CS 374 Spring 2018
Homework 1

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Problem 2 solutions:

1.

- $\rho(\epsilon) = \{(A, A), (B, B), (C, C), (D, D), (E, E), (F, F)\}$
- $\rho(1011) = \{(C, A), (D, B), (E, C), (F, D), (A, E), (B, F)\}$
- $\rho(101) = \{(D, A), (E, B), (F, C), (A, D), (B, E), (C, F)\}$
- $\rho(10110) = \{(B, A), (C, B), (D, C), (E, D), (F, E), (A, F)\}$

2.

For a general DFA, $\rho(w) = \{(q_1, q_2) \mid q_1 \in Q,\ q_2 = \delta^*(q_1, w)\}$

3.

$$\rho(10110) = \{(B, A), (C, B), (D, C), (E, D), (F, E), (A, F)\}$$

We see that $s =' A' \in Q$, $'F' \in A \subseteq Q$, and $(A, F) \in \rho(10110)$. It follows that $w = 10110 \in L'(M_0)$.

$$\rho(101) = \{(D, A), (E, B), (F, C), (A, D), (B, E), (C, F)\}$$

(A, D) is the only pair $(q_1, q_2) \in \rho(101)$ where $q_1 = s$, however, $'D' \notin A \subseteq Q$, so we must say that $w - 101 \notin L'(M_0)$.

4.

$$L'(M_0) = \{w \mid |w| \bmod 6 \in \{1, 5\}\}$$

5.

In general, $L'(M) = L(M)$ because $L'(M)$ describes pairs of states $(q_1, q_2) \in \rho(w)$ where $q_1 = s$ and $q_2 \in A$. Given our definition of $\rho(w)$ above we can see that $L(M) = \{w \mid \delta^*(s, w) \in A\}$ defines the same set as $L'(M)$.