-\alpha)}$

1
point

3.

Define the recursion depth of QuickSort to be the maximum number of successive recursive calls before it hits the base case --- equivalently, the number of the last level of the corresponding recursion tree. Note that the recursion depth is a random variable, which depends on which pivots get chosen. What is the minimum-possible and maximum-possible recursion depth of QuickSort, respectively?

- Minimum: $\Theta(\log(n))$; Maximum: $\Theta(n)$
- Minimum: $\Theta(\log(n))$; Maximum: $\Theta(n\log(n))$
- Minimum: $\Theta(1)$; Maximum: $\Theta(n)$
- Minimum: $\Theta(\sqrt{n})$; Maximum: $\Theta(n)$

1
point

4.

Consider a group of k people. Assume that each person's birthday is drawn uniformly at random from the 365 possibilities. (And ignore leap years.) What is the smallest value of k such that the expected number of pairs of distinct people with the same birthday is at least one?

[Hint: define an indicator random variable for each ordered pair of people. Use linearity of expectation.]

- 20
- 23

- 366
- 28
- 27

1 point

5.

Let X_1, X_2, X_3 denote the outcomes of three rolls of a six-sided die. (I.e., each X_i is uniformly distributed among $1, 2, 3, 4, 5, 6$, and by assumption they are independent.) Let Y denote the product of X_1 and X_2 and Z the product of X_2 and X_3. Which of the following statements is correct?

- Y and Z are independent, but $E[Y * Z] \neq E[Y] * E[Z]$.
- Y and Z are not independent, and $E[Y * Z] \neq E[Y] * E[Z]$.
- Y and Z are independent, and $E[Y * Z] = E[Y] * E[Z]$.
- Y and Z are not independent, but $E[Y * Z] = E[Y] * E[Z]$.

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