Problem 3 lemma

Problem 2

For this problem, A contains B is defined as the collection of all strings in A that have some string in B as a substring.

1. When $A = \{1001, 1111\}$ and $B = \{01, 10\}$, A contains $B = \{1001\}$.This is simply because the string 1001 is the only string in A that has 10 or (in this case, and) 01 as substrings in it.The string 1111 has no substring in it which is a string in B and hence does not belong in the set A contains B.

2. When $A = L((0+1)^*)$ and $B = L(1^*)$, A contains $B = (0+1)^*$.This is because A is the set of all binary strings and B is the set of all strings of 1 including the empty string. Since A contains B can be any binary string containing a 1 or the empty string, A contains B is $(0+1)^*$.

3. We want to prove that if A and B are regular then the language A contains B is also regular. We use previously established closure properties to prove this result.

We will construct a new language $\text{MID}(L)$ as follows:

$$\text{MID}(L) = \{w \mid \exists\, x, z \in \Sigma^*,\ y \in L \text{ such that } w = xyz\}$$

<u>Claim</u>: $\text{MID}(L) = \{w \mid \exists\, x, z \in \Sigma^*,\ y \in L \text{ such that } w = xyz\}$ is regular if L is regular.

<u>Proof</u>: Let us assume that L is regular. We will then prove that $\text{MID}(L)$ is regular. L is regular implies that a DFA $M = (Q, \Sigma, \delta, s, A)$ exists such that $L(M) = L$. Let us construct a new NFA N that will recognize $L(\text{MID}(L))$.

Let us define $N = (Q', \Sigma', \delta', s', A')$.

- $Q' = Q \cup \{s', t\}$
- $\Sigma' = \Sigma$
- $s' = s'$
- $A' = \{t\}$
- Let us define δ':

- $\delta'(q, a) = \delta(q, a)$ if $q \in Q \setminus A$
- $\delta'(s', a) = s$, $a \in \Sigma$
- $\delta'(s, \epsilon) = s'$
- $\delta'(q, a) = t$, $q \in A, a \in \Sigma$
- $\delta'(t, a) = t$, $a \in \Sigma$

We have constructed an NFA N such that $L(N) = L(\text{MID}(L))$, and thus, $L(\text{MID}(L))$ is regular.

$\square$

It follows from the proof above that $\text{MID}(B)$ is regular because B is regular.

Because $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$, we observe that

$$A \text{ contains } B = \{w \in A \mid \exists\, x, y, z.\ y \in B \text{ and } w = xyz\}$$

$$= A \cap \{w \mid \exists\, x, y \in \Sigma^*,\ y \in B \text{ such that } w = xyz\}$$

$$= A \cap \text{MID}(B)$$

Since A contains B is the intersection of two regular languages, A contains B is regular.

$\square$