

2022年11月19日

2021.09.21





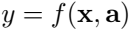


$$x^* = \arg \min_x f(x),$$

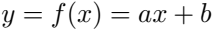








$D = \frac{1}{2} \left(\frac{1}{2} \right) = \frac{1}{4}$







$$o(a) = \sum_{n=1}^N [y_n - f((x_n, a))]^2$$

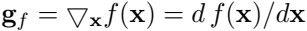


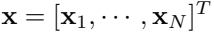
$$o(a) = L(a|\mathcal{D}) = p(\mathcal{D}|a) = \prod_{n=1}^N p(y_n|x_n,a)$$





$$\mathbf{g}_f = \nabla_{\mathbf{x}} f(\mathbf{x}) = \frac{df(\mathbf{x})}{d\mathbf{x}} = \left[\frac{\partial f(\mathbf{x})}{\partial x_1}, \dots, \frac{\partial f(\mathbf{x})}{\partial x_N} \right]^T = \mathbf{0}$$







$$\mathbf{H}_f = \begin{bmatrix} \frac{\partial^2 f(\mathbf{x})}{\partial x_1^2} & \cdots & \frac{\partial^2 f(\mathbf{x})}{\partial x_1 \partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\mathbf{x})}{\partial x_N \partial x_1} & \cdots & \frac{\partial^2 f(\mathbf{x})}{\partial x_N^2} \end{bmatrix}_{\mathbf{x}=\mathbf{x}^*}$$

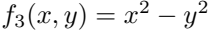






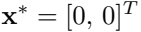
1. $x^2 + x^2$

2020-2021



$$g_{f_1} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}, \quad g_{f_2} = \begin{bmatrix} -2x \\ -2y \end{bmatrix}, \quad g_{f_3} = \begin{bmatrix} 2x \\ -2y \end{bmatrix}$$

$$\mathbf{H}_{f_1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} > 0; \quad \mathbf{H}_{f_2} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} < 0; \quad \mathbf{H}_{f_3} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$



1234567890







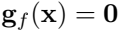


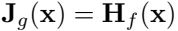


$$\mathbf{g}_f(\mathbf{x}) = \nabla_{\mathbf{x}} f(\mathbf{x}) = \frac{df(\mathbf{x})}{d\mathbf{x}} = \begin{bmatrix} \partial f / \partial x_1 \\ \vdots \\ \partial f / \partial x_N \end{bmatrix} = \mathbf{0}$$



$$x_{n+1} = x_n - \frac{1}{f'(x_n)}$$



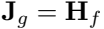




$$x_{n+1} - x_n \leq \sqrt[n]{g(x_n)} - \sqrt[n]{g(x_n)} = x_n - \sqrt[n]{g(x_n)}$$







EXERCISES ON THE

$$f(\mathbf{x}) = \frac{1}{2} \mathbf{g}^T(\mathbf{x}) \mathbf{g}(\mathbf{x}) = \frac{1}{2} ||\mathbf{g}(\mathbf{x})||^2 = \frac{1}{2} \sum_{i=1}^N |g_i(\mathbf{x})|^2$$

$$\frac{\partial f(\mathbf{x})}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{1}{2} \sum_{i=1}^N (g_i(\mathbf{x}))^2 \right) = \sum_{i=1}^N g_i(\mathbf{x}) \frac{\partial g_i(\mathbf{x})}{\partial x_j} = 0, \quad (j = 1, \dots, N)$$



1982

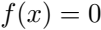
$$\nabla_x f(x) = \frac{d}{dx} \left[\frac{1}{2} g^T(x) g(x) \right] = \frac{d}{dx} g(x) \cdot g(x) = \mathbf{J}_g(x) g(x) = \mathbf{0}$$

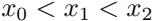












A pixelated, black and white graphic of the text "100% 100%". The text is rendered in a stylized, handwritten font with a dithered or pixelated appearance, giving it a retro, digital feel. The characters are bold and slightly irregular, with varying shades of gray used for shading and texture. The overall composition is simple and centered.

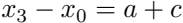


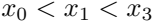




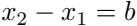


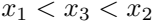
1920-21

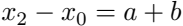




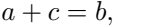
1920-21













$a + b$

b

$=$

$a + c$

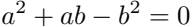
a

$=$

b

a

$$\frac{a + b}{b} = \frac{b}{b - c} = \frac{b}{a}$$



$$a_{1,2} = \frac{-1 \pm \sqrt{5}}{2} b = \begin{cases} 0.618 b \\ -1.618 b \end{cases}$$

$$\frac{a}{b} = 0.618, \quad \frac{b}{a+b} = \frac{b}{0.618b+b} = \frac{1}{1.618} = 0.618$$





2020-01-02





$$x_0^{\text{new}} = x_1^{\text{old}}, \quad x_1^{\text{new}} = x_2^{\text{old}}, \quad x_2^{\text{new}} = x_2^{\text{old}} + c(x_2^{\text{old}} - x_1^{\text{old}})$$













for the first time



$$q(x) = f(a) \frac{(x-b)(x-c)}{(a-b)(a-c)} + f(b) \frac{(x-c)(x-a)}{(b-c)(b-a)} + f(c) \frac{(x-a)(x-b)}{(c-a)(c-b)}$$



$$g'(x) = f(a) \frac{(x-b) + (x-c)}{(a-b)(a-c)} + f(b) \frac{(x-c) + (x-a)}{(b-c)(b-a)} + f(c) \frac{(x-a) + (x-b)}{(c-a)(c-b)} = 0$$



$$f(a)(c-b)(2a-b-c) + f(b)(a-c)(2a-c-a) + f(c)(b-a)(2a-a-b)$$



$$2a[f(a)(c-b) + f(b)(a-c)] - [f(a)(c^2 - b^2) + f(b)(a^2 - c^2)] = 0$$



$$1 \quad \frac{f(a)(c^2 - b^2) + f(b)(a^2 - c^2) + f(c)(b^2 - a^2)}{2}$$

$$2 \quad f(a)(c - b) + f(b)(a - c) + f(c)(b - a)$$

$$b + \frac{1}{2} \frac{f(a)(c-b)(c+b-2b) + f(b)(a-c)(a+c-2b) + f(c)(b-a)(b+a-2b)}{f(a)(c-b) + f(b)(a-c) + f(c)(b-a)}$$

$$b + \frac{1}{2} \frac{f(a)(c-b)^2 + f(b)(a-c)(a+c-2b) - f(c)(b-a)^2}{f(a)(c-b) + f(b)(a-c) + f(c)(b-a)}$$

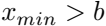
$$b + \frac{1}{2} \frac{[f(a) - f(b)](c - b)^2 - [f(c) - f(b)](b - a)^2}{[f(a) - f(b)](c - b) + [f(c) - f(b)](b - a)}$$

Wormholes

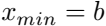




A pixelated, black and white graphic of the word "Wikipedia" in a stylized, blocky font. The letters are composed of various shades of gray, giving it a digital or retro aesthetic. The word is centered horizontally.



$$\left[f(d) - f(d^2) \right] (d - d^2) = \left[f(d) - f(d^2) \right] (d - d^2)$$







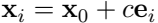


2019.09.27

$$S = \left\{ \sum_{i=0}^N c_i x_i \mid \sum_{i=0}^N c_i = 1, \quad c_i \geq 0 \right\}$$



1999-2000









9 = 12









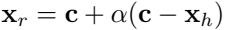
for the first time in the history of the world.



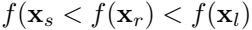




$$\mathbf{c} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$$



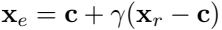








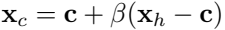
















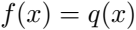


$$f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \cdots + \frac{1}{n!}f^{(n)}(x_0)(x - x_0)^n + \cdots$$



$$f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 = q(x)$$

A pixelated black and white illustration. On the left is a large, stylized letter 'Q' with a thick, dark vertical stroke and a lighter, curved tail. In the center is an equals sign, represented by two horizontal bars of different shades of gray. On the right is another large, stylized letter 'Q', identical to the one on the left. The entire image has a low-resolution, dithered appearance.





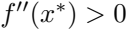
$$\frac{d}{dx}g(x) = \frac{d}{dx}\left[f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2\right]$$

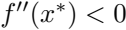
$$\frac{d}{dx} \left(x^2 + \frac{1}{x} \right) = 2x - \frac{1}{x^2}$$

$$x^* = x_0 - \frac{f'(x_0)}{f''(x_0)} = x_0 + \Delta x_0$$











$$x_{n+1} = x_n + \Delta x_n = x_n - \frac{f'(x_n)}{f''(x_n)}$$



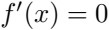








$$x^2 + 1 = x^2 + 2x + 1 = (x + 1)^2$$



$$q(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$$

$$q(x) = \frac{a(x-b)^2}{2} + c$$

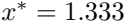
$$\begin{cases} a = f''(x_0) \\ b = x_0 - f'(x_0)/f''(x_0) \\ c = f(x_0) - f'^2(x_0)/2f''(x_0) \end{cases}$$





QWERTY QWERTY

$$f(x) = 2x^3 - 4x^2 + x, \quad f'(x) = 6x^2 - 8x + 1, \quad f''(x) = 12x - 8$$



100%



$$f(x_0) = f(3) = 21, \quad f'(x_0) = f'(3) = 31, \quad f''(x_0) = f''(3) = 28$$

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$$\begin{cases} a = f''(3) = 28 \\ b = 3 - f'(3)/f''(3) = 1.8929 \\ c = f(3) - f'^2(3)/2f''(3) = 3.8393 \end{cases}$$

$\varphi(1) = 1$
 $\varphi(2) = 1$
 $\varphi(3) = 2$
 $\varphi(4) = 2$
 $\varphi(5) = 4$
 $\varphi(6) = 2$
 $\varphi(7) = 6$
 $\varphi(8) = 4$
 $\varphi(9) = 6$
 $\varphi(10) = 4$
 $\varphi(11) = 10$
 $\varphi(12) = 4$
 $\varphi(13) = 12$
 $\varphi(14) = 6$
 $\varphi(15) = 8$
 $\varphi(16) = 8$
 $\varphi(17) = 16$
 $\varphi(18) = 6$
 $\varphi(19) = 18$
 $\varphi(20) = 8$
 $\varphi(21) = 12$
 $\varphi(22) = 10$
 $\varphi(23) = 22$
 $\varphi(24) = 8$
 $\varphi(25) = 20$
 $\varphi(26) = 12$
 $\varphi(27) = 18$
 $\varphi(28) = 12$
 $\varphi(29) = 28$
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 $\varphi(58) = 28$
 $\varphi(59) = 58$
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 $\varphi(62) = 30$
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 $\varphi(66) = 20$
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 $\varphi(69) = 44$
 $\varphi(70) = 24$
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 $\varphi(73) = 72$
 $\varphi(74) = 36$
 $\varphi(75) = 40$
 $\varphi(76) = 36$
 $\varphi(77) = 60$
 $\varphi(78) = 24$
 $\varphi(79) = 78$
 $\varphi(80) = 32$
 $\varphi(81) = 54$
 $\varphi(82) = 40$
 $\varphi(83) = 82$
 $\varphi(84) = 24$
 $\varphi(85) = 64$
 $\varphi(86) = 42$
 $\varphi(87) = 56$
 $\varphi(88) = 40$
 $\varphi(89) = 88$
 $\varphi(90) = 24$
 $\varphi(91) = 80$
 $\varphi(92) = 44$
 $\varphi(93) = 72$
 $\varphi(94) = 46$
 $\varphi(95) = 72$
 $\varphi(96) = 32$
 $\varphi(97) = 96$
 $\varphi(98) = 48$
 $\varphi(99) = 96$
 $\varphi(100) = 40$

for the first time

$x = x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$

A pixelated, black and white graphic of the text "x1=x0+x1+x0". The characters are rendered in a thick, blocky, and slightly irregular font, reminiscent of early digital art or video game text. The "x" characters are formed by intersecting lines, and the "0" characters are solid blocks with a small opening at the top. The entire text is set against a plain white background.

1991-1992



$$f(x) \approx f(x_0) + g_0^T (x - x_0) + \frac{1}{2} (x - x_0)^T \mathbf{H}_0 (x - x_0) =: q(x)$$



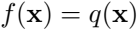




$$\mathbf{g}_f(\mathbf{x}_0) = \frac{d}{d\mathbf{x}} f(\mathbf{x}_0) = \begin{bmatrix} \frac{\partial f(\mathbf{x}_0)}{\partial x_1} \\ \vdots \\ \frac{\partial f(\mathbf{x}_0)}{\partial x_N} \end{bmatrix},$$



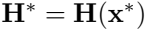
$$\mathbf{H}_f(\mathbf{x}_0) = \frac{d}{d\mathbf{x}} \mathbf{g}(\mathbf{x}_0) = \frac{d^2}{d\mathbf{x}^2} f(\mathbf{x}_0) = \begin{bmatrix} \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_1^2} & \cdots & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_1 \partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_N \partial x_1} & \cdots & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_N^2} \end{bmatrix}$$



$$\frac{d}{dx} \left[f(x_0) + g_0^T (x - x_0) + \frac{1}{2} (x - x_0)^T \mathbf{H}_0 (x - x_0) \right] = g_0 + \mathbf{H}_0 (x - x_0) = 0$$

EXPERIMENTAL
RESULTS





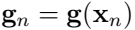


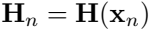


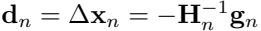


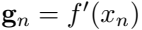


$$x_{n+1} = x_n + \Delta x_n = x_n + d_n$$

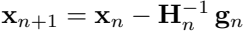








Handwritten text in a stylized, pixelated font, likely representing the word "HAPPY". The letters are formed by thick, dark strokes with a lighter, pixelated outline, giving it a retro, digital appearance. The text is centered horizontally and spans most of the width of the image.





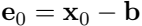


$$x_{n+1} = x_n + \Delta x_n = x_n + \Delta^n x_0$$

$$q(x) = \frac{1}{2}(x-b)^T A(x-b) + c$$

$$\begin{cases} \mathbf{A} = \mathbf{H}_0 \\ \mathbf{b} = \mathbf{x}_0 - \mathbf{H}^{-1} \mathbf{g}_0 \\ c = f(\mathbf{x}_0) - \mathbf{g}_0^T \mathbf{H}_0^{-1} \mathbf{g}_0 / 2 \end{cases}$$

$$B_0 = B/(x_0) = A/(x_0 - b) = Ae_0, \quad H_0 = H/(x_0) = A$$







Q. B. E.



$$q(x, y) = \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} (ax^2 + bxy + cy^2)$$

$$g = \begin{bmatrix} ax + by/2 \\ bx/2 + cy \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}, \quad \det \mathbf{H} = ac - b^2/4$$







11

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WORLD OF

1234567890









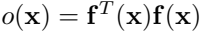






$$\begin{cases} f_1(x_1, x_2, x_3) = 3x_1 - (x_2x_3)^2 - 3/2 \\ f_2(x_1, x_2, x_3) = 4x_1^2 - 625x_2^2 + 2x_2 - 1 \\ f_3(x_1, x_2, x_3) = \exp(-x_1x_2) + 20x_3 + 9 \end{cases}$$

15, 25, 25, 25, 25





n	$\mathbf{x} = [x_1, x_2, x_3]$	$ \mathbf{f}(\mathbf{x}) $
0	0.000000, 0.000000, 0.000000	$1.016120e + 01$
1	0.500000, 0.500000, -0.500000	$1.552502e + 02$
2	0.499550, 0.250800, -0.493801	$3.881300e + 01$
3	0.500096, 0.126206, -0.496852	$9.702208e + 00$
4	0.500025, 0.063914, -0.498405	$2.425198e + 00$
5	0.500010, 0.032778, -0.499181	$6.059054e - 01$
6	0.500005, 0.017231, -0.499570	$1.510777e - 01$
7	0.500003, 0.009498, -0.499763	$3.737330e - 02$
8	0.500002, 0.005712, -0.499857	$8.959365e - 03$
9	0.500001, 0.003968, -0.499901	$1.900145e - 03$
10	0.500001, 0.003326, -0.499917	$2.577603e - 04$
11	0.500001, 0.003206, -0.499920	$8.932714e - 06$
12	0.500001, 0.003202, -0.499920	$1.238536e - 08$
13	0.500001, 0.003202, -0.499920	$2.371437e - 14$

0x111x121210-20

$$\mathbf{x}^* = \begin{bmatrix} 0.50000008539707297 \\ 0.0032017070323056 \\ -0.49999200212218281 \end{bmatrix}$$



11

11

11

11

$x_1 = x_0 + \Delta x_0 = x_0 + \delta x_0$

A pixelated, black and white graphic of the text "L'XO=5dfeXO". The characters are rendered in a blocky, digital font style. The apostrophe in "L'" is a simple single tick. The equals sign is composed of three horizontal bars. The letter "d" has a small loop at the top. The letter "f" has a long, thin vertical stem. The letter "e" is a simple, rounded shape. The letter "X" is composed of two intersecting diagonal lines. The letter "O" is a simple circle. The overall style is reminiscent of early computer graphics or digital art.

1000

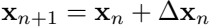
1011

$$f(x_1) \approx f(x_0) + f'(x_0) \Delta x_0 = f(x_0) + f'(x_0) \Delta x_0 / 2 \approx f(x_0)$$

$$x^2 + 1 = x^2 - 2x + 1 = (x - 1)^2$$

A pixelated, black and white representation of the word "EQUO". The letters are thick and blocky, with a jagged, pixelated edge. The 'E' is composed of three horizontal bars. The 'Q' has a small tail at the bottom right. The overall style is reminiscent of early digital art or a low-resolution scan of a printed word.

0123456789



The first chart, 'How often do you use the Internet?', shows that 90% of respondents use the Internet 'often' (black bar) and 10% use it 'sometimes' (grey bar). The second chart, 'How often do you use a mobile phone?', shows that 90% of respondents use a mobile phone 'often' (dark grey bar) and 10% use it 'sometimes' (light grey bar).

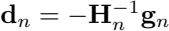
A pixelated, black and white representation of the number 50. The digits are composed of a grid of black and white squares, giving it a low-resolution, digital appearance. The number 50 is centered in the image.

$$f(x_1) = f(x_0) + \Delta x = f(x_0) + g_0^T \Delta x = f(x_0) + \delta \|g\|^2 \leq f(x_0)$$

$$x_{n+1} = x_n - \delta_n g_n = (x_{n-1} - \delta_{n-1} g_{n-1}) - \delta_n g_n = \cdots = x_0 - \sum_{i=0}^n \delta_n g_i$$











$$f(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + c$$



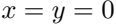




$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad c = 0$$

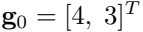
$$f(x_1, x_2) = \frac{1}{2} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{2} (2x_1^2 + 2x_1x_2 + x_2^2)$$

1992-93



A pixelated, grayscale image of the number 10. The digits are rendered in a blocky, low-resolution style using various shades of gray and black against a white background. The '1' is on the left and the '0' is on the right.

A pixelated, grayscale illustration of a person standing on a large, dark, rectangular base. The person is wearing a dark, long-sleeved shirt and dark pants. The background is white with some faint, pixelated gray shapes.

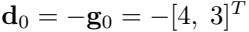




$$g = \begin{bmatrix} 2x_1 + x_2 \\ x_1 + x_2 \end{bmatrix}, \quad \mathbf{H} = \mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{H}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

0-11-1901-12-12

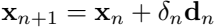
$$\mathbf{x}_1 = \mathbf{x}_0 - \mathbf{H}^{-1} \mathbf{g}_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$$x_1 = x_0 - \delta g_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \delta \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 - \delta 4 \\ 2 - \delta 3 \end{bmatrix}$$









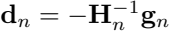








$$\theta = \cos^{-1} \frac{d_n^T g_n}{\|d_n\| \|g_n\|} > \frac{\pi}{2} \text{ i.e., } d_n^T g_n < 0$$

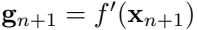


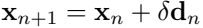
$$d_n^2 g_n = -g_n^2 H_n^{-1} g_n \leq 0, \quad d_n^2 g_n = -g_n^2 g_n = -|g_n|^2 \leq 0$$



$\frac{d}{dx} \left(x^2 + 1 \right) = 2x$

$$\frac{d}{d\delta_n} f(\mathbf{x}_{n+1}) = \frac{d}{d\delta_n} f(\mathbf{x}_n + \delta_n \mathbf{d}_n) = \left(\frac{d f(\mathbf{x}_{n+1})}{d\mathbf{x}} \right)^T \frac{d(\mathbf{x}_n + \delta_n \mathbf{d}_n)}{d\delta_n} = \mathbf{g}_{n+1}^T \mathbf{d}_n = 0$$







$\frac{d}{dx} \left(x^2 + 1 \right) = 2x$



$$f(\mathbf{x}_{n+1}) = f(\mathbf{x}_n + \delta \mathbf{d}_n) \approx [f(\mathbf{x}_n + \delta \mathbf{d}_n)]_{\delta=0} + \delta \left[\frac{d}{d\delta} f(\mathbf{x}_n + \delta \mathbf{d}_n) \right]_{\delta=0} + \frac{\delta^2}{2} \left[\frac{d^2}{d\delta^2} f(\mathbf{x}_n + \delta \mathbf{d}_n) \right]_{\delta=0}$$

1990-2000

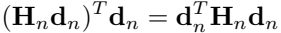


$$\left[\frac{d}{d\delta} f(x_n + \delta d_n) \right]_{\delta=0}$$

$$\left. g(x_n + \delta d_n)^T d_n \right|_{\delta=0} = g_n^T d_n$$

$$\left[\frac{d^2}{d\delta^2} f(x_n + \delta d_n) \right]_{\delta=0}$$

$$\left[\frac{d}{d\delta}g(x_n+\delta\mathbf{d}_n)^T\right]_{\delta=0}\mathbf{d}_n=\left[\frac{d}{dx}g(\mathbf{x})\frac{d}{d\delta}(x_n+\delta\mathbf{d}_n)\right]_{\delta=0}^T\mathbf{d}_n$$





$$f(x_n + \delta d_n) \approx f(x_n) + \delta g_n^T d_n + \frac{\delta^2}{2} d_n^T \mathbf{H}_n d_n$$



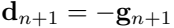
$$\frac{d}{d\delta} f(x_n + \delta \mathbf{d}_n) \approx \frac{d}{d\delta} \left(f(x_n) + \delta \mathbf{g}_n^T \mathbf{d}_n + \frac{\delta^2}{2} \mathbf{d}_n^T \mathbf{H}_n \mathbf{d}_n \right) = \mathbf{g}_n^T \mathbf{d}_n + \delta \mathbf{d}_n^T \mathbf{H}_n \mathbf{d}_n = 0$$

$$\delta_n = - \frac{\mathbf{g}_n^T \mathbf{d}_n}{\mathbf{d}_n^T \mathbf{H}_n \mathbf{d}_n}$$

$$\delta_n = \frac{g_n^T d_n}{d_n^T H_n d_n} = \frac{g_n^T (H_n^{-1} g_n)}{(H_n^{-1} g_n)^T H_n (H_n^{-1} g_n)} = \frac{g_n^T (H_n^{-1} g_n)}{(H_n^{-1} g_n)^T g_n} = 1$$

$$x_{n+1} = x_n + \delta_n \mathbb{I}_{\{x_n \leq x_n - \delta_n\}} \mathbb{I}_{\{x_n \leq x_n - \delta_n\}}$$





$$\delta_n = - \frac{g_n^T d_n}{d_n^T H_n d_n} = \frac{g_n^T g_n}{g_n^T H_n g_n} = \frac{\|g_n\|^2}{g_n^T H_n g_n}$$

POD = 0





$$\left[\frac{d^2}{d\delta^2} f(x + \delta d) \right]_{\delta=0}$$

$$\left[\frac{d}{d\delta} f'(x + \delta d) \right]_{\delta=0} = \lim_{\sigma \rightarrow 0} \frac{f'(x + \sigma d) - f'(x)}{\sigma}$$

$$\frac{g^T(x+\sigma d)d-g^T(x)d}{\sigma}=\frac{g_\sigma^Td-g^Td}{\sigma}$$



2023 + 2024

01234

$$\frac{d}{d\delta} f(x_n + \delta d_n) \approx g_n^T d_n + \frac{\delta}{\sigma} (g_{\sigma n}^T d_n - g_n^T d_n) = 0$$

$$\delta_n = - \frac{\sigma g_n^T d_n}{(g_{\sigma n}^T d_n - g_n^T d_n)} = - \frac{\sigma g_n^T d_n}{(g_{\sigma n} - g_n)^T d_n}$$

0-123456789

$$\delta_n = \frac{\sigma g_n^T d_n}{(g_{\sigma n} - g_n)^T d_n} = \frac{\sigma g_n^T g_n}{(g_{\sigma n} - g_n)^T g_n} = \frac{\sigma ||g_n||^2}{||g_n||^2 - g_{\sigma n}^T g_n}$$

$$x_{n+1} = x_n - \delta_n g_n = x_n + \frac{\sigma \|g_n\|^2}{g_{\sigma n}^T g_n - \|g_n\|^2} g_n$$



n	$\mathbf{x} = [x_1, x_2, x_3]$	$ \mathbf{f}(\mathbf{x}) $
0	0.0000, 0.0000, 0.0000	$1.032500e + 02$
10	0.4246, -0.0073, -0.5002	$1.535939e - 01$
20	0.5015, 0.0064, -0.4998	$2.448241e - 05$
30	0.5009, 0.0057, -0.4998	$9.178209e - 06$
40	0.5006, 0.0052, -0.4998	$4.17587e - 06$
50	0.5004, 0.0049, -0.4999	$2.122594e - 06$
60	0.5003, 0.0047, -0.4999	$1.182466e - 06$
100	0.5001, 0.0043, -0.4999	$1.805871e - 07$
150	0.5000, 0.0041, -0.4999	$2.720744e - 08$
200	0.5000, 0.0041, -0.4999	$4.934027e - 09$
250	0.5000, 0.0040, -0.4999	$9.630085e - 10$
300	0.5000, 0.0040, -0.4999	$1.939747e - 10$
350	0.5000, 0.0040, -0.4999	$3.961646e - 11$
400	0.5000, 0.0040, -0.4999	$8.140393e - 12$
450	0.5000, 0.0040, -0.4999	$1.677968e - 12$
500	0.5000, 0.0040, -0.4999	$3.462695e - 13$
550	0.5000, 0.0040, -0.4999	$7.136628e - 14$
600	0.5000, 0.0040, -0.4999	$1.474205e - 14$

0x111x1211

0x10000000

$$\mathbf{x}^* = \begin{bmatrix} 0.50000013623816102 \\ 0.00400027495837189 \\ -0.499990000311539049 \end{bmatrix}$$

$$f(x_{n+1}) = f(x_n) + \frac{1}{n} f'(x_n) + \frac{1}{n^2} f''(x_n)$$

on + 1d on 2d on

100%

En+1d | 2Bn |







A pixelated, black and white graphic of the text "DREAMS ARE MADE OF STARS". The text is rendered in a stylized, outlined font where each letter is composed of a grid of black and white pixels. The letters are slightly irregular, giving it a hand-drawn or digital-art feel. The words are spaced out evenly across the image.

A pixelated, black and white representation of the mathematical expression "100 = 10 + 100". The numbers and symbols are rendered in a blocky, digital font. The equals sign is composed of two horizontal bars. The plus sign is a simple cross. The entire image has a low-resolution, dithered appearance.

$$\begin{aligned}
 \phi &= \psi_0 = \phi_0 \\
 \phi_0 &= \psi_0
 \end{aligned}$$

$$b = \left. \frac{d}{d\delta} f(x_n + \delta d_n) \right|_{\delta=0} = g_n^T d_n < 0$$

for order

$$I_0(\theta) = I_0(x) + I_0(\sqrt{2})$$

[illegible]

1000

1000

Explanatory



A pixelated, black and white graphic of the text "open up open up". The text is rendered in a stylized, hand-drawn font with a dithered or pixelated appearance. The letters are thick and blocky, with some internal shading or dithering. The spaces between the words are wide, and the overall style is reminiscent of early digital art or video game graphics. The text is centered horizontally and occupies most of the width of the image.

$$f(x_{n+1}) = f(x_n) + d_n \leq f(x_n) + c_1 d_n \leq f(x_n)$$





for an order



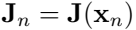


$$f(x, y) = (x - x^2 + y - y^2)$$

1111111



$$x_{n+1} = x_n - 1 f(x_n) = x_n - 1 f_n$$





$$x_{n+1} = x_n \cdot \frac{1}{b(x_n)} = x_n \cdot \frac{1}{b}$$





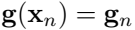


QWERTZ

$$f(x) = f(x_{n+1}) + (x - x_{n+1})^T g_{n+1} + \frac{1}{2} (x - x_{n+1})^T \mathbf{H}_{n+1} (x - x_{n+1}) + O(||x - x_{n+1}||^3)$$

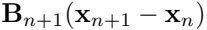
$$\frac{d}{dx} f(x) = g(x) = g_{n+1} + \mathbf{H}_{n+1}(x - x_{n+1}) + O(||x - x_{n+1}||^2)$$













Environ + 1 Environ + 1

1991-1992

1-11-2020

B

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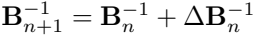
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7























U

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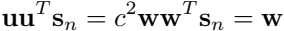


$$E_{n+1}B_n = E_nB_{n+1} + v_1^T B_{n+1}v_n,$$

$$V(V \cap S) = V \cap S$$







equal to the
weight of the
air displaced by the
object. The
weight of the
object is the
weight of the
air displaced by the
object.

$$\mathbf{u} =_{CW} \frac{y_n - \mathbf{B}_n \mathbf{s}_n}{(\mathbf{w}^T \mathbf{s}_n)^{1/2}} = \frac{y_n - \mathbf{B}_n \mathbf{s}_n}{((y_n - \mathbf{B}_n \mathbf{s}_n)^T \mathbf{s}_n)^{1/2}}$$

$$\mathbf{B}_{n+1} = \mathbf{B}_n + \mathbf{u}\mathbf{u}^T = \mathbf{B}_n + \frac{(\mathbf{y}_n - \mathbf{B}_n \mathbf{s}_n)(\mathbf{y}_n - \mathbf{B}_n \mathbf{s}_n)^T}{(\mathbf{y}_n - \mathbf{B}_n \mathbf{s}_n)^T \mathbf{s}_n}$$

A pixelated, grayscale version of the number 12. The digits are composed of various shades of gray, with the '1' being a solid dark gray and the '2' having a lighter gray outline and a darker gray fill. The overall style is reminiscent of early digital art or a low-resolution scan.

A large, pixelated cross shape composed of various shades of gray and black, centered on a white background. The cross is formed by a horizontal bar and a vertical bar intersecting at a central point. The central intersection is the darkest, appearing as a solid black square. The bars are made of discrete pixels, with some pixels being a medium gray and others a darker charcoal gray. The overall effect is a low-resolution, digital-style representation of a cross.

B

—

1

π

+

1

$$\left(B_n + uu^T\right)^{-1} = B_n^{-1} - \frac{B_n^{-1}uu^TB_n^{-1}}{1 + u^TB_n^{-1}u}$$

$$\mathbf{B}_n^{-1} + \frac{(\mathbf{s}_n - \mathbf{B}_n^{-1} \mathbf{y}_n)(\mathbf{s}_n - \mathbf{B}_n^{-1} \mathbf{y}_n)^T}{(\mathbf{s}_n - \mathbf{B}_n^{-1} \mathbf{y}_n)^T \mathbf{y}_n}$$

B

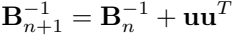
—

1

π

+

1



$$B_n^{-1}x_n = B_n^{-1}x_n + \|x_n\| = \|x_n\|$$

$$w_n(B_n^{-1}v_n) = B_n^{-1}v_n \Rightarrow 0$$

$$B_{n+1} = B_n + \alpha v v^T + \beta v v^T$$

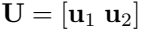
$$B_n + I S_n = (B_n + \alpha u u^T + \beta v v^T) S_n = B_n S_n + u (\alpha u^T S_n) + v (\beta v^T S_n) = y_n$$

$v_0 v_1 + v_2 v_3 = x_1 x_2$

$$\mathbf{u} = \mathbf{y}_n, \quad \alpha = \frac{1}{\mathbf{s}_n^T \mathbf{y}_n}, \quad \mathbf{v} = \mathbf{B}_n \mathbf{s}_n, \quad \beta = -\frac{1}{\mathbf{v}^T \mathbf{s}_n} = -\frac{1}{\mathbf{s}_n^T \mathbf{B}_n \mathbf{s}_n}$$

$$B_{x+1} = B_x + \alpha v_{x+1} + \alpha v_{x+1}^2$$

$$\mathbf{B}_{n+1} = \mathbf{B}_n + \alpha \mathbf{u} \mathbf{u}^T + \beta \mathbf{v} \mathbf{v}^T = \mathbf{B}_n + \frac{\mathbf{y}_n \mathbf{y}_n^T}{\mathbf{y}_n^T \mathbf{S}_n} - \frac{\mathbf{B}_n \mathbf{S}_n \mathbf{S}_n^T \mathbf{B}_n}{\mathbf{S}_n^T \mathbf{B}_n \mathbf{S}_n}$$





$$\mathbf{u}_1 = \mathbf{v}_1 = \frac{\mathbf{y}_n}{(\mathbf{s}_n^T \mathbf{y}_n)^{1/2}}, \quad \mathbf{u}_2 = -\mathbf{v}_2 = \frac{\mathbf{B}_n \mathbf{s}_n}{(\mathbf{s}_n^T \mathbf{B}_n \mathbf{s}_n)^{1/2}}$$

$$\mathbb{E}_{n+1} = \mathbb{E}_n + v_1 v_1^T + v_2 v_2^T = \mathbb{E}_n + U U^T$$

$$(B_n + U^T)^{-1} = B_n^{-1} U (I + U^T B_n^{-1} U)^{-1} U^T B_n^{-1}$$

$$B_n^{-1} B_n^{-1} U C^{-1} V^T B_n^{-1} = B_n^{-1} B_n^{-1} [v_1 v_2] C^{-1} (B_n^{-1} [v_1 v_2])^T$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \mathbf{I} + \mathbf{V}^T \mathbf{B}_n^{-1} \mathbf{U} = \mathbf{I} + [\mathbf{v}_1 \ \mathbf{v}_2]^T \mathbf{B}_n^{-1} [\mathbf{u}_1 \ \mathbf{u}_2]$$

$$c_{11} = 1 + v_1^T B_n^{-1} u_1 = 1 + \frac{y_n^T B_n^{-1} y_n}{s_n^T y_n}$$

$$c_{22} = 1 + v_2 B_n^{-1} u_2 = 1 - \frac{s_n^T B_n B_n^{-1} B_n s_n}{s_n^T B_n s_n} = 0$$

$$c_{12} = v_1^T B_n^{-1} u_2 = \frac{y_n^T B_n^{-1} B_n s_n}{(s_n^T y_n)^{1/2} (s_n^T B_n s_n)^{1/2}} = \frac{(s_n^T y_n)^{1/2}}{(s_n^T B_n s_n)^{1/2}}$$

Q11 - 1112

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ -c_{12} & 0 \end{bmatrix}, \quad \mathbf{C}^{-1} = \begin{bmatrix} 0 & -1/c_{12} \\ 1/c_{12} & c_{11}/c_{12}^2 \end{bmatrix}$$



$$\mathbf{B}_n^{-1} - [\mathbf{B}_n^{-1} \mathbf{u}_1 \quad \mathbf{B}_n^{-1} \mathbf{u}_2] \begin{bmatrix} 0 & -1/c_{12} \\ 1/c_{12} & c_{11}/c_{12}^2 \end{bmatrix} [\mathbf{B}_n^{-1} \mathbf{v}_1 \quad \mathbf{B}_n^{-1} \mathbf{v}_2]^T$$

$$B_n^{-1} - \frac{1}{c_{12}} \left[B_n^{-1} u_2 v_1^T B_n^{-1} - B_n^{-1} u_1 v_2^T B_n^{-1} \right] - \frac{c_{11}}{c_{12}^2} B_n^{-1} u_2 v_2^T B_n^{-1}$$

$$\frac{1}{c_{12}} \mathbf{B}_n^{-1} \mathbf{u}_2 \mathbf{v}_1^T \mathbf{B}_n^{-1} = \frac{(s_n^T \mathbf{B}_n s_n)^{1/2}}{(s_n^T \mathbf{y}_n)^{1/2}} \mathbf{B}_n^{-1} \frac{\mathbf{B}_n s_n}{(s_n^T \mathbf{B}_n s_n)^{1/2}} \frac{\mathbf{y}_n^T}{(s_n^T \mathbf{y}_n)^{1/2}} \mathbf{B}_n^{-1} = \frac{s_n \mathbf{y}_n^T \mathbf{B}_n^{-1}}{s_n^T \mathbf{y}_n}$$

$$-\frac{1}{c_{12}}\mathbf{B}_n^{-1}\mathbf{u}_1\mathbf{v}_2^T\mathbf{B}_n^{-1}=\frac{(s_n^T\mathbf{B}_ns_n)^{1/2}}{(s_n^Ty_n)^{1/2}}\mathbf{B}_n^{-1}\frac{y_n}{(s_n^Ty_n)^{1/2}}\frac{s_n^T\mathbf{B}_n}{(s_n^T\mathbf{B}_ns_n)^{1/2}}\mathbf{B}_n^{-1}=\frac{\mathbf{B}_n^{-1}y_ns_n^T}{s_n^Ty_n}$$

$$\frac{C_{11}}{C_{12}} \mathbf{B}_n^{-1} \mathbf{U}_2 \mathbf{V}_2^T \mathbf{B}_n^{-1}$$

$$\begin{aligned}
 & \frac{1}{s_n^T y_n} \left(1 + \frac{y_n^T B_n^{-1} y_n}{s_n^T y_n} \right) \frac{s_n^T B_n s_n}{s_n^T y_n} B_n^{-1} \frac{B_n s_n}{(s_n^T B_n s_n)^{1/2}} \frac{s_n^T B_n}{(s_n^T B_n s_n)^{1/2}} B_n^{-1}
 \end{aligned}$$

$$-\left(1 + \frac{y_n^T B_n^{-1} y_n}{s_n^T y_n}\right) \frac{s_n s_n^T}{s_n^T y_n}$$

$$\mathbf{B}_{n+1}^{-1} = \mathbf{B}_n^{-1} - \frac{\mathbf{B}_n^{-1} y_n s_n^T + s_n y_n^T \mathbf{B}_n^{-1}}{s_n^T y_n} + \left(1 + \frac{y_n^T \mathbf{B}_n^{-1} y_n}{s_n^T y_n} \right) \frac{s_n s_n^T}{s_n^T y_n}$$

B

—

1

7

$$E_{n+1} = E_n + \alpha v_n^T + \alpha v_n$$







$$B_n^{-1} y_n = (B_n^{-1} + \alpha u u^T + \beta v v^T) y_n = B_n^{-1} y_n + u(\alpha u^T y_n) + v(\beta v^T y_n) = s_n$$

$$v(vv^{\top} + vv^{\top}) = B^{-1}v$$

$$\mathbf{u} = \mathbf{s}_n, \quad \alpha = \frac{1}{\mathbf{s}_n^T \mathbf{y}_n}, \quad \mathbf{v} = \mathbf{B}_n^{-1} \mathbf{y}_n, \quad \beta = -\frac{1}{\mathbf{v}^T \mathbf{y}_n} = -\frac{1}{\mathbf{y}_n^T \mathbf{B}_n^{-1} \mathbf{y}_n}$$

$$E_{n+1} = E_n + \alpha v v^T + \beta v v^T$$

$$\mathbf{B}_{n+1}^{-1} = \mathbf{B}_n^{-1} + \alpha \mathbf{u} \mathbf{u}^T + \beta \mathbf{v} \mathbf{v}^T = \mathbf{B}_n^{-1} + \frac{\mathbf{s}_n \mathbf{s}_n^T}{\mathbf{s}_n^T \mathbf{y}_n} - \frac{\mathbf{B}_n^{-1} \mathbf{y}_n \mathbf{y}_n^T \mathbf{B}_n^{-1}}{\mathbf{y}_n^T \mathbf{B}_n^{-1} \mathbf{y}_n}$$

$$\mathbf{B}_{n+1} = \mathbf{B}_n - \frac{\mathbf{B}_n \mathbf{s}_n \mathbf{y}_n^T + \mathbf{y}_n \mathbf{s}_n^T \mathbf{B}_n}{\mathbf{y}_n^T \mathbf{s}_n} + \left(1 + \frac{\mathbf{s}_n^T \mathbf{B}_n \mathbf{s}_n}{\mathbf{y}_n^T \mathbf{s}_n} \right) \frac{\mathbf{y}_n \mathbf{y}_n^T}{\mathbf{y}_n^T \mathbf{s}_n}$$

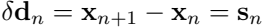
1234567



Ben+1

02 Ben

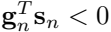
1 on 20



$$(B_{n+1} - c_2 B_n)^T B_n = B_n^T (B_{n+1} - c_2 B_n) \geq 0$$

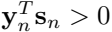
$$\sqrt[n]{a} = \sqrt[n]{a^{n+1} - a^n} = \sqrt[n]{a^{n+1} - a^n}$$

$$\sqrt[n]{a_n} - (a_{n+1} a_n)^{-1/n} = (a_2 - 1) a_n^{-1/n} > 0$$





$$v_n \leq \frac{1}{n} \leq \frac{1}{n+1} \leq v_{n+1}$$









$$a^T a = z^T B_n z, \quad b^T b = s^T B_n s, \quad a^T b = z^T B_n s,$$

$$z^T \left[B_n - \frac{B_n s_n s_n^T B_n}{s_n^T B_n s_n} \right] z = z^T B_n z - \frac{(z^T B_n s_n)^2}{s_n^T B_n s_n} = a^T a - \frac{(a^T b)^2}{b^T b} \geq 0$$

9 1 2 3 4 5 6

$$\mathbf{z}^T \left(\frac{\mathbf{s}_n \mathbf{s}_n^T}{\mathbf{s}_n^T \mathbf{y}_n} \right) \mathbf{z} \geq 0$$

$$\mathbf{z}^T \mathbf{B}_{n+1} \mathbf{z} = \mathbf{z}^T \left(\mathbf{B}_n - \frac{\mathbf{B}_n \mathbf{s}_n \mathbf{s}_n^T \mathbf{B}_n}{\mathbf{s}_n^T \mathbf{B}_n \mathbf{s}_n} \right) \mathbf{z} + \mathbf{z}^T \left(\frac{\mathbf{y}_n \mathbf{y}_n^T}{\mathbf{s}_n^T \mathbf{y}_n} \right) \mathbf{z} \geq 0$$

$$f(x) = \frac{1}{2} x^T A x - b^T x + c$$



$$g(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \left(\frac{1}{2} x^T A x - b^T x + c \right) = Ax - b$$

$$\mathbf{H}(x) = \frac{d^2}{dx^2} f(x) = \frac{d}{dx} g = \frac{d}{dx} (Ax - b) = A$$

9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.



$$f(x^*) = \frac{1}{2}(A^{-1}b)^T A (A^{-1}b) - b^T (A^{-1}b) + c = -\frac{1}{2}b^T A^{-1}b + c$$







$\sqrt{Av} = \sqrt{Av} = 0$



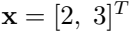


1.1 = 0



$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$u_1 = v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad u_2 = v_2 - \frac{u_1^T A v_2}{u_1^T A u_1} u_1 = \begin{bmatrix} -1/3 \\ 1 \end{bmatrix}$$







$$\mathbf{p}_{v_1}(\mathbf{x}) = \frac{\mathbf{v}_1^T \mathbf{x}}{\mathbf{v}_1^T \mathbf{v}_1} \mathbf{v}_1 = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad \mathbf{p}_{v_2}(\mathbf{x}) = \frac{\mathbf{v}_2^T \mathbf{x}}{\mathbf{v}_2^T \mathbf{v}_2} \mathbf{v}_2 = 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$



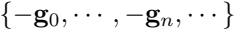


$$p_{u_1}(x) = \frac{u_1^T A x}{u_1^T A u_1} u_1 = 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad p_{u_2}(x) = \frac{u_2^T A x}{u_2^T A u_2} u_2 = 3 \begin{bmatrix} -1/3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

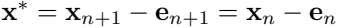
$$2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2v_1 + 3v_2 = p_{v_1}(x) + p_{v_2}(x)$$

$$3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} -1/3 \\ 1 \end{bmatrix} = 3u_1 + 3u_2 = p_{u_1}(x) + p_{u_2}(x)$$



[illegible]

$$x_{n+1} = x_n + \delta_n d_n = \dots = x_0 + \sum_{i=0}^n \delta_i d_i$$



$$e_{n+1} = e_n + \delta_n d_n = \cdots = e_0 + \sum_{i=0}^n \delta_i d_i$$

1821



$$e_i = f(x_i) = Ax_i + b = A(x_i^* + e_i) + b = Ax_i^* + b + Ae_i = Ae_i$$



$$\frac{\frac{\mathbf{g}_i^T \mathbf{d}_i}{\mathbf{d}_i^T \mathbf{A} \mathbf{d}_i}}{= \frac{\frac{\mathbf{e}_i^T \mathbf{A} \mathbf{d}_i}{\mathbf{d}_i^T \mathbf{A} \mathbf{d}_i}}{= \frac{\frac{\mathbf{d}_i^T \mathbf{A} \mathbf{e}_i}{\mathbf{d}_i^T \mathbf{A} \mathbf{d}_i}}{= \frac{\mathbf{d}_i^T \mathbf{A} \left(\mathbf{e}_0 + \sum_{j=0}^{i-1} \delta_j \mathbf{d}_j \right)}}{\mathbf{d}_i^T \mathbf{A} \mathbf{d}_i}}$$

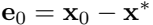
$$\frac{d_i^T Ae_0 + \sum_{j=0}^{i-1} \delta_j d_i^T Ad_j}{d_i^T Ad_i} = - \frac{d_i^T Ae_0}{d_i^T Ad_i}$$





1.1 = 0

$$e_{n+1} = e_n + \delta_n d_n = e_n - \left(\frac{d_n^T A e_0}{d_n^T A d_n} \right) d_n$$



$$e_0 = \sum_{i=0}^{N-1} c_i d_i = \sum_{i=0}^{N-1} p_{d_i}(e_0) = \sum_{i=0}^{N-1} \left(\frac{d_i^T A e_0}{d_i^T A d_i} \right) d_i$$





$$p_{d_i}(e_0) = c_i d_i = \left(\frac{d_i^T A e_0}{d_i^T A d_i} \right) d_i$$



$$c_i = \frac{d_i^T A e_0}{d_i^T A d_i} = -\delta_i$$



$$\mathbf{e}_{n+1} = \mathbf{e}_0 + \sum_{i=0}^n \delta_i \mathbf{d}_i = \sum_{i=0}^{N-1} c_i \mathbf{d}_i - \sum_{i=0}^n c_i \mathbf{d}_i = \sum_{i=n+1}^{N-1} c_i \mathbf{d}_i = \sum_{i=n+1}^{N-1} p_{\mathbf{d}_i}(\mathbf{e}_0)$$





Real World







1234567890

$$\mathbf{d}_k^T \mathbf{A} \mathbf{e}_{n+1} = \mathbf{d}_k^T \mathbf{g}_{n+1} = \sum_{i=n+1}^{N-1} c_i \mathbf{d}_k^T \mathbf{A} \mathbf{d}_i = 0$$













91

A pixelated, black and white representation of the text "WAVES". The letters are thick and blocky, with a jagged, pixelated edge. The 'W' and 'V' are particularly stylized, with the 'V' having a sharp, pointed bottom. The 'A' is a simple, wide character. The 'S' is composed of several small, curved segments. The overall effect is that of a low-resolution digital graphic or a retro video game title.

$$\mathbf{d}_n = \mathbf{v}_n - \sum_{j=0}^{n-1} \mathbf{P}_{\mathbf{d}_j}(\mathbf{v}_n) = \mathbf{v}_n - \sum_{m=0}^{n-1} \left(\frac{\mathbf{d}_m^T \mathbf{A} \mathbf{v}_n}{\mathbf{d}_m^T \mathbf{A} \mathbf{d}_m} \right) \mathbf{d}_m = \mathbf{v}_n - \sum_{m=0}^{n-1} \beta_{nm} \mathbf{d}_m$$

Adaptation

QWERTY







$$d_n = -g_n - \sum_{m=0}^{n-1} \beta_{nm} d_m$$

$$\rho_{nm} = \frac{d_m^T A v_n}{d_m^T A d_m} = - \frac{d_m^T A g_n}{d_m^T A d_m} \quad (m < n)$$

$$g_n = \sum_{i=0}^n a_i d_i$$

97 + 1

$$\mathbf{g}_{n+1}^T \mathbf{g}_k = \mathbf{g}_{n+1}^T \left(\sum_{i=0}^k a_i \mathbf{d}_i \right) = \sum_{i=0}^k a_i \mathbf{g}_{n+1}^T \mathbf{d}_i = 0$$

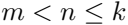
9 + 10 = 0



0123456789

$$\mathbf{g}_k^T \mathbf{d}_n = -\mathbf{g}_k^T \mathbf{g}_n - \sum_{m=0}^{n-1} \beta_{mn} \mathbf{g}_k^T \mathbf{d}_m = -\mathbf{g}_k^T \mathbf{g}_n = \begin{cases} -||\mathbf{g}_n||^2 & n=k \\ 0 & n < k \end{cases}$$

2021 = 0



$$\delta_n = - \frac{\mathbf{g}_n^T \mathbf{d}_n}{\mathbf{d}_n^T \mathbf{A} \mathbf{d}_n} = \frac{\|\mathbf{g}_n\|^2}{\mathbf{d}_n^T \mathbf{A} \mathbf{d}_n}$$

$$g_{m+1} = A x_{m+1} - b = A(x_m + \delta_m d_m) - b = (A x_m - b) + \delta_m A d_m = g_m + \delta_m A d_m$$





Ben + 1 Ben Add Ben Add







$$g_n^T \text{Ad}_m = \frac{1}{\delta_m} g_n^T g_{m+1} = \begin{cases} ||g_n||^2 / \delta_{n-1} & m = n-1 \\ 0 & m < n-1 \end{cases}$$

$$\theta_{nm} = -\frac{d_m^T A g_n}{d_m^T A d_m} = \begin{cases} -||g_n||^2/\delta_{n-1} d_{n-1}^T A d_{n-1} & m = n-1 \\ 0 & m < n-1 \end{cases}$$





2020

on 1 = 1/2nd 1A on 1



$$\rho_n = - \frac{||g_n||^2}{||g_n-1||^2}$$



$$\delta_n = \frac{\|g_n\|^2}{d_n^T A d_n},$$

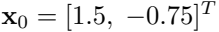
$$x_{n+1} = x_n + \delta_n d_n$$

$$g_{n+1} = \frac{d}{dx} f(x_{n+1})$$

$$\rho_{n+1} = \frac{\|g_{n+1}\|^2}{\|g_n\|^2}$$

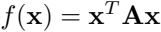
0123456789

$$f(x,y)=x^T Ax=[x_1,x_2]\begin{bmatrix}3&1\\1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$



| n | $\mathbf{x} = [x_1, x_2]$ | $f(\mathbf{x})$ |
|-----|---------------------------|------------------|
| 0 | 1.500000, -0.750000 | 2.812500 |
| 1 | 0.250000, -0.750000 | $0.468750e - 01$ |
| 2 | 0.250000, -0.125000 | $7.812500e - 02$ |
| 3 | 0.041667, -0.125000 | $1.302083e - 02$ |
| 4 | 0.041667, -0.020833 | $2.170139e - 03$ |
| 5 | 0.006944, -0.020833 | $3.616898e - 04$ |
| 6 | 0.006944, -0.003472 | $6.028164e - 05$ |
| 7 | 0.001157, -0.003472 | $1.004694e - 05$ |
| 8 | 0.001157, -0.000579 | $1.674490e - 06$ |
| 9 | 0.000193, -0.000579 | $2.790816e - 07$ |
| 10 | 0.000193, -0.000096 | $4.651361e - 08$ |
| 11 | 0.000032, -0.000096 | $7.752268e - 09$ |
| 12 | 0.000032, -0.000016 | $1.292045e - 09$ |
| 13 | 0.000005, -0.000016 | $2.153408e - 10$ |

| n | $\mathbf{x} = [x_1, x_2]$ | $f(\mathbf{x})$ |
|-----|---------------------------|-----------------|
| 0 | 1.5000000, -0.7500000 | 2.812500e + 00 |
| 1 | 0.2500000, -0.7500000 | 4.687500e - 01 |
| 2 | 0.0000000, -0.0000000 | 1.155558e - 33 |



$$\mathbf{A} = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

A pixelated, black and white graphic of the text "XO 1981". The characters are rendered in a bold, blocky font with a dithered or pixelated texture. The "X" and "O" are on the left, followed by a space, then "1981". The "1" is a simple vertical bar. The "9" has a curved top. The "8" is composed of two loops. The "1" at the end is a simple vertical bar. The overall style is reminiscent of early digital art or video game graphics.

$$x_1 = [3.548e-06, -7.4471e-06, 4.6180e-06]^T$$

542011

| n | $\mathbf{x} = [x_1, x_2, x_3]$ | $f(\mathbf{x})$ |
|-----|----------------------------------|-------------------|
| 0 | 1.0000000, 2.0000000, 3.0000000 | $4.5000000e + 01$ |
| 1 | -0.734716, -0.106441, 1.265284 | $2.809225e + 00$ |
| 2 | 0.123437, -0.209498, 0.136074 | $3.584736e - 02$ |
| 3 | -0.0000000, 0.0000000, 0.0000000 | $3.949119e - 31$ |



9

5

10

-

10

101

2

10

-

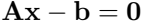
10

100% 100% 100%

| n | $\mathbf{x} = [x_1, x_2, x_3]$ | $o(\mathbf{x})$ |
|-----|--------------------------------|------------------|
| 0 | 0.0000, 0.0000, 0.0000 | $1.032500e + 02$ |
| 1 | 0.0113, 0.0050, -0.5001 | $3.160163e + 00$ |
| 2 | 0.0188, -0.0021, -0.5004 | $3.095894e + 00$ |
| 3 | 0.5009, -0.0018, -0.5004 | $7.268252e - 05$ |
| 4 | 0.5009, -0.0017, -0.5000 | $1.051537e - 05$ |
| 5 | 0.5008, -0.0012, -0.5000 | $6.511151e - 06$ |
| 6 | 0.5001, -0.0005, -0.5000 | $6.365321e - 07$ |
| 7 | 0.5001, -0.0005, -0.5000 | $5.667357e - 07$ |
| 8 | 0.5002, -0.0004, -0.5000 | $2.675128e - 07$ |
| 9 | 0.5001, -0.0003, -0.5000 | $1.344218e - 07$ |
| 10 | 0.5001, -0.0002, -0.5000 | $1.241196e - 07$ |
| 11 | 0.5000, -0.0001, -0.5000 | $2.120969e - 08$ |
| 12 | 0.5000, -0.0001, -0.5000 | $1.541814e - 08$ |
| 13 | 0.5000, -0.0001, -0.5000 | $7.282025e - 09$ |
| 14 | 0.5000, -0.0001, -0.5000 | $4.801781e - 09$ |
| 15 | 0.5000, -0.0000, -0.5000 | $4.463926e - 09$ |

1821-22

0123456789 ABCDEFGH



0123456789

A pixelated, black and white graphic of the text "1A10101010". The characters are rendered in a thick, blocky, and slightly irregular font, reminiscent of early digital or video game typography. The "1"s are simple vertical strokes, while the "A" and "0"s have more complex, multi-segmented structures. The overall aesthetic is that of a low-resolution digital display or a retro-style logo.

$$\mathbf{x} = \sum_{i=1}^N c_i \mathbf{d}_i$$

$$\mathbf{b} = \mathbf{A}\mathbf{x} = \mathbf{A} \left[\sum_{i=1}^N c_i \mathbf{d}_i \right] = \sum_{i=1}^N c_i \mathbf{A} \mathbf{d}_i$$



$$\mathrm{d}_j^T \mathbf{b} = \sum_{i=1}^N c_i \mathrm{d}_j^T \mathrm{Ad}_i = c_j \mathrm{d}_j^T \mathrm{Ad}_j$$



$$c_j = \frac{d_j^T b}{d_j^T A d_j}$$

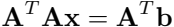
$$\mathbf{x} = \sum_{i=1}^N c_i \mathbf{d}_i = \sum_{i=1}^N \left(\frac{\mathbf{d}_i^T \mathbf{b}}{\mathbf{d}_i^T \mathbf{A} \mathbf{d}_i} \right) \mathbf{d}_i$$



$$p_{d_i}(x) = \left(\frac{d_i^T A x}{d_i^T A d_i} \right) d_i$$









A pixelated, black and white graphic of the text "A B C D E F G H I J K L M N O P Q R S T U V W X Y Z" in a stylized, blocky font. The letters are arranged in a single row, with each letter being a distinct, pixelated shape. The overall style is reminiscent of early digital art or computer graphics.





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1999-2000

optimize

$$f(\mathbf{x}) = f(x_1, \cdots, x_N)$$

subject to:

$$\begin{cases} h_i(\mathbf{x}) = 0, & (i = 1, \cdots, m) \\ g_j(\mathbf{x}) \leq 0, & (j = 1, \cdots, n) \end{cases}$$







$$\begin{array}{ll} \text{maximize/minimize} & f(\mathbf{x}) = f(x_1, \cdots, x_N) \\ \text{subject to:} & h_i(\mathbf{x}) = h_i(x_1, \cdots, x_N) = 0, \quad (i = 1, \cdots, m) \end{array}$$

1995



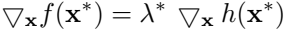












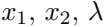




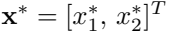
$$\frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \left(\frac{dx}{dt} \right)^2 \right) = m \frac{dx}{dt} \frac{d^2x}{dt^2} = m v \frac{d^2x}{dt^2}$$

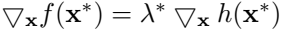
$$\begin{cases} \nabla_x f(x) = \lambda \nabla_x h(x) \\ \nabla_\lambda L(x, \lambda) = \partial L(x, \lambda) / \partial \lambda = -h(x) = 0 \end{cases}$$

123456789



$$\frac{\partial f(x)}{\partial x_i} = \lambda \frac{\partial h(x)}{\partial x_i} \quad (i = 1, 2), \quad h(x) = 0$$



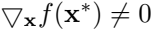


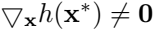






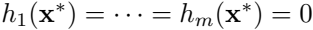








Waxhaw, NC 28091



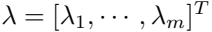






$$\nabla_x f(x^*) = \sum_{i=1}^m \lambda_i^* \nabla_x h_i(x^*)$$

$$L(x, \lambda) = f(x) - \sum_{i=1}^m \lambda_i h_i(x)$$

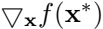


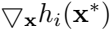
$$\nabla_{x,\lambda} L(x,\lambda) = \nabla_{x,\lambda} \left[f(x) - \sum_{i=1}^m \lambda_i h_i(x) \right] = 0$$

$$\nabla_x f(x) = \sum_{i=1}^m \lambda_i \nabla_x h_i(x)$$

$$\frac{\partial f(\mathbf{x})}{\partial x_j} = \sum_{i=1}^m \lambda_i \frac{\partial h_i(\mathbf{x})}{\partial x_j} \quad (j = 1, \dots, N),$$

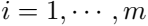
$$\frac{\partial L(x, \lambda)}{\partial \lambda_i} = h_i(x) = 0 \quad (i = 1, \dots, m)$$







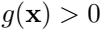


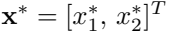


$$\nabla_x f(x^*) = \sum_{i=1}^m \lambda_i \nabla_x h_i(x) = 0$$

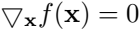
$$\begin{array}{ll} \text{maximize/minimize} & f(\mathbf{x}) = f(x_1, \cdots, x_N) \\ \text{subject to:} & g_j(\mathbf{x}) = g_j(x_1, \cdots, x_N) \leq \text{ or } \geq 0, \quad (j = 1, \cdots, n) \end{array}$$

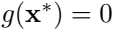


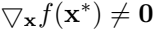




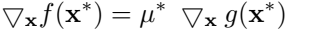
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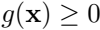




$\sqrt{x} \cdot \sqrt{x} = x$





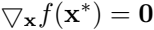


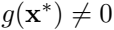




| | | |
|----------------------|----------------------------------------------------------------|----------------------------------------------------------------|
| | $g(\mathbf{x}) \geq 0$ | $g(\mathbf{x}) \leq 0$ |
| $\max f(\mathbf{x})$ | $\nabla f(\mathbf{x}_1) = \mu \nabla g(\mathbf{x}_1), \mu < 0$ | $\nabla f(\mathbf{x}_3) = \mu \nabla g(\mathbf{x}_3), \mu > 0$ |
| $\min f(\mathbf{x})$ | $\nabla f(\mathbf{x}_2) = \mu \nabla g(\mathbf{x}_2), \mu > 0$ | $\nabla f(\mathbf{x}_4) = \mu \nabla g(\mathbf{x}_4), \mu < 0$ |

$$\mu g(x) \begin{cases} \leq 0 & \text{for maximization} \\ \geq 0 & \text{for minimization} \end{cases}$$

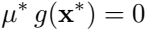












Q. X. = O. S. S. S.

$$L(x, \mu) = f(x) - \sum_{i=1}^n \mu_i g_i(x)$$

$$\nabla_{x,\mu} L(x,\mu) = \nabla_{x,\mu} \left[f(x) - \sum_{i=1}^n \mu_i g_i(x) \right] = 0$$

$$\nabla_x f(x) = \sum_{i=1}^n \mu_i \nabla_x g_i(x)$$

$$\frac{\partial I(x, \mu)}{\partial \mu_i} = g_i(x) = 0 \quad (i = 1, \dots, n)$$









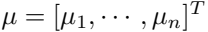
$$\begin{array}{ll}
 \text{maximize/minimize} & f(\mathbf{x}) = f(x_1, \cdots, x_N) \\
 \text{subject to:} & \begin{cases} h_i(\mathbf{x}) = 0, & (i = 1, \cdots, m) \\ g_j(\mathbf{x}) \leq 0 \text{ or } g_j(\mathbf{x}) \geq 0, & (j = 1, \cdots, n) \end{cases}
 \end{array}$$

$$\begin{array}{ll}
 \text{maximize/minimize} & f(\mathbf{x}) \\
 \text{subject to:} & \begin{cases} \mathbf{h}(\mathbf{x}) = \mathbf{0} \\ g(\mathbf{x}) \leq 0 \text{ or } g(\mathbf{x}) \geq 0 \end{cases}
 \end{array}$$

Визначення рівняння лінійної функції

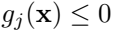
Q1X2 Q1X2

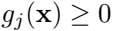
$$L(\mathbf{x}, \lambda, \mu) = f(\mathbf{x}) - \sum_{i=1}^m \lambda_i h_i(\mathbf{x}) - \sum_{j=1}^n \mu_j g_j(\mathbf{x}) = f(\mathbf{x}) - \lambda^T \mathbf{h}(\mathbf{x}) - \mu^T \mathbf{g}(\mathbf{x})$$



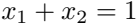


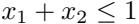


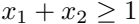




$w_j(x) = 0, \quad w_j = 1, \quad j = 1, 2, 3$







$$\frac{d}{dx} \left(x^2 \right) = 2x$$

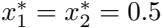
minimize/maximize: $f(x_1, x_2) = x_1^2 + x_2^2$

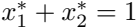
subject to: $h(x_1, x_2) = x_1 + x_2 - 1 = 0$

$$L(x_1, x_2, \lambda) = f(x_1, x_2) - \lambda g(x_1, x_2) = x_1^2 + x_2^2 - \lambda(x_1 + x_2 - 1)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - \lambda = 0, \quad \frac{\partial L}{\partial x_2} = 2x_2 - \lambda = 0, \quad \frac{\partial L}{\partial \lambda} = x_1 + x_2 - 1 = 0$$







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Q. 1. 2.

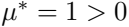
$v_{0.5} = v_{0.5} = 1, 1$

$$\begin{array}{ll} \text{minimize:} & f(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to:} & g(x_1, x_2) = x_1 + x_2 - 1 \geq 0 \end{array}$$

$$\begin{array}{ll} \text{maximize:} & f(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to:} & g(x_1, x_2) = x_1 + x_2 - 1 \leq 0 \end{array}$$

$$L(x_1, x_2, \mu) = f(x_1, x_2) - \mu g(x_1, x_2) = x_1^2 + x_2^2 - \mu(x_1 + x_2 - 1)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = 2x_1 - \mu = 0, \quad \frac{\partial \mathcal{L}}{\partial x_2} = 2x_2 - \mu = 0, \quad \frac{\partial \mathcal{L}}{\partial \mu} = x_1 + x_2 - 1 = 0$$



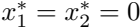
01-02-2025

$$\begin{array}{ll} \text{minimize:} & f(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to:} & g(x_1, x_2) = x_1 + x_2 - 1 \leq 0 \end{array}$$

$$\begin{array}{ll} \text{maximize:} & f(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to:} & g(x_1, x_2) = x_1 + x_2 - 1 \geq 0 \end{array}$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = 2x_1 = 0,$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 2x_2 = 0$$



minimize

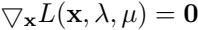
subject to:

$$\begin{cases} f_p(\mathbf{x}) \\ \mathbf{h}(\mathbf{x}) = \mathbf{0} \\ g(\mathbf{x}) \leq 0 \text{ or } g(\mathbf{x}) \geq 0 \end{cases}$$

$\frac{d}{dx} \left(x^2 \right) = 2x$

1000000

$$f_d(\lambda, \mu) = \inf_x \mathcal{L}(x, \lambda, \mu) = \inf_x [f_p(x) - \lambda^T h(x) - \mu^T g(x)]$$



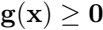
A pixelated, black and white representation of the word "Euler" in a stylized, handwritten font. The letters are composed of a grid of black and gray pixels, giving it a digital or retro aesthetic. The word is centered horizontally and occupies the middle portion of the image.

$$\text{maximize } f_d(\lambda, \mu) = \inf_x \mathcal{L}(x, \lambda, \mu)$$

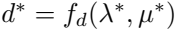
$$\text{subject to: } \mu \leq 0 \quad \text{or} \quad \mu \geq 0$$

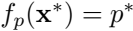
100%

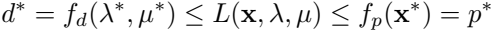


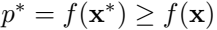


$$f_d(x, \mu) \leq L(x, \mu) = f_p(x) - \mu^T h(x) \leq f_p(x) - \mu^T g(x) \leq f_p(x)$$

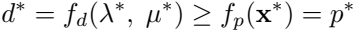


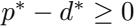


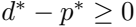








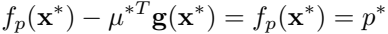








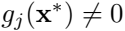
$$f(\lambda^*, \mu^*) = L(x^*, y^*) = f_p(x^*) - \mu^* \nabla g(x^*)$$



$$\left\{ \begin{array}{ll} \nabla_{\mathbf{x}} L(\mathbf{x}^*, \lambda, \mu) = \nabla_{\mathbf{x}} f(\mathbf{x}^*) - \sum_{i=1}^m \lambda_i \nabla_{\mathbf{x}} h_i(\mathbf{x}^*) - \sum_{j=1}^n \mu_j \nabla_{\mathbf{x}} g_j(\mathbf{x}^*) = \mathbf{0} & \text{(stationarity)} \\ \left\{ \begin{array}{l} h_i(\mathbf{x}^*) = 0, \quad (i = 1, \dots, m) \\ g_j(\mathbf{x}^*) \leq 0 \text{ or } g_j(\mathbf{x}^*) \geq 0, \quad (j = 1, \dots, n) \end{array} \right. & \text{(primal feasibility)} \\ \mu_j^* \leq 0, \text{ or } \mu_j^* \geq 0, \quad (j = 1, \dots, n) & \text{(dual feasibility)} \\ \mu_j^* g_j(\mathbf{x}^*) = 0, \quad (j = 1, \dots, n) & \text{(complementarity)} \end{array} \right.$$





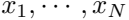








$$f(x_1, \dots, x_n) = \sum_{i=1}^n x_i$$



$$P(x_1, \dots, x_N) = \int \mathcal{D}x \mathcal{D}x_1$$

subject to:

$$\begin{cases} \sum_{i=1}^n a_{1i}x_i \leq b_1 \\ \dots\dots\dots \\ \sum_{i=1}^N a_{Mi}x_i \leq b_M \\ x_1 \geq 0, \dots, x_N \geq 0 \end{cases}$$







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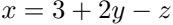
5

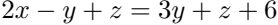
6

0

123

$$\begin{array}{ll} \text{maximize} & f(x, y, z) = 2x - y + 3z \\ \text{subject to:} & \begin{cases} x - 2y + z = 3 \\ 3x - y + 4z = 10 \\ y \geq 0, \quad z \geq 0 \end{cases} \end{array}$$







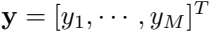
$$\text{maximize} \quad f(y, z) = 3y + z + 6$$

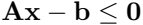
$$\text{subject to:} \quad \begin{cases} 5y + z = 1 \\ y \geq 0, \quad z \geq 0 \end{cases}$$

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & \begin{cases} \mathbf{Ax} - \mathbf{b} \leq \mathbf{0} \\ \mathbf{x} \geq \mathbf{0} \end{cases} \end{array}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_M \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MN} \end{bmatrix}$$

$$L(x) = c^T x - y^T (Ax - b) = (c - A^T y)^T x + y^T b$$



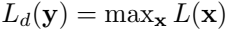






$$f_d(y) = \max_x L(x) = \max_x [c^T x - y^T (Ax - b)] = \max_x [(c - A^T y)^T x + y^T b]$$

$$\nabla_x \mathcal{L}(x) = \nabla_x [c - A^T y] + \nabla_x [b] = c - A^T y = 0$$





Q _ A 1 2 3 4

Q E A X V I X E O







$$f(y) = b^T y \geq l(x) = c^T x - b^T x = p(x)$$

$$\begin{array}{ll} \text{minimize} & f_d(\mathbf{y}) = \mathbf{b}^T \mathbf{y} \\ \text{subject to} & \begin{cases} \mathbf{A}^T \mathbf{y} - \mathbf{c} \geq \mathbf{0} \\ \mathbf{y} \geq \mathbf{0} \end{cases} \end{array}$$

$$\begin{cases} \max : & c^T x \\ \text{s.t.} & Ax - b \leq 0, \quad x \geq 0 \end{cases} \Longleftrightarrow \begin{cases} \min & b^T y \\ \text{s.t.} & Ay - c \geq 0, \quad y \geq 0 \end{cases}$$

$$\begin{cases} \min : & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} : & \mathbf{A}\mathbf{x} - \mathbf{b} \leq \mathbf{0}, \quad \mathbf{x} \geq \mathbf{0} \end{cases} \longleftrightarrow \begin{cases} \max & \mathbf{b}^T \mathbf{y} \\ \text{s.t.} & \mathbf{A}\mathbf{y} - \mathbf{c} \leq \mathbf{0}, \quad \mathbf{y} \leq \mathbf{0} \end{cases} \longleftrightarrow \begin{cases} \max & -\mathbf{b}^T \mathbf{y} \\ \text{s.t.} & \mathbf{A}\mathbf{y} + \mathbf{c} \geq \mathbf{0}, \quad \mathbf{y} \geq \mathbf{0} \end{cases}$$





minimize

$$\mathbf{b}^T \mathbf{y}$$

subject to

$$\mathbf{A}^T \mathbf{y} - \mathbf{c} \leq \mathbf{0}$$



$$\begin{array}{ll}\text{minimize} & \mathbf{b}^T \mathbf{y} \\ \text{subject to} & \begin{cases} \mathbf{A}^T \mathbf{y} - \mathbf{s} = \mathbf{c} \\ \mathbf{s} \geq 0 \end{cases}\end{array}$$





$$0 \leq (Ax) \leq Ax = x \leq (Ax) \leq b$$



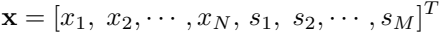
$$\sum_{i=1}^N a_i x_i \leq b \quad \Rightarrow \quad \sum_{i=1}^N a_i x_i + s = b, \quad (s \geq 0)$$

$$\begin{array}{ll}
\text{maximize} & z = f(\mathbf{x}) = \mathbf{c}^T \mathbf{x} = \sum_{j=1}^N c_j x_j \\
\text{subject to:} & \left\{ \begin{array}{llll}
\sum_{i=1}^N a_{1i} x_i & + s_1 & & = b_1 \\
\sum_{i=1}^N a_{2i} x_i & & + s_2 & = b_2 \\
\cdots & \cdots & \cdots & \cdots \\
\sum_{i=1}^N a_{Mi} x_i & & & + s_M = b_M \\
x_1 \geq 0, & \cdots, & x_N \geq 0, & s_1 \geq 0, \cdots s_M \geq 0
\end{array} \right.
\end{array}$$



1919-1919

1990-1991





$$\mathbf{A} = \left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1N} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{1N} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{MN} & 0 & \cdots & 0 & 1 \end{array} \right]_{M \times (N+M)} = \left[\mathbf{A}_{M \times N} \mid \mathbf{I}_{M \times M} \right]$$

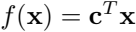




$$\begin{array}{ll}
 \text{maximize} & \mathbf{c}^T \mathbf{x} \\
 \text{subject to} & \begin{cases} \mathbf{Ax} + \mathbf{s} = \mathbf{b} \\ \mathbf{x} \geq \mathbf{0}, \mathbf{s} \geq \mathbf{0} \end{cases}
 \end{array}
 \qquad \text{or} \qquad
 \begin{array}{ll}
 \text{maximize} & \mathbf{c}^T \mathbf{x} \\
 \text{subject to} & \begin{cases} \mathbf{Ax} = \mathbf{b} \\ \mathbf{x} \geq \mathbf{0} \end{cases}
 \end{array}$$









2 = [01, 01, 01, 01]

$$\sum_{j=1}^N a_{ij}x_j = \mathbf{a}_j^T \mathbf{x} = b_i, \quad (i=1, \cdots, M)$$







$$C_{M+N}^N = \frac{(M+N)!}{N!M!}$$







x = x + e
v
e
v
e
v

$$f(x) = c^T x = c^T (x^* + e/c/c) = c^T x^* + e/c/c = c^T x^* = f(x^*)$$



$$\begin{array}{ll}
 \text{maximize} & f_p(x_1, x_2) = 2x_1 + 3x_2 \\
 \text{subject to:} & \begin{cases} 2x_1 + x_2 \leq 18 \\ 6x_1 + 5x_2 \leq 60 \\ 2x_1 + 5x_2 \leq 40 \\ x_1 \geq 0, \quad x_2 \geq 0 \end{cases}
 \end{array}
 \qquad \text{or} \qquad
 \begin{array}{ll}
 \text{maximize} & f_p(\mathbf{x}) = \mathbf{c}^T \mathbf{x} \\
 \text{subject to:} & \begin{cases} \mathbf{Ax} \leq \mathbf{b} \\ \mathbf{x} \geq \mathbf{0} \end{cases}
 \end{array}$$

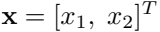
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 18 \\ 60 \\ 40 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 2 & 5 \end{bmatrix}$$





QWERTY QWERTY

$$p(x_1, x_2) = \int_0^{\infty} \int_0^{\infty} C_1 x_1 + C_2 x_2$$

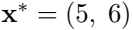




025

11

10



$$p(x_1, x_2) = 2x_1 + 3x_2 = 20$$

0x*|c|_|=203.01=777

| | (x_1, x_2) | | objective function (normalized) |
|----|--------------|------------|---------------------------------|
| 1 | 5.00, 6.00 | feasible | 28 (7.77) |
| 2 | 6.25, 5.50 | infeasible | 29 (8.04) |
| 3 | 7.50, 3.00 | feasible | 24 (6.66) |
| 4 | 20.00, 0.00 | infeasible | 40 (11.1) |
| 5 | 10.00, 0.00 | infeasible | 20 (5.55) |
| 6 | 9.00, 0.00 | feasible | 18 (4.10) |
| 7 | 0.00, 8.00 | feasible | 24 (6.66) |
| 8 | 0.00, 12.00 | infeasible | 36 (9.98) |
| 9 | 0.00, 18.00 | infeasible | 54 (14.99) |
| 10 | 0.00, 0.00 | feasible | 0 (0.00) |

$$\begin{array}{ll}
 \text{maximize} & f_p(x_1, x_2) = 2x_1 + 3x_2 \\
 \text{subject to:} & \begin{cases} 2x_1 + x_2 \leq 18 \\ 6x_1 + 5x_2 \leq 60 \\ 2x_1 + 5x_2 \leq 40 \\ x_1 \geq 0, \quad x_2 \geq 0 \end{cases}
 \end{array}
 \implies
 \begin{array}{ll}
 \text{maximize} & f_p(x_1, x_2) = 2x_1 + 3x_2 \\
 \text{subject to:} & \begin{cases} 2x_1 + x_2 + s_1 = 18 \\ 6x_1 + 5x_2 + s_2 = 60 \\ 2x_1 + 5x_2 + s_3 = 40 \\ x_1 \geq 0, \quad x_2 \geq 0, \quad s_1 \geq 0, \quad s_2 \geq 0, \quad s_3 \geq 0 \end{cases}
 \end{array}$$



$$\begin{array}{ll}
 \text{minimize} & f_d(y_1, y_2, y_3) = 18y_1 + 60y_2 + 40y_3 \\
 \text{subject to:} & \begin{cases} 2y_1 + 6y_2 + 2y_3 \geq 2 \\ y_1 + 5y_2 + 5y_3 \geq 3 \\ y_1 \geq 0, \ y_2 \geq 0, \ y_3 \geq 0 \end{cases}
 \end{array}
 \qquad \text{or} \qquad
 \begin{array}{ll}
 \text{minimize} & f_d(\mathbf{y}) = \mathbf{b}^T \mathbf{y} \\
 \text{subject to:} & \begin{cases} \mathbf{A}^T \mathbf{y} \leq \mathbf{b} \\ \mathbf{y} \geq \mathbf{0} \end{cases}
 \end{array}$$













| | (y_1, y_2, y_3) | objective function | |
|----|-------------------|--------------------|----|
| 1 | $(1, 0, 0)$ | infeasible | 18 |
| 2 | $(3, 0, 0)$ | feasible | 54 |
| 3 | $(0, 1/3, 0)$ | infeasible | 20 |
| 4 | $(0, 3/5, 0)$ | feasible | 36 |
| 5 | $(0, 0, 1)$ | feasible | 40 |
| 6 | $(0, 0, 3/5)$ | infeasible | 24 |
| 7 | $(0, 1/5, 2/5)$ | feasible | 28 |
| 8 | $(1/2, 0, 1/2)$ | feasible | 29 |
| 9 | $(-2, 1, 0)$ | infeasible | 24 |
| 10 | $(0, 0, 0)$ | infeasible | 0 |

015250

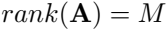
$$1234 = 1921 + 602 + 402$$



$p(x_1, x_2) = 2x_1 + x_2$



Adventures
in
the
Wild
West
of
the
American
Frontier







$$Ax = \begin{bmatrix} A_n & I \end{bmatrix} \begin{bmatrix} x_n \\ x_0 \end{bmatrix} = A_n x_n + I x_0 = b$$









$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_n \\ \mathbf{x}_b \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}$$

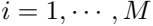


1919-2019







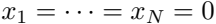


12

+

12

$$f(x) = \int_0^x f(t) dt = 1 - \cos x$$



QWERTYUIOP

12

+

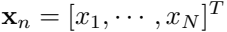
12

+

12



| basic variables | x_1 | x_2 | \cdots | x_N | s_1 | s_2 | \cdots | s_M | basic solution |
|-----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------------|
| s_1 | a_{11} | a_{12} | \cdots | a_{1N} | 1 | 0 | \cdots | 0 | b_1 |
| s_2 | a_{21} | a_{22} | \cdots | a_{2N} | 0 | 1 | \cdots | 0 | b_2 |
| \vdots | \vdots | \vdots | \ddots | \vdots | \vdots | \vdots | \ddots | \vdots | \vdots |
| s_M | a_{M1} | a_{M2} | \cdots | a_{MN} | 0 | 0 | \cdots | 1 | b_M |
| z | $-c_1$ | $-c_2$ | \cdots | $-c_M$ | 0 | 0 | \cdots | 0 | 0 |





$$x_0 = [s_1, \dots, s_n]^T = b, \quad x_n = [x_1, \dots, x_n]^T = 0$$

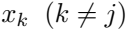
1234567890



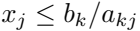


Q = 1000





$\nabla^2 \psi = 1$ on $\partial \Omega$

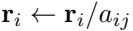






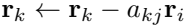
























EMMA + NINA = 1000



| | x_1 | x_2 | s_1 | s_2 | s_3 | | objective function (normalized) |
|----|-------|-------|-------|-------|-------|------------|---------------------------------|
| 1 | 5 | 6 | 2 | 0 | 0 | feasible | 28 (7.77) |
| 2 | 6.25 | 5.5 | 0 | −5 | 0 | infeasible | 29 (8.04) |
| 3 | 7.5 | 3 | 0 | 0 | 10 | feasible | 24 (6.66) |
| 4 | 20 | 0 | −2 | −60 | 20 | infeasible | 40 (11.1) |
| 5 | 10 | 0 | −2 | 0 | 20 | infeasible | 20 (5.55) |
| 6 | 9 | 0 | 0 | 6 | 22 | feasible | 18 (4.10) |
| 7 | 0 | 8 | 10 | 20 | 0 | feasible | 24 (6.66) |
| 8 | 0 | 12 | 6 | 0 | −20 | infeasible | 36 (9.98) |
| 9 | 0 | 18 | 0 | −30 | −50 | infeasible | 54 (14.99) |
| 10 | 0 | 0 | 18 | 60 | 40 | feasible | 0 (0.00) |

$$\begin{array}{ll}
\text{maximize} & f(\mathbf{x}) = 2x_1 + 3x_2 \\
\text{subject to:} & \left\{ \begin{array}{l} 2x_1 + x_2 + s_1 = 18 \\ 6x_1 + 5x_2 + s_2 = 60 \\ 2x_1 + 5x_2 + s_3 = 40 \\ x_1 \geq 0, \ x_2 \geq 0, \ s_1 \geq 0, \ s_2 \geq 0, \ s_3 \geq 0 \end{array} \right.
\end{array}$$













| | x_1 | x_2 | s_1 | s_2 | s_3 | b |
|-------|-------|-------|-------|-------|-------|-----|
| s_1 | 2 | 1 | 1 | 0 | 0 | 18 |
| s_2 | 6 | 5 | 0 | 1 | 0 | 60 |
| s_3 | 2 | 5 | 0 | 0 | 1 | 40 |
| z | -2 | -3 | 0 | 0 | 0 | 0 |















DESIGN 405 = 0

A pixelated, grayscale version of the word "DREAM". The letters are thick and blocky, with a jagged, pixelated edge. The color is a dark gray, and the background is white. The font style is reminiscent of early digital art or video game titles.

1919 BOB 405

12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100



A pixelated, black and white graphic of the text "1949, 1950, 1951" in a stylized, blocky font. The text is rendered in a high-contrast, dithered style, giving it a retro, digital appearance. The numbers are large and bold, with the years separated by commas. The overall aesthetic is reminiscent of early computer graphics or video game titles.





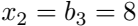
0x2 = 0x2 = 12





| | x_1 | x_2 | s_1 | s_2 | s_3 | b |
|-------|-------|-------|-------|-------|-------|-----|
| s_1 | 1.6 | 0 | 1 | 0 | -0.2 | 10 |
| s_2 | 4 | 0 | 0 | 1 | -1 | 20 |
| x_2 | 0.4 | 1.0 | 0 | 0 | 0.2 | 8 |
| z | -0.8 | 0 | 0 | 0 | 0.6 | 24 |



















2021-2021-21

0.001

10/10/25, 20/45, 20/45, 20

12 = 14,000,000,000

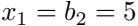


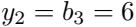
v2 = [1, 0, 0, 25, -0, 25, 5]



QWERTYUIOPASDFGHJKLZXCVBNM

| | x_1 | x_2 | s_1 | s_2 | s_3 | b |
|-------|-------|-------|-------|---------|---------|-----|
| s_1 | 0 | 0 | 1 | -0.4 | 0.2 | 2 |
| x_1 | 1 | 0 | 0 | 0.25 | -0.25 | 5 |
| x_2 | 0 | 1 | 0 | $-0, 1$ | 0.3 | 6 |
| z | 0 | 0 | 0 | 0.2 | 0.4 | 28 |











QWERTY UIOPASDFGHJKLZXCVBNM











$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) = \frac{1}{2}[\mathbf{x} - \mathbf{m}]^T \mathbf{Q} [\mathbf{x} - \mathbf{m}] = \frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{c}^T \mathbf{x} + c \\ \text{subject to:} & \mathbf{A} \mathbf{x} \leq \mathbf{b} \end{array}$$



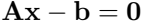




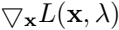




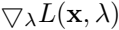


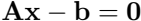


$$L(x, \lambda) = f(x) + \lambda^T (Ax - b) = \frac{1}{2} x^T Q x + c^T x + \lambda^T (Ax - b)$$



$$\nabla_x f(x) + \nabla_x \lambda(Ax - b) = 0x + c + A^T \lambda = 0$$





$$\begin{bmatrix} Q & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -c \\ b \end{bmatrix}$$



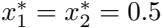


$$f(x_1, x_2) = x_1^2 + x_2^2 = \frac{1}{2} [x_1, x_2] \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{2} x^T Q x$$

$$Ax = \begin{bmatrix} 1, & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 + x_2 = b = 1$$

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad A = [1, 1], \quad c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad b = 1;$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$





1234567890





minimize

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{c}^T\mathbf{x}$$

subject to:

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to:} & \left\{ \begin{array}{l} \mathbf{h}(\mathbf{x}) = \mathbf{0} \\ \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \quad \text{or} \quad \mathbf{g}(\mathbf{x}) \geq \mathbf{0} \\ \mathbf{x} \geq \mathbf{0} \end{array} \right. \end{array}$$

$$g(x) \leq 0 \implies g(x) + s = 0, \quad g(x) \geq 0 \implies g(x) - s = 0$$



$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to:} & \begin{cases} \mathbf{h}(\mathbf{x}) = \mathbf{0} \\ \mathbf{x} \geq \mathbf{0} \end{cases} \end{array}$$

$$\frac{d}{dx} \left(x^2 \frac{dy}{dx} \right) = x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx}$$

$$\begin{cases} \nabla_{\mathbf{x}} L(\mathbf{x}, \lambda, \mu) = \mathbf{g}_f(\mathbf{x}) + \mathbf{J}_h^T(\mathbf{x})\lambda - \mu = \mathbf{0} & \text{(stationarity)} \\ \mathbf{h}(\mathbf{x}) = \mathbf{0}, \quad \mathbf{x} \geq \mathbf{0} & \text{(primal feasibility)} \\ \mu \geq \mathbf{0} & \text{(dual feasibility)} \\ \mu_j x_j = 0, \quad (j = 1, \dots, N) \quad \text{or} \quad \mathbf{XM}\mathbf{1} = \mathbf{0} & \text{(complementarity)} \end{cases}$$





$$\mathbf{X} = \begin{bmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{bmatrix}_{N \times N}, \quad \mathbf{M} = \begin{bmatrix} \mu_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mu_N \end{bmatrix}_{N \times N}, \quad \mathbf{J}_h(\mathbf{x}) = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \cdots & \frac{\partial h_m}{\partial x_n} \end{bmatrix}_{M \times N}$$

$$I(x) = \begin{cases} 0 & x \geq 0 \\ \infty & x < 0 \end{cases}$$



$$\begin{array}{ll}
 \text{minimize} & f(x) + \sum_{i=1}^N I(x_i) \\
 \text{subject to} & h(x) = 0
 \end{array}$$





$$-\frac{1}{t} \ln(x)$$

$$t \rightarrow \infty$$

$$I(x)$$

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) - \frac{1}{t} \sum_{i=1}^N \ln x_i \\ \text{subject to} & h(\mathbf{x}) = 0 \end{array}$$

$$L(\mathbf{x}, \lambda, \mu) = f(\mathbf{x}) + \lambda^T \mathbf{h}(\mathbf{x}) - \frac{1}{t} \sum_{j=1}^N \ln x_j$$

$$\nabla_x \mathcal{L}(x, \lambda, \mu) = g_f(x) + J_h^T(x) \lambda - \frac{1}{t} x^{-1} \mathbf{1}$$

$$4 = \frac{x}{x+1} = \frac{1}{1-x} - 1,$$

xw

$=$

$xm1$

$=$

1
 $-$
 j

$$\begin{cases} \mathbf{g}_f(\mathbf{x}) + \mathbf{J}_h^T(\mathbf{x})\lambda - \mu = \mathbf{0} & \text{(stationarity)} \\ \mathbf{h}(\mathbf{x}) = \mathbf{0} & \text{(primal feasibility)} \\ \mathbf{X}\mathbf{M}\mathbf{I} - \mathbf{1}/t = \mathbf{0} & \text{(complementarity)} \end{cases}$$

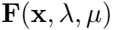






1999-2000





$$\mathbf{F}(\mathbf{x}, \lambda, \mu) = \begin{bmatrix} \mathbf{g}_f(\mathbf{x}) + \mathbf{J}_h^T(\mathbf{x})\lambda - \mu \\ \mathbf{h}(\mathbf{x}) \\ \mathbf{X}\mathbf{M}\mathbf{I}1 - 1/t \end{bmatrix}, \quad \mathbf{J}_F = \begin{bmatrix} \mathbf{W}(\mathbf{x}) & \mathbf{J}_h^T(\mathbf{x}) & -\mathbf{I} \\ \mathbf{J}_h(\mathbf{x}) & \mathbf{0} & \mathbf{0} \\ \mathbf{M} & \mathbf{0} & \mathbf{X} \end{bmatrix}$$

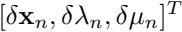


$$\sqrt{x} \left[\ln(x) + \frac{1}{x} \right] + \ln(x) \left[\ln(x) + \frac{1}{x} \right] - \ln(x)$$

$$\nabla_{\mathbf{x}} g_f(\mathbf{x}) - \nabla_{\mathbf{x}} \left[\sum_{i=1}^M \lambda_i \frac{\partial h_i}{\partial x_1}, \dots, \sum_{i=1}^M \lambda_i \frac{\partial h_i}{\partial x_N} \right]^T = \mathbf{H}_f(\mathbf{x}) - \sum_{i=1}^M \lambda_i \mathbf{H}_{h_i}(\mathbf{x})$$

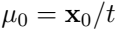
1990-1991

$$\begin{bmatrix} x_{n+1} \\ \lambda_{n+1} \\ \mu_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ \lambda_n \\ \mu_n \end{bmatrix} + \alpha \begin{bmatrix} \delta x_n \\ \delta \lambda_n \\ \delta \mu_n \end{bmatrix}$$



$$\mathbf{J}_F(\mathbf{x}_n, \lambda_n, \mu_n) \begin{bmatrix} \delta \mathbf{x}_n \\ \delta \lambda_n \\ \delta \mu_n \end{bmatrix} = \begin{bmatrix} \mathbf{W}(\mathbf{x}_n) & \mathbf{J}_h^T(\mathbf{x}_n) & -\mathbf{I} \\ \mathbf{J}_h(\mathbf{x}_n) & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_n & \mathbf{0} & \mathbf{X}_n \end{bmatrix} \begin{bmatrix} \delta \mathbf{x}_n \\ \delta \lambda_n \\ \delta \mu_n \end{bmatrix}$$

$$-\mathbf{F}(x_n, \lambda_n, \mu_n) = - \begin{bmatrix} g_f(x_n) + \mathbf{J}_h^T(x_n) \lambda_n - \mu_n \\ h(x_n) \\ \mathbf{X}_n \mathbf{M}_n \mathbf{1} - \mathbf{1}/t \end{bmatrix}$$







$$x^{-1} \nabla \partial x + \partial x = x^{-1} \nabla 1 + x^{-1} \nabla x = \partial x + x^{-1} \nabla x$$

$$W(x) + J_B(x) = -B(x) + J_B(x) + \mu$$

$$(N + X^{-1}NI) \delta x + J_H^T(x) \delta \lambda = g_f(x) - J_H^T(x) + X^{-1}1/t = \nabla_x L(x, \lambda)$$





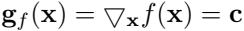
$$\begin{bmatrix} \mathbf{W} + \mathbf{X}^{-1} \mathbf{M} \mathbf{J}_h^T(\mathbf{x}) \\ \mathbf{J}_h(\mathbf{x}) \end{bmatrix} \begin{bmatrix} \delta \mathbf{x} \\ \delta \lambda \end{bmatrix} = - \begin{bmatrix} \nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda, \mu) \\ \mathbf{h}(\mathbf{x}) \end{bmatrix}$$



A pixelated, black and white graphic of the mathematical expression "ln(x) = ln(1)". The characters are rendered in a blocky, digital font style. The "x" and "1" are slightly larger than the other characters. The equals sign is composed of three horizontal bars. The entire expression is centered horizontally.

A pixelated, black and white graphic of the text "0x11/110x" in a monospace font. The characters are composed of black pixels on a white background, with some gray shading visible on the right side of the image. The font is a standard monospace typeface, and the text is centered horizontally.

$$\begin{array}{ll} \text{minimize} & f(x) = c^T x \\ \text{subject to:} & h(x) = Ax - b = 0, \quad x \geq 0 \end{array}$$



Wiederherstellung

$$\begin{cases} \nabla_{\mathbf{x}} L(\mathbf{x}, \lambda, \mu) = \mathbf{g}_f(\mathbf{x}) + \mathbf{J}_h^T(\mathbf{x})\lambda - \mu = \mathbf{c} + \mathbf{A}^T\lambda - \mu = \mathbf{0} \\ \mathbf{h}(\mathbf{x}) = \mathbf{A}\mathbf{x} - \mathbf{b} = \mathbf{0} \\ \mathbf{x}^T \mathbf{M} \mathbf{1} - 1/t = 0 \end{cases}$$

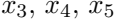
$$\begin{bmatrix} 0 & A^T & -I \\ A & 0 & 0 \\ M & 0 & X \end{bmatrix} \begin{bmatrix} \delta x \\ \delta \lambda \\ \delta \mu \end{bmatrix} = - \begin{bmatrix} c + A^T \lambda - \mu \\ Ax - b \\ XM1 - 1/t \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9

$$\begin{array}{ll}
 \text{maximize} & f(x_1, x_2) = 2x_1 + 3x_2 \\
 \text{subject to:} & \begin{cases} 2x_1 + x_2 \leq 18 \\ 6x_1 + 5x_2 \leq 60 \\ 2x_1 + 5x_2 \leq 40 \\ x_1 \geq 0, \quad x_2 \geq 0 \end{cases}
 \end{array}
 \implies
 \begin{array}{ll}
 \text{minimize} & f(x_1, x_2, x_3, x_4, x_5) = -2x_1 - 3x_2 \\
 \text{subject to:} & \begin{cases} 2x_1 + x_2 + x_3 = 18 \\ 6x_1 + 5x_2 + x_4 = 60 \\ 2x_1 + 5x_2 + x_5 = 40 \\ x_i \geq 0, \quad (i = 1, \dots, 5) \end{cases}
 \end{array}$$

$$\begin{array}{ll} \text{minimize} & f(x) = c^T x \\ \text{subject to} & h(x) = Ax - b = 0, \quad x \geq 0 \end{array}$$

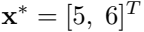
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 \\ 6 & 5 & 0 & 1 & 0 \\ 2 & 5 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 18 \\ 60 \\ 40 \end{bmatrix}$$



Exercises 1213







2021 + 2021 = 2021

| | $(x_1 \quad x_2)$ | | $f(\mathbf{x})$ | error |
|---|--------------------|--------------------|-----------------|-----------|
| 1 | $(1.0000000e + 00$ | $2.0000000e + 00)$ | -8.0000000 | 52.413951 |
| 2 | $(4.654514e + 00$ | $6.346354e + 00)$ | -28.348090 | 3.080204 |
| 3 | $(5.040828e + 00$ | $5.946973e + 00)$ | -27.922575 | 0.213509 |
| 4 | $(4.997282e + 00$ | $6.001764e + 00)$ | -27.999856 | 0.004262 |
| 5 | $(4.999906e + 00$ | $5.999962e + 00)$ | -27.999697 | 0.000303 |
| 6 | $(4.999989e + 00$ | $5.999996e + 00)$ | -27.999966 | 0.000034 |
| 7 | $(4.999999e + 00$ | $6.0000000e + 00)$ | -27.999996 | 0.000004 |
| 8 | $(5.0000000e + 00$ | $6.0000000e + 00)$ | -28.0000000 | 0.000000 |

1992-1993

$$\begin{array}{ll}
 \text{minimize} & f(x) = \frac{1}{2}x^T Q x + c^T x \\
 \text{subject to:} & h(x) = Ax - b = 0, \quad x \geq 0
 \end{array}$$

BRUNNEN

Wiederherstellung

$$L(x, \lambda, \mu) = \frac{1}{2} x^T Q x + c^T x + \lambda^T (Ax - b) - \mu^T x$$

$$\begin{bmatrix} Q & A^T & -I \\ A & 0 & 0 \\ M & 0 & X \end{bmatrix} \begin{bmatrix} \delta x \\ \delta \lambda \\ \delta \mu \end{bmatrix} = - \begin{bmatrix} Qx + c + A^T \lambda - \mu \\ Ax - b \\ XM1 - 1/t \end{bmatrix}$$

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) = [\mathbf{x} - \mathbf{m}]^T \mathbf{Q} [\mathbf{x} - \mathbf{m}] = \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{c}^T \mathbf{x} + d \\ \text{subject to} & \mathbf{h}(\mathbf{x}) = \mathbf{A} \mathbf{x} - \mathbf{b} = 0, \quad \mathbf{x} \geq 0 \end{array}$$

$$Q = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\mathbf{m} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}, \begin{bmatrix} 9 \\ 5 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

2011