CS 374 Spring 2018
Homework 4

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Problem 1

1.

$$L = \{w \in \{0,1\}^* \mid w \neq w^R\}$$

Let us define a context free grammar $G = (\Sigma, \Gamma, R, S)$ such that $L(G) = L$.

- $\Sigma = \{0, 1\}$
- $\Gamma = \{S, M\}$
- $S = S$
- Let R be defined as follows:
  - $S \rightarrow 0S0 \mid 1S1 \mid 1M0 \mid 0M1$
  - $M \rightarrow 1M1 \mid 1M0 \mid 0M0 \mid 0M1 \mid 0 \mid 1 \mid \epsilon$

Notice that the only way to terminate $w \in L(G)$ is to go to the rule M. For a word w, notice that $w[i] = w^R[k-i]$ where k is the length of w (and the 'w[i]' notation is indexed notation for the symbol at location i in string w. The only way ot go to rule M is to write different symbols in the i and $k-i$ positions. The only way $w^R = w$ is if $w[i] = w^R[i]$, but since $w[i] \neq w[k-i] \Rightarrow w^R[k-i] \neq w[k-i]$, $w \neq w^R$.

2.

$$L = \{a^i b^j c^k d^\ell \mid i + l = j + k\}$$

Let us define a context free grammar $G = \{\Sigma, \Gamma, R, S\}$ such that $L(G) = L$.

- $\Sigma = \{a, b, c, d\}$
- $\Gamma = \{S, A, B, C, D\}$
- $S = S$
- Let R be defined as follows:
  - $S \to AD \mid BD \mid BC \mid \epsilon$
  - $A \to aAc \mid aBc \mid \epsilon$
  - $B \to aBb \mid \epsilon$
  - $C \to bCd \mid bDd \mid \epsilon$
  - $D \to cDd \mid \epsilon$

First let us note that the rules A, B, C, D have no significance relating to their specific letters; let them just be generic placeholder non-terminals. The letters a, b, c, d must appear in alphabetical order, so essentially, what this construction exploits is splitting up the word w into 2 parts $w_1 w_2$. The first part, w_1 continas all i of the $a's$ and the next i characters in the string (which must be in $\{b, c\}$). The second part of the word, w_2 contains ℓ characters (also from $\{b, c\}$ followed by ℓ $d's$. This insures that no inbalance is achieved because all of our rules ensure that the balance $i + \ell = j + k$ is maintained by only writing characters of 'opposite' values.