MATH 444 HW 9

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5.1.12

Suppose that $f(x) \neq 0$ for $x \in \mathbb{R} \setminus \mathbb{Q}$. But, in 4.1.15, we explored the function $g : \mathbb{R} \to \mathbb{R}$, where $g(x) = x$ if x is rational and $g(x) = c$ if x is irrational. We proved that if $c \neq 0$, that there exists no limit at point c. We can translate this to this problem by seeing that if $f(x) \neq 0$ for all irrationals, that there exists no limit of f at x, and therefore, f is not continuous at point x. This is a contradiction because f is given to be continuous, so therefore $f(x) = 0$ for every $x \in \mathbb{R}$.

5.1.13

g will be continuous only where $2x = x + 3$, which is $x = 3$.

5.2.1

a.

$$f(x) = \frac{x^2 + 2x + 1}{x^2 + 1} = \frac{(x+1)(x+1)}{(x+1)(x-1)}$$

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = x$ is continuous, so it is trivial that $x + 1$ and $x - 1$ are continuous. By theorem 5.2.2a, we can see that $h(x) = (x + 1)(x + 1)$ is continuous and that $g(x) = (x+1)(x-1)$ is continuous. Also, by theorem 5.5.5b, we see that $\frac{h}{g}$ is continuous, because h and g are continuous, but only when $g(x) \neq 0$. So we must omit $x = -1, 1$. It follows that $fracx^2 + 2x + 1x^2 + 1$ is continuous at every point on the real line except -1 and 1.