

# CS 374 Problem Set 1

## Problem 2

1.

- $\rho(\epsilon) = \{(A, A), (B, B), (C, C), (D, D), (E, E), (F, F)\}$
- $\rho(1011) = \{(C, A), (D, B), (E, C), (F, D), (A, E), (B, F)\}$
- $\rho(101) = \{(D, A), (E, B), (F, C), (A, D), (B, E), (C, F)\}$
- $\rho(10110) = \{(B, A), (C, B), (D, C), (E, D), (F, E), (A, F)\}$

2.

For a general DFA,  $\rho(w) = \{(q_1, q_2) \mid q_1 \in Q, q_2 = \delta^*(q_1, w)\}$

3.

$$\rho(10110) = \{(B, A), (C, B), (D, C), (E, D), (F, E), (A, F)\}$$

We see that  $s = 'A' \in Q$ ,  $'F' \in A \subseteq Q$ , and  $(A, F) \in \rho(10110)$ . It follows that  $w = 10110 \in L'(M_0)$ .

$$\rho(101) = \{(D, A), (E, B), (F, C), (A, D), (B, E), (C, F)\}$$

$(A, D)$  is the only pair  $(q_1, q_2) \in \rho(101)$  where  $q_1 = s$ , however,  $'D' \notin A \subseteq Q$ , so we must say that  $w = 101 \notin L'(M_0)$ .

4.

$$L'(M_0) = \{w \mid |w| \bmod 6 \in \{1, 5\}\}$$

5.

In general,  $L'(M) = L(M)$  because  $L'(M)$  describes pairs of states  $(q_1, q_2) \in \rho(w)$  where  $q_1 = s$  and  $q_2 \in A$ . Given our definition of  $\rho(w)$  above we can see that  $L(M) = \{w \mid \delta^*(s, w) \in A\}$  defines the same set as  $L'(M)$ .