









As 10, 24, 69, 10, 1, 10, 1, 1, 10,





RA

0 1 2 3 4 5 6

PEQUER

1995

ASAP

PARADISE 101

Q A P











1999



Pravda v pravdu

$$\int_{-\infty}^{u_1} \cdots \int_{-\infty}^{u_N} p(\xi_1, \cdots, \xi_N) d\xi_1 \cdots d\xi_N$$





$$P(A \cap B) = P(A \cup B) - P(A \cup B) = P(A \cup B) = P(A \cup B)$$







exp = exp

1999

$$\int_{-\infty}^u \int_{-\infty}^v p(\xi, \eta) d\xi d\eta = \int_{-\infty}^u \int_{-\infty}^v p(\xi) p(\eta) d\xi d\eta$$

$$\int_{-\infty}^u p(\xi) d\xi \int_{-\infty}^v p(\eta) d\eta = P(x < v) P(y < v)$$

$$p(x_1, \dots, x_n) = p(x_1)p(x_2, \dots, x_n)$$

PARAB

PARADE



PARAB + PARAB



PARABOLAS + PARABOLAS



$AB \cup B = A \cup B$

$$P(A|B) = \frac{P(A, B)}{P(B)},$$

$$P(B|A) = \frac{P(A, B)}{P(B)}$$

$$\left. \begin{aligned} P(A|B)P(B|C) &= P(B|A)P(A|C) \\ P(A|B,C)P(B|C) &= P(B|A,C)P(A|C) \end{aligned} \right\} = P(A,B|C)$$

PARABRA

PERIPHERAL

PARADE PARADE

$$P(A=a) = \int P(A=a, B=b) db = \int P(a|b)P(b) db$$

1 A 1 2 3 4 5 6 7 8 9 10 11 12



$$\bigcup_{i=1}^n A_i = S,$$

Ai

n

Aj

=

0

xi

x

xi



$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^n P(B|A_i)P(A_i)}$$

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(B|A_i)P(A_i)}{P(B)}$$

$$P(B) = P(B \cap S) = P(B \cap (\bigcup_{i=1}^n A_i)) = P(\bigcup_{i=1}^n (B \cap A_i)) = \sum_{i=1}^n P(B \cap A_i)$$



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$$F_x(\xi) = \int_{-\infty}^{\xi} p(x) dx$$

$$p(x) = \frac{d}{d\varepsilon} F_x(\varepsilon)$$

$$P(a \leq x < b) = F_x(b) - F_x(a) = \int_a^b p(x) dx$$

$$\int_{-\infty}^{\infty} p(x) dx = 1$$

199

25

1999

2000

$P(x) = P(x) = 1, \dots, x$

$$0 \leq P_i \leq 1,$$

$$\sum_{i=1}^n P_i = 1$$

$$F_x(\xi) = P(x < \xi) = \sum_{x_i < \xi} P(x_i) = \sum_{i=1}^k P_i$$





$$\mu_x = E(x) = \sum_{i=1}^n x_i P(x_i) = \sum_{i=1}^n x_i P_i$$

$$\mu_x = E(x) = \int_{-\infty}^{\infty} x p(x) dx$$

1999-2000

1993

$$\hat{\mu}_x = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$



$$x = \begin{cases} x_{(n+1)/2} & \text{if } n \text{ is odd} \\ \frac{x_{n/2} + x_{n/2+1}}{2} & \text{if } n \text{ is even} \end{cases}$$

$$\int_{-\infty}^{\tilde{x}}$$

$$p(x)dx$$

$$=$$

$$\int_{\tilde{x}}^{\infty}$$

$$p(x)dx$$

$$=$$

$$0.5$$

$$x = \arg \max_p (x),$$

x

$p(x) =$

$\max_{x \in \mathcal{X}}$
 $p(x)$



12345

$$\bar{x} = \frac{1}{8}(1+2+3+3+4+4+4+5) = 1 \times \frac{1}{8} + 2 \times \frac{1}{8} + 3 \times \frac{2}{8} + 4 \times \frac{3}{8} + 5 \times \frac{1}{8} = \frac{26}{8} = 3.25$$







$$\sigma^2 = \text{Var}(x) \triangleq E[(x - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$$

$$\sigma^2 = \text{Var}(x) = E[(x - \mu)^2] = \sum_{i=1}^n (x_i - \mu)^2 P_i$$



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$$E(x_1, x_2) = E(x_1, x_2) + x_2$$

$$E(x^2) - E(x)^2 = E(x^2) - E(x)^2$$

$$\mathcal{N}(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

$$\int_{-\infty}^{\infty} \mathcal{N}(x, \mu, \sigma) dx = 1$$

$$E(x) = \int_{-\infty}^{\infty} x \mathcal{N}(x, \mu, \sigma) dx = \mu$$

$$\text{Var}(x) = \int_{-\infty}^{\infty} (x - \mu)^2 \mathcal{N}(x, \mu, \sigma) dx = \sigma^2$$



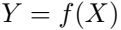


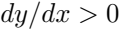
$$F_y(\psi) = P(y < \psi) = P(f(x) < \psi) = P(x < f^{-1}(\psi)) = \int_{-\infty}^x p_X(x) dx$$

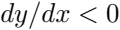




$$p_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} \int_{-\infty}^x p_X(\xi) d\xi = \left[p_X(x) \frac{dx}{dy} \right]_{x=f^{-1}(y)}$$







$$F_Y(y) = P(Y \leq y) = P(f(X) \leq y) = P(X \geq f^{-1}(y)) = \int_x^{\infty} p_X(\xi) d\xi$$

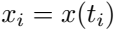
$$p_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} \int_x^\infty p_X(\xi) d\xi = \left[-p_X(x) \frac{dx}{dy} \right]_{x=f^{-1}(y)}$$

$$p_Y(y) = \left[p_X(x) \left| \frac{dx}{dy} \right| \right]_{x=f^{-1}(y)}$$



1999-2000

A pixelated, black and white representation of the text "Xavier's Law". The font is a stylized, blocky, and slightly irregular typeface, reminiscent of early digital art or video game titles. The letters are composed of various shades of gray and black pixels, giving it a textured, digital appearance. The text is centered horizontally and occupies the middle portion of the image.







$$\mu_i = E(x_i) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \xi_i p(\xi_1, \cdots, \xi_N) d\xi_1 \cdots d\xi_N$$

$$\mathbf{m} = E(x) = [E(x_1), \dots, E(x_N)]^T = [\mu_1, \dots, \mu_N]^T$$





$$Cov(x_i, x_j) = E[(x_i - \mu_i)(x_j - \mu_j)] = E[x_i x_j] - E[x_i] \mu_j - \mu_i E[x_j] + \mu_i \mu_j$$

$$E(x_i x_j) - \mu_i \mu_j = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \xi_i \xi_j p(\xi_1, \cdots, \xi_N) d\xi_1 \cdots d\xi_N - \mu_i \mu_j$$



$$\begin{aligned}
 \mathcal{L}_{\text{KL}} &= \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \left[-\log p(\mathbf{z}) \right] \\
 &= \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \left[-\log \left(\frac{1}{\sqrt{2\pi}} \exp \left(-\frac{\mathbf{z}^T \mathbf{z}}{2} \right) \right) \right] \\
 &= \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \left[\frac{1}{2} \mathbf{z}^T \mathbf{z} + \log \sqrt{2\pi} \right] \\
 &= \frac{1}{2} \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} [\mathbf{z}^T \mathbf{z}] + \log \sqrt{2\pi} \\
 &= \frac{1}{2} \text{tr}(\mathbf{I}) + \log \sqrt{2\pi} \\
 &= \frac{1}{2} \text{tr}(\mathbf{I}) + \log \sqrt{2\pi}
 \end{aligned}$$



Exercises in Algebra

$$\Sigma_x = E[(\mathbf{x} - \mathbf{m})(\mathbf{x} - \mathbf{m})^T] = E(\mathbf{x}\mathbf{x}^T) - \mathbf{m}\mathbf{m}^T = \begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1N}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{N1}^2 & \cdots & \sigma_{NN}^2 \end{bmatrix},$$

2020-2021

$$\text{tr} \Sigma = \sum_{i=1}^N \sigma_i^2 = \sum_{i=1}^N \lambda_i$$











$$\sum^{\top} E \left[(x - \Pi) (x - \Pi)^{\top} \right] z = E \left[z^{\top} (x - \Pi) (x - \Pi)^{\top} z \right]$$

1992-1993



1992-1993

$$p(x_1, \dots, x_n) = p(x_1, \dots, x_n)$$







$$\rho_{ij} = \frac{\sigma_{ij}^2}{\sigma_i \sigma_j}$$



$$\sqrt{x_1, x_2} / 2 = \sigma_{x_1}^2 \leq \sqrt{x_1, x_2} = \sigma_{x_1}^2 \sigma_{x_2}^2,$$

$$\left(\frac{\sigma_{ij}^2}{\sigma_i\sigma_j}\right)^2 = \rho^2 \leq 1$$

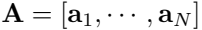


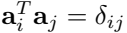
$$\mathbf{R}_x = E(\mathbf{x}\mathbf{x}^T) = \Sigma_x + \mathbf{m}\mathbf{m}^T = \begin{bmatrix} r_{11}^2 & \cdot & \cdot & \cdot & r_{1N}^2 \\ \vdots & \cdot & \cdot & \cdot & \vdots \\ r_{N1}^2 & \cdot & \cdot & \cdot & r_{NN}^2 \end{bmatrix}$$

$$r_j = E(x_j) = \sigma_j^2 + \mu_j \quad (i, j = 1, \dots, N)$$











$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \mathbf{A}^T \mathbf{x} = \begin{bmatrix} \mathbf{a}_1^T \\ \vdots \\ \mathbf{a}_N^T \end{bmatrix} \mathbf{x},$$

$$y = \int_0^x x = 1, \dots, x$$



$$\mathbf{x} = \mathbf{A}\mathbf{y} = [\mathbf{a}_1, \cdots, \mathbf{a}_N] \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \sum_{i=1}^N y_i \mathbf{a}_i$$

$$||\mathbf{x}||^2 = \mathbf{x}^T \mathbf{x} = \left(\sum_{i=1}^N y_i \mathbf{a}_i \right)^T \left(\sum_{j=1}^N y_j \mathbf{a}_j \right) = \sum_{i=1}^N \sum_{j=1}^N y_i y_j \mathbf{a}_i^T \mathbf{a}_j = \sum_{i=1}^N y_i^2 = ||\mathbf{y}||^2$$







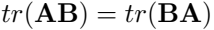


$$\begin{aligned}
 & \text{III} = \text{IV} \left(\text{A}^{\text{I}} \text{X} \right) = \text{A}^{\text{I}} \text{IV} \left(\text{X} \right) = \text{A}^{\text{I}} \text{III}
 \end{aligned}$$



$$E(vv^T) - vv^T = E(Ax x^T A) - A v v^T A$$

$$A^T E(x) A - A^T n_1 n_1^T A = A^T [E(x) - n_1 n_1^T] A = A^T \Sigma_x A$$



12/12/2020





परिचयः

1991-1992



$$\hat{\mathbf{m}} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k$$





$$\frac{1}{K} \sum_{k=1}^K (\mathbf{x}_k - \hat{\mathbf{m}})(\mathbf{x}_k - \hat{\mathbf{m}})^T = \frac{1}{K} \sum_{k=1}^K (\mathbf{x}_k \mathbf{x}_k^T - \mathbf{x}_k \hat{\mathbf{m}}^T - \hat{\mathbf{m}} \mathbf{x}_k^T + \hat{\mathbf{m}} \hat{\mathbf{m}}^T)$$

$$\frac{1}{K-1} \left(\sum_{k=1}^K \mathbf{x} \mathbf{x}^T - K \hat{\mathbf{m}} \hat{\mathbf{m}}^T \right) = \hat{\mathbf{R}} - \frac{K}{K-1} \hat{\mathbf{m}} \hat{\mathbf{m}}^T$$

$$\hat{R} = \frac{1}{K-1} \sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^T$$





$$\hat{\Sigma} \approx \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^T - \hat{\mathbf{m}} \hat{\mathbf{m}}^T, \quad \hat{\mathbf{R}} \approx \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^T$$









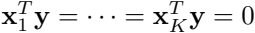
$$\sum_{k=1}^K (x_k - \hat{m}) = \sum_{k=1}^K x_k - K \hat{m} = 0$$







$$y^T \Sigma y = y^T \left(\frac{1}{K} \sum_{k=1}^K x_k x_k^T \right) y = \frac{1}{K} \sum_{k=1}^K y^T x_k x_k^T y = \frac{1}{K} \sum_{k=1}^K (x_k^T y)^2 \geq 0$$







$$y^T \hat{\Sigma} y = \frac{1}{K} \sum_{k=1}^K (x_k^T y)^2 = 0$$

$$y = \sum_{i=1}^K a_i x_i$$

$$y^T y = \sum_{i=1}^K a_i x_i^T y = 0$$







$$\text{tr} \, \Sigma = \sum_{i=1}^N \lambda_i > 0, \quad \det \, \Sigma = \prod_{i=1}^N \lambda_i > 0$$



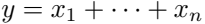




ΕΙΣ ΤΗΝ ΕΙΡΗΝΗΝ



$$E(y^2) - \mu_y^2 = E[(c x)^2] - (c \mu_x)^2 = c^2 E(x^2) - c^2 (\mu_x^2) = c^2 (E(x^2) - \mu_x^2) = c^2 \sigma_x^2$$



$$\mu_y = E(y) = E\left(\sum_{i=1}^n x_i\right) = \sum_{i=1}^n E(x_i) = \sum_{i=1}^n \mu_{x_i}$$

$$E(y^2) = E\left(\sum_{i=1}^n x_i\right)^2 = E\left(\sum_{i=1}^n \sum_{j=1}^n x_i x_j\right) = \sum_{i=1}^n \sum_{j=1}^n E(x_i x_j)$$

$$(E(y))^2 = \left(\sum_{i=1}^n E(x_i) \right)^2 = \sum_{i=1}^n \sum_{j=1}^n E(x_i) E(x_j)$$

$$Var(y) = E(y^2) - (E(y))^2 = \sum_{i=1}^n \sum_{j=1}^n E(x_i x_j) - \sum_{i=1}^n \sum_{j=1}^n E(x_i) E(x_j) = \sum_{i=1}^n \sum_{j=1}^n Cov(x_i, x_j)$$



$$\sigma_y^2 = \text{Var}(y) = \sum_{i=1}^n \sum_{j=1}^n \text{Cov}(x_i, x_j) = \sum_{i=1}^n \text{Cov}(x_i, x_i) = \sum_{i=1}^n \text{Var}(x_i) = \sum_{i=1}^n \sigma_{x_i}^2$$



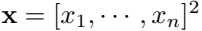
$$y = f(x) = f(\mu_x) + f'(\mu_x)(x - \mu_x) + \frac{f''(\mu_x)}{2}(x - \mu_x)^2 + \dots \approx f(\mu_x) + f'(\mu_x)(x - \mu_x)$$

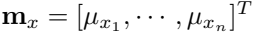


$$N_v = E(v) + f(v(x) - v(x)) = f(v(x))$$

$$\text{Var}(y) \approx \text{Var}[f(\mu_x) + f'(\mu_x)(x - \mu_x)] = \text{Var}[f'(\mu_x)(x - \mu_x)] = f'^2(\mu_x) \text{Var}(x) = f'^2(\mu_x) \sigma_x^2$$

১৭১০ খ্রিঃ





1999

$$f(\mu x_1, \dots, \mu x_n) + \sum_{i=1}^n \frac{\partial f}{\partial x_i} (x_i - \mu x_i) + \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j} (x_i - \mu x_i)(x_j - \mu x_j) + \dots$$



$$f(\mu_{x_1}, \dots, \mu_{x_n}) + \sum_{i=1}^n \frac{\partial f}{\partial x_i} (x_i - \mu_{x_i})$$

$$f(\mathbf{x}) = f(\mathbf{m}_x) + g(\mathbf{m}_x)^T (\mathbf{x} - \mathbf{m}_x) + \frac{1}{2} (\mathbf{x} - \mathbf{m}_x)^T \mathbf{H}(\mathbf{m}_x) (\mathbf{x} - \mathbf{m}_x) + \cdots \approx f(\mathbf{m}_x) + g(\mathbf{m}_x)^T (\mathbf{x} - \mathbf{m}_x)$$

$$\mathbf{g}(\mathbf{m}_x) = \begin{bmatrix} \frac{\partial f(\mathbf{x})}{\partial x_1} \\ \vdots \\ \frac{\partial f(\mathbf{x})}{\partial x_n} \end{bmatrix}_{\mathbf{m}_x}, \quad \mathbf{H}(\mathbf{m}_x) = \begin{bmatrix} \frac{\partial^2 f(\mathbf{x})}{\partial x_1^2} & \cdots & \frac{\partial^2 f(\mathbf{x})}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\mathbf{x})}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f(\mathbf{x})}{\partial x_n^2} \end{bmatrix}_{\mathbf{m}_x}$$



$$E f(x) \approx E \left[f(\mu_{x_1}, \dots, \mu_{x_n}) + \sum_{i=1}^n \frac{\partial f}{\partial x_i} (x_i - \mu_{x_i}) \right]$$

$$f(\mu x_1, \dots, \mu x_n) + \sum_{i=1}^n \frac{\partial f}{\partial x_i} E(x_i - \mu x_i) = f(\mu x_1, \dots, \mu x_n)$$



$$\text{Var}[f(x_1, \dots, x_n)] \approx \text{Var}\left[f(\mu_{x_1}, \dots, \mu_{x_n}) + \sum_{i=1}^n \frac{\partial f}{\partial x_i} (x_i - \mu_{x_i})\right]$$

$$\text{Var} \left[\sum_{i=1}^n \frac{\partial f}{\partial x_i} (x_i - \mu_{x_i}) \right] = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2$$

$$\sigma_y = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2}$$

$$p(x) = \mathcal{N}(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

2023-2024

$$p(\mathbf{x}) = \mathcal{N}(\mathbf{x}, \mathbf{m}, \Sigma) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m}) \right]$$

$$\exp\left(-\frac{1}{2}(n\log(2\pi) + \log|\Sigma|)\right) \exp\left[-\frac{1}{2}(\mathbf{x}^T \Sigma^{-1} \mathbf{x} - 2\mathbf{m}^T \Sigma^{-1} \mathbf{x} + \mathbf{m}^T \Sigma^{-1} \mathbf{m})\right]$$

$$\exp\left(-\frac{1}{2}(n\log(2\pi)+\log|\Sigma|+\mathbf{m}^T\Sigma^{-1}\mathbf{m})\right)\exp\left[-\frac{1}{2}(\mathbf{x}^T\Sigma^{-1}\mathbf{x}-2\mathbf{m}^T\Sigma^{-1}\mathbf{x})\right]$$





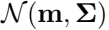
$$\psi(x) = \log \mathcal{N}(x, \mathbf{m}, \Sigma) = -\frac{1}{2}(x - \mathbf{m})^T \Sigma^{-1}(x - \mathbf{m}) - \frac{n}{2} \log(2\pi) - \frac{1}{2} \log |\Sigma|$$

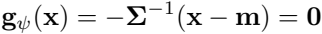


$$\frac{d}{dx} \psi(x) = -\Sigma^{-1}(x - \mu)$$



$$\frac{d^2}{dx^2} \psi(x) = -\Sigma^{-1} \psi(x)$$



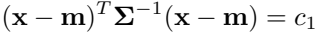






1992











$$\Sigma^{-1} = \begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix}$$

$$(\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m}) = [x_1 - \mu_1, x_2 - \mu_2] \begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix} \begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{bmatrix}$$

$$A(x_1 - \mu_1)^2 + B(x_1 - \mu_1)(x_2 - \mu_2) + C(x_2 - \mu_2)^2$$

$$A\alpha_1^2 + B\alpha_1\alpha_2 + C\alpha_2^2 - (2A\mu_1 + B\mu_2)\alpha_1 - (2C\mu_2 + B\mu_1)\alpha_2 + (A\mu_1^2 + B\mu_1\mu_2 + C\mu_2^2) = c_1$$



2-1-2024-09

LABORERS





$$f(x,y) = Ax^2 + Bxy + Cy^2 + Dx + Ey = 0$$



$$\left\{ \begin{array}{ll} \text{an ellipse} & \text{if } \Delta < 0 \\ \text{a parabola} & \text{if } \Delta = 0 \\ \text{a hyperbola} & \text{if } \Delta > 0 \end{array} \right.$$





1992

$$\Sigma = \textit{diag}[\sigma_1^2, \cdots, \sigma_n^2] = \begin{bmatrix} \sigma_1^2 & \cdot & \cdot & \cdot & 0 \\ \vdots & \cdot & \cdot & \cdot & \vdots \\ 0 & \cdot & \cdot & \cdot & \sigma_n^2 \end{bmatrix}$$

$$(\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m}) = \sum_{i=1}^n \frac{(x_i - \mu_i)^2}{\sigma_i^2} = c_1$$

Wiederholungsfrage

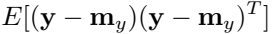
$$D(x_1 + x_2) = M(x_1 + x_2, z_1 + z_2)$$

Waxhaw, N.C.

$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$

W = Ax + b, z = Ax

$$m_x = E(Ax + b) = AEx + b = Am_x + b$$



$$E[(Ax-b)(Ax-b)^T] = E[Ax(Ax-b)^T] + E[b(Ax-b)^T]$$

$$E[Ax - Ax - Ax]^T = A^T E[x - mx]^T A^T$$





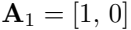
$\frac{d}{dx} \left(b \frac{dx}{dt} \right) = \frac{d}{dt} \left(b \frac{dx}{dt} \right)$

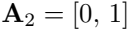


WAXES IN THE AIR

$$D(x) = N[x_1, \dots, x_n], \quad d[x_1, \dots, x_n]$$

$$p(x_1, x_2) = p(\mathbf{x}) = p\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_2^2 \end{bmatrix}\right)$$









$$p(x_1) = \mathcal{N}(x_1; \mu_1, \sigma_1^2)$$

$$p(x_2) = \mathcal{N}(x_2; \mu_2, \sigma_2^2)$$



$$\mathcal{N}(\mathbf{x}, \mathbf{m}_i, \Sigma_i) = \frac{1}{(2\pi)^{n/2} |\Sigma_i|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{x} - \mathbf{m}_i)^T \Sigma_i^{-1} (\mathbf{x} - \mathbf{m}_i) \right]$$

$$\frac{1}{(2\pi)^{n/2} |\Sigma_i|^{1/2}} \exp \left[-\frac{1}{2} (x^T \Sigma_i^{-1} x - 2x^T \Sigma_i^{-1} \mathbf{m}_i + \mathbf{m}_i^T \Sigma_i^{-1} \mathbf{m}_i) \right]$$

$$\exp\left[-\frac{1}{2}\left[n\log(2\pi)+\log|\Sigma_i|+\mathbf{m}_i^T\Sigma_i^{-1}\mathbf{m}_i\right]\right]\exp\left[-\frac{1}{2}\left(\mathbf{x}^T\Sigma_i^{-1}\mathbf{x}-2\mathbf{x}^T\Sigma_i^{-1}\mathbf{m}_i\right)\right]$$

$$\exp\left[-\frac{1}{2}\left[x^T\Sigma_i^{-1}x-2x^T\Sigma_i^{-1}m_i+c_i\right]\right] \quad (i=1,2)$$

$$\cos \left(\log \left(2\pi \right) + \log \left| z \right| + i \left(\arg z - 1 \right) \right)$$

11222

www.2020.2020.2020

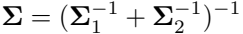
$$\exp\left[-\frac{1}{2}\left[x^T\Sigma_1^{-1}x-2x^T\Sigma_1^{-1}\mathbf{m}_1+c_1+x^T\Sigma_2^{-1}x-2x^T\Sigma_2^{-1}\mathbf{m}_2+c_2\right]\right]$$

$$\exp\left[-\frac{1}{2}\left[x^T(\Sigma_1^{-1}+\Sigma_2^{-1})x-2x^T(\Sigma_1^{-1}\mathbf{m}_1+\Sigma_2^{-1}\mathbf{m}_2)+c_1+c_2\right]\right]$$

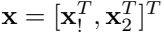
$$\exp\left[-\frac{1}{2}\left[x^T\Sigma^{-1}x-2x^T\Sigma^{-1}\mathbf{m}+c+(c_1+c_2-c)\right]\right]$$

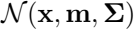
$$M(x, y, z) \exp(c_1 + c_2) \exp(x, y, z)$$

$$c = n \log(2\pi) + \log |Z| + \prod Z - 1$$



$$m_1(z_1) + m_2(z_2) = (z_1 + z_2) m_1(z_1 + z_2)$$









$$p\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=p(\mathbf{x})=\mathcal{N}(\mathbf{x},\mathbf{m},\Sigma)=\mathcal{N}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix},\begin{bmatrix}\mathbf{m}_1\\\mathbf{m}_2\end{bmatrix},\begin{bmatrix}\Sigma_{11}&\Sigma_{12}\\\Sigma_{21}&\Sigma_{22}\end{bmatrix}\right)$$



2020年12月20日





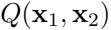
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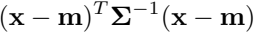
$$\begin{cases} m_{i/j} = m_i + \Sigma_{ij} \Sigma_{jj}^{-1} (x_j - m_j) \\ \Sigma_{i/j} = \Sigma_{ii} - \Sigma_{ij} \Sigma_{jj}^{-1} \Sigma_{ji} \end{cases}$$



$$p(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m}) \right]$$

$$\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left[-\frac{1}{2}Q(x_1,x_2)\right]$$





$$\begin{bmatrix} (x_1 - m_1)^T, (x_2 - m_2)^T \end{bmatrix} \begin{bmatrix} \Sigma^{11} & \Sigma^{12} \\ \Sigma^{21} & \Sigma^{22} \end{bmatrix} \begin{bmatrix} x_1 - m_1 \\ x_2 - m_2 \end{bmatrix}$$

$$(x_1 - m_1)^2 \Sigma^{11} (x_1 - m_1)^2 \Sigma^{12} (x_2 - m_2)^2 \Sigma^{22} (x_2 - m_2)$$

$$\Sigma^{-1} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \Sigma^{11} & \Sigma^{12} \\ \Sigma^{21} & \Sigma^{22} \end{bmatrix}$$



$$\begin{pmatrix} \Sigma_1 & \Sigma_2 & \Sigma_3 & \Sigma_4 & \Sigma_5 & \Sigma_6 & \Sigma_7 & \Sigma_8 & \Sigma_9 & \Sigma_{10} & \Sigma_{11} & \Sigma_{12} \\ \Sigma_1 & \Sigma_2 & \Sigma_3 & \Sigma_4 & \Sigma_5 & \Sigma_6 & \Sigma_7 & \Sigma_8 & \Sigma_9 & \Sigma_{10} & \Sigma_{11} & \Sigma_{12} \end{pmatrix}$$



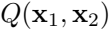
[illegible]







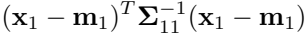




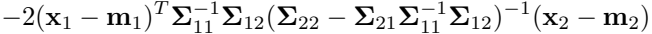
$$(x_1 - m_1)^T [\Sigma_1^{-1} + \Sigma_1^{-1} \Sigma_2^{-1} \Sigma_2^{-1} \Sigma_1^{-1}] (x_1 - m_1)$$

$$2(x_1 - 10)^2 + 2(x_1 - 1)^2 + 2(x_2 - 1)^2 + 2(x_2 - 10)^2$$

$$+ (x_2 - m_2)^2 - (x_2 - m_2)^2 - 1 (x_2 - m_2)^2 - 1 (x_2 - m_2)^2$$



$$+ \left(x_1 - \frac{1}{2} \right) \left(x_2 - \frac{1}{2} \right) + \left(x_1 - \frac{1}{2} \right)^2 + \left(x_2 - \frac{1}{2} \right)^2$$



$$+ [\sum_{i=1}^n (x_i - m_i) \sum_{j=1}^n (x_j - m_j) - \sum_{i=1}^n (x_i - m_i) \sum_{j=1}^n (x_j - m_j)]$$







$vAv + 2vAv + vAv = vAv + vAv + vAv$

$$v^T A(v-v) = v^T A(v-v) = v^T A(v-v)$$

www.Awv-Awv-Awv

$$\begin{cases} \mathbf{b} = \mathbf{m}_2 + \Sigma_{21} \Sigma_{11}^{-1} (\mathbf{x}_1 - \mathbf{m}_1) \\ \mathbf{A} = \Sigma_{22} - \Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12} \end{cases}$$

$$\begin{cases} Q_1(\mathbf{x}_1) &= (\mathbf{x}_1 - \mathbf{m}_1)^T \Sigma_{11}^{-1} (\mathbf{x}_1 - \mathbf{m}_1) \\ Q_2(\mathbf{x}_1, \mathbf{x}_2) &= [(\mathbf{x}_2 - \mathbf{m}_2) - \Sigma_{21} \Sigma_{11}^{-1} (\mathbf{x}_1 - \mathbf{m}_1)]^T (\Sigma_{22} - \Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12})^{-1} [(\mathbf{x}_2 - \mathbf{m}_2) - \Sigma_{21} \Sigma_{11}^{-1} (\mathbf{x}_1 - \mathbf{m}_1)] \\ &= (\mathbf{x}_2 - \mathbf{b})^T \mathbf{A}^{-1} (\mathbf{x}_2 - \mathbf{b}) \end{cases}$$

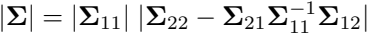
$$Q(x_1, x_2) = Q(x_1, x_2) + Q(x_1, x_2)$$

$$p(x_1, x_2) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp \left[-\frac{1}{2} Q(x_1, x_2) \right]$$

$$\frac{1}{(2\pi)^{n/2}|\Sigma_{11}|^{1/2}|\Sigma_{22}-\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}|^{1/2}}\exp\left[-\frac{1}{2}Q_1(x_1)\right]\exp\left[-\frac{1}{2}Q_2(x_1,x_2)\right]$$

$$\frac{1}{(2\pi)^{p/2}|\Sigma_{11}|^{1/2}}\exp\left[-\frac{1}{2}(\mathbf{x}_1-\mathbf{m}_1)^T\Sigma_{11}^{-1}(\mathbf{x}_1-\mathbf{m}_1)\right]\frac{1}{(2\pi)^{q/2}|\mathbf{A}|^{1/2}}\exp\left[-\frac{1}{2}(\mathbf{x}_2-\mathbf{b})^T\mathbf{A}^{-1}(\mathbf{x}_2-\mathbf{b})\right]$$

Waxayaa ahayd inay noqotay





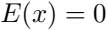
$$\int p(x_1, x_2) dx_2 = \mathcal{N}(x_1, \mathbf{m}_1, \Sigma_{11}) \int \mathcal{N}(x_2, \mathbf{b}, A) dx_2$$

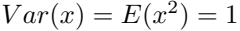
$$\mathcal{N}(\mathbf{x}_1, \mathbf{m}_1, \Sigma_{11}) = \frac{1}{(2\pi)^{p/2} |\Sigma_{11}|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{x}_1 - \mathbf{m}_1)^T \Sigma_{11}^{-1} (\mathbf{x}_1 - \mathbf{m}_1) \right]$$

$$p_{2|1}(x_2|x_1) = \frac{p(x_1, x_2)}{p_1(x_1)} = \mathcal{N}(x_2, \mathbf{b}, \mathbf{A}) = \frac{1}{(2\pi)^{q/2} |\mathbf{A}|^{1/2}} \exp \left[-\frac{1}{2} (x_2 - \mathbf{b})^T \mathbf{A}^{-1} (x_2 - \mathbf{b}) \right]$$

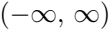
$$\begin{cases} \mathbf{b} = \mathbf{m}_{2/1} = \mathbf{m}_2 + \Sigma_{21} \Sigma_{11}^{-1} (\mathbf{x}_1 - \mathbf{m}_1) \\ \mathbf{A} = \Sigma_{2/1} = \Sigma_{22} - \Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12} \end{cases}$$

2023-09-11





0.1



W E W E W E W E W E

For $x \in \mathbb{R}$ and $y \in \mathbb{R}$



2023-2024

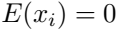


$$E(x) = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = 1$$

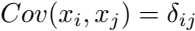
WORLD

$$[x - x]^2 = E(x^2) = 0^2 E(x^2) = 0^2$$





WORLDWIDE

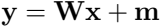


WAVELENGTH









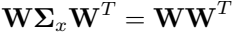


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UWAVE







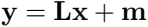


$$WW^T = (VB^{1/2})^T(VB^{1/2})^T = VB^T V^T = \Sigma$$

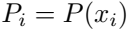








WAVELENGTH



1000

1000

1000

1000



1990-1991





$$H(P) = E\left(\log \frac{1}{P_i}\right) = \sum_i P_i \left(\log \frac{1}{P_i}\right) = -\sum_i P_i \log_2 P_i$$





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4. I have a question about the
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9. I have a question about the
10. I have a question about the

$$\frac{\partial}{\partial P_j} \left[- \sum_{i=1}^N P_i \log P_i + \lambda \left(1 - \sum_{i=0}^N P_i \right) \right] = 0$$

$$-\frac{\partial}{\partial P_j} P_j \log P_j - \lambda = -(\log P_j + 1) - \lambda = 0$$

$$P_1 = 2 - 1 = 1 \quad \text{and} \quad P_2 = 1 \cdot 1 \cdot 1 = 1$$

$$\sum_{j=1}^N P_j = \sum_{j=1}^N 2^{-\lambda-1} = 2^{-\lambda-1} N = 1$$





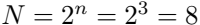


$$P_j = 2^{-\lambda-1} = 2^{-\log N} = \frac{1}{N}, \quad \mathcal{C} = 1, \dots, N)$$

$$H_{max}(E_1, \dots, E_N) = - \sum_{i=1}^N \frac{1}{N} \log_2 \frac{1}{N} = \log_2 N = n$$

$$\log_2 \frac{1}{P_i} = \log_2 \left(\frac{1}{1/2^n} \right) = \log_2 2^n = n$$

$$-\sum_{i=1}^N P_i \log_2 P = -\sum_{i=1}^N \frac{1}{N} \log_2 \frac{1}{2^n} = \sum_{i=1}^N \frac{1}{N} \log_2 2^n = n$$



123456789









100% 100%

1-10-2020





$$A = 0.1 \log 0.1 + 0.9 \log 0.9 = 0.47$$





$$A = (0.2 \times 0.8) = 0.16$$





$$A = 0.3 \log 0.7 = 0.88$$





$$A = (0.41 + 0.61) = 0.97$$

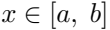
123456789



$$H = -\frac{1}{2}\log_2 -\frac{1}{2}\log_2 = -\frac{1}{2} + \frac{1}{2} = 1$$

$$H(p) = - \int p(x) \log p(x) \, dx$$







$$L(p) = H(p) + \lambda \left(1 - \int_a^b p(x) \, dx \right) = \int_a^b \left[-p(x) \log p(x) - \lambda p(x) \right] \, dx + \lambda$$

$$\frac{d\mathcal{I}(p)}{dp(x)} = -(1 + \log p(x)) - \lambda = 0$$

2021-11-11

$$\int_{-\infty}^{\infty} p(x) dx = \int_a^b e^{-\lambda-1} dx = e^{-\lambda-1} (b-a) = 1$$

$$p(x) = e^{-\lambda - 1} = \frac{1}{0 - a}$$



$$H(p) = - \int_a^b \frac{1}{b-a} \log \left(\frac{1}{b-a} \right) = \log(b-a)$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$$



$$L(p) = - \int_{-\infty}^{\infty} p(x) \log p(x) \, dx + \lambda_1 \left(1 - \int_{-\infty}^{\infty} p(x) \, dx \right) + \lambda_2 \left(\sigma^2 - \int_{-\infty}^{\infty} (x - \mu)^2 p(x) \, dx \right)$$

$$\frac{dI(p)}{dp(x)} = -(\log p(x) + 1) - \lambda_1 - \lambda_2 (x - \mu)^2 = 0$$

$$p(x) = e^{-(\lambda_1 + \lambda_2)(x - \mu)^2 + 1} = e^{-(\lambda_1 + 1)} e^{-\lambda_2(x - \mu)^2}$$

$$1 = \int_{-\infty}^{\infty} p(x) dx = e^{-\lambda_1 - 1} \int_{-\infty}^{\infty} e^{-\lambda_2 (x - \mu)^2} dx = e^{-(\lambda_1 + 1)} \sqrt{\frac{\pi}{\lambda_2}}$$



$$\int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx = e^{-\lambda_1 - 1} \int_{-\infty}^{\infty} e^{-\lambda_2 (x - \mu)^2} (x - \mu)^2 dx$$

$$e^{-\lambda_1-1}\frac{1}{2}\sqrt{\frac{\pi}{\lambda_2}}=e^{-\lambda_1-1}\frac{1}{2\lambda_2}\sqrt{\frac{\pi}{\lambda_2}}$$

λ_2

=

$\frac{1}{202}$

$$E = \sqrt{2\pi\sigma^2} = 1.41$$

$$e^{-(\lambda_1+1)} = \frac{1}{\sqrt{2\pi\sigma^2}}$$

19202

$$p(x) = e^{-(\lambda_1 + 1)} e^{-\lambda_2 (x - \mu)^2} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x - \mu)^2}{2\sigma^2}} = \mathcal{N}(x, \mu, \sigma^2)$$

199

$$-\int_{-\infty}^{\infty} g(x) \log g(x) \, dx = -\int_{-\infty}^{\infty} g(x) \log \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} \right) \, dx$$

$$-\int_{-\infty}^{\infty} g(x) \left(\log \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(x-\mu)^2}{2\sigma^2} \right) dx$$

$$\frac{1}{2} \log(2\pi\sigma^2) + \frac{1}{2\sigma^2} \int_{-\infty}^{\infty} (x - \mu)^2 g(x) dx$$

$$-\frac{1}{2}\log(2\pi\sigma^2) + \frac{1}{2\sigma^2}\sigma^2 = -\frac{1}{2}\log(2\pi\sigma^2) + \frac{1}{2}\log(e)$$

$$\frac{1}{2} \log(2\pi e \sigma^2) = \frac{1}{2} \log(2\pi e) + \log \sigma$$

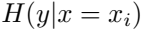
02-12-2020

Aspirin *logopin*

$$H(x,y) = - \sum_i \sum_j P(x_i,y_j) \log P(x_i,y_j)$$

$$H(x_1, \dots, x_n) = - \sum_{x_1} \dots \sum_{x_n} P(x_1, \dots, x_n) \log P(x_1, \dots, x_n)$$







$$H(y|x=x_i)=-\sum_j P(y_j|x_i)\log P(y_j|x_i)$$



$$E_x(H(y|x=x_i))=\sum_i P(x_i)H(y|x=x_i)$$

$$-\sum_i P(x_i) \sum_j P(y_j|x_i) \log P(y_j|x_i) = -\sum_i \sum_j P(x_i) P(y_j|x_i) \log P(y_j|x_i)$$

$$-\sum_i \sum_j P(x_i, y_j) \log P(y_j | x_i)$$

Pravda

$$-\sum_i \sum_j P(x_i, y_j) \log P(y_j | x_i) = \sum_i \sum_j P(x_i, y_j) \log \frac{P(x_i)}{P(x_i, y_j)}$$

$$-\sum_i \sum_j P(x_i, y_j) \log P(x_i, y_j) + \sum_i \sum_j P(x_i, y_j) \log P(x_i)$$

$$H(x, y) + \sum_i P(x_i) \log P(x_i) = H(x, y) - H(x)$$

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Handwritten text in a cursive script, likely a signature or name, rendered in a pixelated, black and white style. The text is split into three segments by vertical bars.

Handwritten text in a cursive script, likely a signature or name, rendered in a pixelated, black and white style. The text is split into three segments by vertical bars.

$$H(x,y) = H(x|y) + H(y)$$

$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$





$$\sum_i \sum_j P(x_i, y_j) \log \frac{P(x_i, y_j)}{P(x_i)P(y_j)}$$

$$\sum_i \sum_j P(x_i, y_j) \log \frac{P(x_i, y_j)}{P(x_i)} - \sum_i \sum_j P(x_i, y_j) \log P(y_j)$$

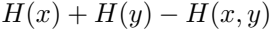
$$\sum_i \sum_j P(y_j|x_i)P(x_i) \log P(y_j|x_i) - \sum_j \log P(y_j) \left(\sum_i P(x_i,y_j) \right)$$

$$\sum_i P(x_i) \sum_j P(y_j | x_i) \log P(y_j | x_i) - \sum_j P(y_j) \log P(y_j)$$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

1820 1821 1822 1823 1824 1825 1826 1827 1828 1829 1830 1831 1832 1833 1834 1835 1836 1837 1838 1839 1840 1841 1842 1843 1844 1845 1846 1847 1848 1849 1850 1851 1852 1853 1854 1855 1856 1857 1858 1859 1860 1861 1862 1863 1864 1865 1866 1867 1868 1869 1870 1871 1872 1873 1874 1875 1876 1877 1878 1879 1880 1881 1882 1883 1884 1885 1886 1887 1888 1889 1890 1891 1892 1893 1894 1895 1896 1897 1898 1899 1900

Handwritten text in a cursive script, likely a signature or name, rendered in a pixelated, grayscale style. The text is split into two groups by a vertical line, both reading "Handwritten".



$$I(x_1, \cdots, x_N) = \sum_{i=1}^N H(x_i) - H(x_1, \cdots, x_N)$$



$$H(P, Q) = \sum_i P_i \log_2 \frac{1}{Q_i} = - \sum_i P_i \log Q_i$$



REPOSE

$$H(P) = - \sum_i P_i \log P_i \leq H(P, Q) = - \sum_i P_i \log Q_i$$

THE WORLD IS A VILLAGE



1000000

$$\sum_i P_i \log \left(\frac{Q_i}{P_i} \right) \leq \sum_i P_i \left(\frac{Q_i}{P_i} - 1 \right) = \sum_i Q_i - \sum_i P_i = 0$$

$$\sum_i P_i \log \left(\frac{Q_i}{P_i} \right) = \sum_i P_i \log \left(\frac{1}{P_i} \right) + \sum_i P_i \log Q_i = H(P) - H(P, Q) \leq 0$$

PERIOD

$$H(P,Q) - H(P) = - \sum_i P_i \log Q_i - (- \sum_i P_i \log P_i)$$

$$\sum_i P_i (\log P_i - \log Q_i) = \sum_i P_i \log \left(\frac{P_i}{Q_i} \right) \geq 0$$

PERIOD

REPO: REPO

$$I(x, y) = \sum_i \sum_j P(x_i, y_j) \log \frac{P(x_i, y_j)}{P(x_i)P(y_j)} = KL(P(x, y) || P(x)P(y))$$



1999

$$P(x,y) = P(x|y), \quad P(x|y) = P(y|x), \quad P(y|x) = P(y)$$



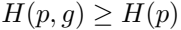
100%

$$-\int p(x) \log g(x) \, dx = -\int p(x) \left(\log \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(x-\mu)^2}{2\sigma^2} \right) dx$$

$$\frac{1}{2} \log(2\pi\sigma^2) + \frac{1}{2\sigma^2} \int (x - \mu)^2 p(x) dx = \frac{1}{2} \log(2\pi\sigma^2) + \frac{1}{2\sigma^2} \sigma^2$$

$$\frac{1}{2} \log(2\pi\sigma^2) + \frac{1}{2} = \frac{1}{2} \log(2\pi\sigma^2) + \frac{1}{2} \log e = \frac{1}{2} \log(2\pi e\sigma^2) = H(g)$$

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1999-2000

W E W O W

$$P(A, B) = P(A|B)P(B) = P(B|A)P(A), \quad p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

$$P(B) = \sum_{i=1}^n P(B|A_i) P(A_i), \quad P(A_i|B) = \frac{P(B|A_i) P(A_i)}{P(B)} = \frac{P(B|A_i) P(A_i)}{\sum_{i=1}^n P(B|A_i) P(A_i)} \quad (i = 1, \dots, n)$$

PARADE



REAR

PARALLEL



likelihood \times prior

evidence

$$p(H|D) = \frac{p(D|H)p(H)}{p(D)} \propto p(D|H)p(H)$$

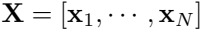




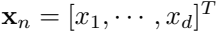




0123456789





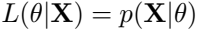




$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta) p(\theta)}{p(\mathbf{x})} = \frac{p(\mathbf{x}|\theta) p(\theta)}{\sum_{\theta} p(\mathbf{x}|\theta) p(\theta)} \propto p(\mathbf{x}|\theta) p(\theta)$$

0000



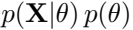


$$p(\mathbf{X}|\theta) = \prod_{n=1}^N p(x_n|\theta)$$

POSTERIOR PROBABILITY









$$\theta_{MLE} = \arg \max_{\theta} p(X|\theta)$$

$$\theta_{MAP} = \underset{\theta}{\operatorname{arg\,max}} \, p(\theta | X) \propto p(X | \theta) p(\theta)$$

2020-2021

$$\theta_{MLE} = \arg \max_{\theta} \log p(\mathbf{X}|\theta) = \arg \max_{\theta} \log \left(\prod_{n=1}^N p(\mathbf{x}_n|\theta) \right) = \arg \max_{\theta} \left(\sum_{n=1}^N \log p(\mathbf{x}_n|\theta) \right)$$

$$\theta_{MAP} = \arg \max_{\theta} \log \left(p(\mathbf{X}|\theta) p(\theta) \right) = \arg \max_{\theta} \left(\sum_{n=1}^N \log p(\mathbf{x}_n|\theta) + \log p(\theta) \right)$$

Q. Q. Q.



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A pixelated, grayscale image of a stylized letter 'V' or 'W' shape. The shape is composed of black and gray squares on a white background. The top part of the shape is a horizontal bar, followed by two diagonal strokes that meet at a point in the middle. The bottom part of the shape is a horizontal bar. The overall appearance is that of a low-resolution, digital drawing.

The image displays two horizontal bar charts, one for 'How often do you use the Internet?' and one for 'How often do you use a mobile phone?'. Each chart has two rows of bars representing different age groups: 18-24 and 25-34. The bars are color-coded: black for 'Daily', dark grey for 'Several times a week', medium grey for 'Once a week', light grey for 'A few times a month', and white for 'Less than once a month'.

Question	Age Group	Daily	Several times a week	Once a week	A few times a month	Less than once a month
How often do you use the Internet?	18-24	85%	10%	5%	0%	0%
	25-34	80%	15%	5%	0%	0%
How often do you use a mobile phone?	18-24	90%	10%	0%	0%	0%
	25-34	85%	15%	0%	0%	0%

A pixelated, grayscale illustration of a stylized figure, possibly a character or a logo, composed of various shades of gray and black pixels. The figure has a broad, flat top, a central vertical column, and two large, rounded, wing-like or arm-like structures extending outwards. The right side of the figure is partially cut off by the edge of the image. The style is reminiscent of early digital art or a low-resolution scan of a drawing.

A pixelated, black and white graphic of a stylized letter 'G'. The 'G' is formed by a thick, blocky line. A long, curved tail extends from the bottom right of the 'G', curving upwards and then downwards again, ending in a small hook-like shape. The entire graphic is composed of large, square pixels, giving it a retro, digital appearance.



$$L(\theta|\mathcal{D}) = p(\mathcal{D}|\theta) = \prod_{i=1}^N \Pr(y = y_i | x = x_i; \theta)$$

A pixelated, black and white graphic of the text "100% 100%". The text is rendered in a bold, blocky font with a dithered or pixelated appearance. The first "100%" is followed by a vertical bar, and the second "100%" is also followed by a vertical bar. The overall style is reminiscent of early digital art or low-resolution computer graphics.

$$\log \left(\prod_{i=1}^N \Pr(y_i = y_i | x_i; \theta) \right)$$

$$\sum_{i=1}^N \log \Pr(y = y_i | x = x_i; \theta)$$

100%





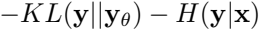




$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \log \Pr(y = y_i | \mathbf{x} = \mathbf{x}_i; \theta) = \sum_i \sum_j p(\mathbf{x}_i, y_j) \log p(y_j | \mathbf{x}_i; \theta)$$

$$\sum_i \sum_j p(x_i, y_j) \log \left[\frac{p(y_j | x_i; \theta)}{p(y_j | x_i)} \right] = \sum_i \sum_j p(x_i, y_j) \left[-\log \frac{p(y_j | x_i)}{p(y_j | x_i; \theta)} + \log(y_j | x_i) \right]$$

$$-\sum_i \sum_j p(x_i, y_j) \log \frac{p(y_j | x_i)}{p(y_j | x_i; \theta)} + \sum_i \sum_j p(x_i, y_j) \log(y_j | x_i)$$





123456789

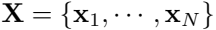




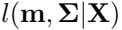
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$$\mathcal{N}(\mathbf{x}, \mathbf{m}, \Sigma) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{x} - \mathbf{m})^T \Sigma^{-1} (\mathbf{x} - \mathbf{m}) \right]$$



$$L(\theta|\mathbf{X}) = L(\mathbf{m}, \Sigma|\mathbf{X}) = p(\mathbf{X}|\mathbf{m}, \Sigma) = \prod_{i=1}^N \mathcal{N}(\mathbf{x}_i|\mathbf{m}, \Sigma)$$



$$\log L(\mathbf{m}, \Sigma | \mathbf{X}) = \log \left[\prod_{i=1}^N \mathcal{N}(\mathbf{x}_i | \mathbf{m}, \Sigma) \right] = \sum_{i=1}^N \log \mathcal{N}(\mathbf{x}_i | \mathbf{m}, \Sigma)$$

$$\sum_{i=1}^N \left[-\frac{d}{2} \log 2\pi - \frac{1}{2} \log |\Sigma| - \frac{1}{2} (\mathbf{x}_i - \mathbf{m})^T \Sigma^{-1} (\mathbf{x}_i - \mathbf{m}) \right]$$

$$\frac{\partial}{\partial \mathbf{m}} \ell(\mathbf{m}, \Sigma | \mathbf{X})$$

$$-\frac{1}{2}\sum_{i=1}^N\frac{\partial}{\partial \mathbf{m}}\left((\mathbf{x}_i-\mathbf{m})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}_i-\mathbf{m})\right)$$

$$-\frac{1}{2}\sum_{i=1}^N\Sigma^{-1}(\mathbf{x}_i-\mathbf{m})=0$$



$$\sum_{i=1}^N (\mathbf{x}_i - \mathbf{m}) = \mathbf{0}$$

$$\hat{m} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\frac{\partial}{\partial \Sigma} \ell(\mathbf{m}, \Sigma | \mathbf{X})$$

$$-\frac{N}{2}\frac{\partial}{\partial \Sigma}\log|\Sigma|-\frac{1}{2}\sum_{i=1}^N\frac{\partial}{\partial \Sigma}\left((\mathbf{x}_i-\mathbf{m})^T\Sigma^{-1}(\mathbf{x}_i-\mathbf{m})\right)$$

$$-\frac{N}{2}\Sigma^{-1}+\frac{1}{2}\sum_{i=1}^N\Sigma^{-1}(\mathbf{x}_i-\mathbf{m})(\mathbf{x}_i-\mathbf{m})^T\Sigma^{-1}=0$$

$$\hat{\Sigma} = \frac{1}{N} \sum_{i=1}^N (\mathbf{x}_i - \mathbf{m})(\mathbf{x}_i - \mathbf{m})^T$$

$$\frac{d}{dA} \log |A| = (A^{-1})^T, \quad \frac{d}{dA} (a^T A^{-1} b) = -(A^{-1})^T a b^T (A^{-1})^T$$





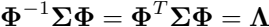
1999-2000

$$\mathbb{E} \left[\exp \left(\int_0^T \left(\frac{1}{2} \sigma^2(s) - \frac{1}{2} \sigma^2(s) \right) ds \right) \right] = 1$$

$$\lambda_N \geq \cdots \geq \lambda_1 \geq 0, \quad \phi_i^T \phi_j = \delta_{ij} = \begin{cases} 0 & i > j \\ 1 & i = j \end{cases} \quad (i, j = 1, \cdots, N)$$







$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



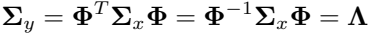
$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \Phi^T \mathbf{x} = \begin{bmatrix} \phi_1^T \\ \vdots \\ \phi_N^T \end{bmatrix} \mathbf{x}$$





$$\mathbf{x} = \Phi \mathbf{y} = [\phi_1, \cdots, \phi_N] \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \sum_{n=1}^N y_n \phi_n$$





$$\begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1N}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{N1}^2 & \cdots & \sigma_{NN}^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_N \end{bmatrix}$$



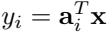




$$\sigma_{ij}^2 = \begin{cases} \lambda_i & i = j \\ 0 & i \neq j \end{cases}$$



$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \mathbf{A}^T \mathbf{x} = \begin{bmatrix} \mathbf{a}_1^T \\ \vdots \\ \mathbf{a}_N^T \end{bmatrix} \mathbf{x}$$





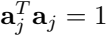
$$\text{tr} \Sigma_y = \sum_{i=1}^N \sigma_{y_i}^2 = \sum_{i=1}^N \sigma_{x_i}^2 = \text{tr} \Sigma_x$$



$$E_M(A) = \sum_{i=1}^M \sigma_{y_i}^2 = \sum_{i=1}^M \mathbf{a}_i^T \Sigma_x \mathbf{a}_i$$



$$\begin{cases} \text{maximize:} & \epsilon_M(A) = \sum_{i=1}^M a_i^T \Sigma_x a_i \\ \text{subject to:} & a_j^T a_j = 1 \quad (j = 1, \dots, N) \end{cases}$$



$$\frac{\partial}{\partial \mathbf{a}_i} \left[S_M(\mathbf{A}) - \sum_{j=1}^M \lambda_j (\mathbf{a}_j^T \mathbf{a}_j - 1) \right] = \frac{\partial}{\partial \mathbf{a}_i} \left[\sum_{j=1}^M (\mathbf{a}_j^T \boldsymbol{\Sigma}_x \mathbf{a}_j - \lambda_j \mathbf{a}_j^T \mathbf{a}_j + \lambda_j) \right]$$

$$\frac{\partial}{\partial a_i} (a_i^T \Sigma_x a_i - \lambda_i a_i^T a_i) \stackrel{*}{=} 2 (\Sigma_x a_i - \lambda_i a_i) = 0, \quad (i = 1, \cdots, N)$$





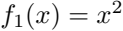


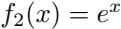
$$\epsilon_M(A) = \epsilon_M(\Phi) = \sum_{i=1}^M \phi_i^T \Sigma_x \phi_i = \sum_{i=1}^M \lambda_i$$

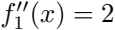


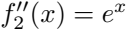


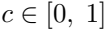
HELLO



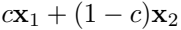






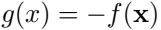


$$f(x_1+x_2) \leq f(x_1) + f(x_2)$$









001+1001+1001+1001

$$f\left(\sum_{i=1}^n \lambda_i x_i\right) \leq \sum_{i=1}^n \lambda_i f(x_i)$$




$$g \left(\sum_{i=1}^n \lambda_i x_i \right) \geq \sum_{i=1}^n \lambda_i g(x_i)$$











$$f\left(\sum_{i=1}^k \lambda_i x_i\right) \leq \sum_{i=1}^k \lambda_i f(x_i)$$



$$f\left(\sum_{i=1}^{k+1}\lambda_ix_i\right)=f\left(\lambda_ax_1+(1-\lambda_1)\sum_{i=2}^{k+1}\frac{\lambda_i}{1-\lambda_1}x_i\right)\leq\lambda_1f(x_1)+(1-\lambda_1)f\left(\sum_{i=2}^{k+1}\frac{\lambda_i}{1-\lambda_1}x_i\right)$$

$$\sum_{i=2}^{k+1} \frac{\lambda_i}{1-\lambda_1} = 1$$

$$f\left(\sum_{i=2}^{k+1}\frac{\lambda_i}{1-\lambda_1}x_i\right)\leq\sum_{i=2}^{k+1}\frac{\lambda_i}{1-\lambda_1}f(x_i)$$

$$f\left(\sum_{i=1}^{k+1}\lambda_ix_i\right)\leq\lambda_1f(x_1)+(1-\lambda_1)\left(\sum_{i=2}^{k+1}\frac{\lambda_i}{1-\lambda_1}f(x_i)\right)=\sum_{i=1}^{k+1}\lambda_if(x_i)$$

THE UNIVERSITY OF CHICAGO

WORLDWIDE

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \sim \mathcal{N}(\mu, \sigma^2/N)$$





2013





$$z = \frac{x - \mu}{\sigma / \sqrt{N}} \sim \mathcal{N}(0, 1)$$



$$S^2 = \frac{1}{N-1} \sum_{i=1}^N e_i^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 = \frac{1}{N-1} \left[\sum_{i=1}^N x_i^2 - N\bar{x}^2 \right]$$





$$t = \frac{\bar{x} - \mu}{S/\sqrt{N}} \sim T_\nu(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}$$



$$T_1(t) = \frac{1}{\pi(1+t^2)}, \quad T_2(t) = \frac{1}{(2+t^2)^{3/2}}, \quad T_3(t) = \frac{6\sqrt{3}}{\pi(3+t^2)^2}, \quad T_\infty(t) = \frac{1}{\sqrt{2\pi}}e^{t^2/2}$$

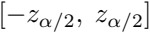


100%

[illegible]



PROPOSED



$$P(-z_{\alpha/2} \leq z \leq z_{\alpha/2}) = \int_{-z_{\alpha/2}}^{z_{\alpha/2}} \mathcal{N}(z, 0, 1) dz = 1 - \alpha,$$

$$\int_{-\infty}^{z-a/2} \mathcal{N}(z,0,1) \, dz = \int_{z_{a/2}}^{\infty} \mathcal{N}(z,0,1) \, dz = a/2$$

202

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1992-1993

$$P\left(-z_{\alpha/2} \leq \frac{\bar{x} - \mu}{\sigma/\sqrt{N}} \leq z_{\alpha/2}\right)$$

$$P\left(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{N}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{N}}\right) = 1 - \alpha$$



$$L = \bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{N}},$$

$$U = \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{N}}$$

1234567890



$$x \pm z\alpha/\sqrt{2} \frac{0}{\sqrt{N}} = x \pm z\alpha/\sqrt{2} E$$

DESIGNER



$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = \bar{x} \pm t_{\alpha/2} ESE$$



$$P(-t_{\alpha/2} \leq t \leq t_{\alpha/2}) = \int_{-t_{\alpha/2}}^{t_{\alpha/2}} f_v(t) dt = 1 - \alpha$$





$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$



A pixelated, black and white graphic of the text "2020-2020". The characters are rendered in a blocky, digital font style. The first "2020" is on the left, followed by a hyphen, and then another "2020" on the right. The entire graphic is composed of black and white pixels on a white background.

100525100

1991-1992

Wesley

