

CS 374 Spring 2018

Homework 3

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Problem 2

For this problem, A contains B is defined as the collection of all strings in A that have some string in B as a substring.

1. When $A = \{1001, 1111\}$ and $B = \{01, 10\}$, A contains $B = \{1001\}$. This is simply because the string 1001 is the only string in A that has 10 or (in this case, and) 01 as substrings in it. The string 1111 has no substring in it which is a string in B and hence does not belong in the set A contains B .
2. When $A = L((0 + 1)^*)$ and $B = L(1^*)$, A contains $B = (0 + 1)^*$. This is because A is the set of all binary strings and B is the set of all strings of 1 including the empty string. Since A contains B can be any binary string containing a 1 or the empty string, A contains B is $(0 + 1)^*$.
3. We want to prove that if A and B are regular then the language A contains B is also regular. We use previously established closure properties to prove this result.

We will construct a new language $\text{MID}(L)$ as follows:

$$\text{MID}(L) = \{w \mid \exists x, z \in \Sigma^*, y \in L \text{ such that } w = xyz\}$$

Claim: $\text{MID}(L) = \{w \mid \exists x, z \in \Sigma^*, y \in L \text{ such that } w = xyz\}$ is regular if L is regular.

Proof: Let us assume that L is regular. We will then prove that $\text{MID}(L)$ is regular. L is regular implies that a DFA $M = (Q, \Sigma, \delta, s, A)$ exists such that $L(M) = L$. Let us construct a new NFA N that will recognize $L(\text{MID}(L))$.

Let us define $N = (Q', \Sigma', \delta', s', A')$.

- $Q' = Q \cup \{s', t\}$
- $\Sigma' = \Sigma$
- $s' = s'$

- $A' = \{t\}$
- Let us define δ' :
 - $\delta'(q, a) = \delta(q, a)$ if $q \in Q \setminus A$
 - $\delta'(s', a) = s, a \in \Sigma$
 - $\delta'(s, \epsilon) = s'$
 - $\delta'(q, a) = t, q \in A, a \in \Sigma$
 - $\delta'(t, a) = t, a \in \Sigma$

We have constructed an NFA N such that $L(N) = L(\text{MID}(L))$, and thus, $L(\text{MID}(L))$ is regular. □

It follows from the proof above that $\text{MID}(B)$ is regular because B is regular.

Because $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$, we observe that

$$\begin{aligned}
 A \text{ contains } B &= \{w \in A \mid \exists x, y, z. y \in B \text{ and } w = xyz\} \\
 &= A \cap \{w \mid \exists x, y \in \Sigma^*, y \in B \text{ such that } w = xyz\} \\
 &= A \cap \text{MID}(B)
 \end{aligned}$$

Since A contains B is the intersection of two regular languages, A contains B is regular. □