

Fluctuations of ultralight dark matter

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Particle Physics-Astrophysics-Cosmology Webinar, June 21, 2023

In this talk

ultralight dark matter (ULDM) = wave dark matter

denotes any bosonic dark matter candidate whose mass is

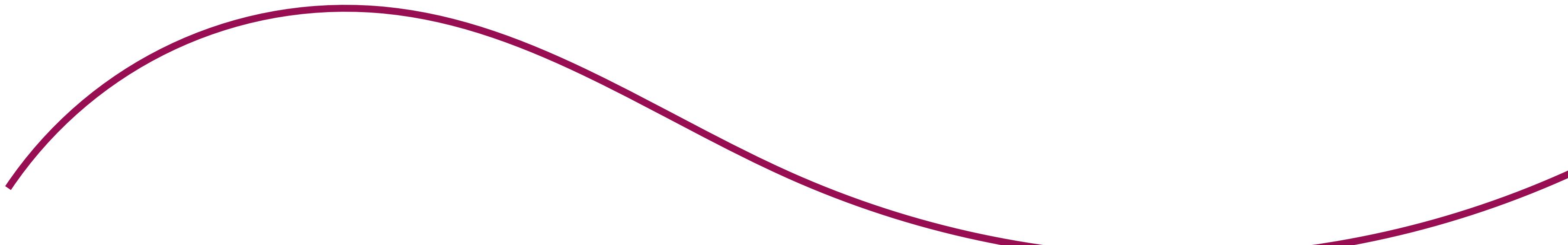
$$m \lesssim 10 \text{ eV}$$

*What's special about wave dark matter is
the occupation number (particle number per wavelength) is much larger than 1*



$$N_{\text{occ}} \sim n_{\text{dm}} \lambda^3 \sim \left(\frac{10 \text{ eV}}{m} \right)^4$$
$$n_{\text{dm}} = \frac{\rho_{\text{dm}}}{m}$$
$$\rho_{\text{dm}} \sim \text{GeV/cm}^3$$

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$$\rho_{\text{dm}} \sim \text{GeV/cm}^3$$

For $m < 10$ eV, the occupation number takes a gigantic value

this dark matter candidate is conveniently described by a collection of classical waves

*it interacts with SM through **gravitational interaction***

*it **may or may not** interact with SM through **non-gravitational interaction***

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*it **may or may not** interact with SM through **non-gravitational interaction***

how it might interact with SM through non-gravitational interaction

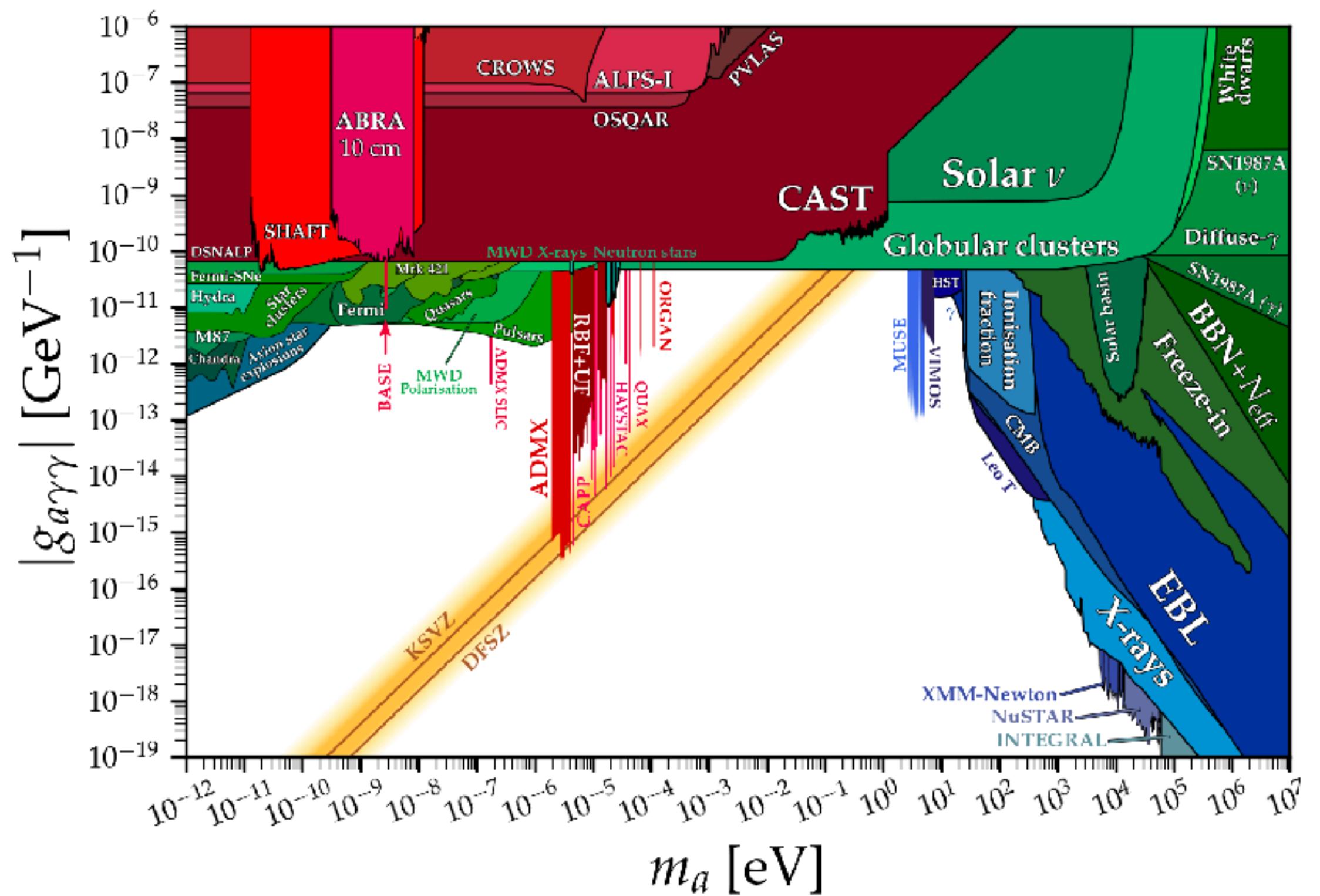
depends on underlying theoretical nature of wave dark matter

For instance for axions and axion-like particles

$$(\phi/f)F\tilde{F} + (\partial_\mu\phi/f)\bar{\psi}\gamma^\mu\gamma_5\psi + (\phi/f)\bar{N}\sigma^{\mu\nu}\gamma_5NF_{\mu\nu}$$

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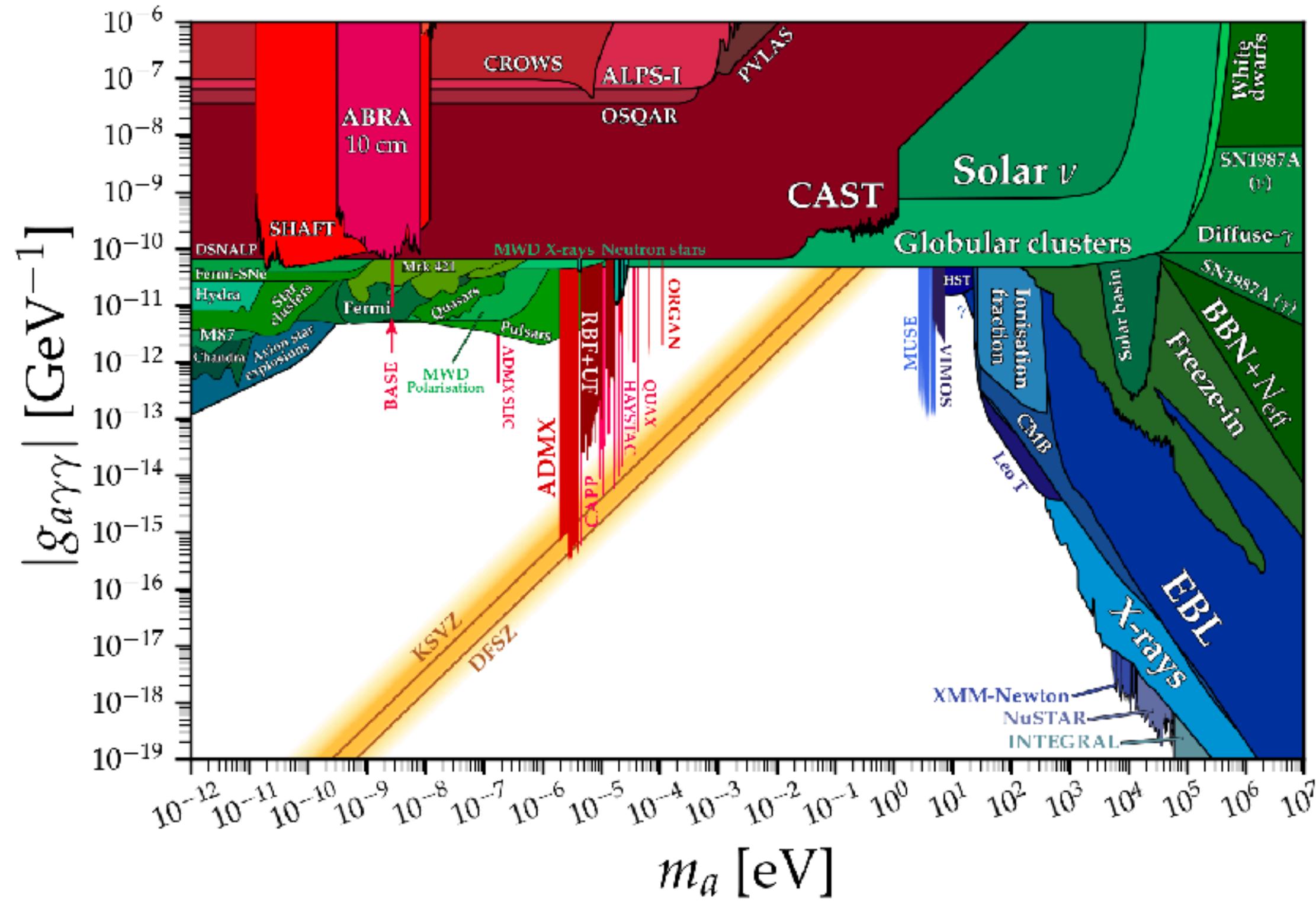
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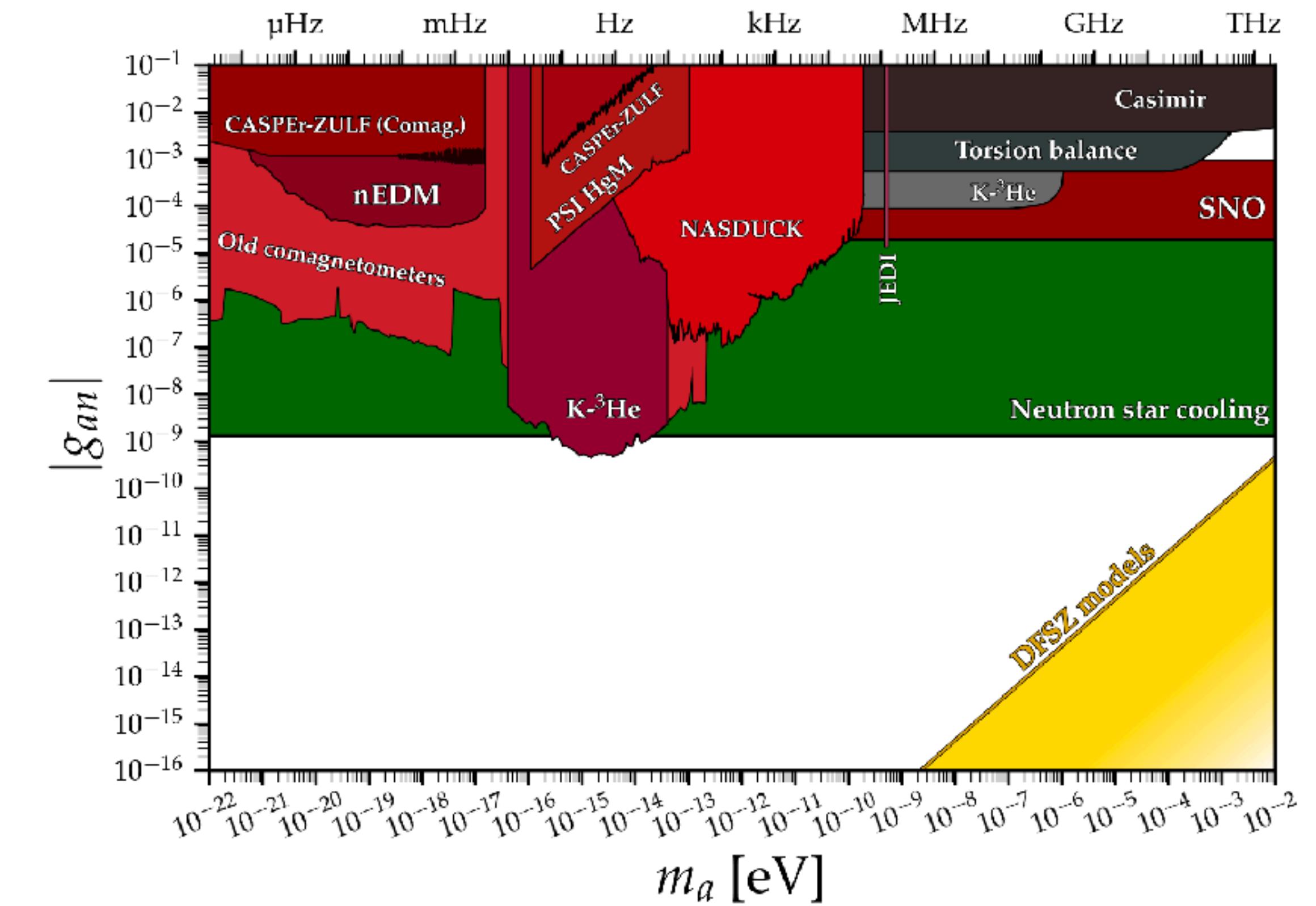
[O'Hare, AxionLimits]

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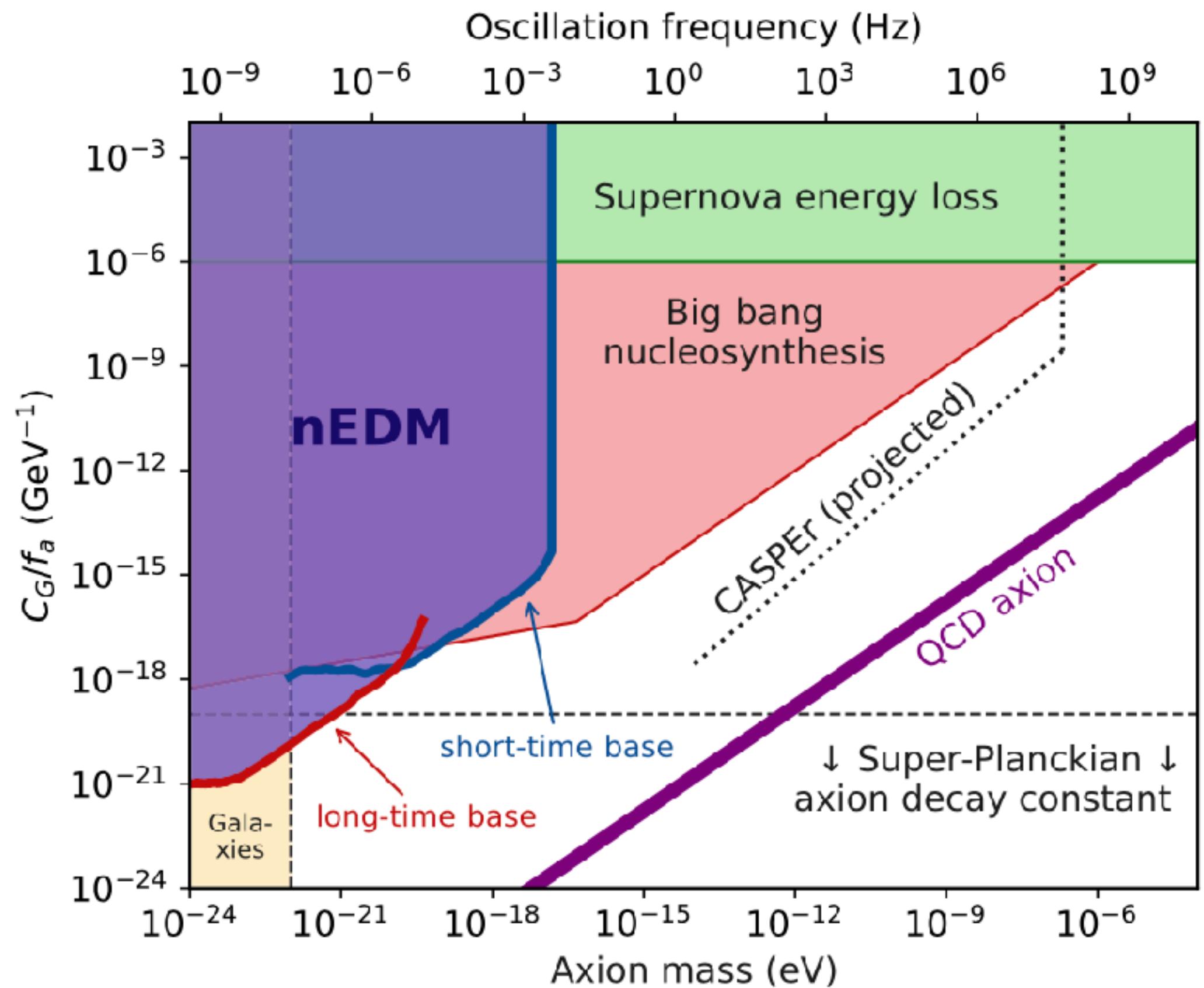


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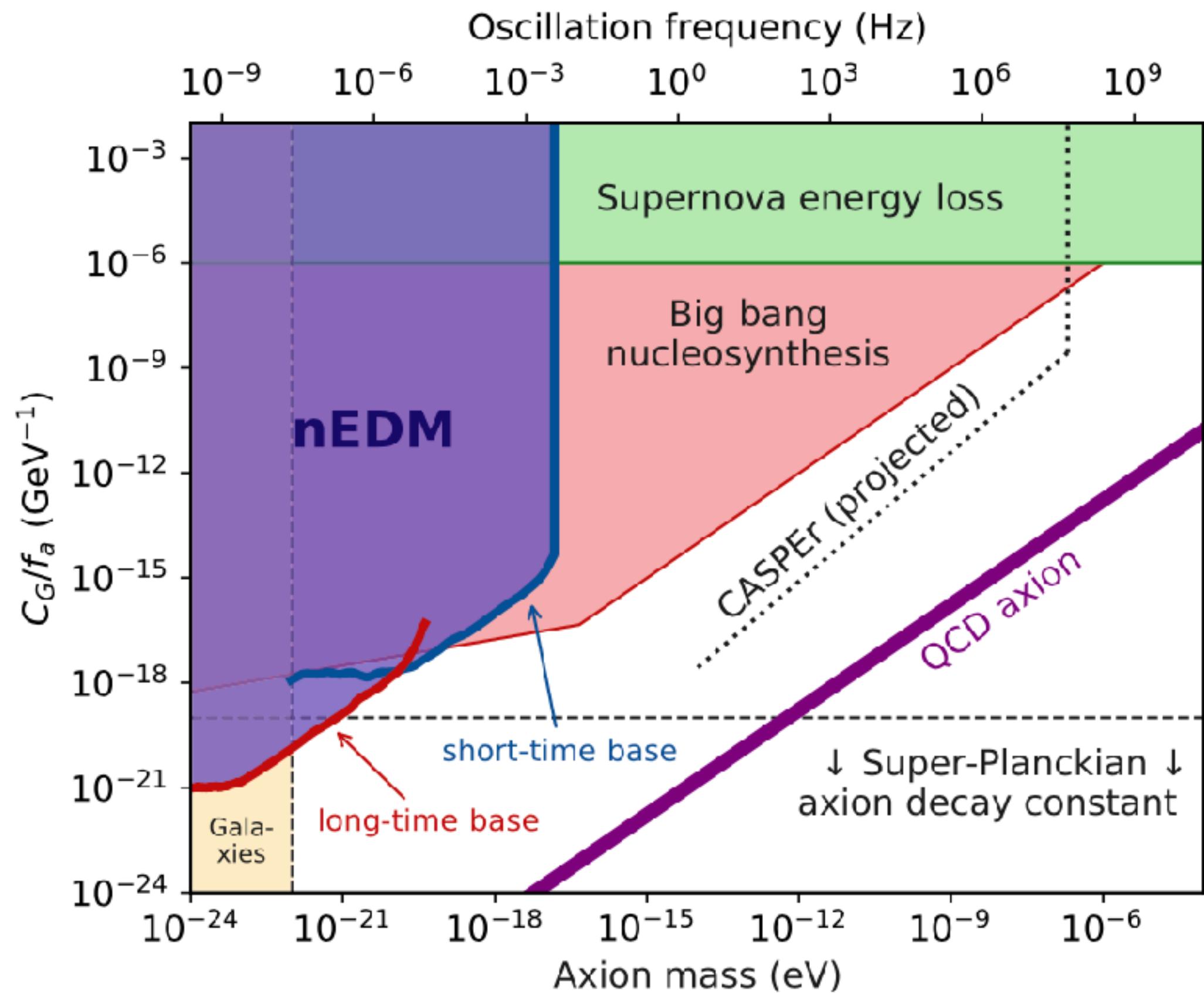
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[Abel et al (17)]

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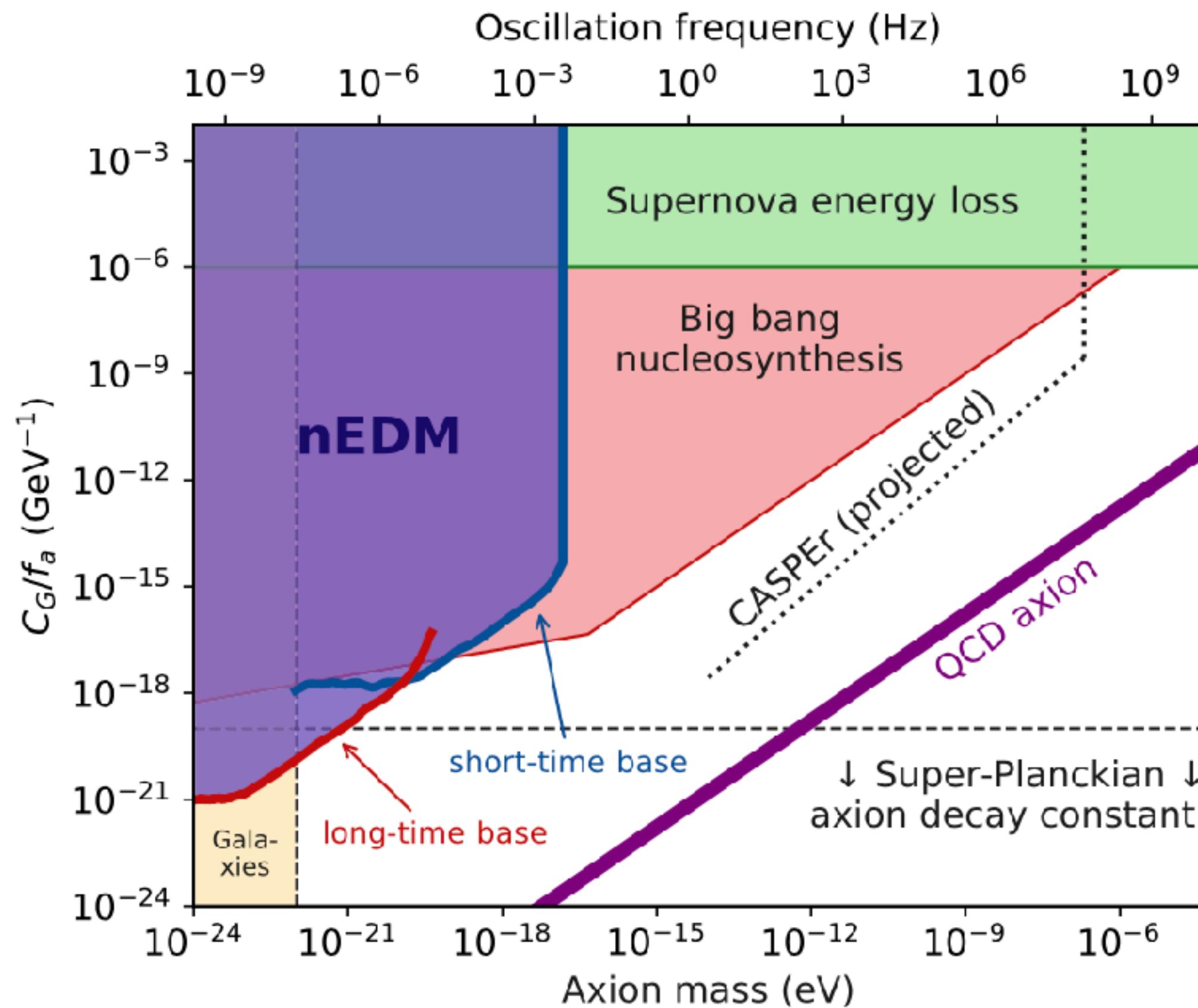
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$$+\phi^2\pi^2 + \phi^2\bar{N}N + \dots$$

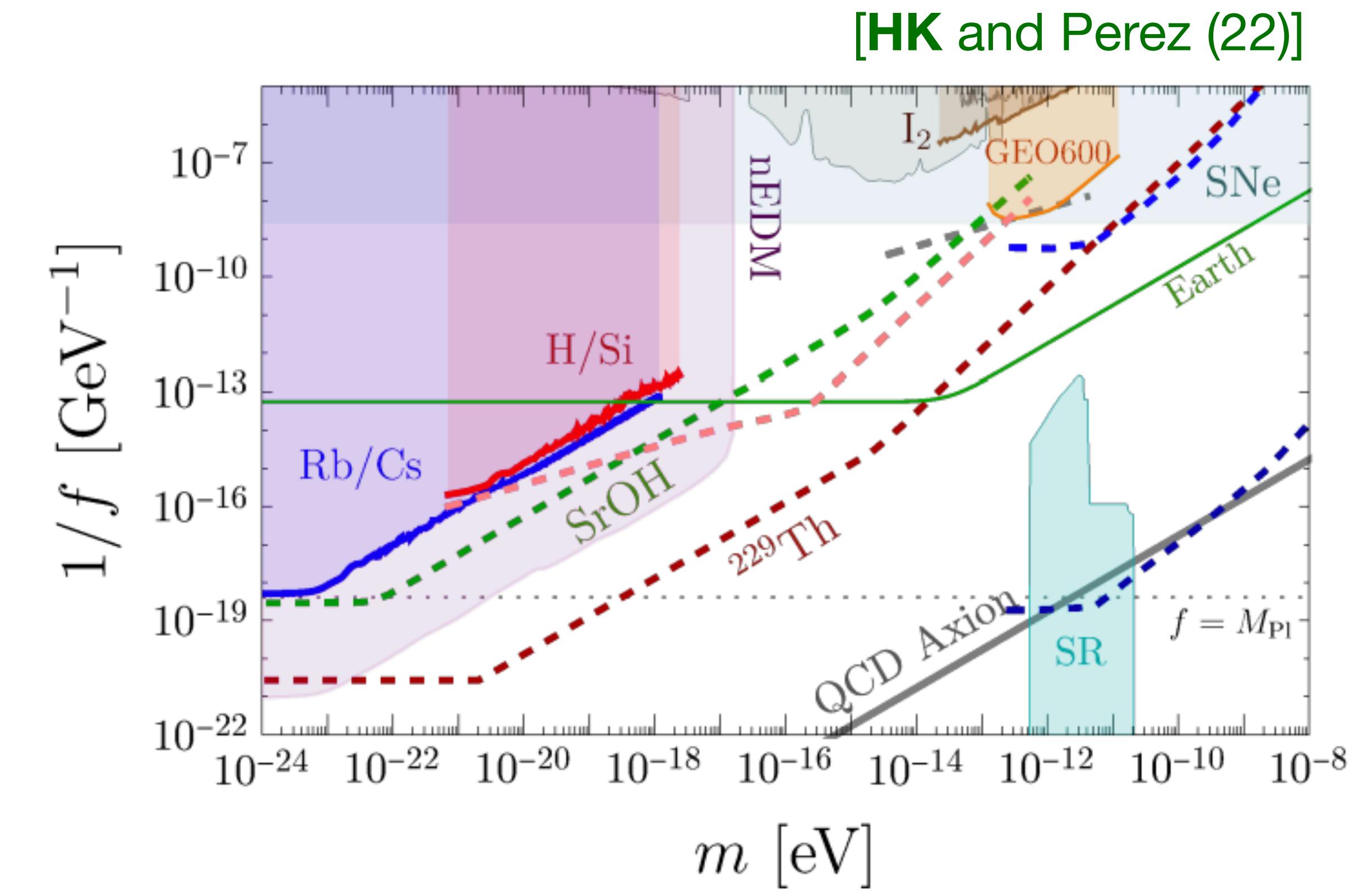
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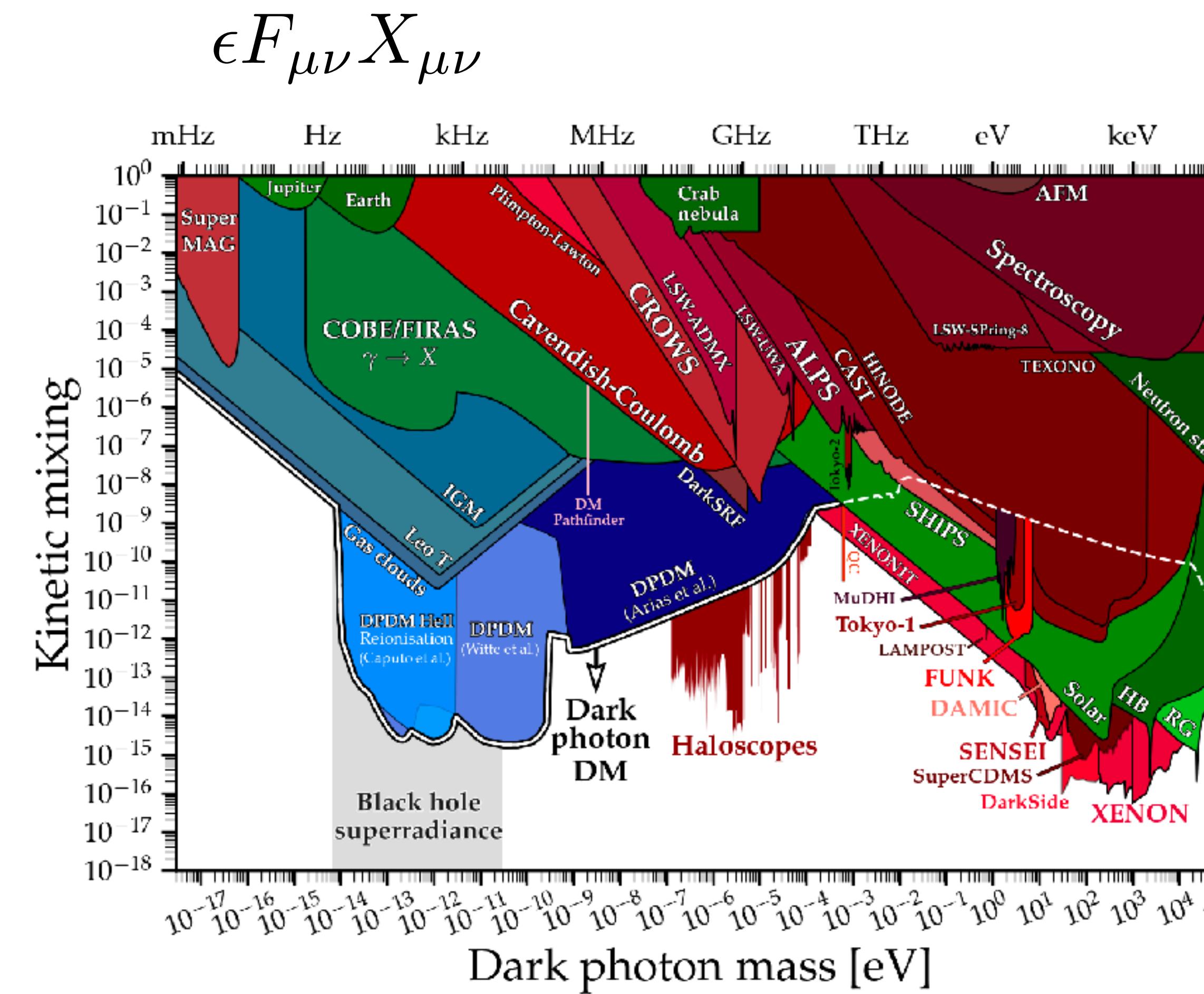


[Abel et al (17)]

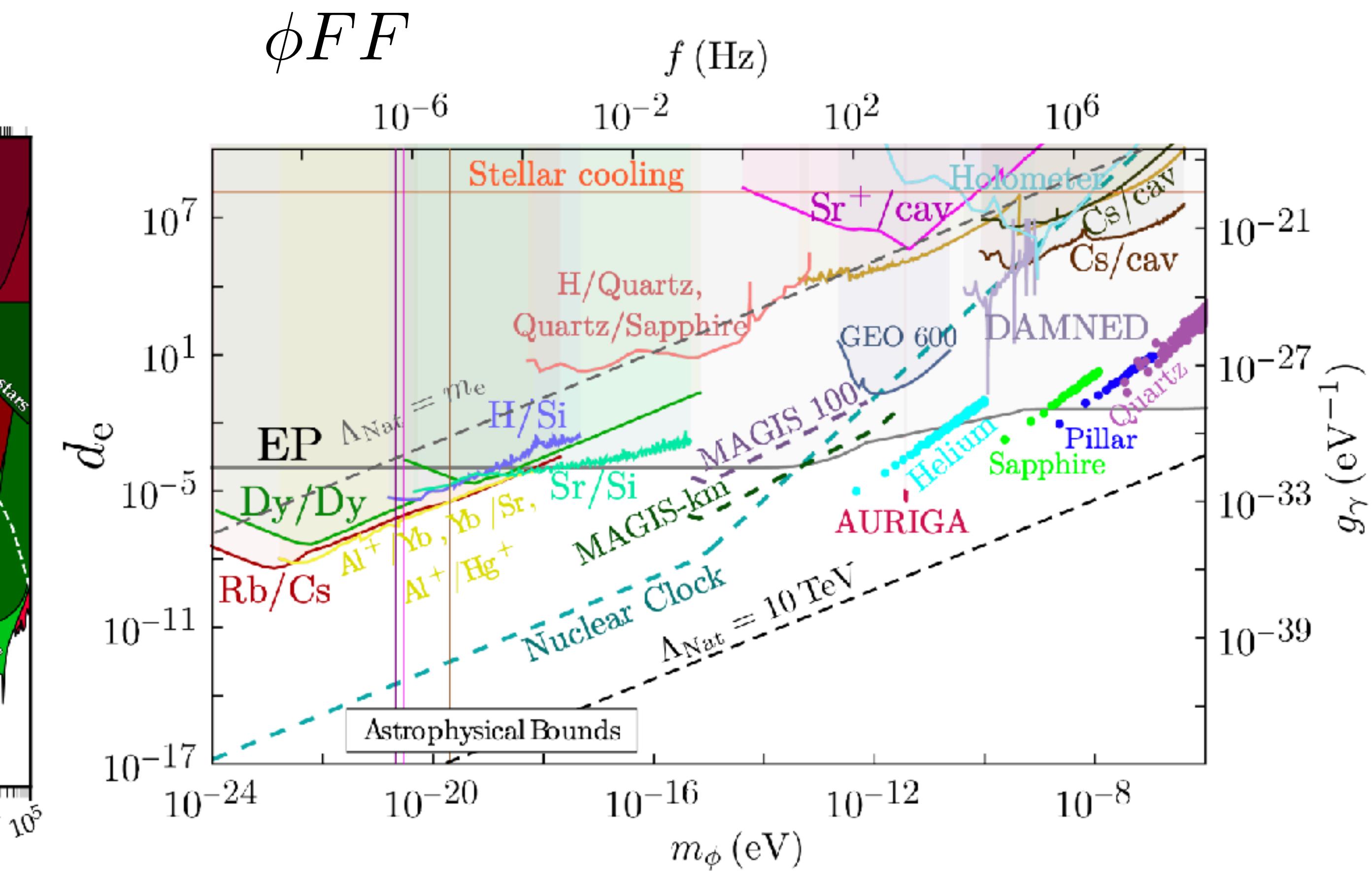
$$+\phi^2\pi^2 + \phi^2\bar{N}N + \dots$$



for dark photon dark matter or scalar dark matter



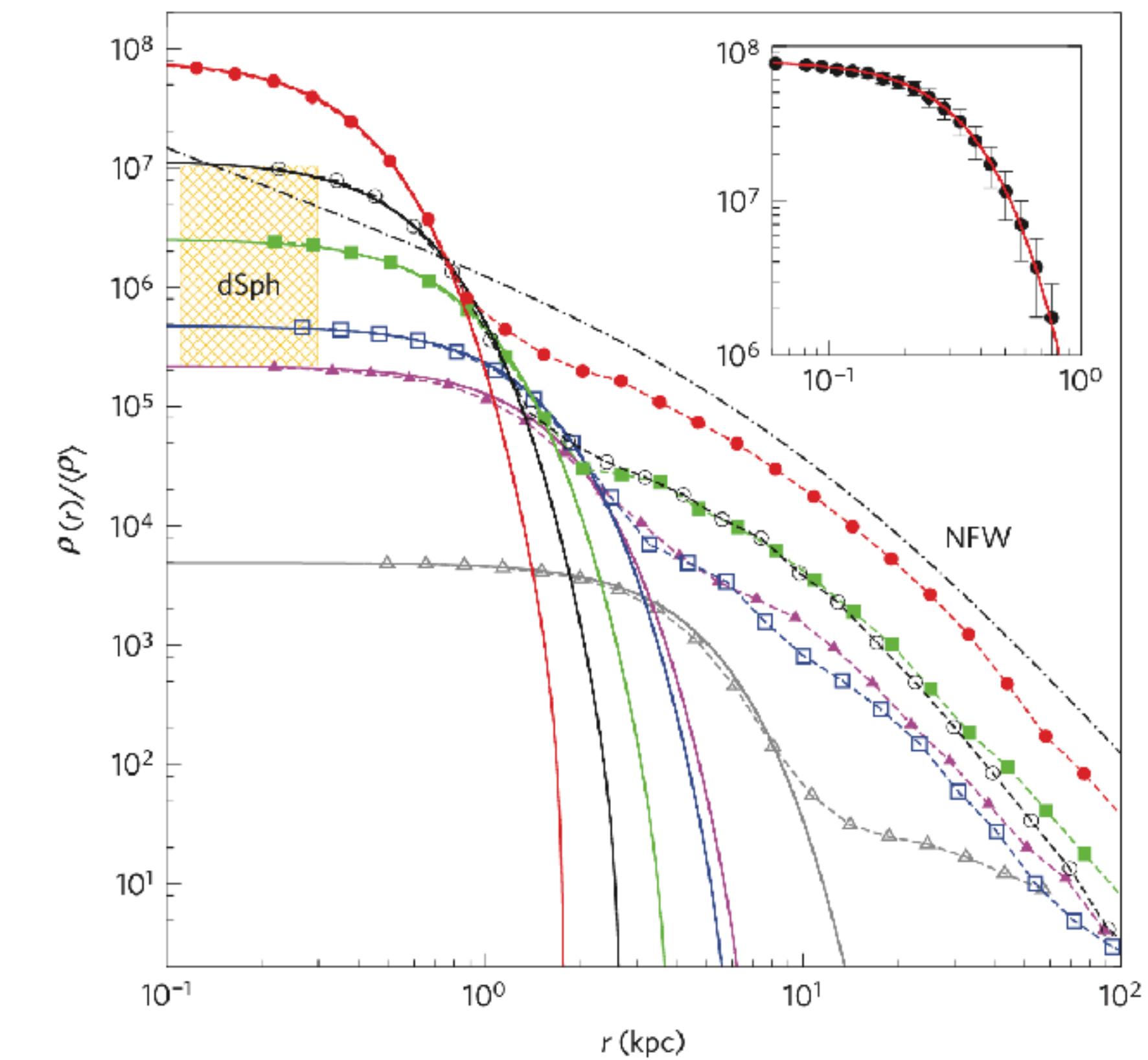
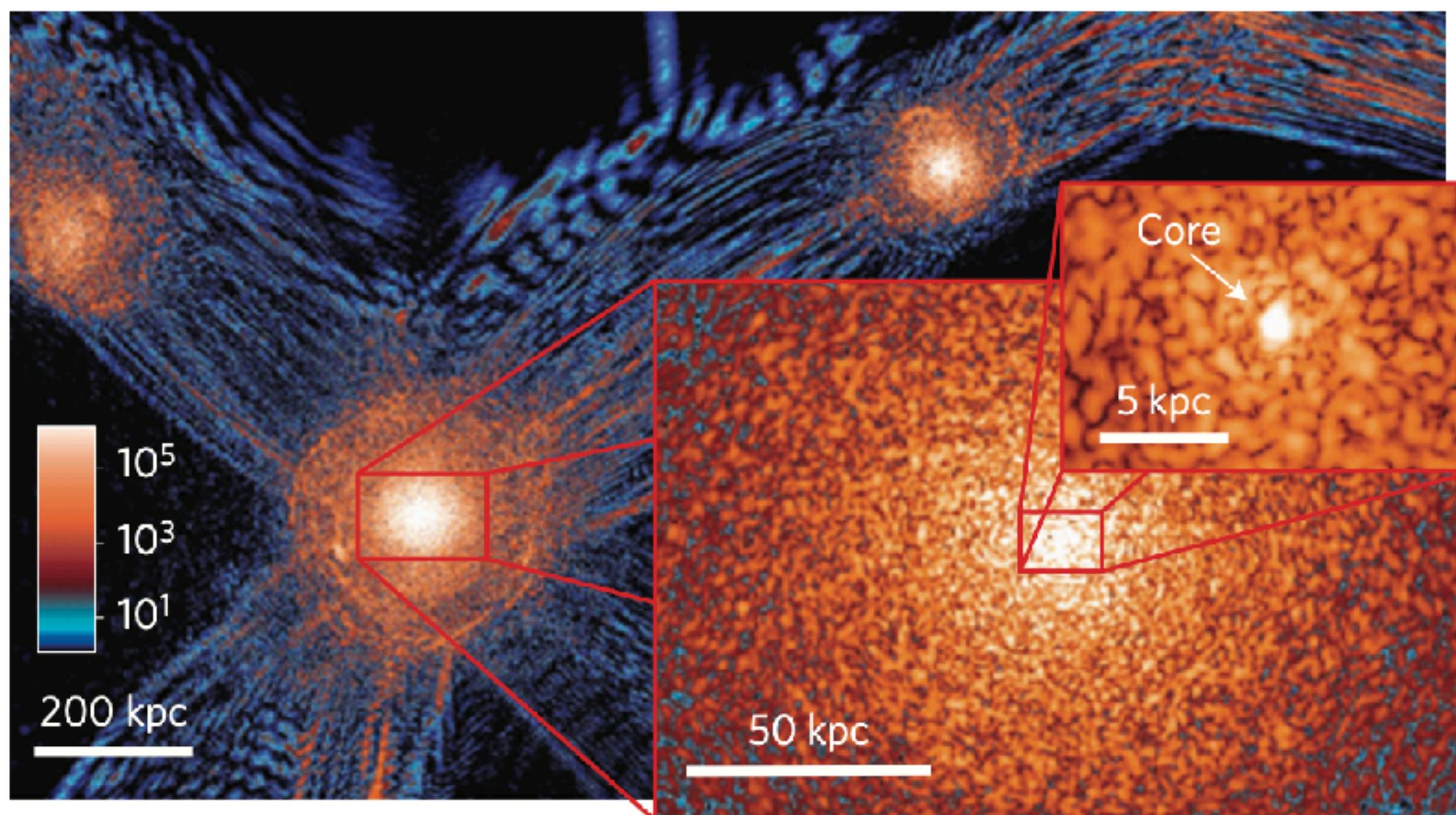
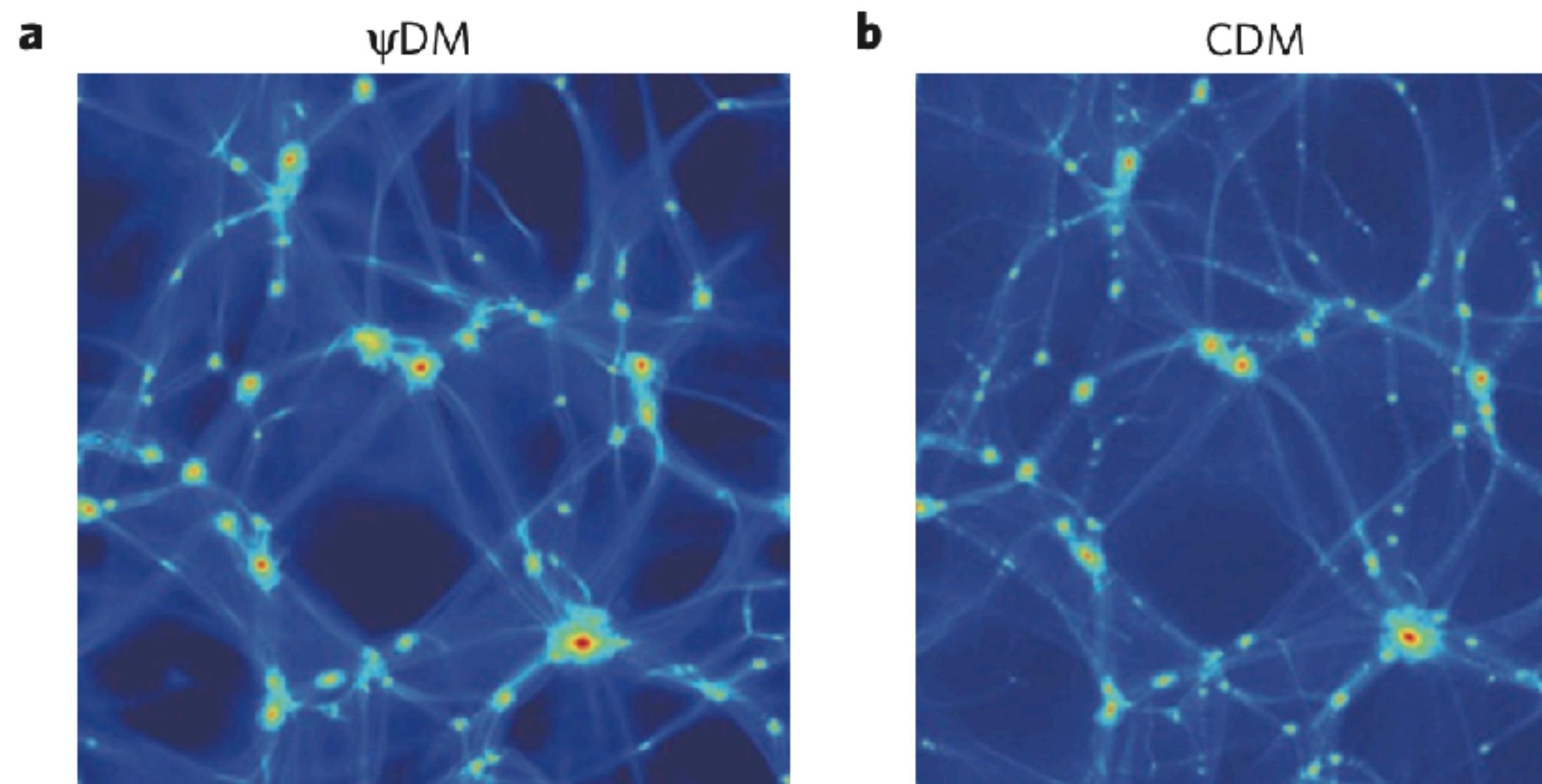
[Caputo et al (21)]



[Antypas et al, Snowmass (21)]

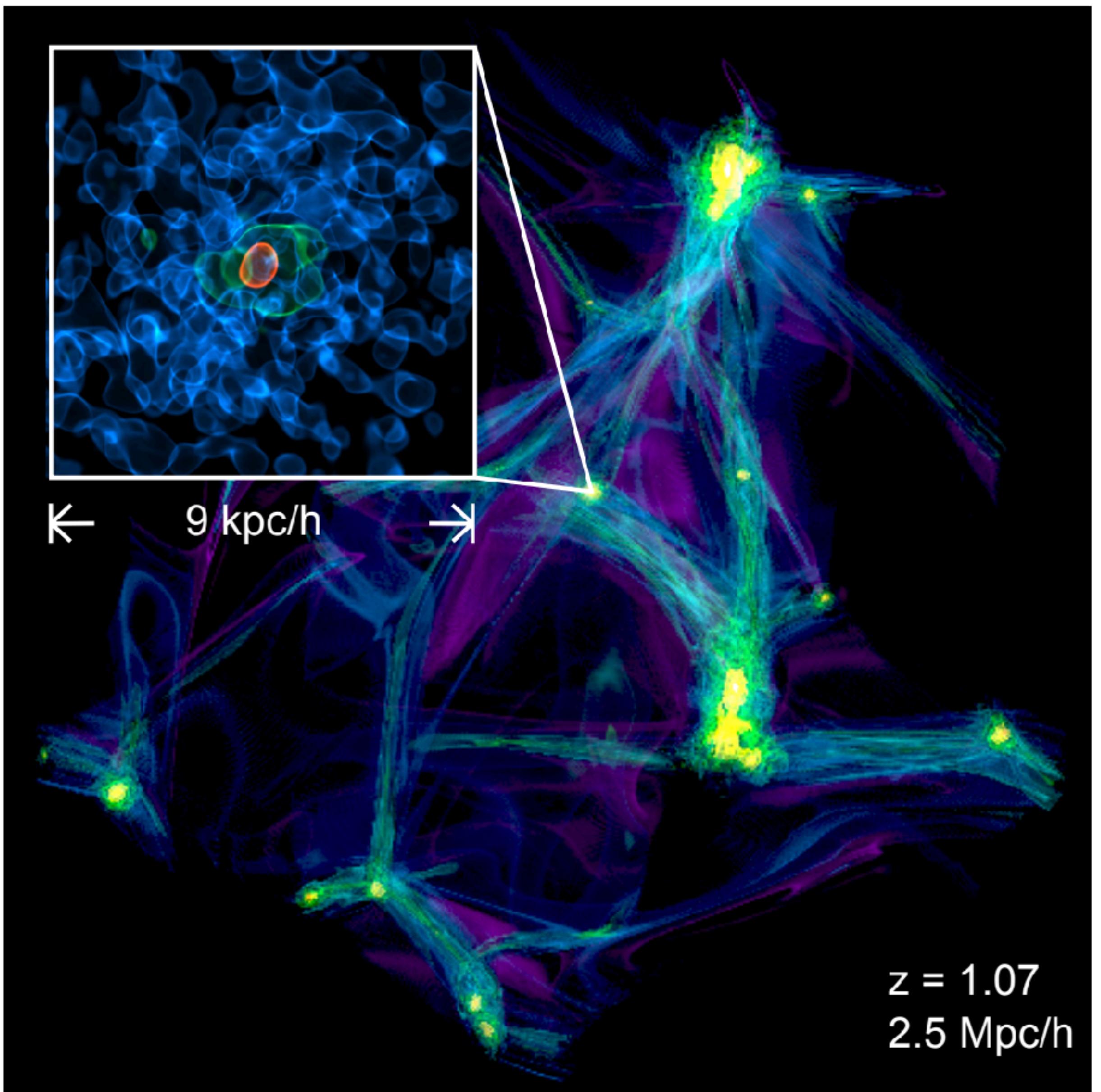
*this summarizes wave DM searches based on **non-gravitational interaction***

*let us now switch gears and discuss **gravitational interaction**
and **the behavior of wave DM on (sub)galactic scales***

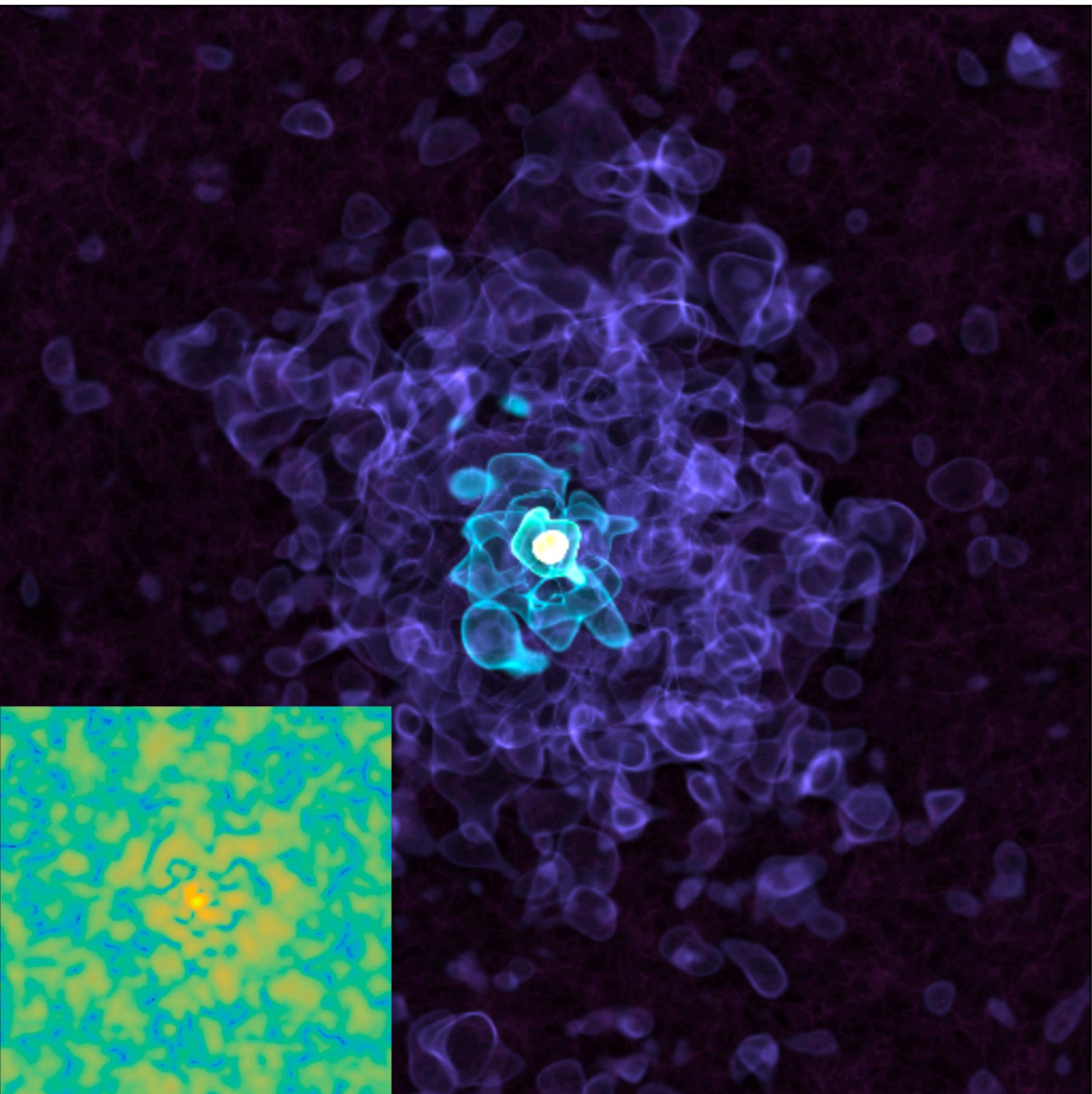


- * characteristic *soliton* at the center has been observed
- * small scale structures are erased

Mocz et al (17)



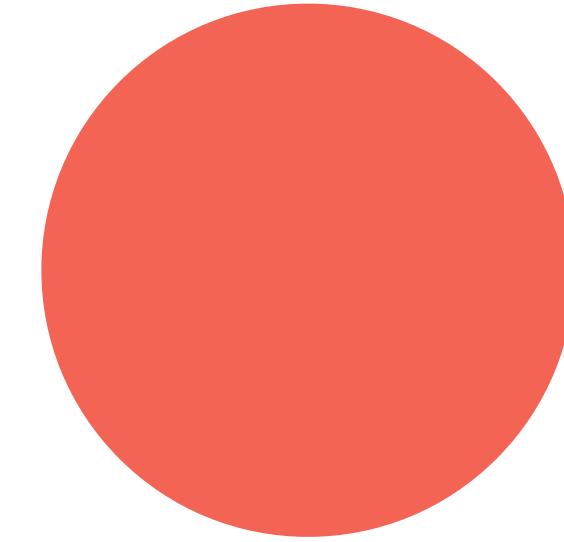
Veltmaat, Niemeyer, Schwabe (18)



*this density fluctuation is another characteristic feature of ultralight dark matter
and some of interesting phenomenology on (sub)galactic scales follow from these fluctuations*

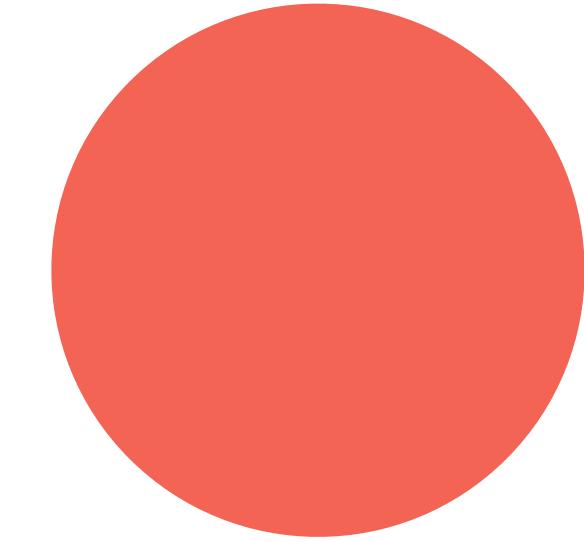
*One of the most intuitive ways to understand this fluctuation is
to think of them as a sort of particles say, **quasiparticles***

Hui et al (17)



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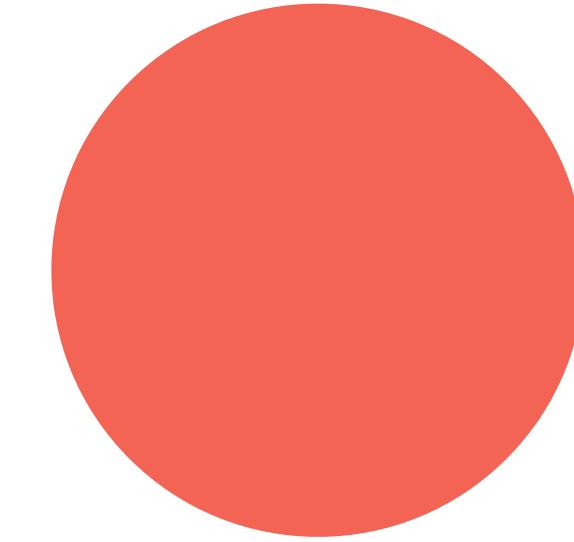


the size of the quasiparticle is given by its wavelength

$$\ell \sim \lambda = \frac{1}{mv}$$

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Hui et al (17)

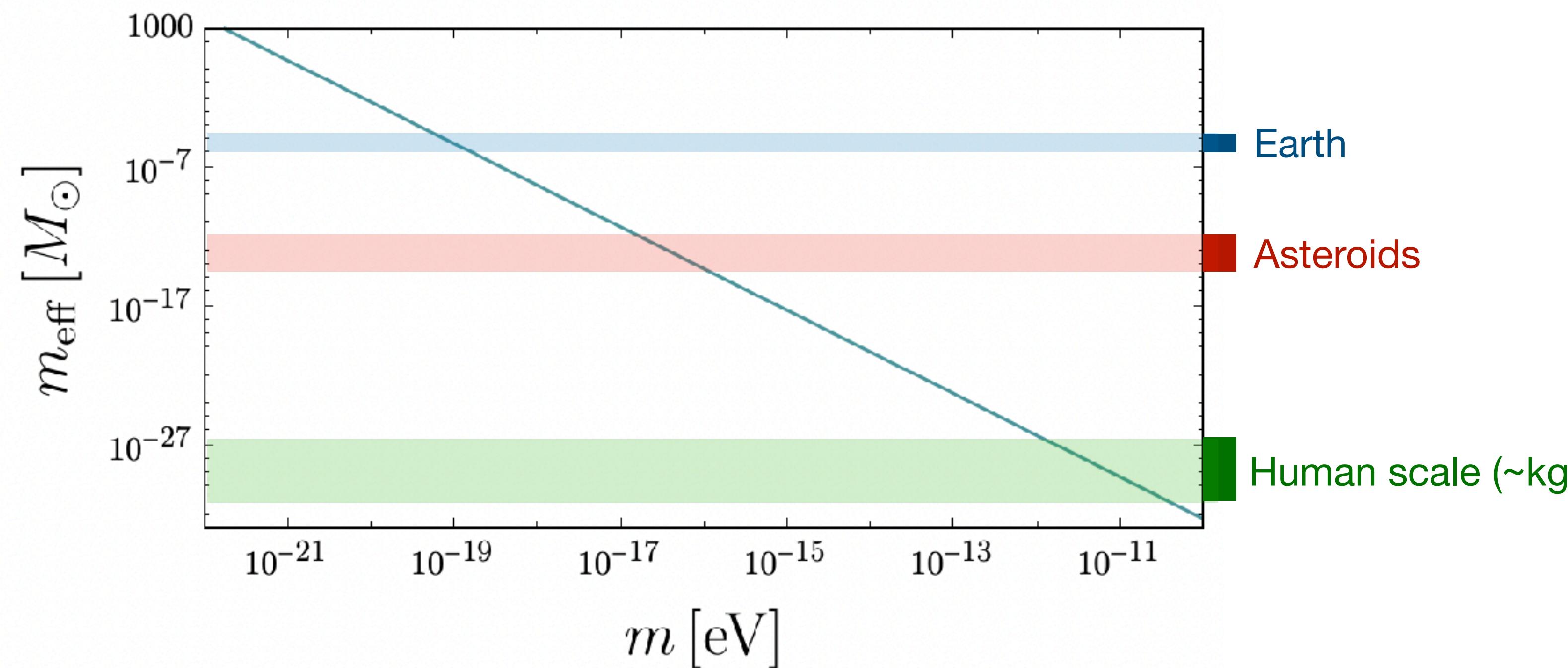
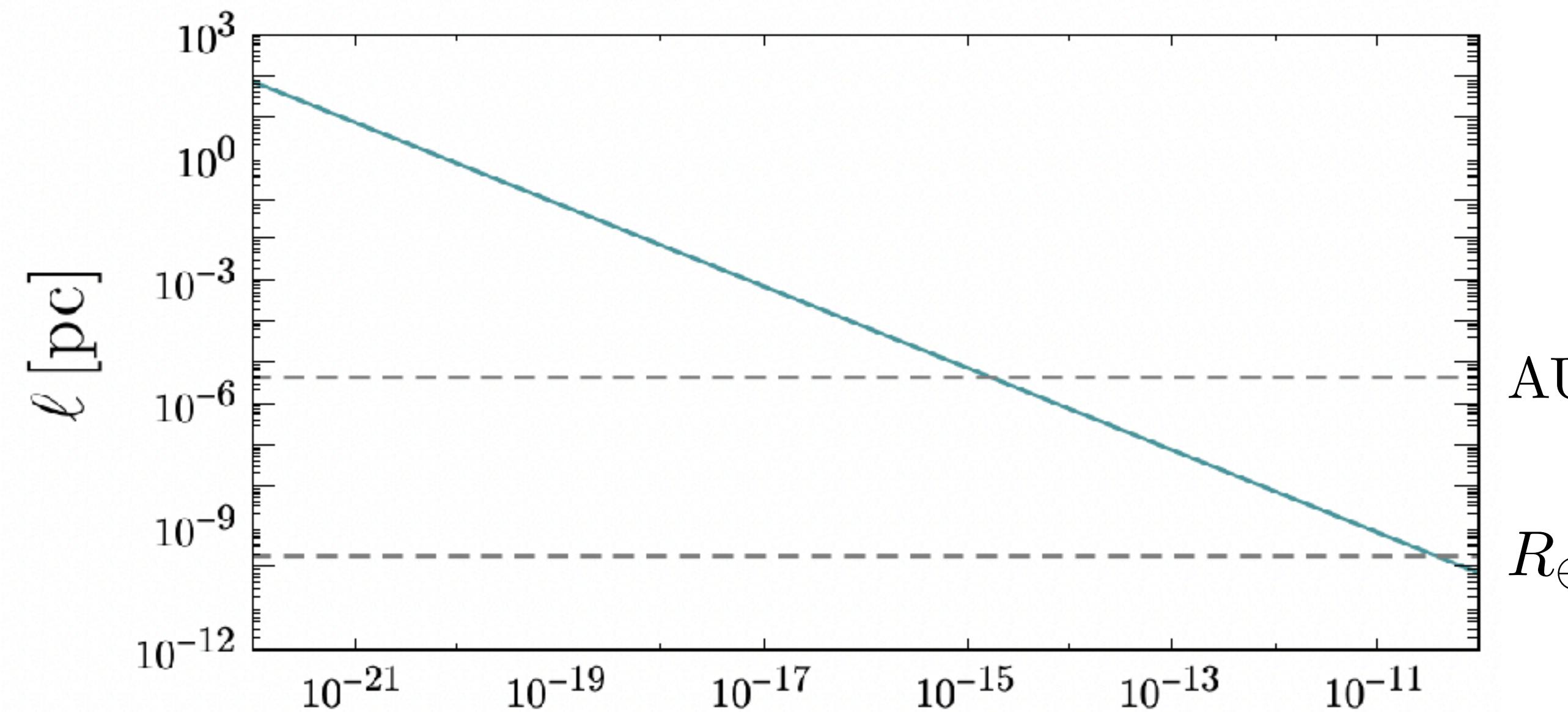


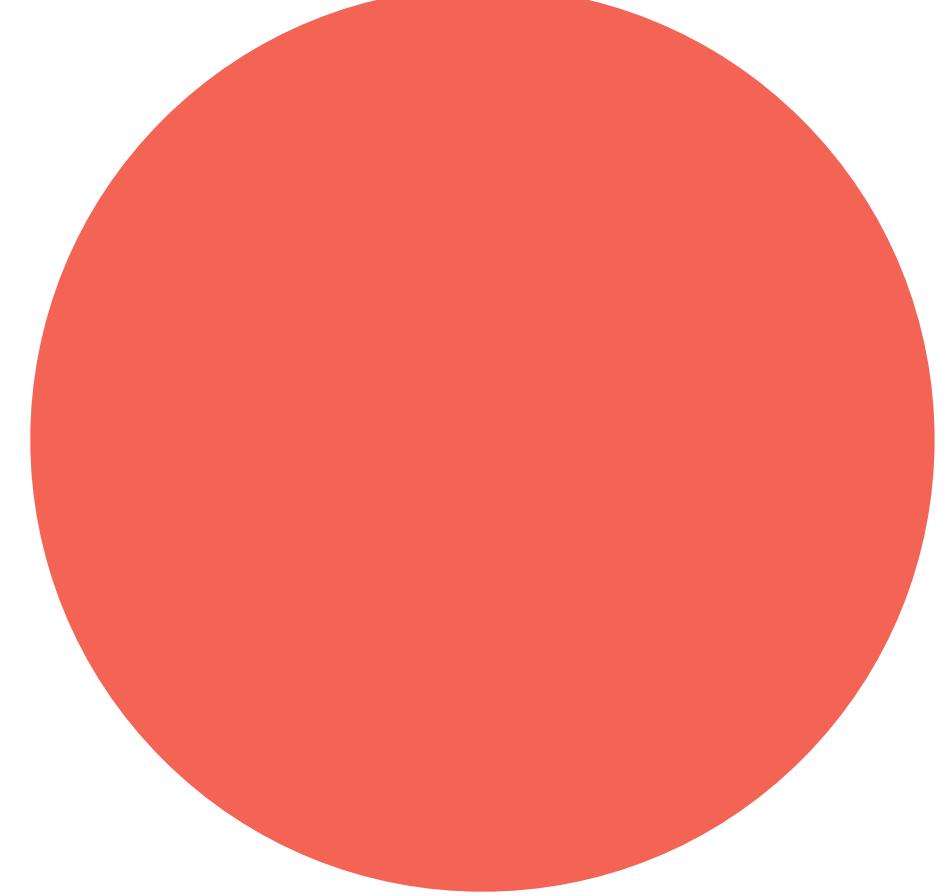
the size of the quasiparticle is given by its wavelength

$$\ell \sim \lambda = \frac{1}{mv}$$

the effective mass of the quasiparticle is given by

$$m_{\text{eff}} \sim \rho_{\text{DM}} \ell^3$$

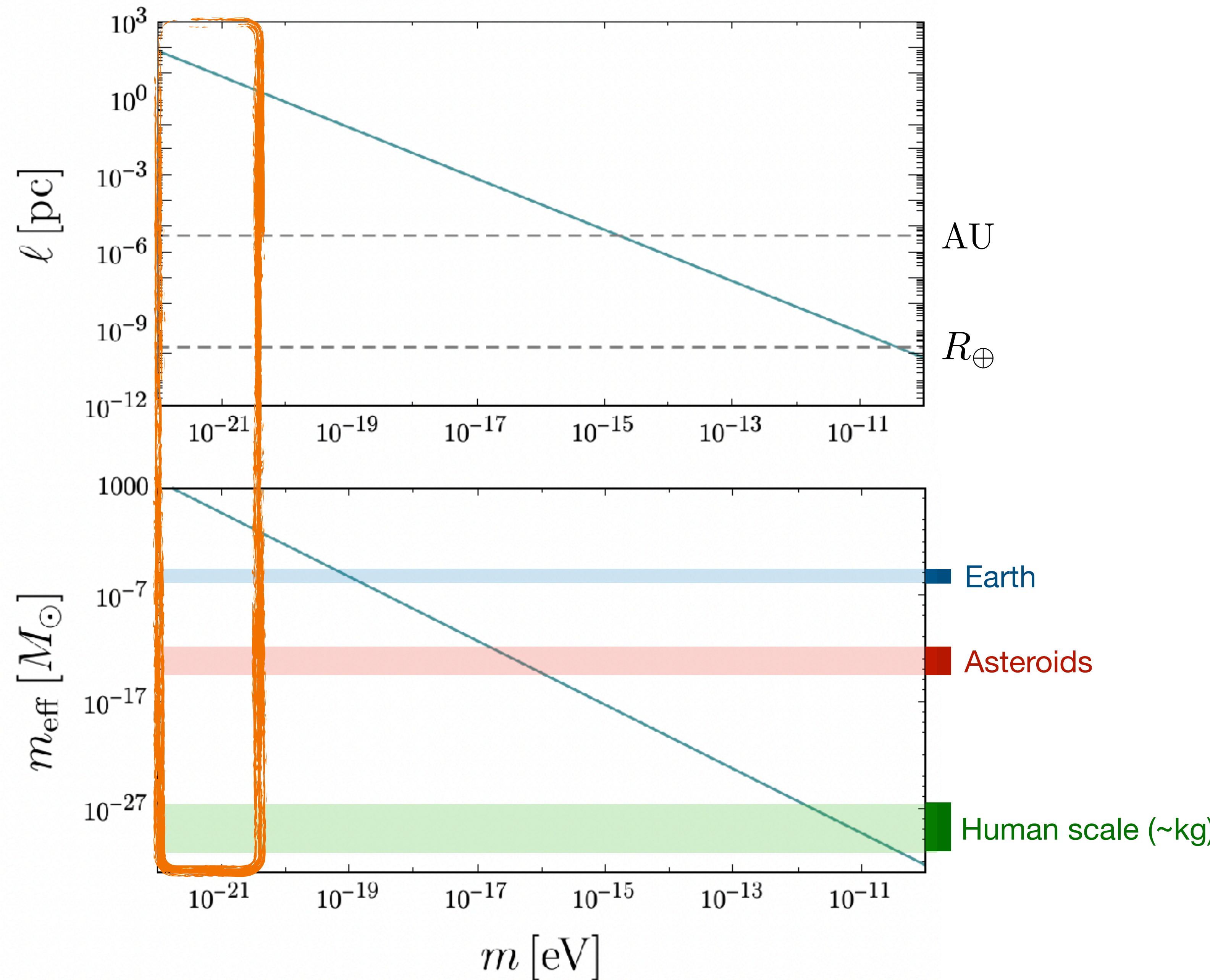




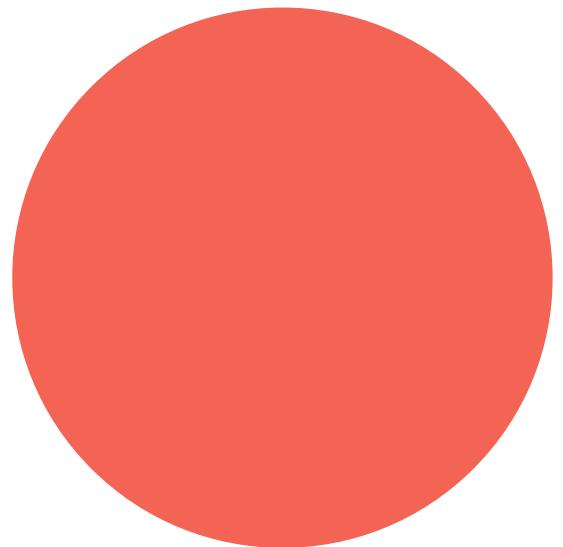
$$m \sim 10^{-22} \text{ eV}$$

$$m_{\text{eff}} \sim \mathcal{O}(10^4) M_{\odot}$$

$$\ell \sim \mathcal{O}(10^2) \text{ pc}$$

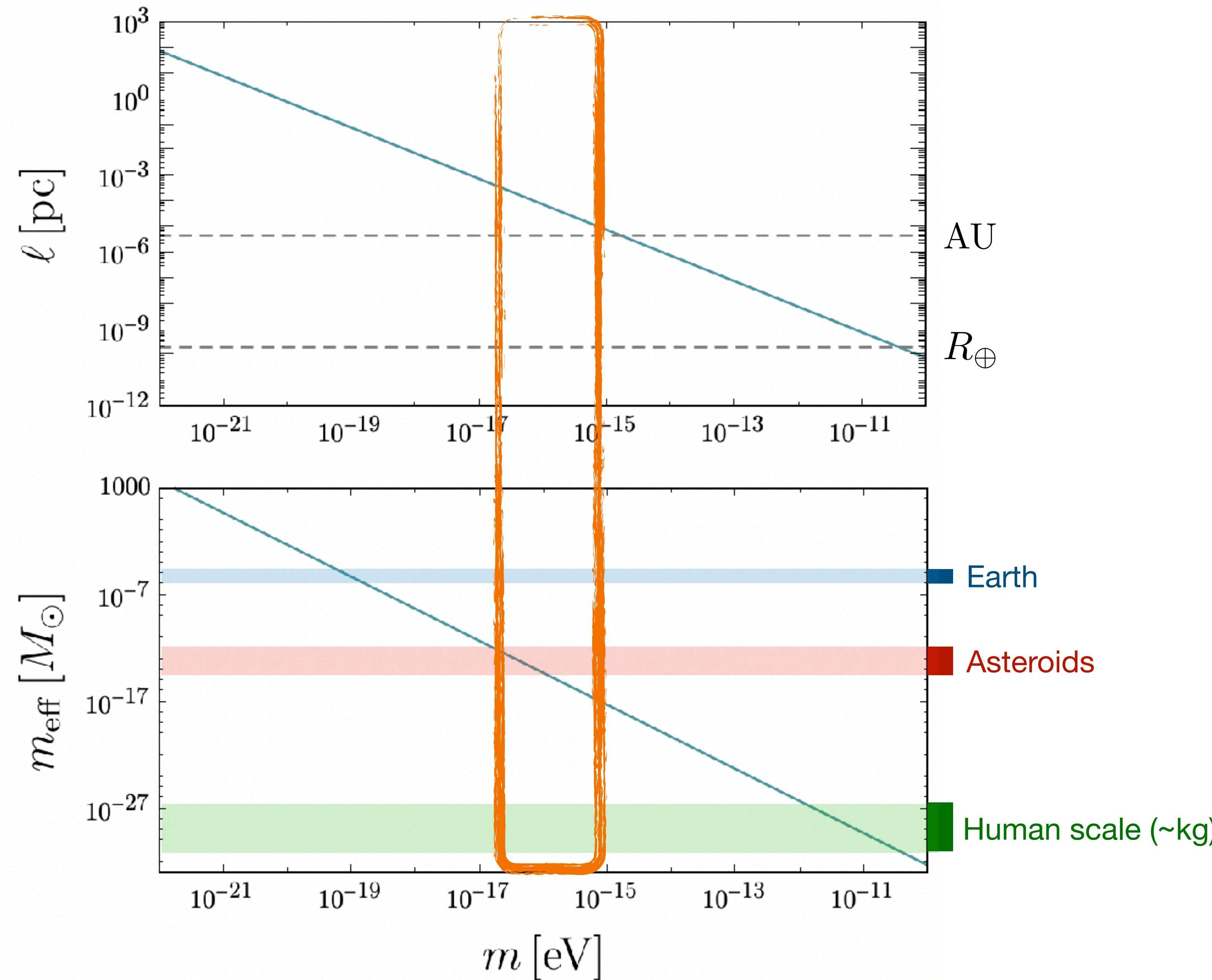


$$m \sim 10^{-16} \text{ eV}$$

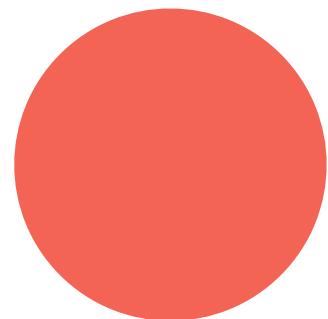


$$m_{\text{eff}} \sim \mathcal{O}(10^{17}) \text{ kg}$$

$$\ell \sim \mathcal{O}(10) \text{ AU}$$

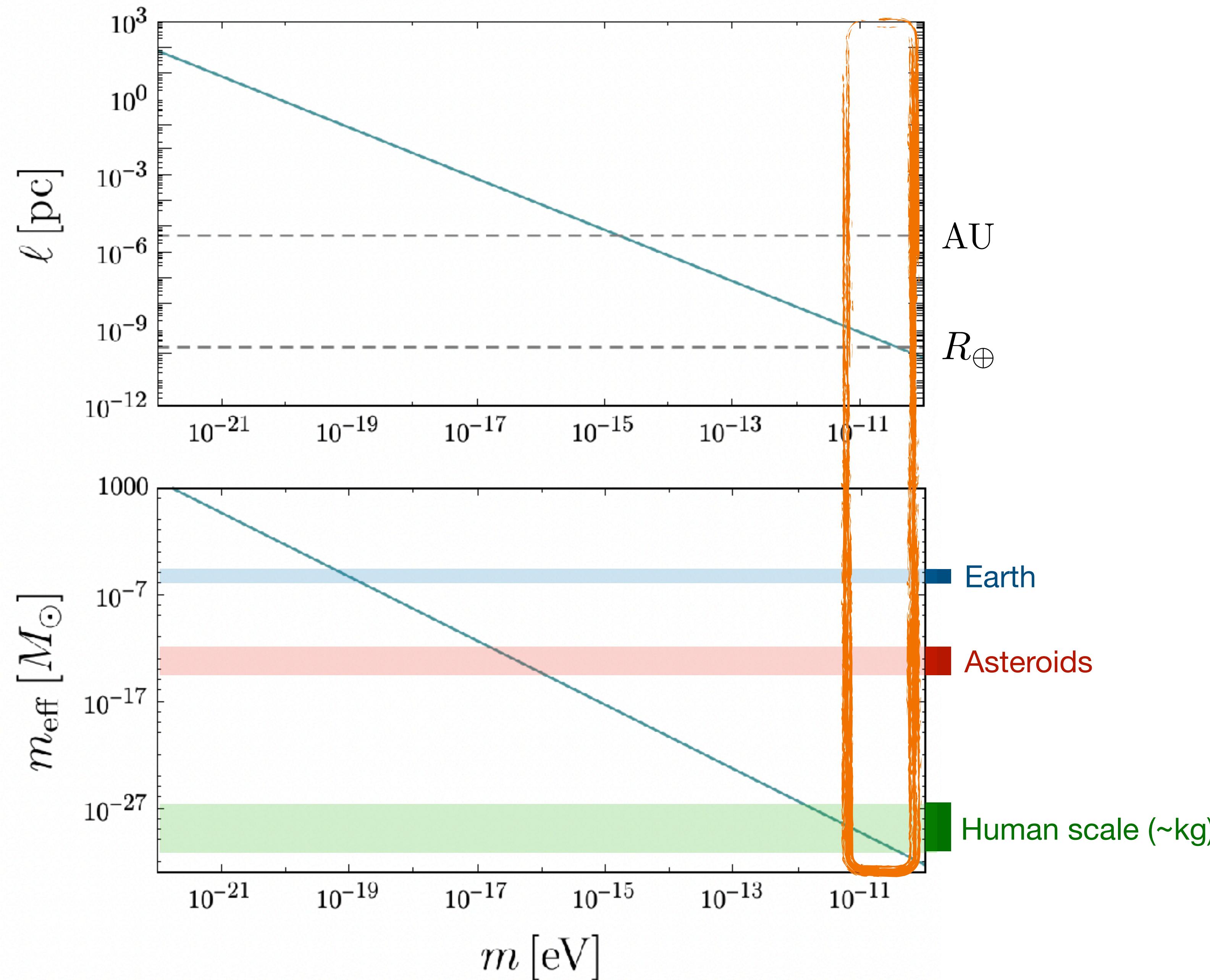


$$m \sim 10^{-11} \text{ eV}$$



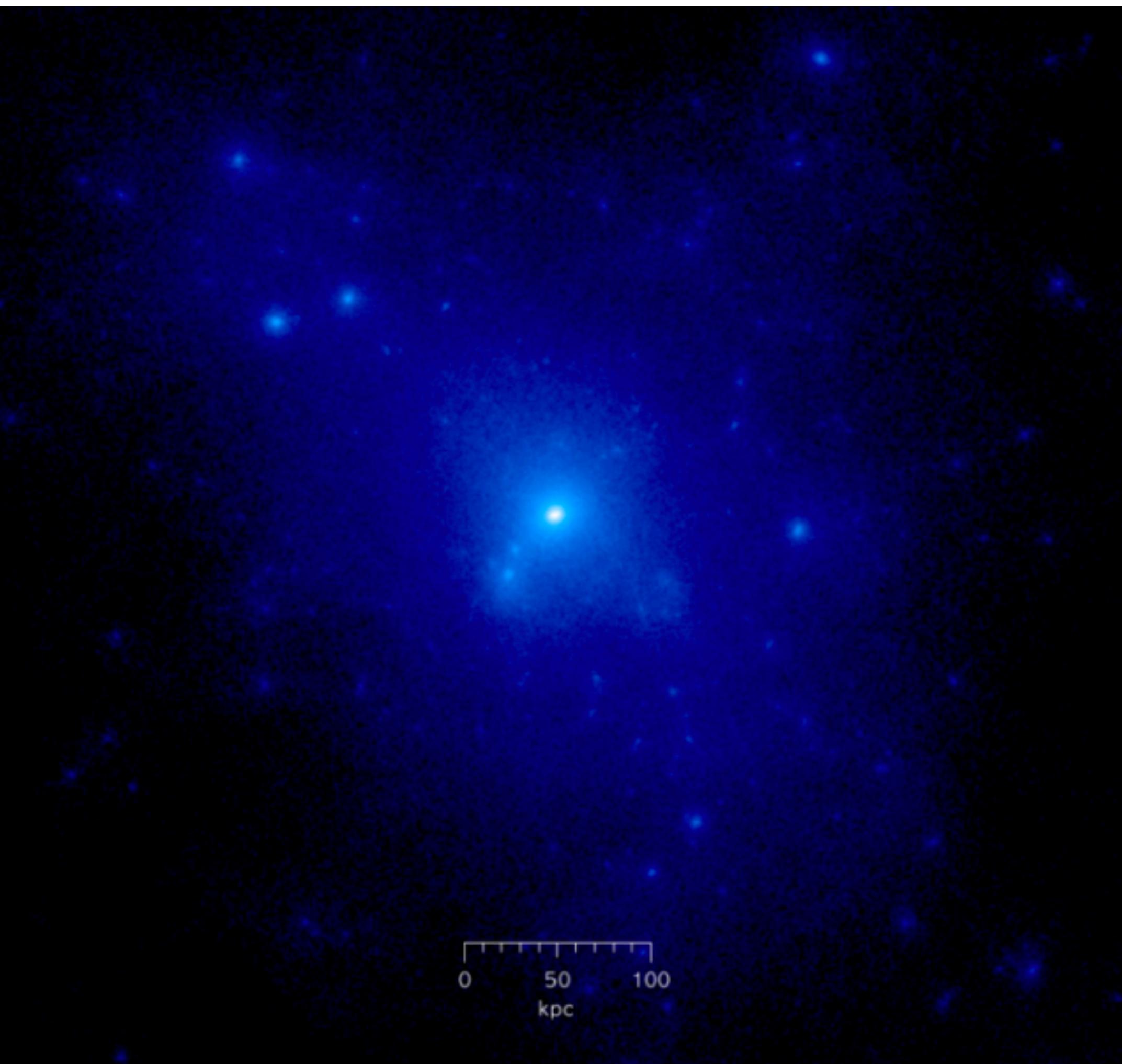
$$m_{\text{eff}} \sim \mathcal{O}(10^3) \text{ kg}$$

$$\ell \sim \mathcal{O}(10^3) \text{ km}$$

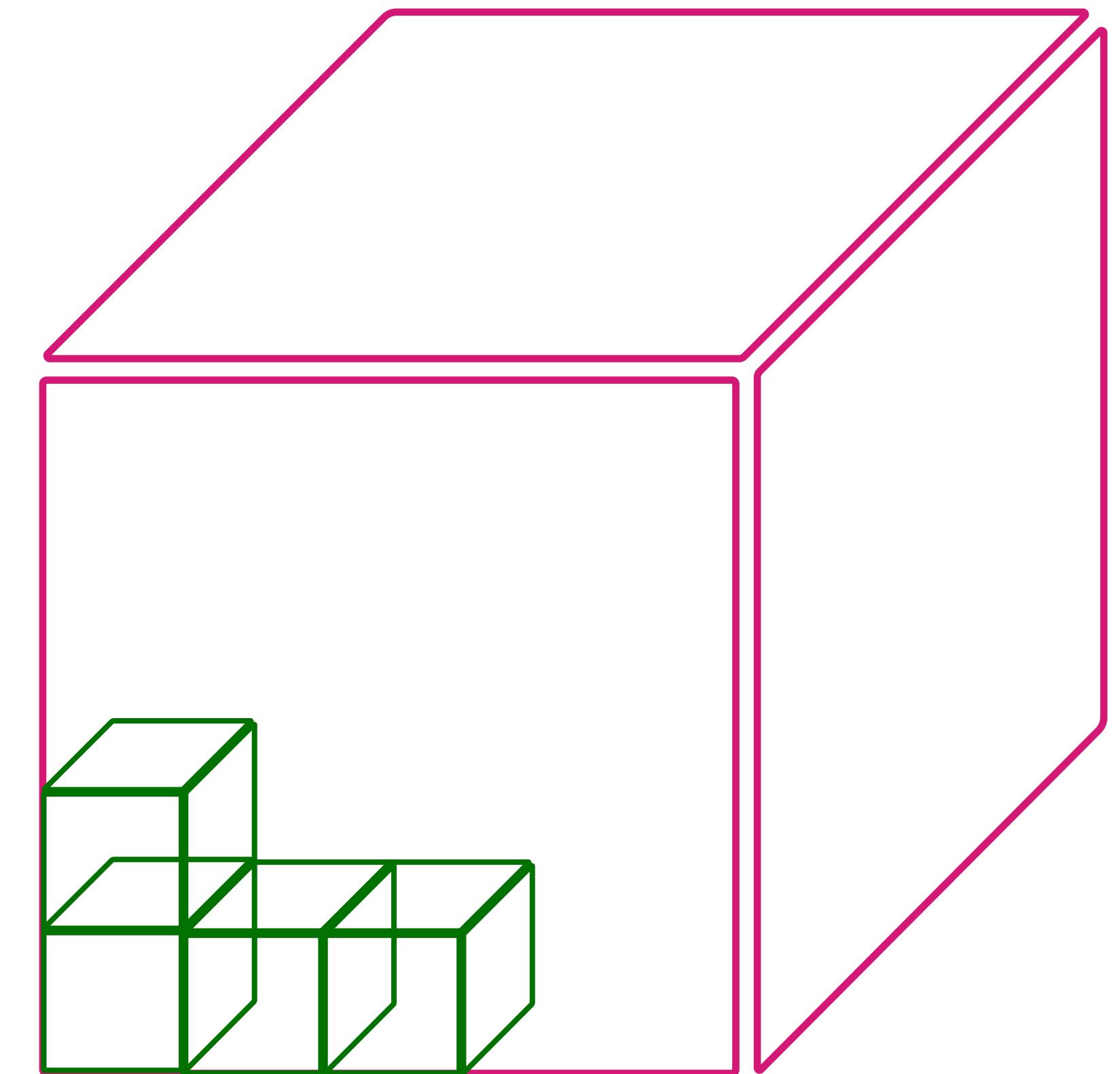
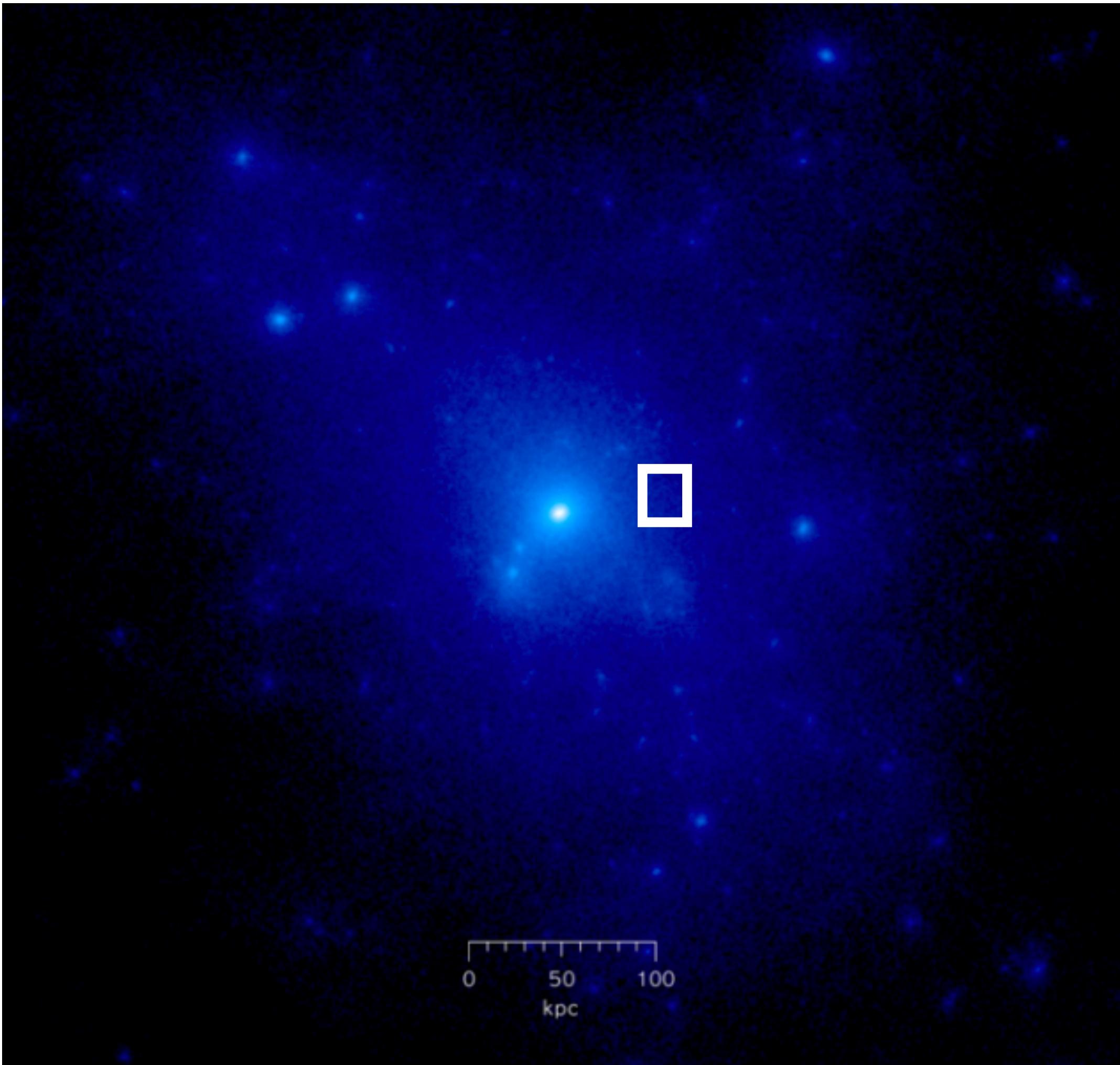


*a concept of quasiparticle seems quite naive
but it provides a useful thinking tool to understand
the behavior of wave DM halo as well as the interaction with stellar objects*

*a large order one density fluctuation can naturally be understood
in the particle dark matter halo*



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in the particle dark matter halo*

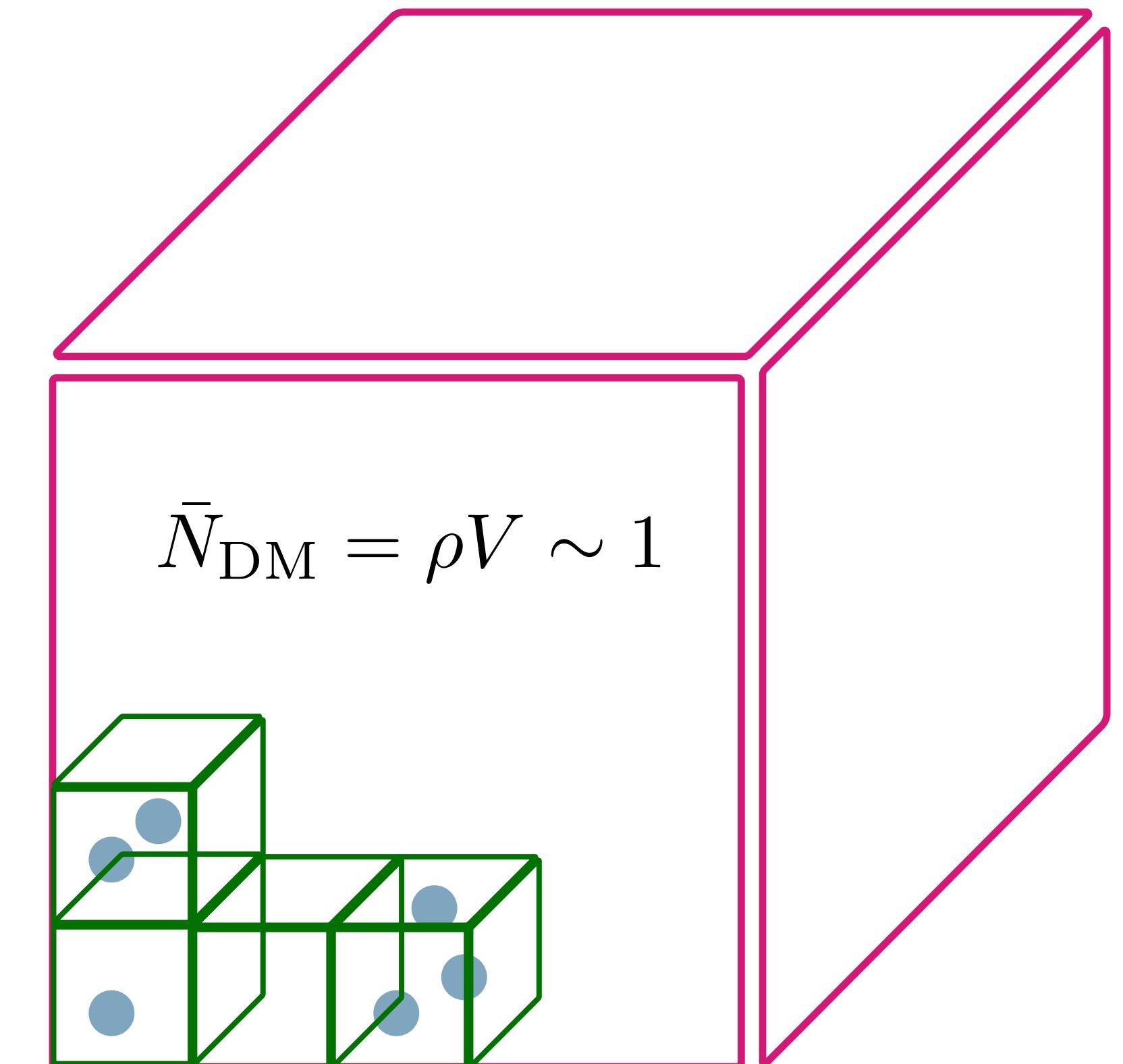
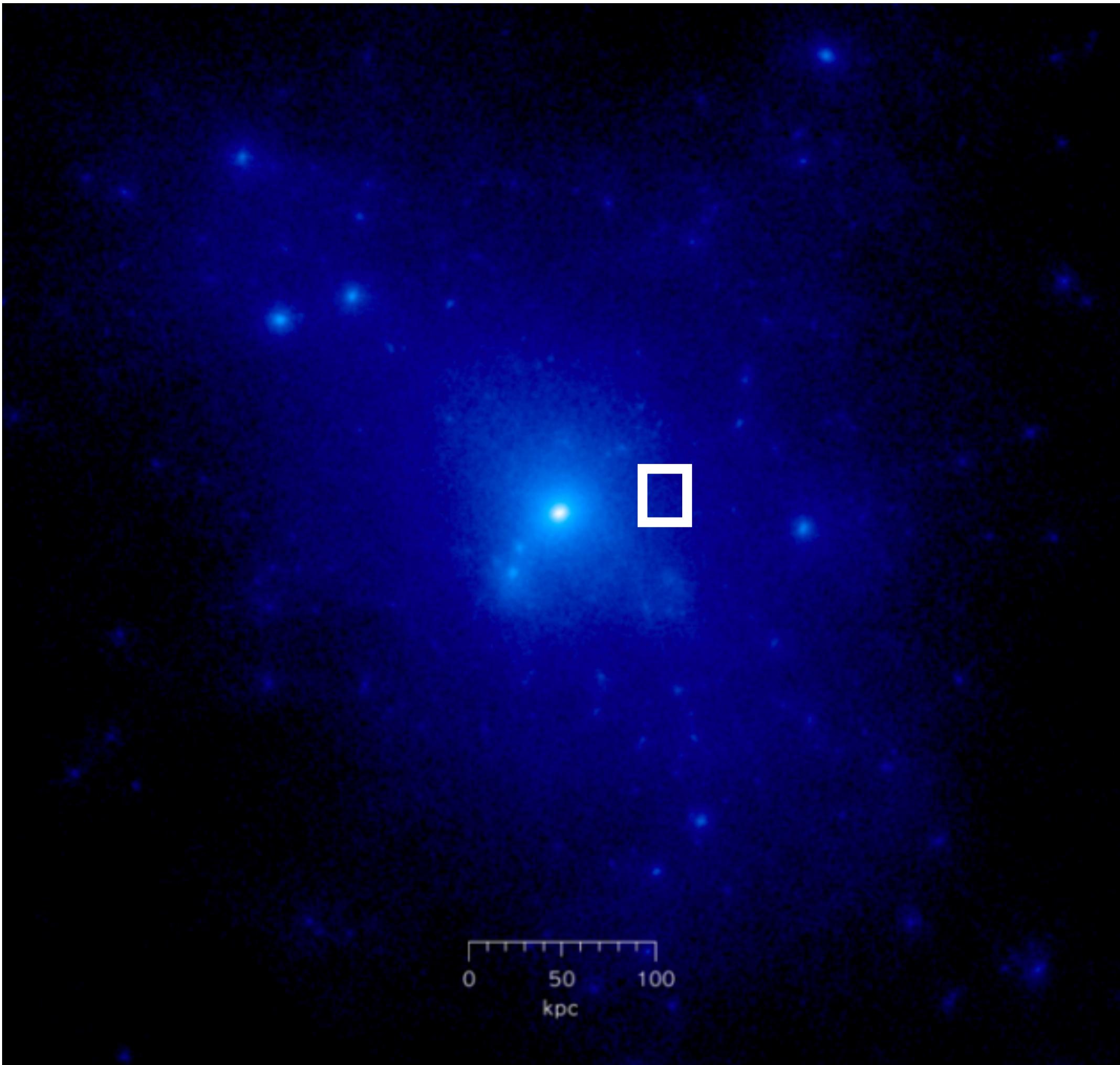


$$\ell \sim \text{cm}$$

$$\rho \sim \text{GeV/cm}^3$$

$$m \sim \text{GeV}$$

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$$\ell \sim \text{cm}$$

$$\begin{aligned} \rho &\sim \text{GeV/cm}^3 \\ m &\sim \text{GeV} \end{aligned}$$

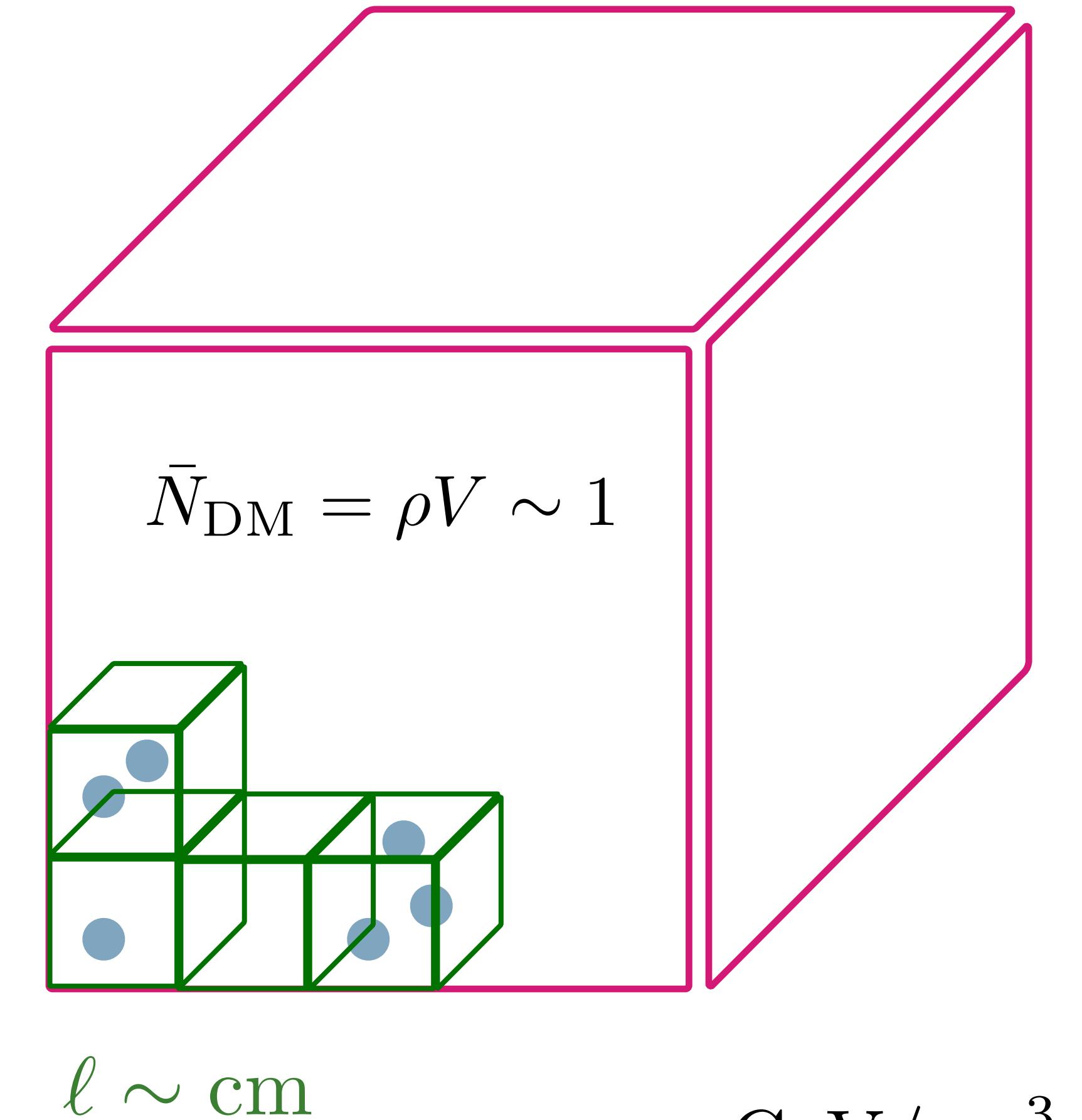
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$$\langle N_{\text{DM}} \rangle = \bar{N}_{\text{DM}}$$

$$\langle \Delta N_{\text{DM}}^2 \rangle = \bar{N}_{\text{DM}}$$

and therefore in this small volume

$$\frac{\delta \rho}{\rho} = \frac{1}{\sqrt{\bar{N}_{\text{DM}}}} \sim \mathcal{O}(1)$$



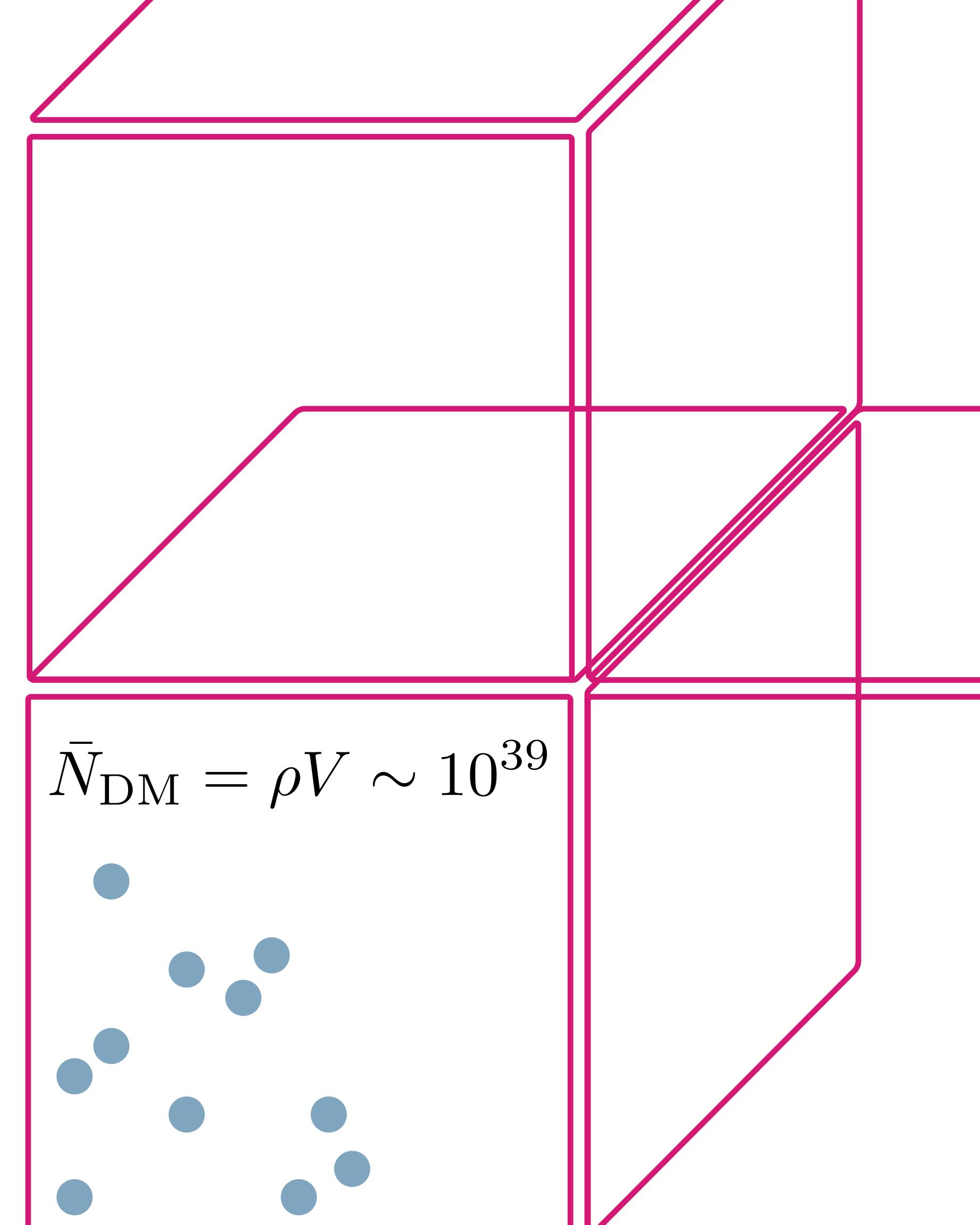
if we instead choose larger volume

$$\langle N_{\text{DM}} \rangle = \bar{N}_{\text{DM}} \gg 1$$

$$\langle \Delta N_{\text{DM}}^2 \rangle = \bar{N}_{\text{DM}}$$

and therefore in this large volume

$$\frac{\delta \rho}{\rho} = \frac{1}{\sqrt{\bar{N}_{\text{DM}}}} \ll 1$$



$$\ell \sim \text{AU}$$

$$\rho \sim \text{GeV/cm}^3$$

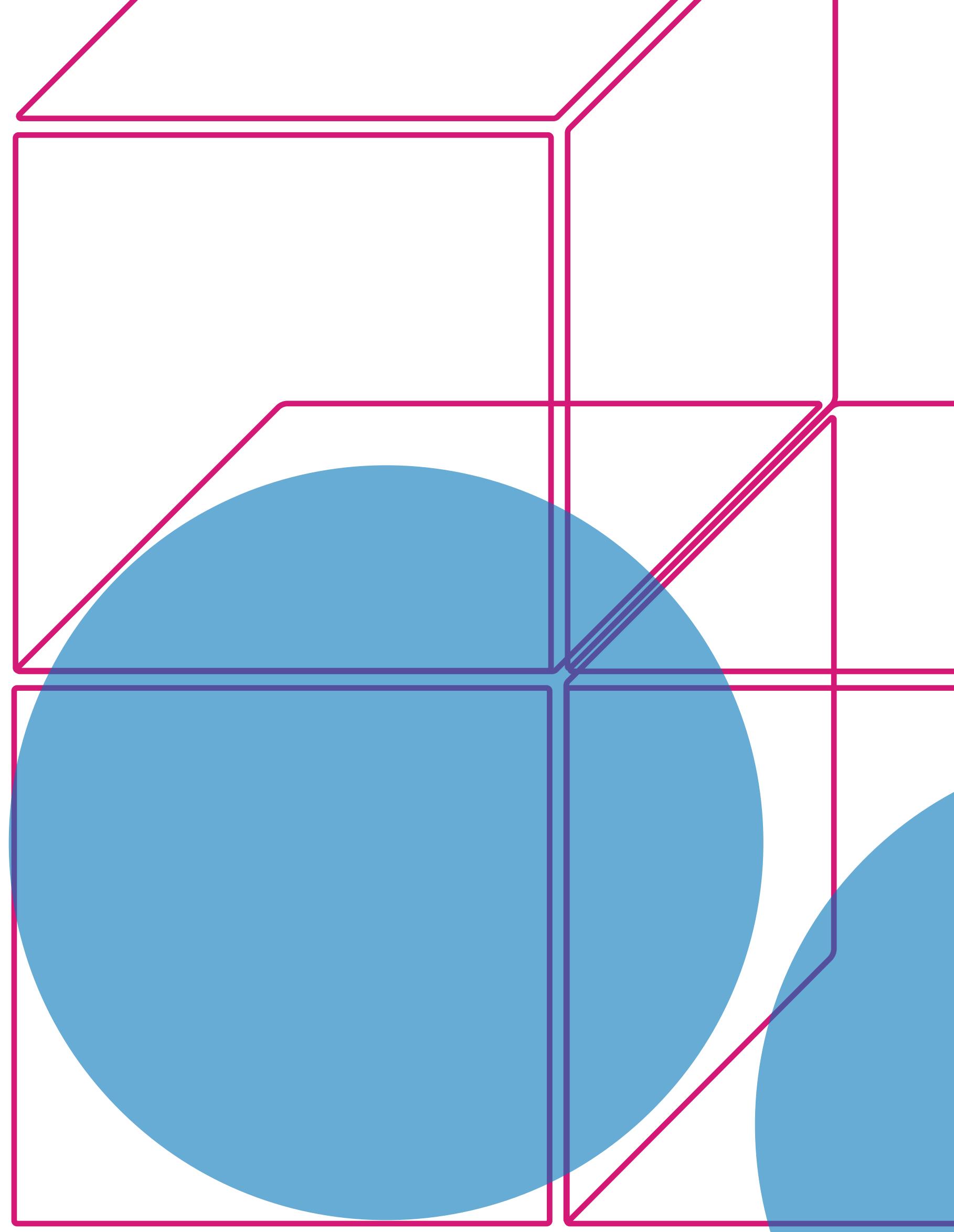
$$m \sim \text{GeV}$$

*we may apply the same argument for DM quasiparticles
in particular over the scale of the wavelength
there's $O(1)$ number of quasiparticle*

$$\frac{\delta\rho}{\rho} \sim 1$$

(for $\ell \sim \lambda$)

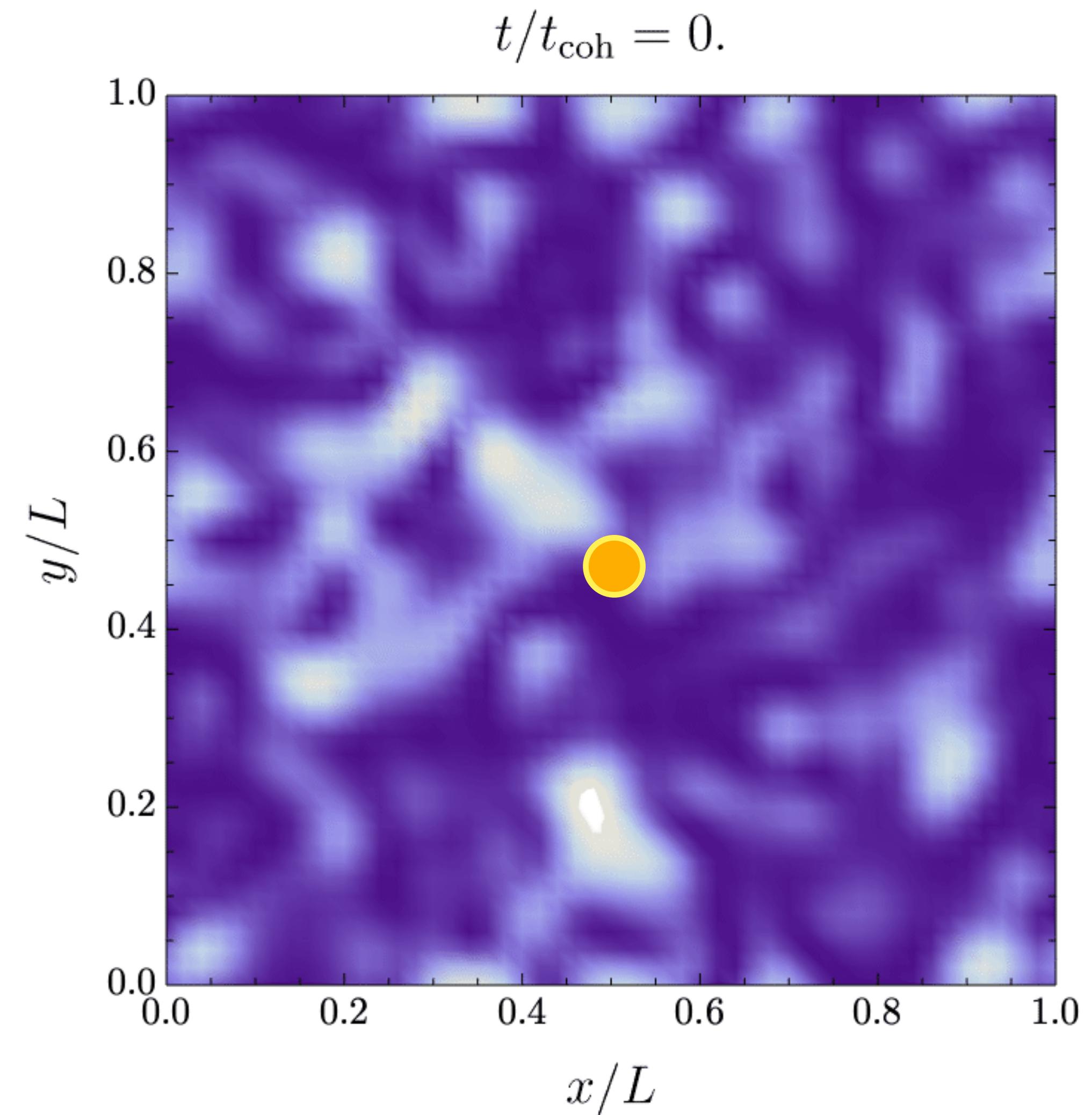
*since the wavelength could be astronomical scales
it's natural to expect an order-one density fluctuation over
such large spatial scales*



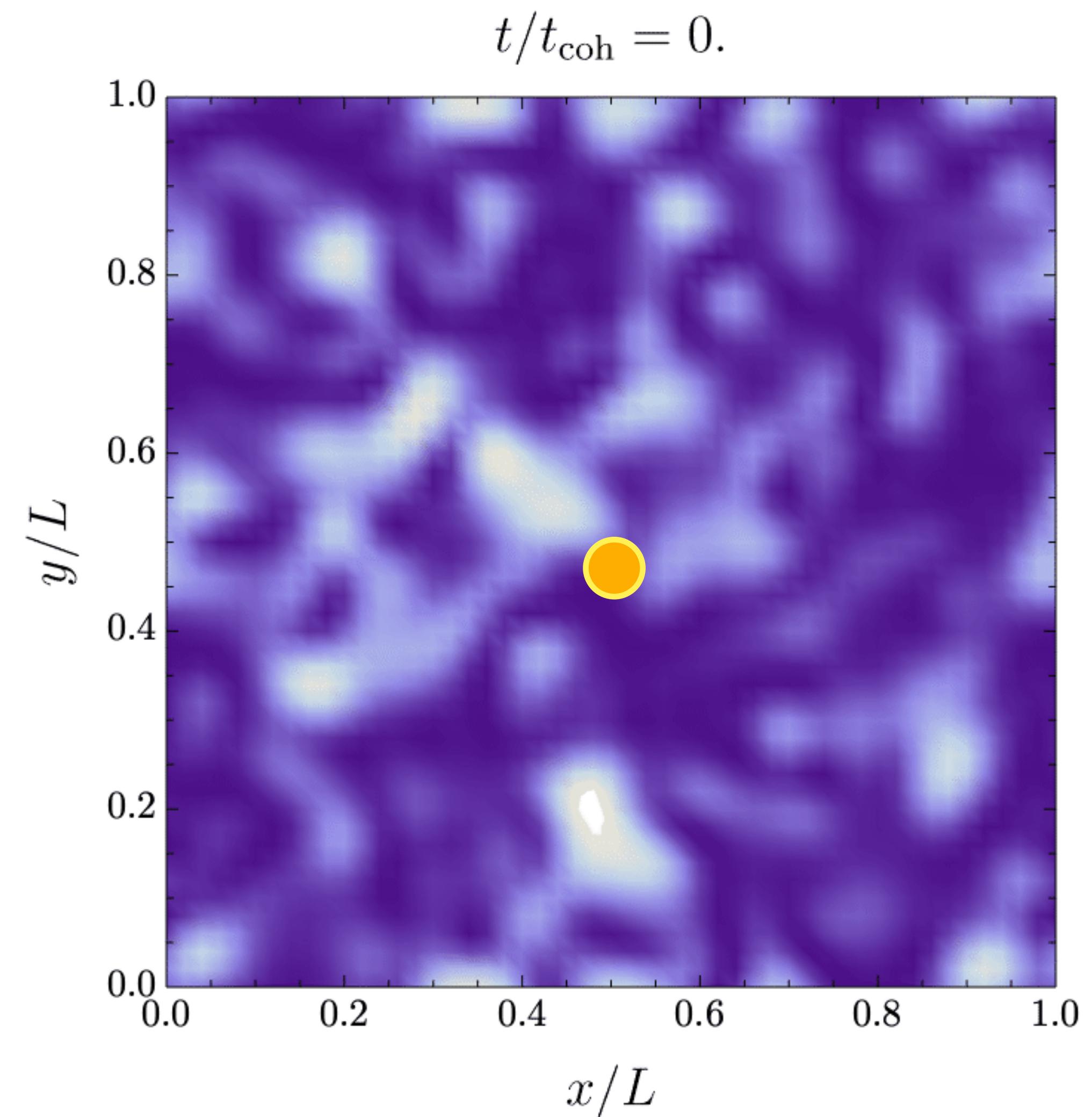
$$\ell \sim \text{AU}$$

$$m \sim 10^{-15} \text{ eV}$$

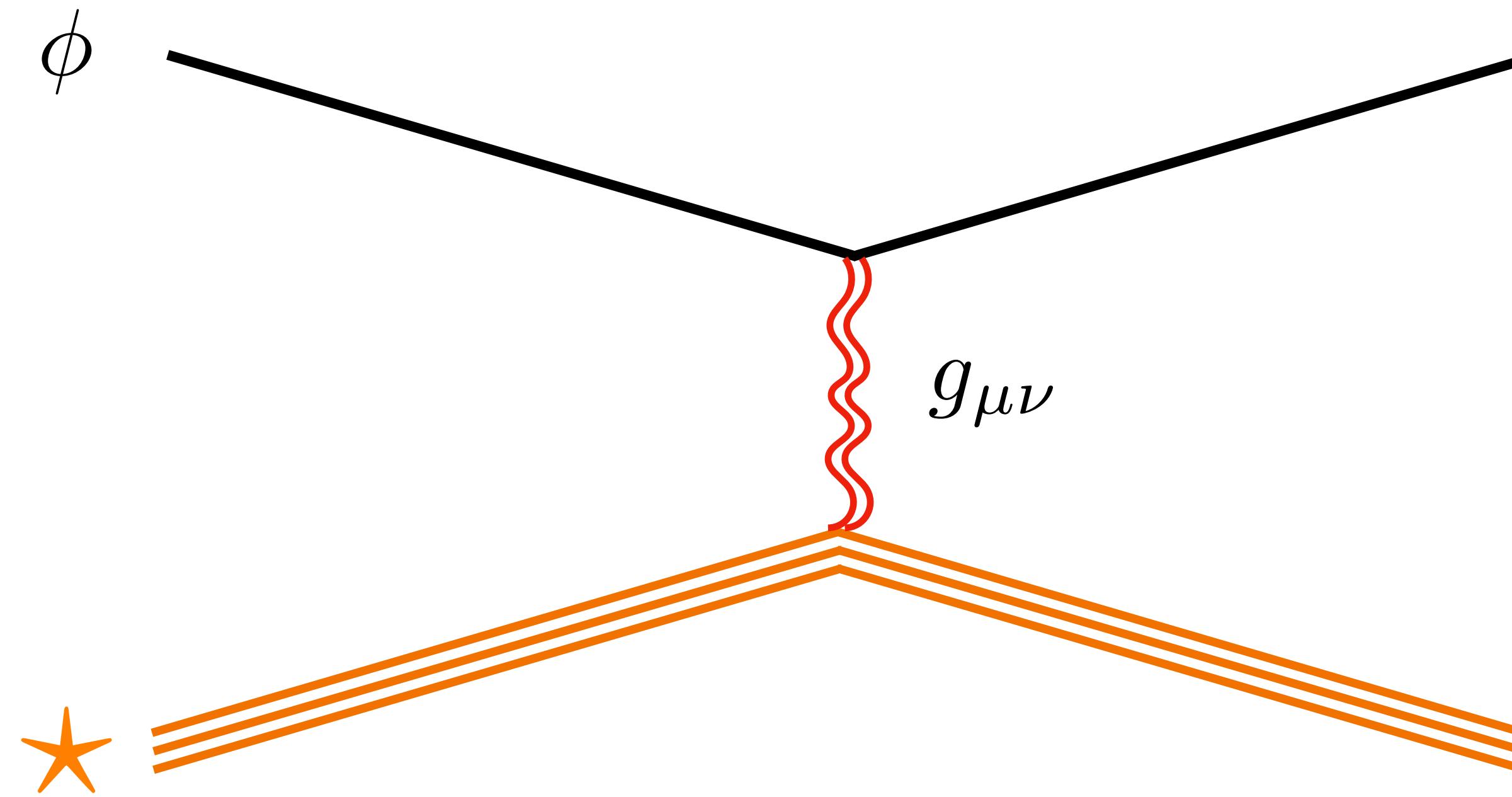
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it is also useful to understand the interaction
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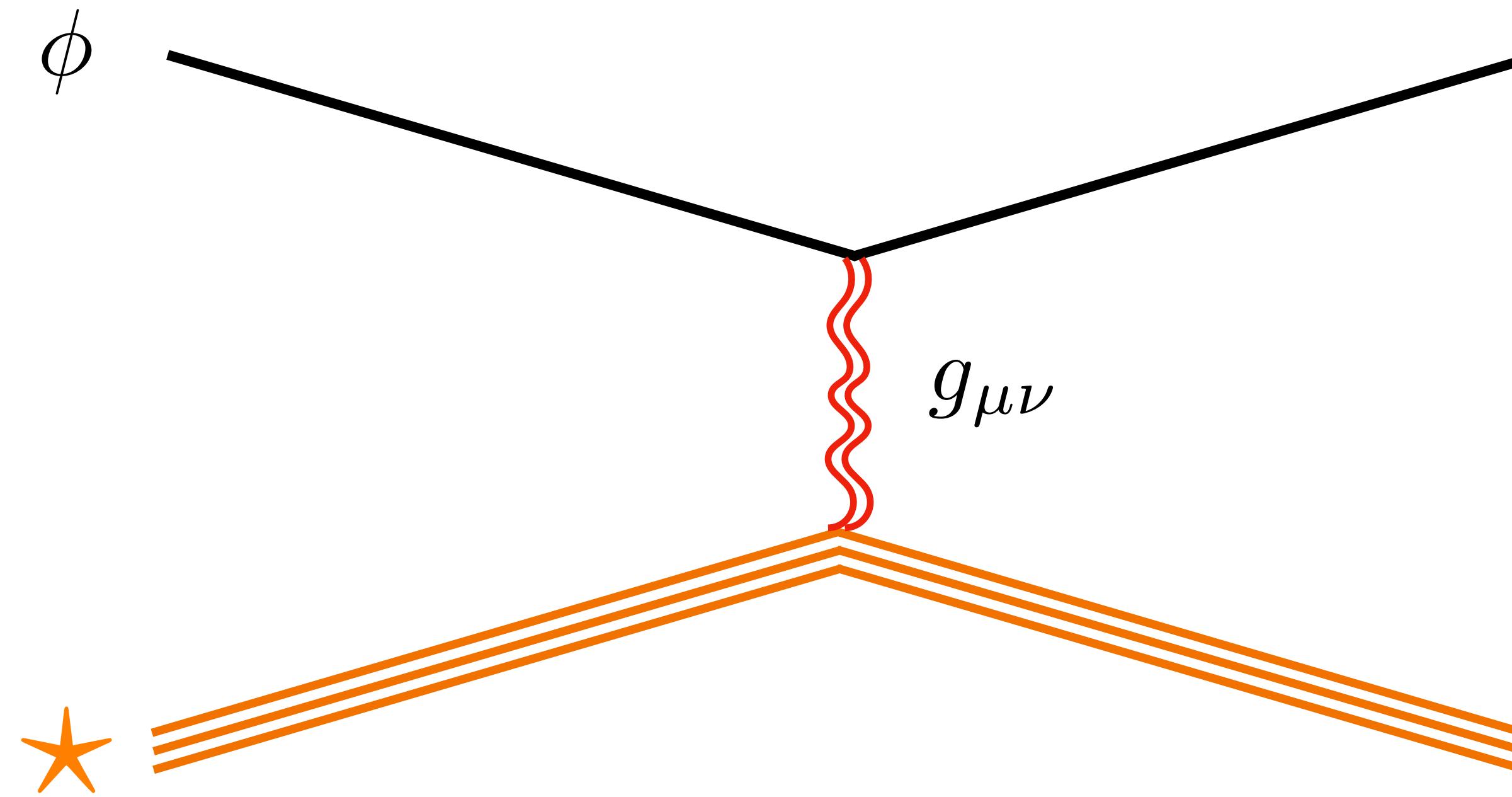


*at the fundamental level
the interaction between wave DM and star might be represented by*



$$\mathcal{M} \sim 16\pi G \frac{m^2 M_\star^2}{q^2}$$

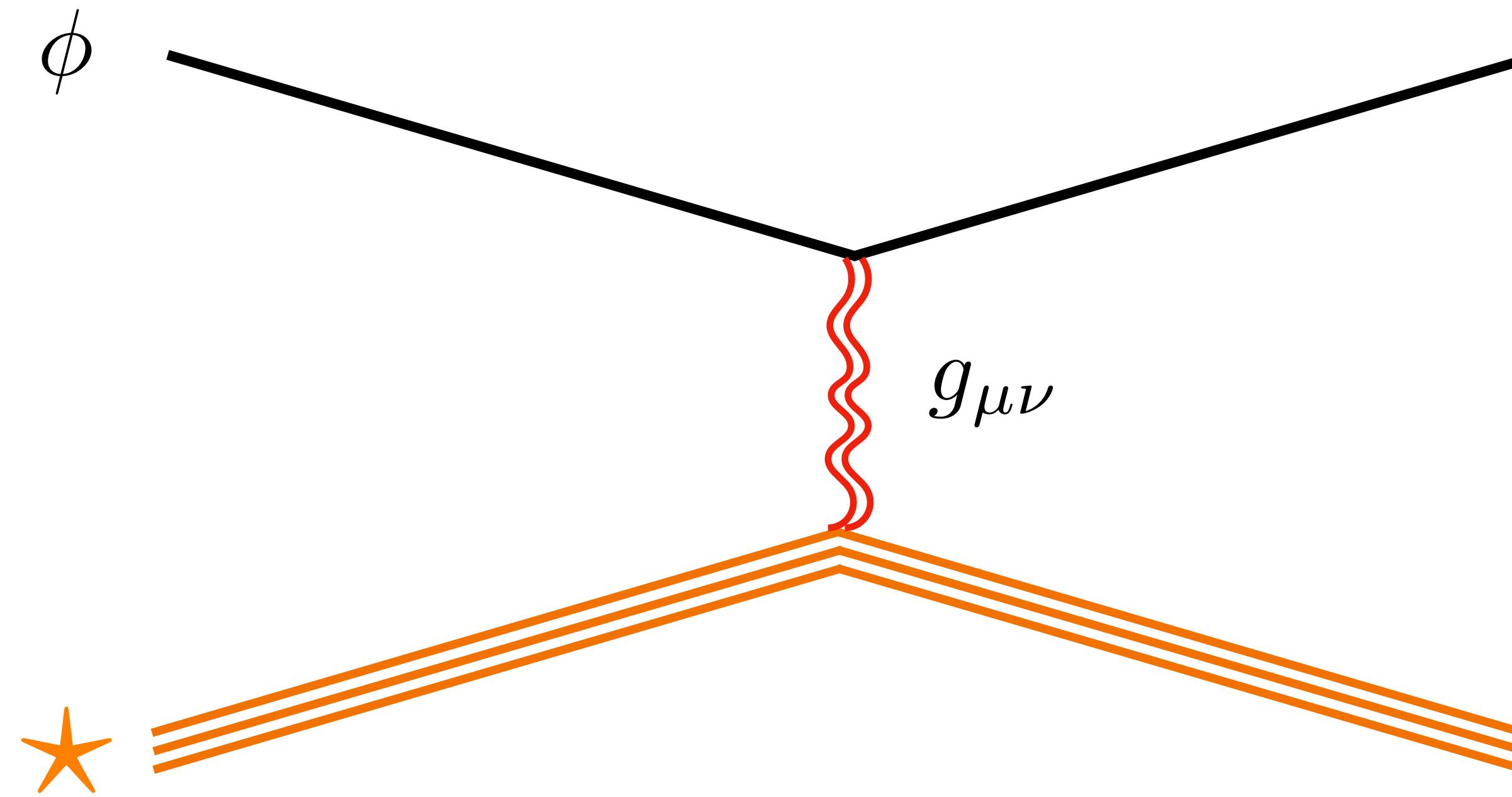
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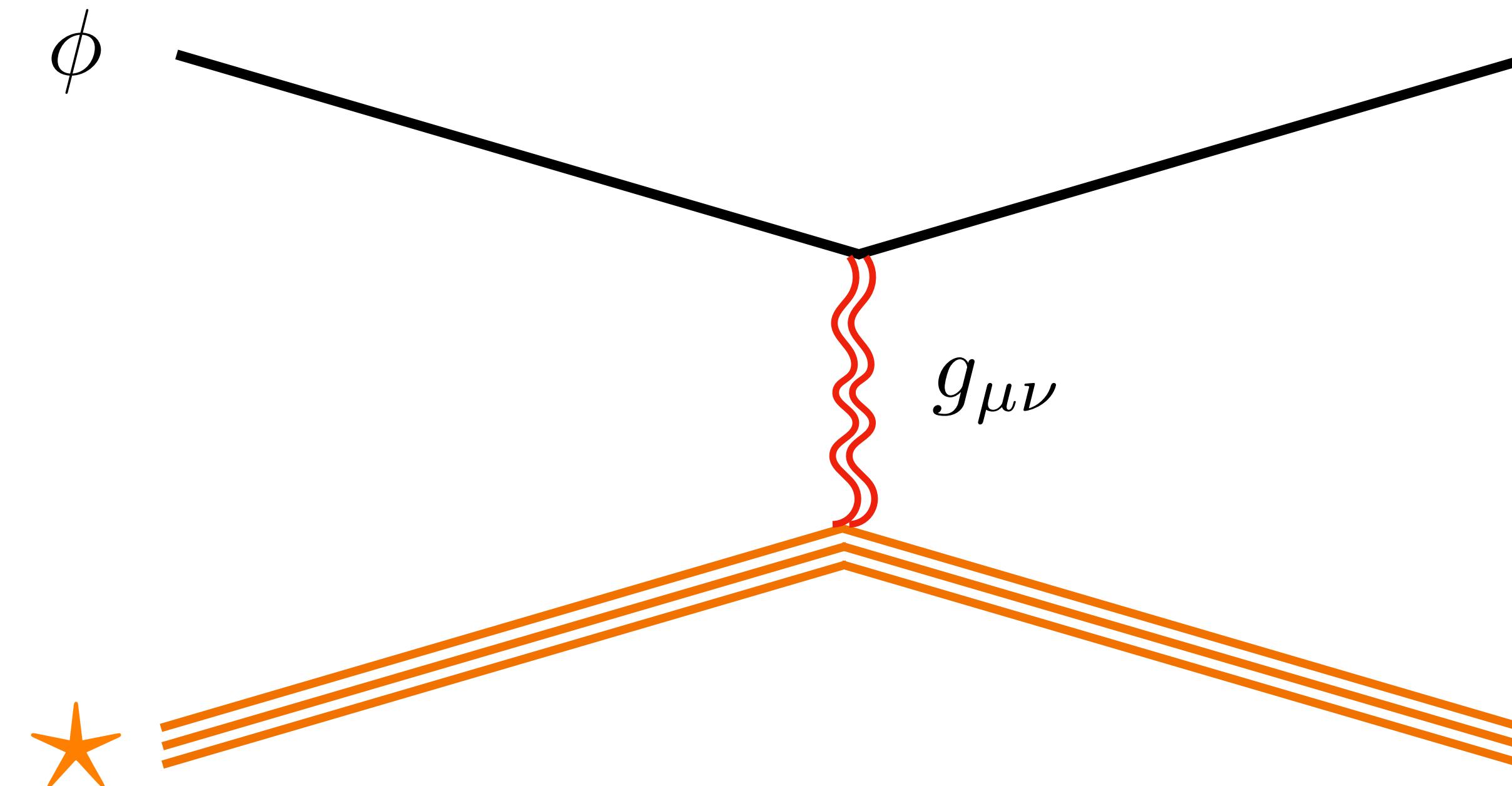
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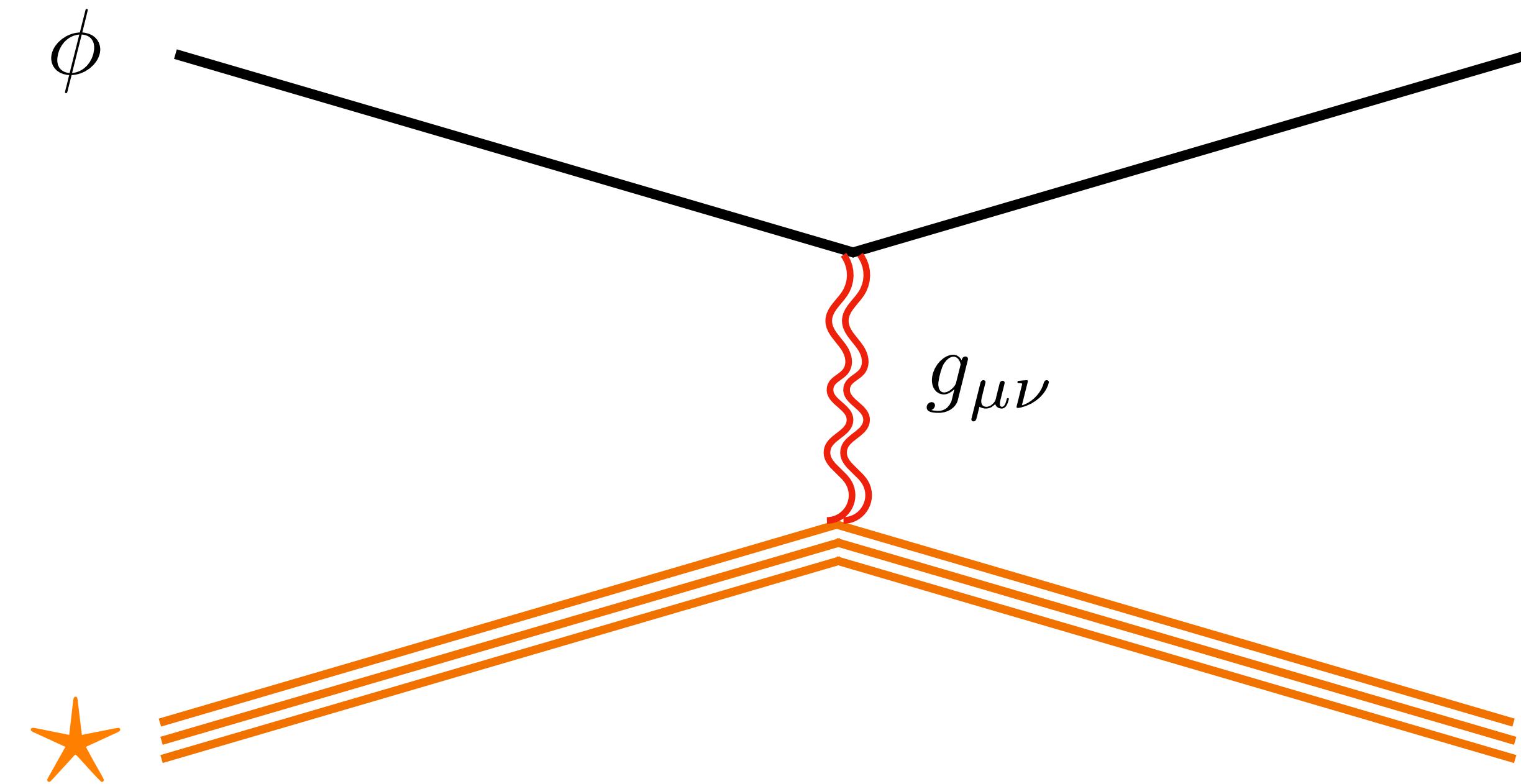
$$\mathcal{M} \sim 16\pi G \frac{m^2 M_\star^2}{q^2} N_{\text{occ}}$$

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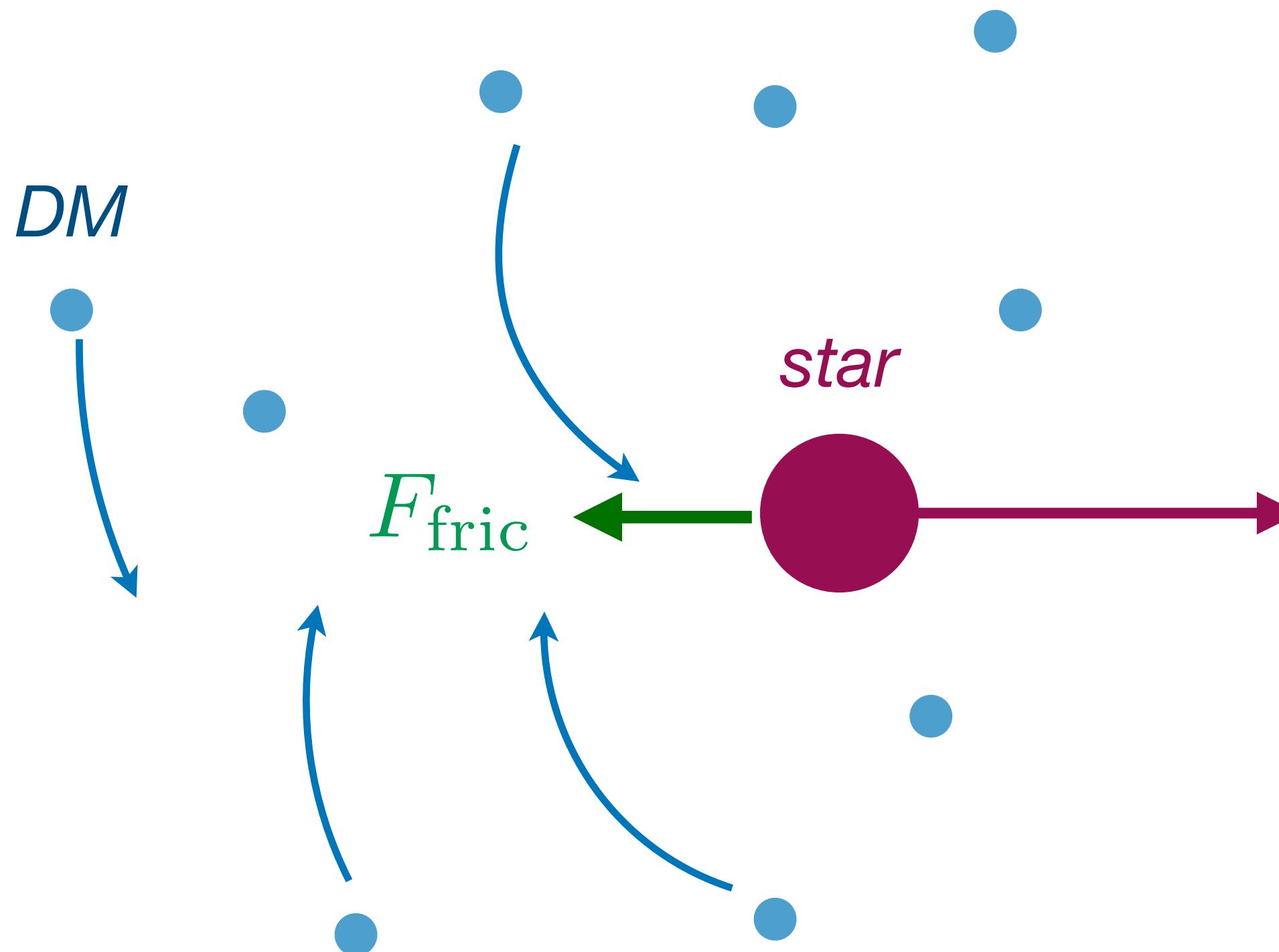
$$V = \frac{G m M_\star}{r} N_{\text{occ}} = \frac{G M_\star m_{\text{eff}}}{r}$$

*while our argument are still at the heuristic level
it can be shown more rigorously using kinetic equations (Boltzmann)
that this intuition generally holds for wave DM*

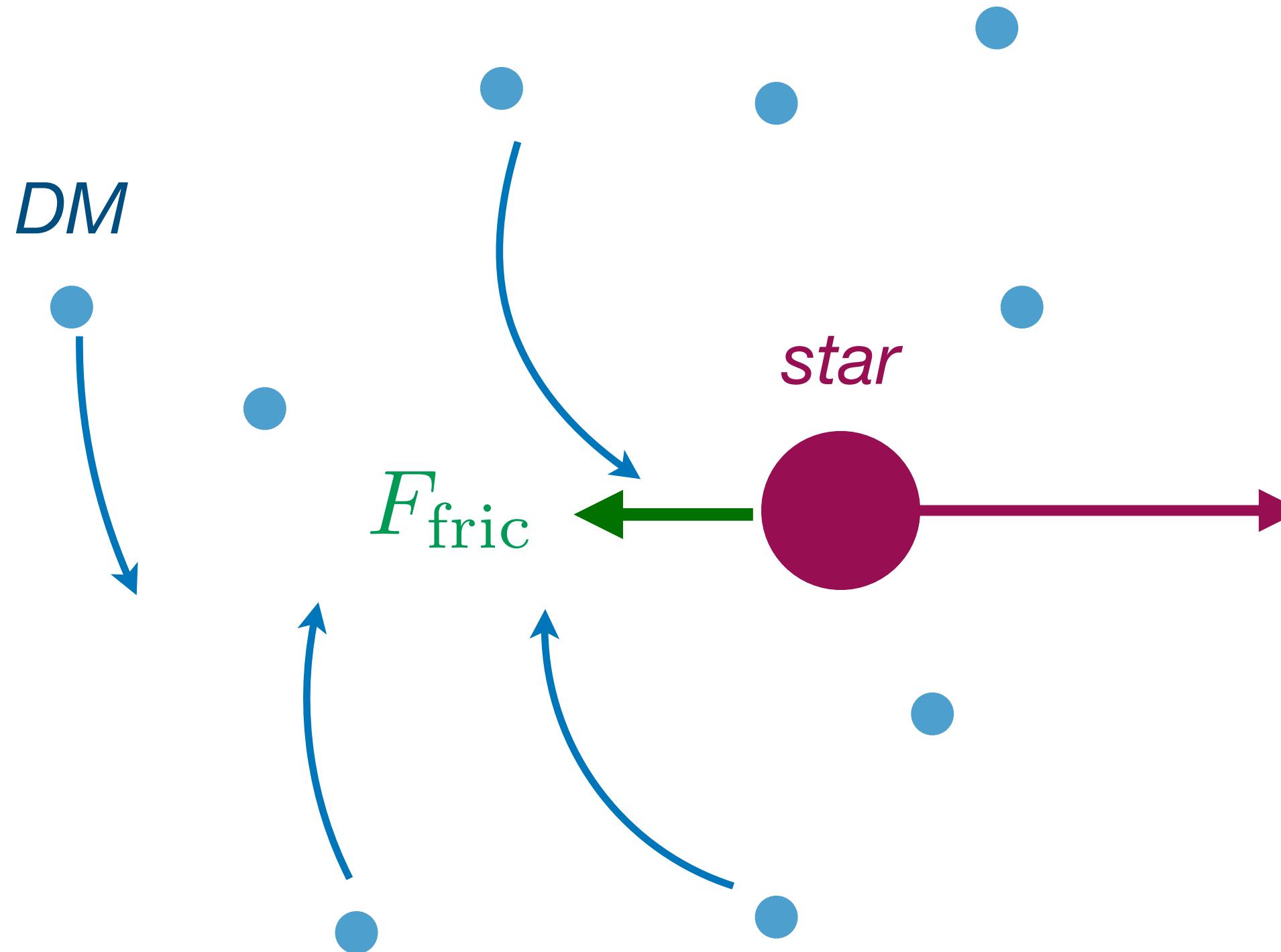
[Bar-Or et al (19, 21); Chavanis (21); Bar, Blas, Blum, **HK** (21)]

phenomenologically, this interaction of wave DM may leave some interesting features in the motion of stars;

*the interaction between DM and stars
slows down stars, causing orbital decays*

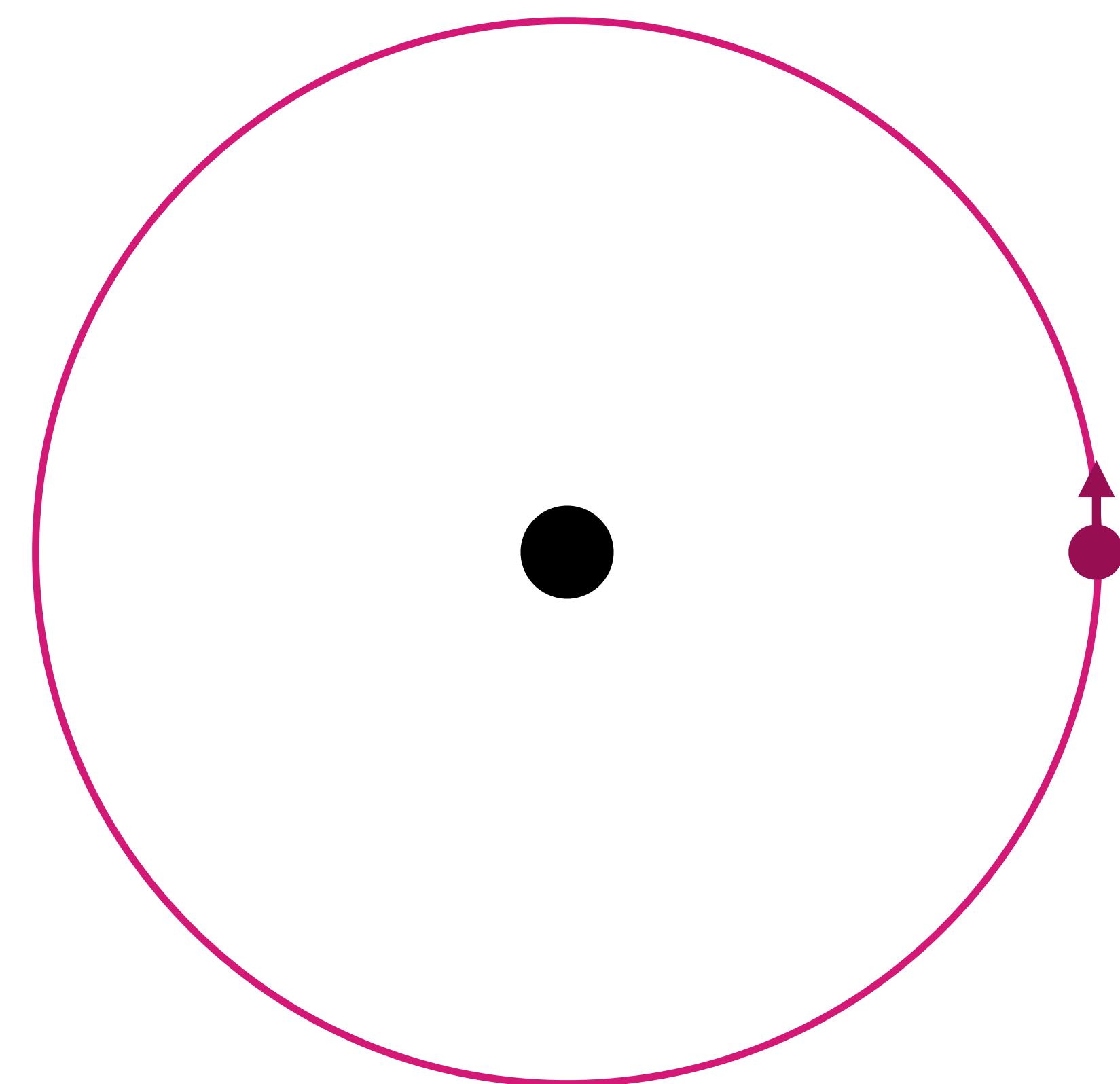


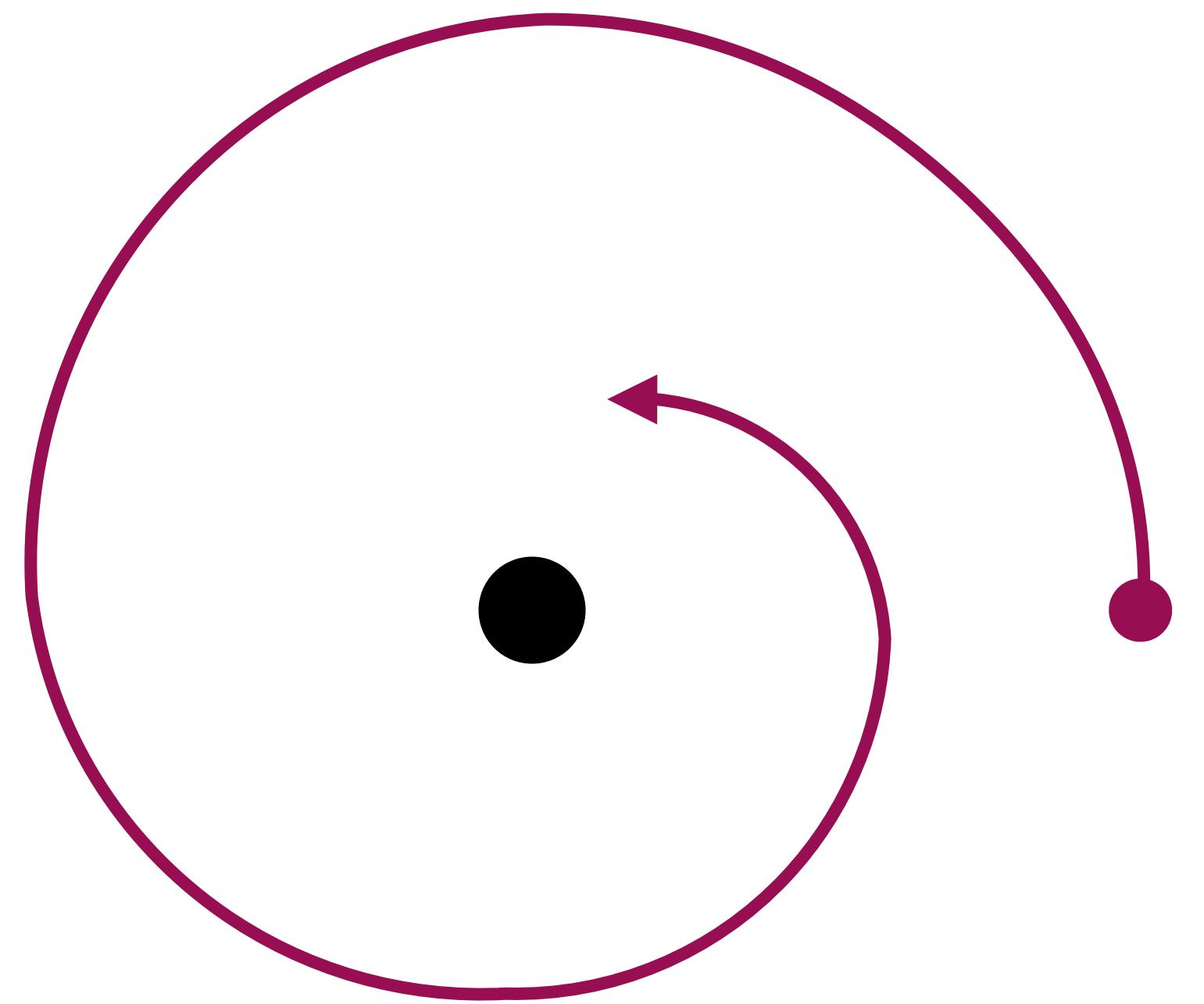
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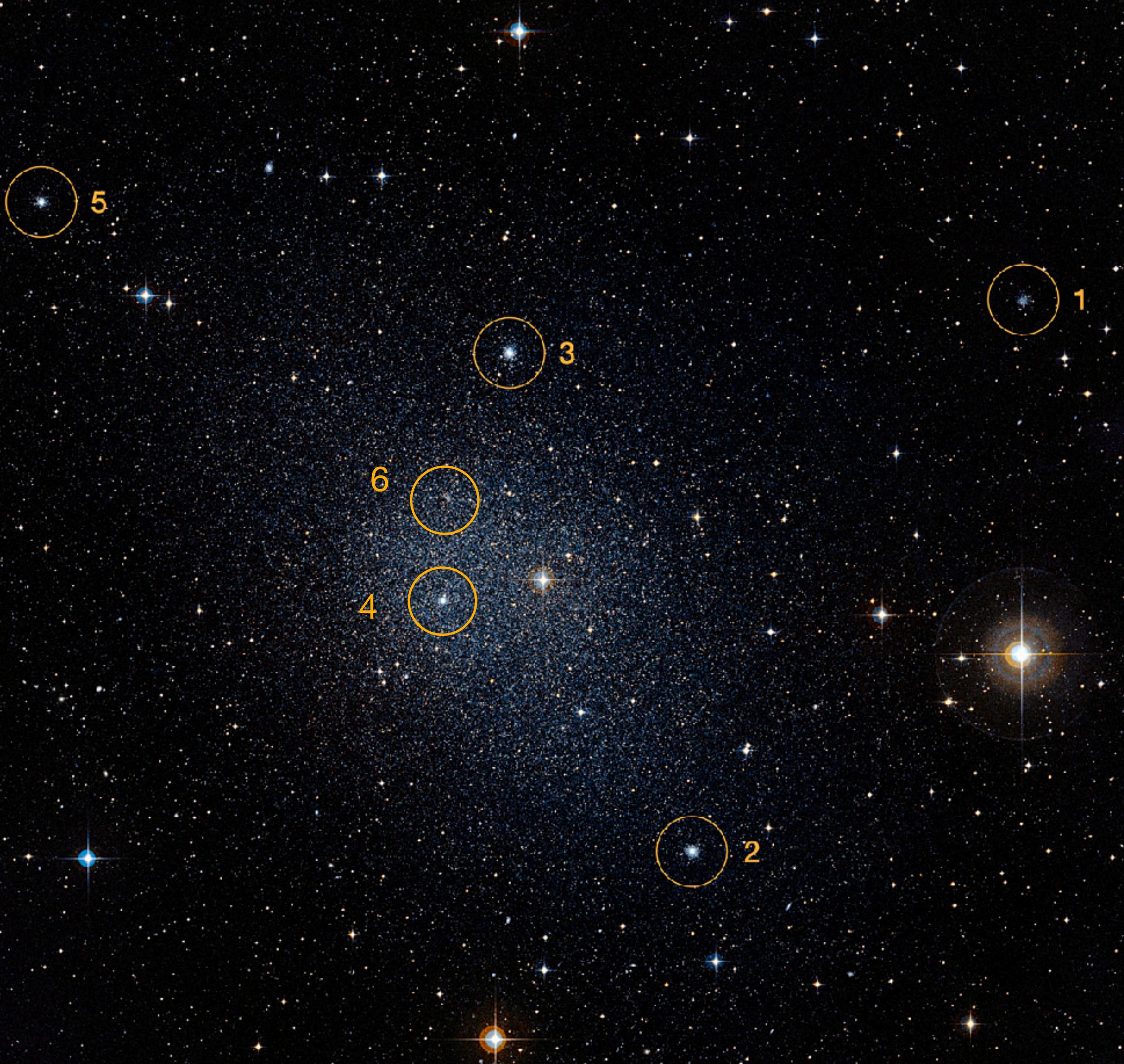


$$F \sim -\frac{4\pi G^2 \rho (m_{\text{DM}} + m_{\star})}{\sigma^2} \ln(b_{\max}/b_{\min}) G(X)$$

(classical) Dynamical friction
[Chandrasekhar (42)]



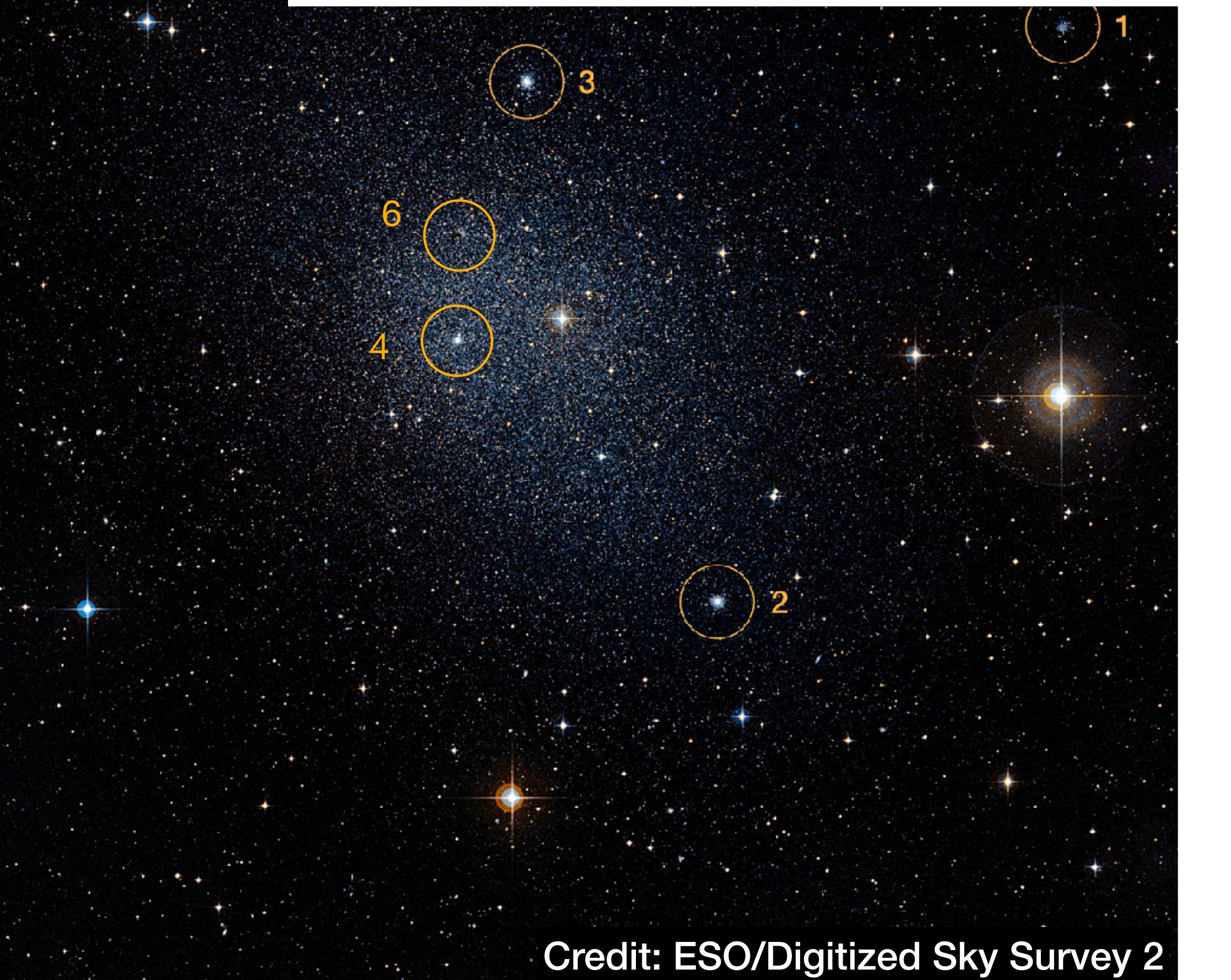




Credit: ESO/Digitized Sky Survey 2

	$m_\star [10^5 M_\odot]$	$r_\perp [\text{kpc}]$	$\Delta v_r [\text{km/s}]$	$r_{c/h} [\text{pc}]$	Refs.	$\tau_{\text{CDM}} [\text{Gyr}]$	$\tau_{\text{DDM}}^{(135)} [\text{Gyr}]$	$\tau_{\text{SIDM}} [\text{Gyr}]$
GC1	0.42 ± 0.10	1.73 ± 0.05	3.54 ± 1.18	10.8 ± 0.3	[18, 19, 56–58]	119	122	79.3
GC2	1.54 ± 0.28	0.98 ± 0.03	3.9 ± 0.7	6.2 ± 0.2	[18, 19, 58, 59]	14.7	7.12	8.82
GC3	4.98 ± 0.84	0.64 ± 0.02	4.94 ± 0.66	1.7 ± 0.1	[18, 19, 60, 61]	2.63	1.48	2.21
GC4	0.76 ± 0.15	0.154 ± 0.014	-8.26 ± 0.64	1.9 ± 0.2	[18, 19, 60, 61]	0.91	10.7	14.8
GC5	1.86 ± 0.24	1.68 ± 0.05	3.93 ± 0.77	1.5 ± 0.1	[18, 19, 56, 60, 61]	32.2	30.1	20
GC6	~ 0.29	0.254 ± 0.015	-1.56 ± 1.36	12.0 ± 1.4	[15, 50]	5.45	16.1	22

5

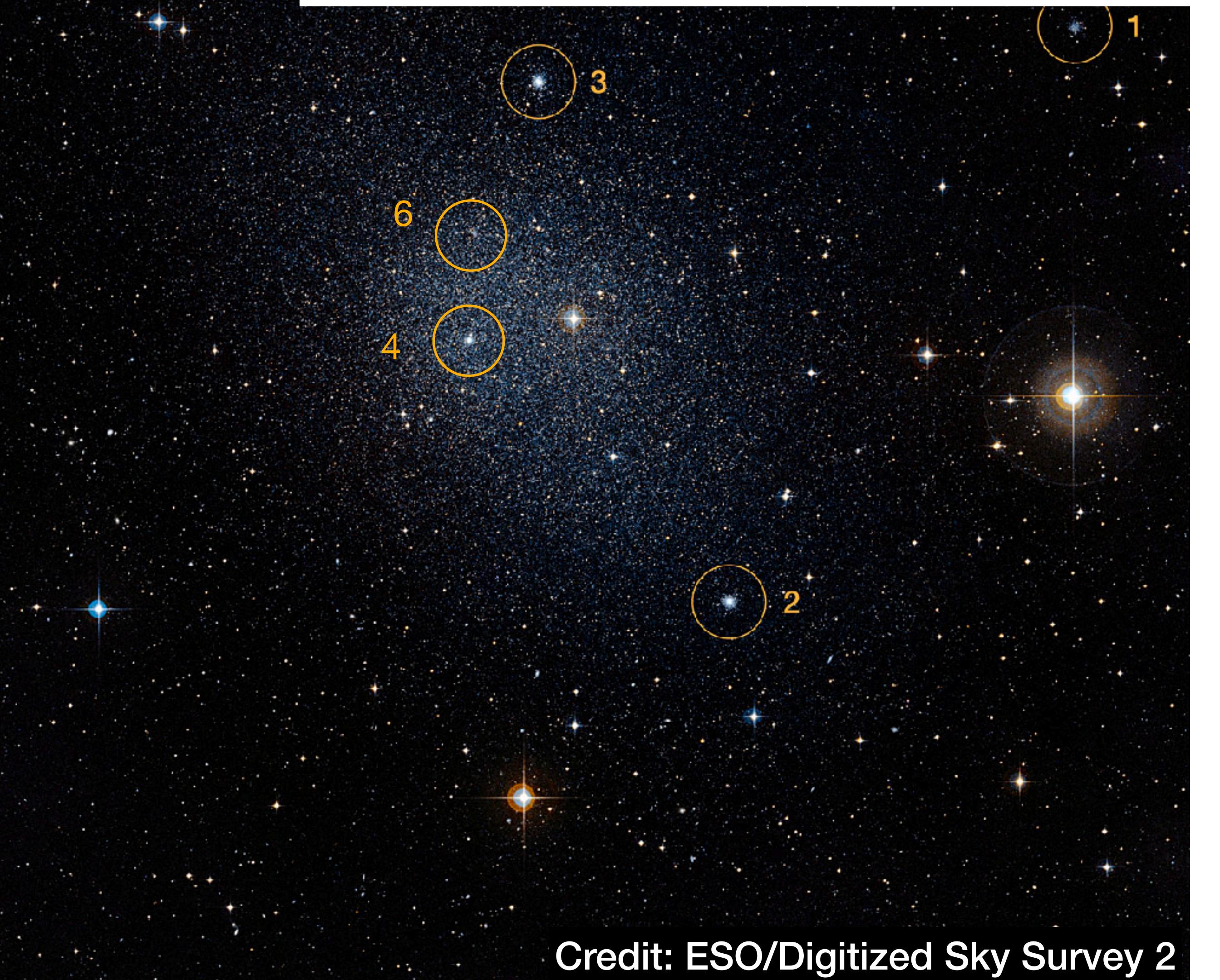


[Bar, Blas, Blum, HK (21)]

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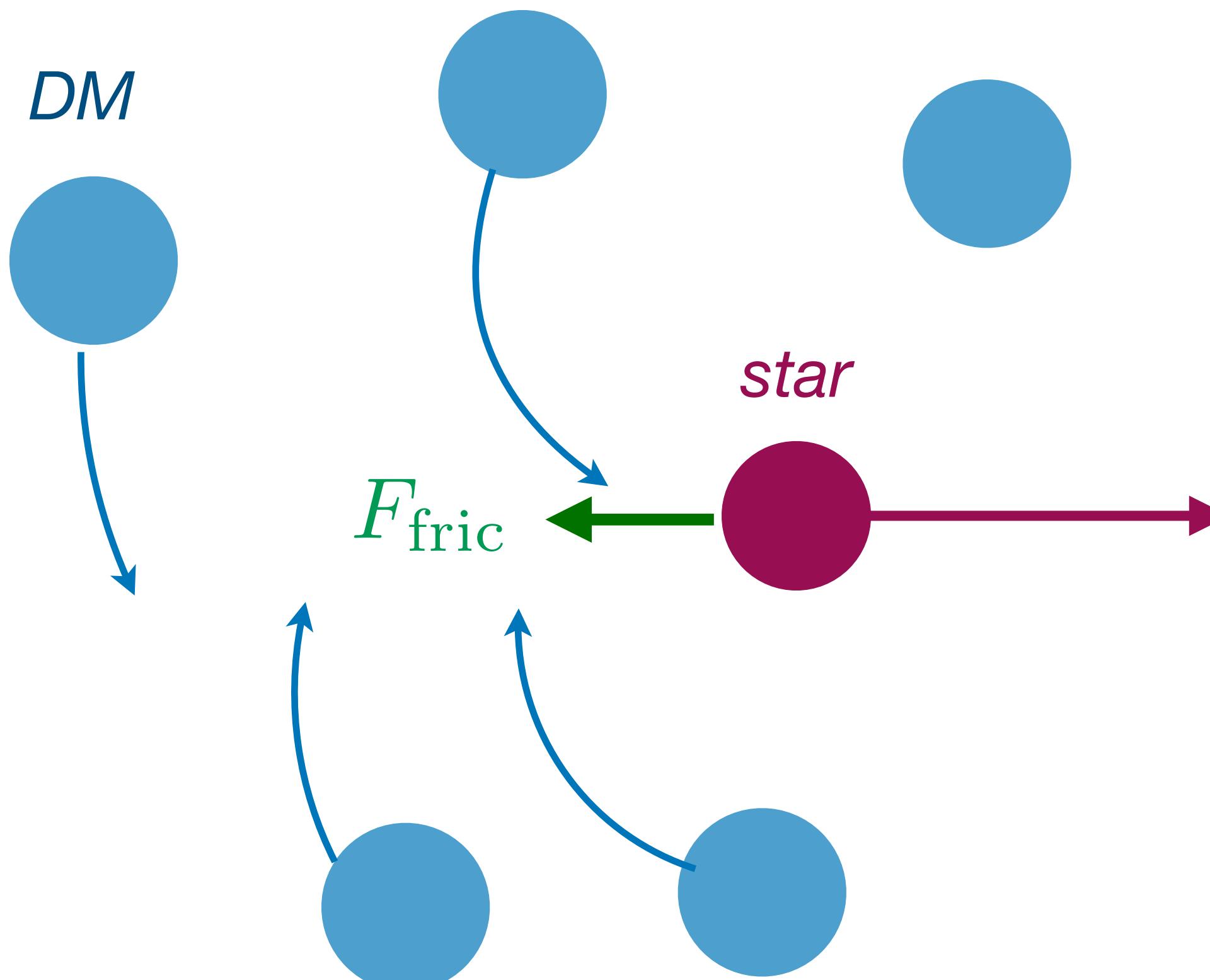
[Bar, Blas, Blum, HK (21)]

Fornax GC timing problem

[Tremaine (76)]

5

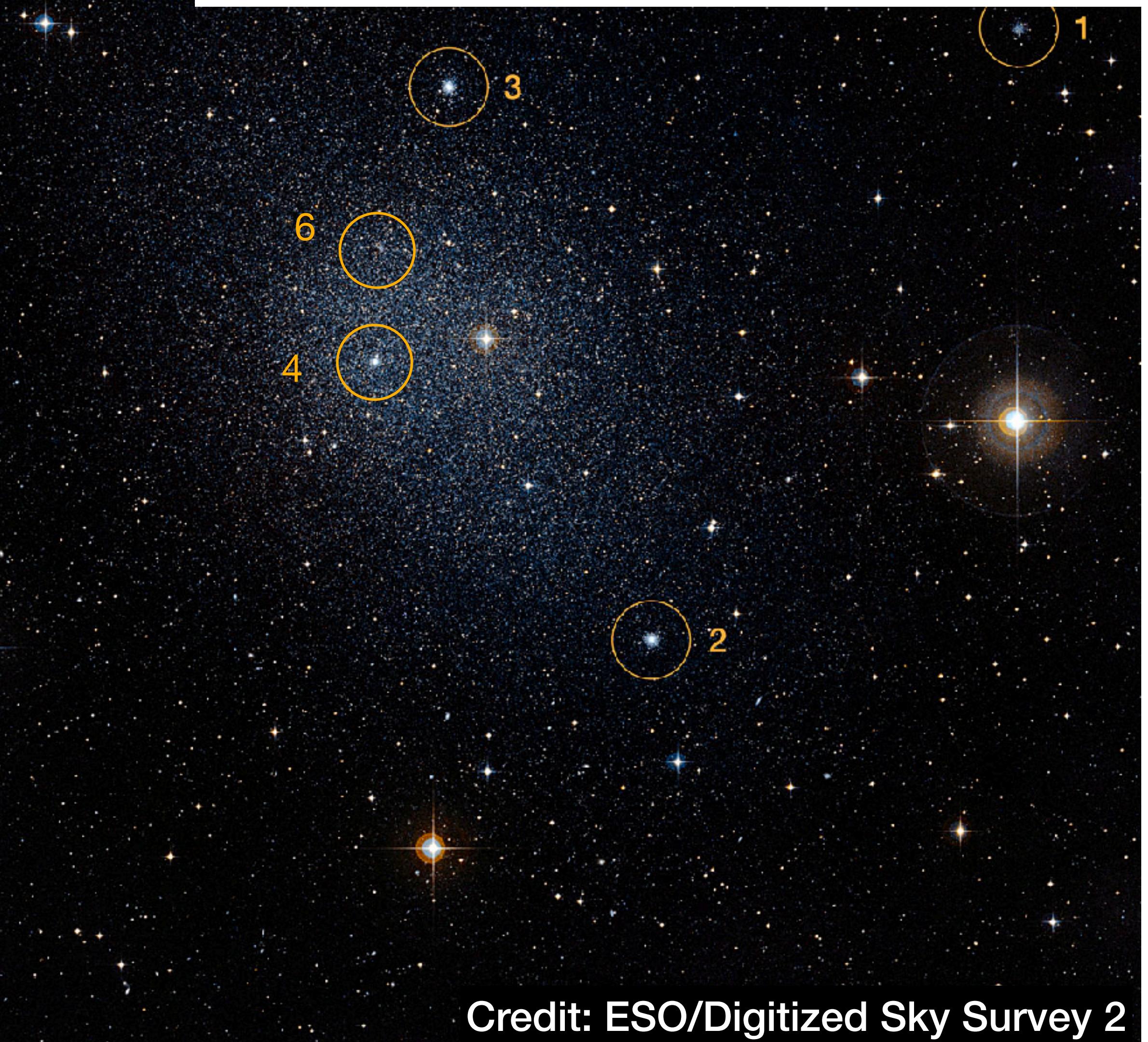
*similarly wave DM (QP) particles will interact with stars
and exert dragging force . . .*



$$F \sim -\frac{4\pi G^2 \rho (m_{\text{eff}} + m_\star)}{\sigma^2} \ln(b_{\max}/\lambda) G(X)$$

	$m_\star [10^5 M_\odot]$	$r_\perp [\text{kpc}]$	$\Delta v_r [\text{km/s}]$	$r_{c/h} [\text{pc}]$	Refs.	$\tau_{\text{CDM}} [\text{Gyr}]$	$\tau_{\text{DDM}}^{(135)} [\text{Gyr}]$	$\tau_{\text{SIDM}} [\text{Gyr}]$
GC1	0.42 ± 0.10	1.73 ± 0.05	3.54 ± 1.18	10.8 ± 0.3	[18, 19, 56–58]	119	122	79.3
GC2	1.54 ± 0.28	0.98 ± 0.03	3.9 ± 0.7	6.2 ± 0.2	[18, 19, 58, 59]	14.7	7.12	8.82
GC3	4.98 ± 0.84	0.64 ± 0.02	4.94 ± 0.66	1.7 ± 0.1	[18, 19, 60, 61]	2.63	1.48	2.21
GC4	0.76 ± 0.15	0.154 ± 0.014	-8.26 ± 0.64	1.9 ± 0.2	[18, 19, 60, 61]	0.91	10.7	14.8
GC5	1.86 ± 0.24	1.68 ± 0.05	3.93 ± 0.77	1.5 ± 0.1	[18, 19, 56, 60, 61]	32.2	30.1	20
GC6	~ 0.29	0.254 ± 0.015	-1.56 ± 1.36	12.0 ± 1.4	[15, 50]	5.45	16.1	22

5



Credit: ESO/Digitized Sky Survey 2

[Bar, Blas, Blum, HK (21)]

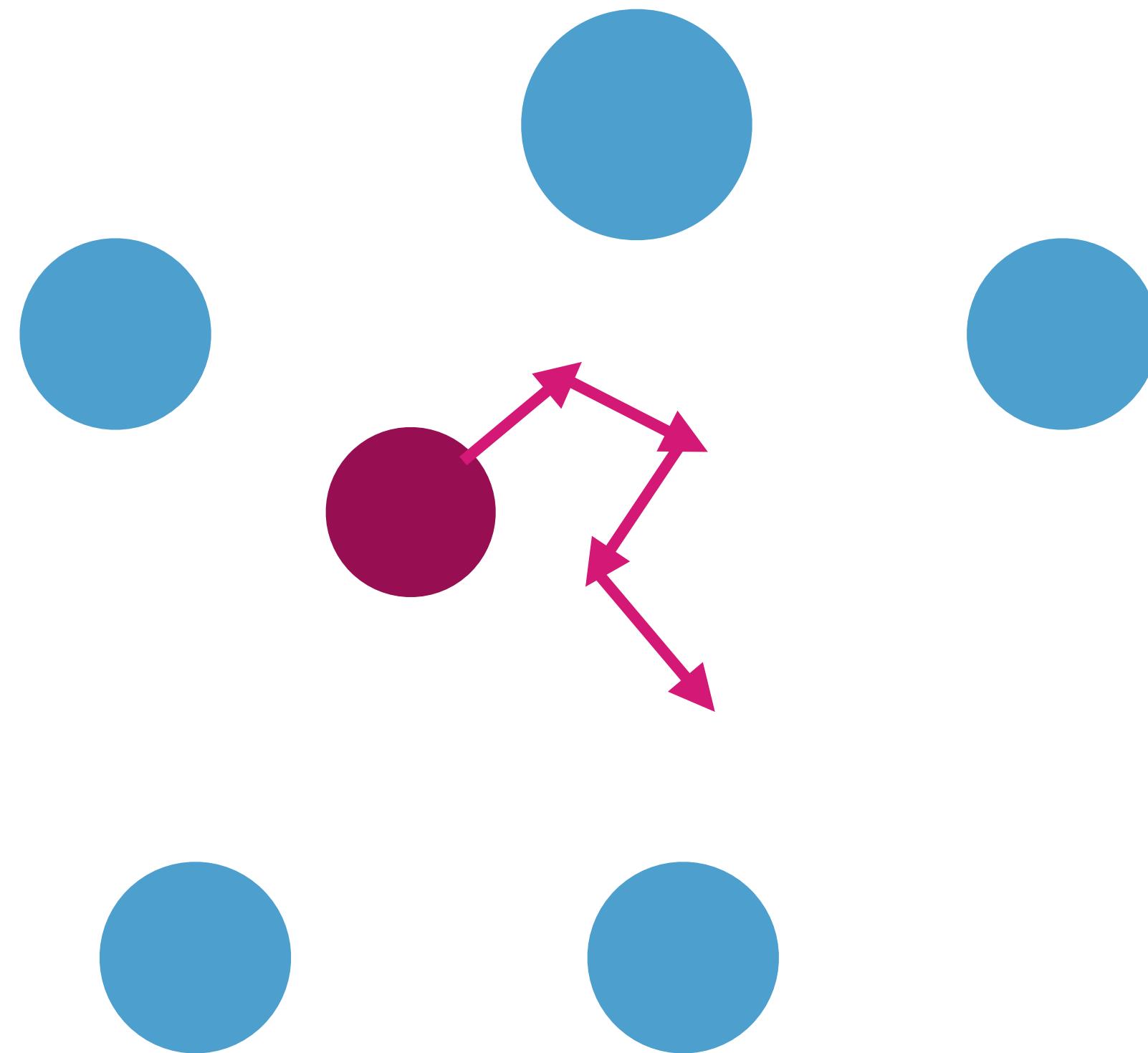
Fornax GC timing problem

[Tremaine (76)]

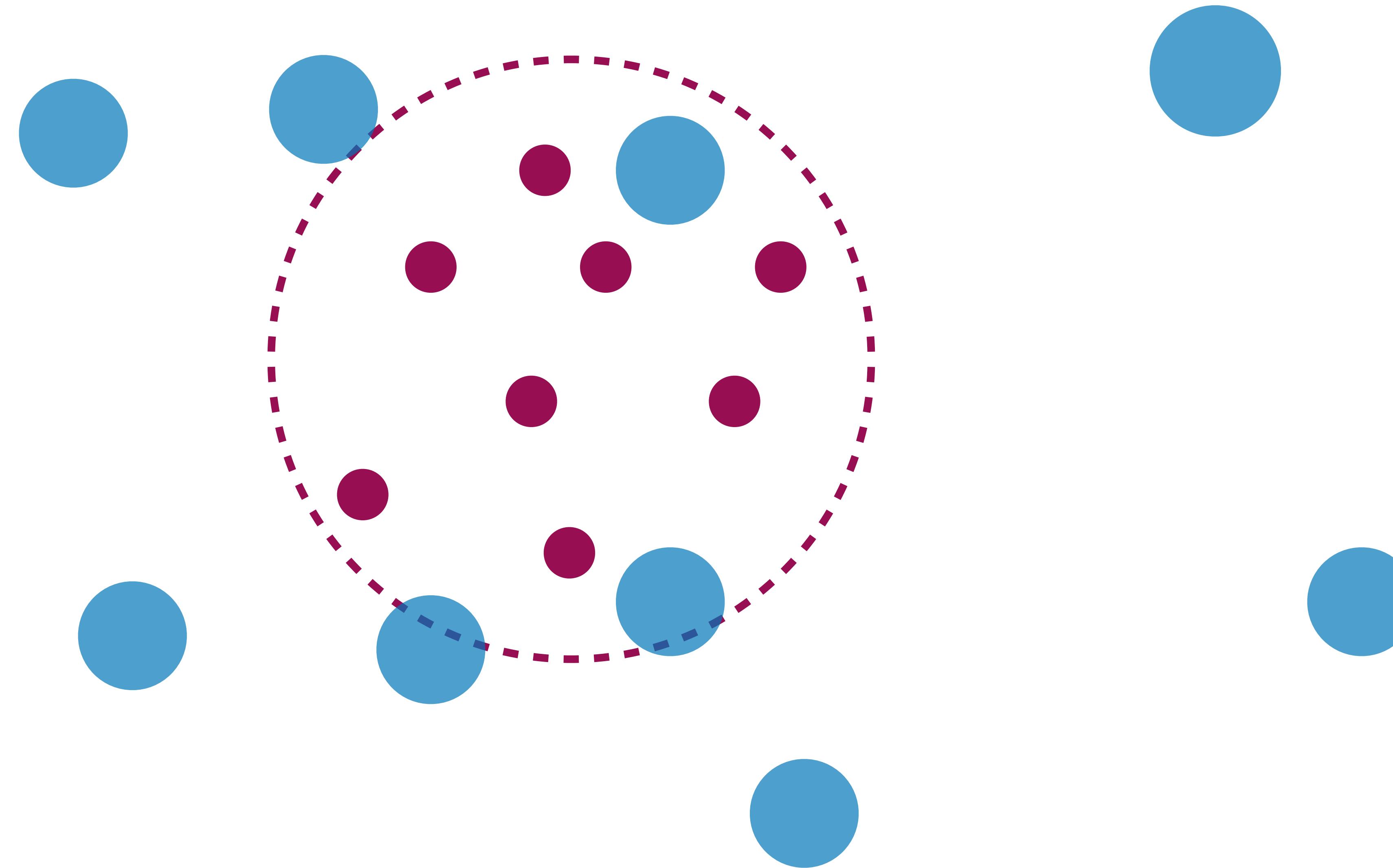
n	$r_\perp (\text{kpc})$	$m_{\text{cl}} (M_\odot)$	Projected radius		Cluster mass		CDM		FDM	
			C	$\tau (\text{Gyr})$	kr	C	$\tau (\text{Gyr})$	kr	C	$\tau (\text{Gyr})$
1	1.6	3.7×10^4	4.29	112	8.90	2.46	215			
2	1.05	1.82×10^5	3.32	9.7	5.04	1.88	12			
3	0.43	3.63×10^5	2.45	0.62	0.97	0.29	2.2			
4	0.24	1.32×10^5	2.50	0.37	0.31	0.033	10			
5	1.43	1.78×10^5	3.46	21.3	7.79	2.32	31			

[Hui et al (17)]

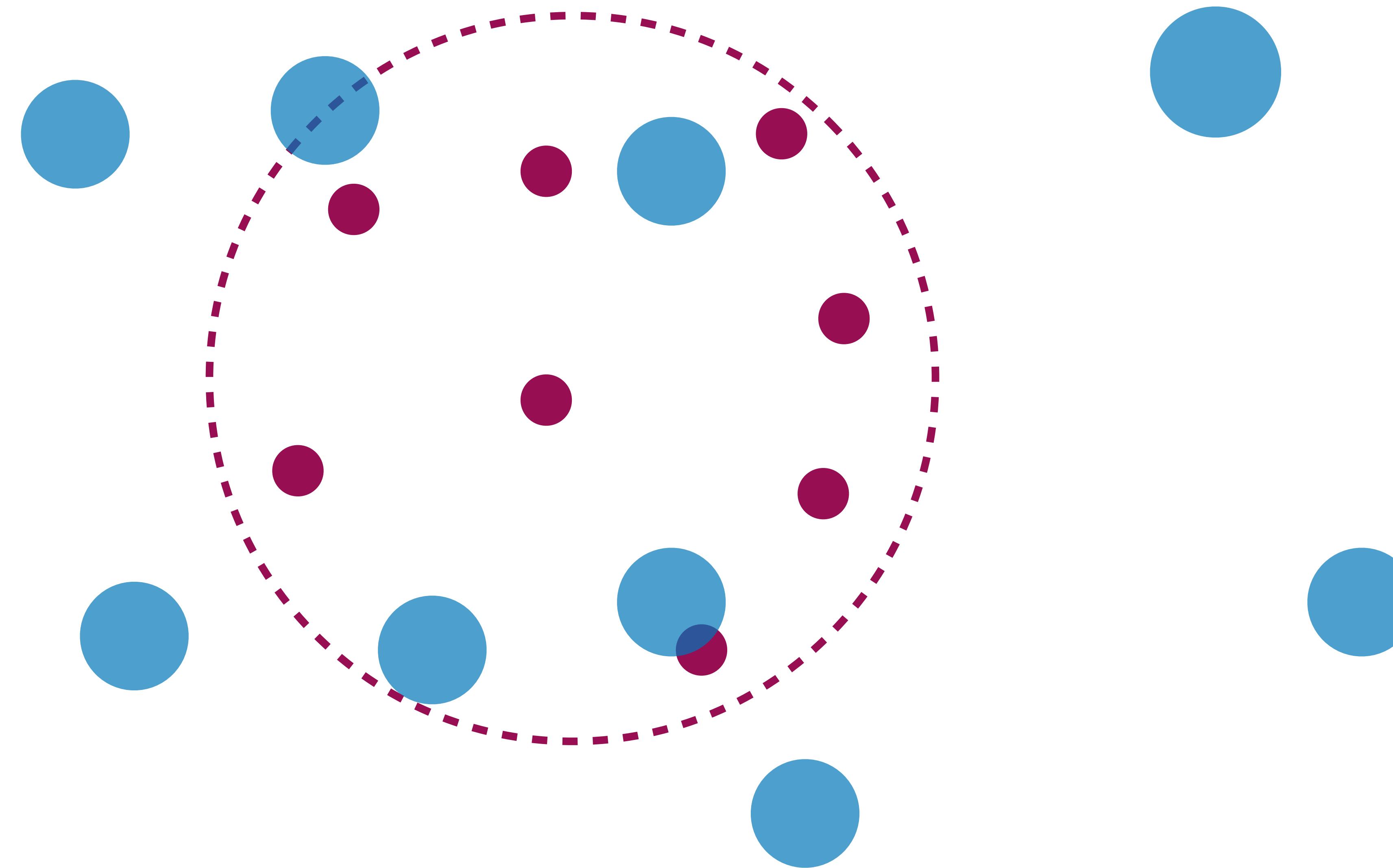
*sometimes quasiparticles bombard stars
injecting additional energies into a stellar population*



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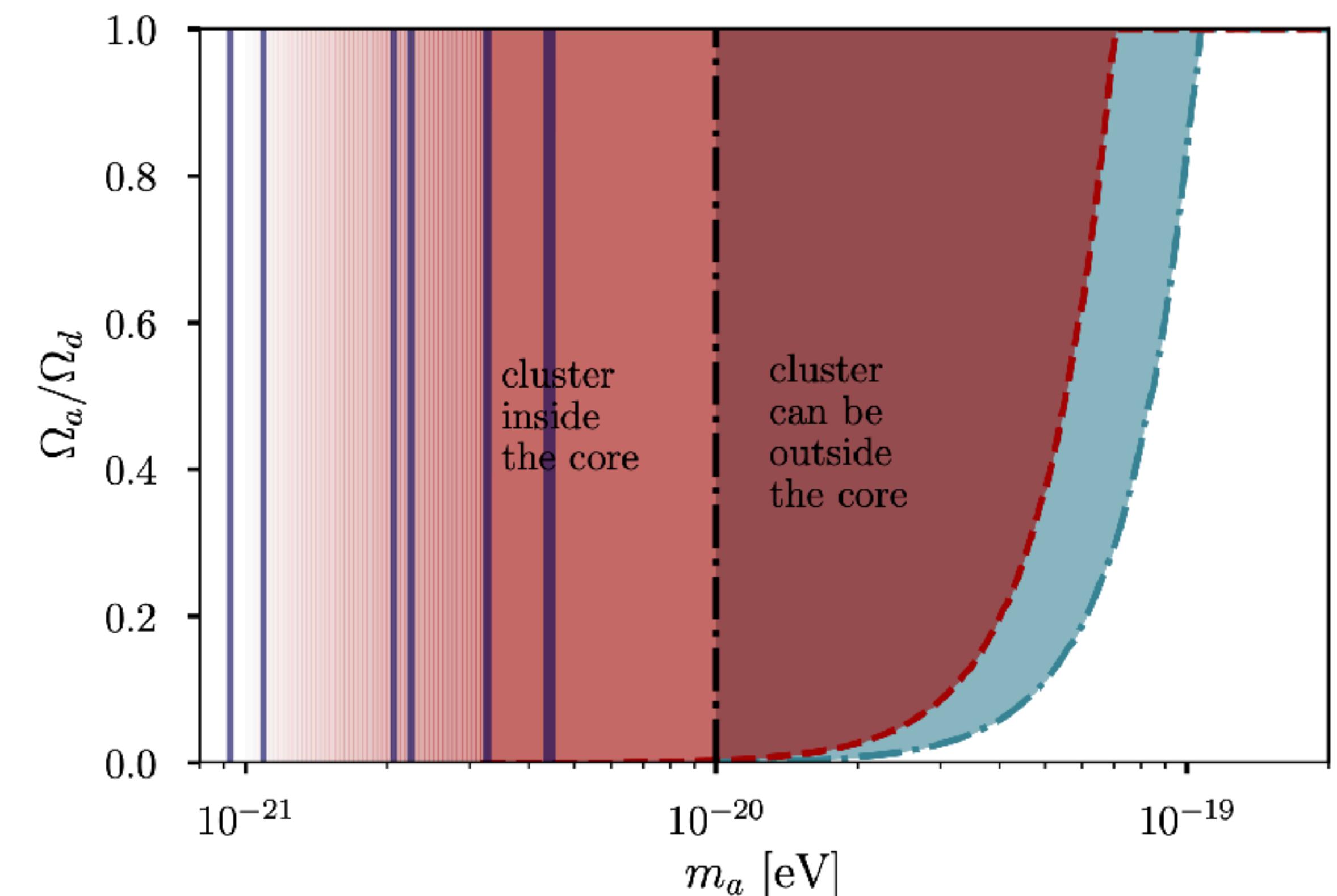


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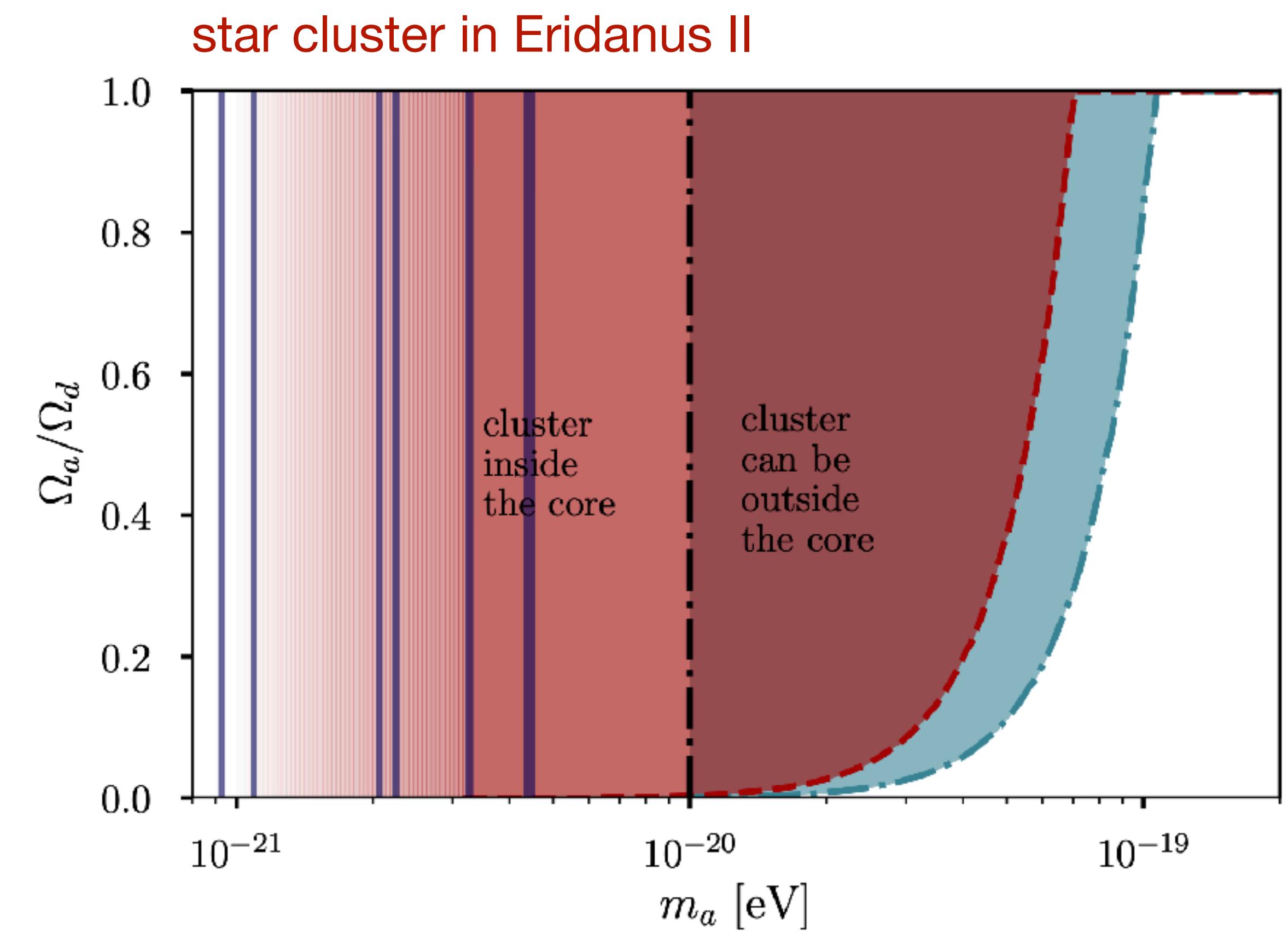


*This ‘heating’ by quasiparticles may dissolve
star clusters, change velocity dispersion of stars,
perturb galactic disc as well as other stellar substructures ...*

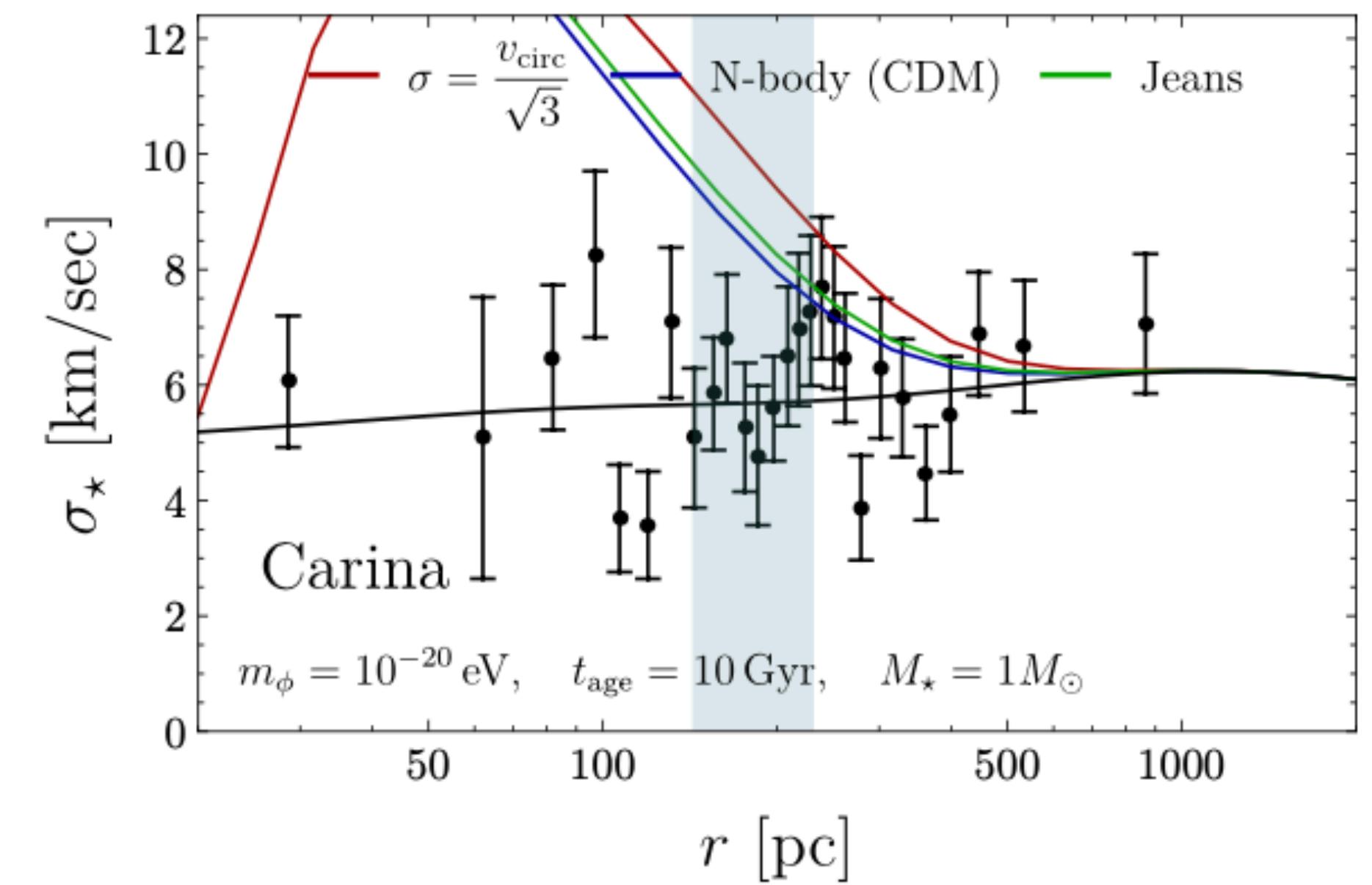
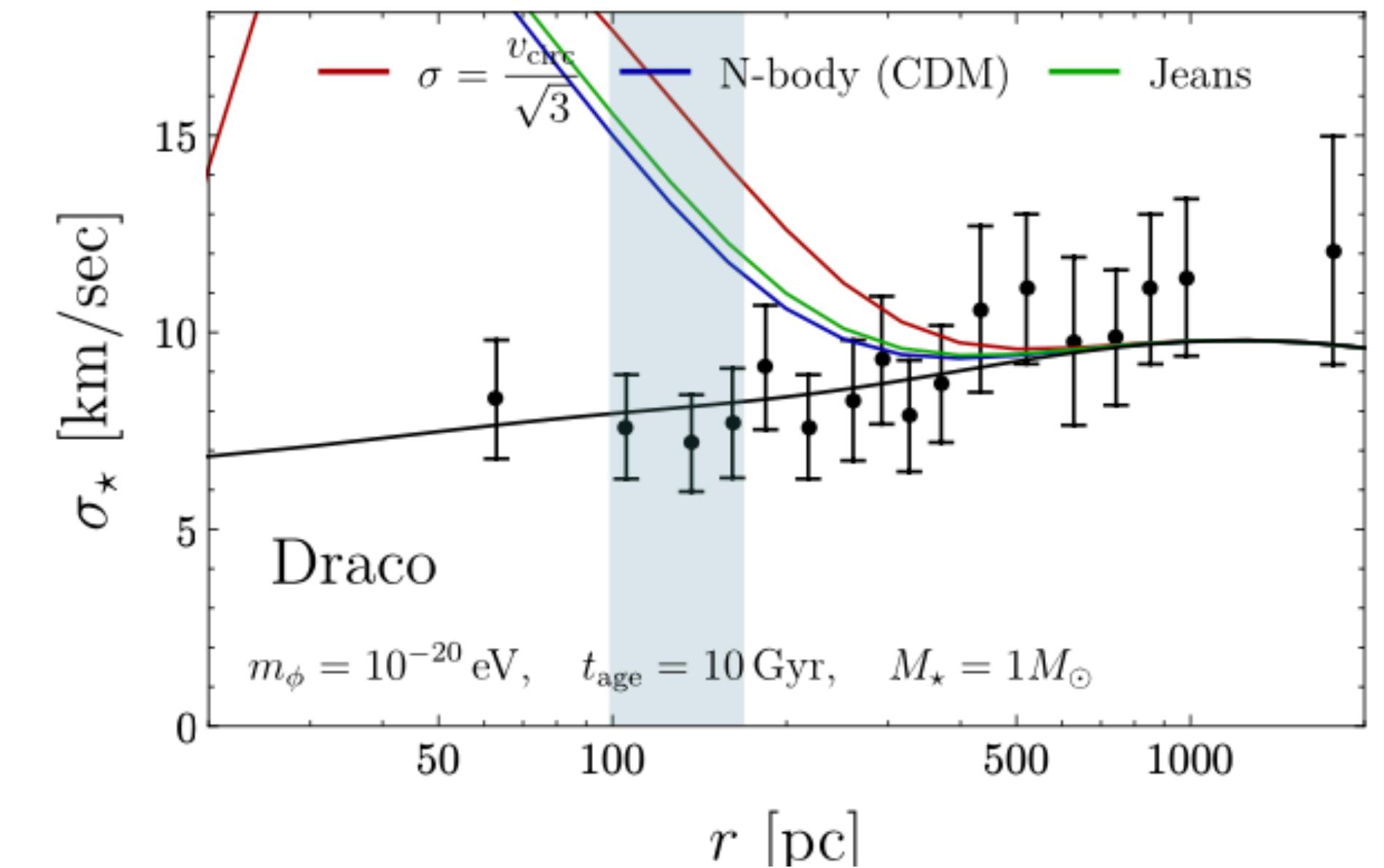
star cluster in Eridanus II



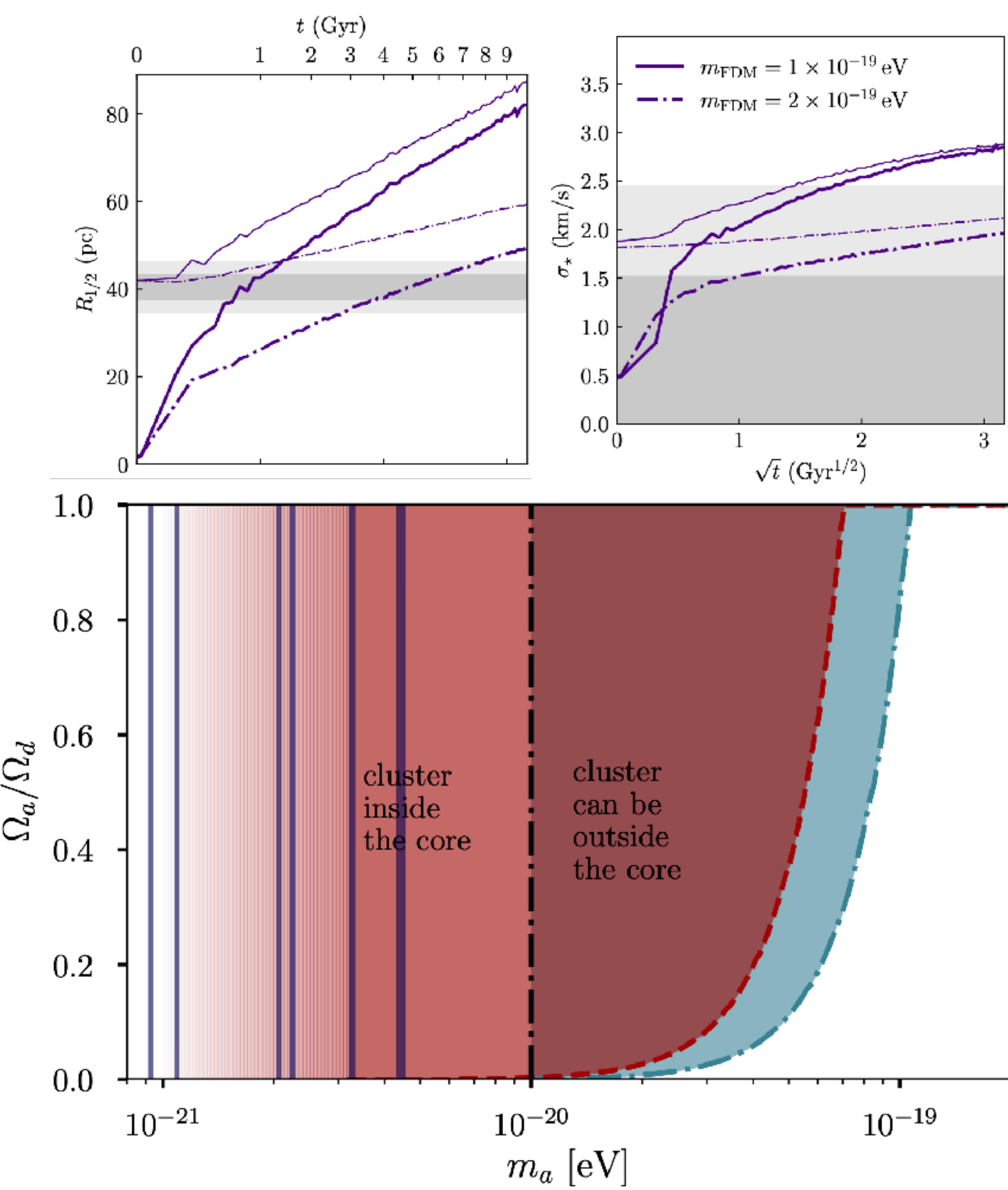
[Marsh and Niemeyer (19)]



[Marsh and Niemeyer (19)]

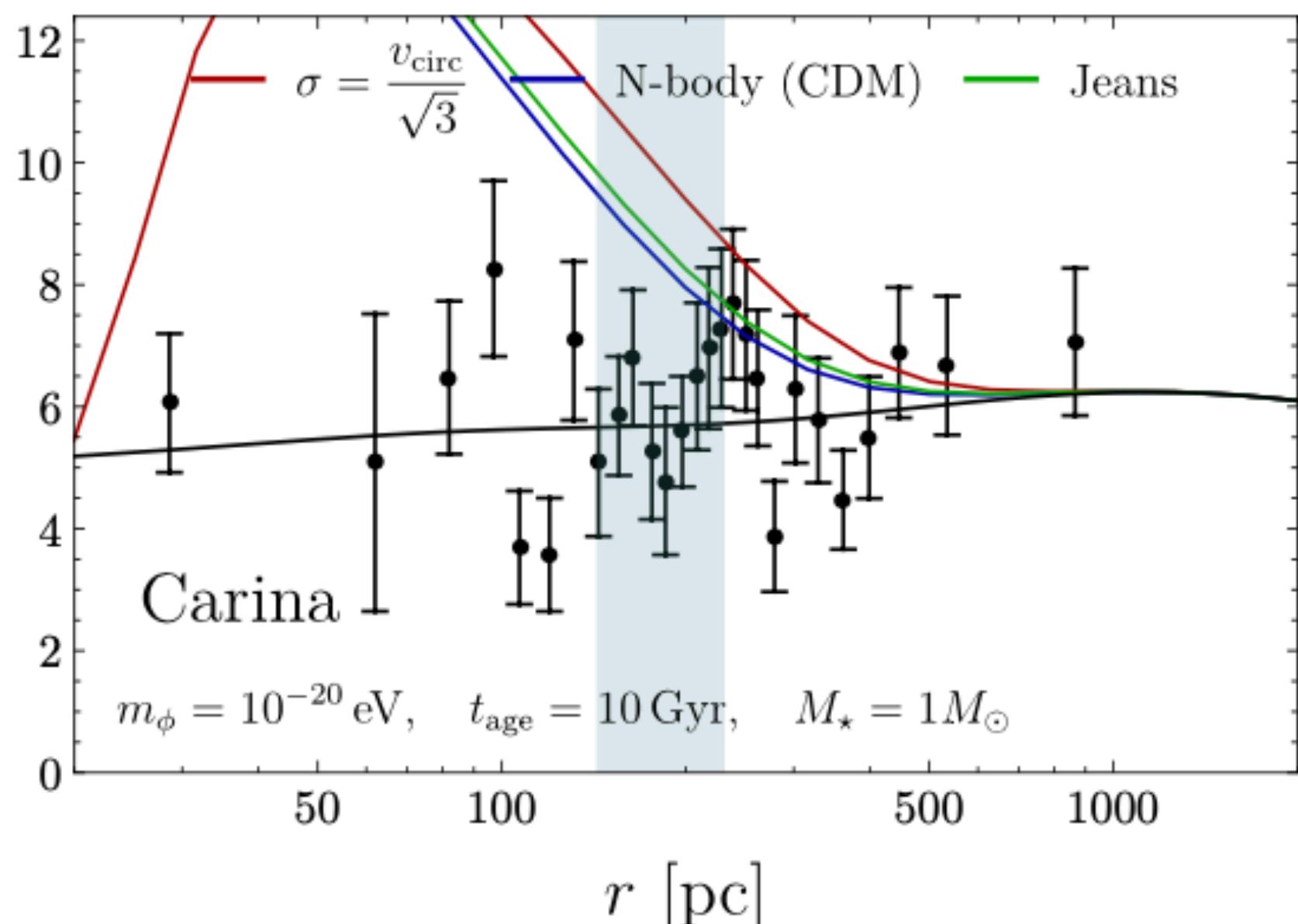
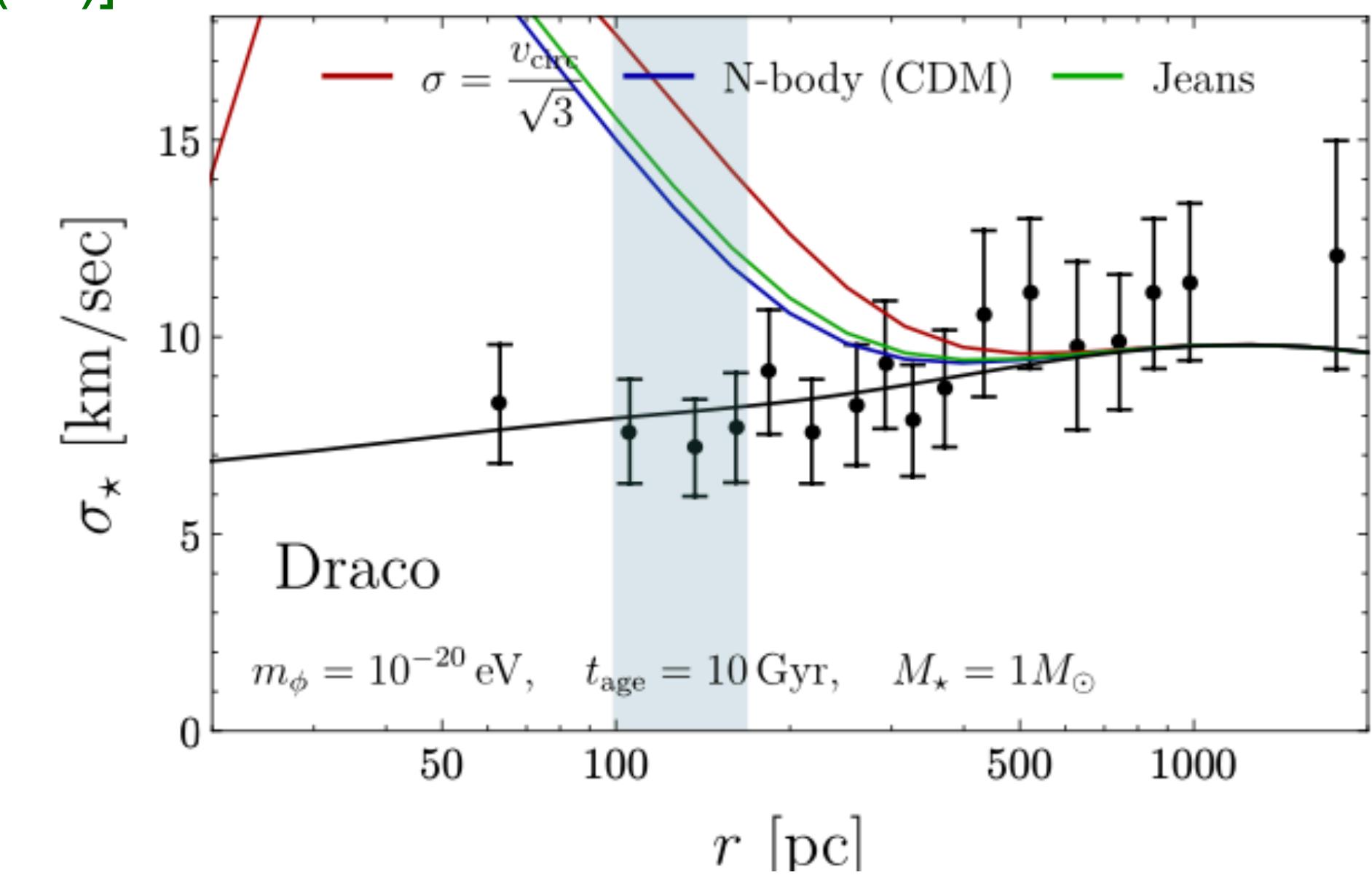


[Blum, Eby, HK (unpublished)]



[Marsh and Niemeyer (19)]

[Dalal, Kravtsov (22)]



[Blum, Eby, HK (unpublished)]

*a deeper understanding of wave DM / QP
can be achieved by studying statistical properties of wave DM*

let us begin with the dark matter field itself

$$\hat{\phi} = \sum_i \frac{1}{\sqrt{2\omega_i}} [a_i \psi_i + a_i^\dagger \psi_i^*]$$

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Operators

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Quantum #

- **Operators**
- **Wave func.**

*the wave function can be obtained
by solving the wave equation in the non-relativistic limit*

$$\epsilon_i \Psi_i = H \Psi_i = \left[-\frac{\nabla^2}{2m} + \Phi \right] \Psi_i$$

$$\psi_i = e^{-i(m+\epsilon_i)t} \Psi_i$$

let us begin with the dark matter field itself

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Quantum #

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*what to do with these operators?
to describe finite density system
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$$\hat{\rho}_i = \int d^2\alpha_i P(\alpha_i) |\alpha_i\rangle\langle\alpha_i|$$

[HK and Lenoci 21]

let us begin with the dark matter field itself

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quasi-probability distribution

$$P(\alpha_i) = \frac{1}{\pi f_i} \exp \left[-\frac{|\alpha_i|^2}{f_i} \right]$$

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$$a_i |\alpha_i\rangle = \alpha_i |\alpha_i\rangle$$

$$\alpha_i \in \mathbb{C}$$

$$\langle \hat{O} \rangle = \text{Tr}(\hat{\rho} \hat{O})$$

let us begin with the dark matter field itself

$$\hat{\phi} = \sum_i \frac{1}{\sqrt{2\omega_i}} [a_i \psi_i + a_i^\dagger \psi_i^*]$$

Quantum #

- **Operators**
- **Wave func.**

*more simplistically or conveniently
we can forget about this operators by replacing them with
complex numbers*

$$(a_i, a_i^\dagger) \rightarrow (\alpha_i, \alpha_i^*)$$

quasi-probability distribution

$$P(\alpha_i) = \frac{1}{\pi f_i} \exp \left[-\frac{|\alpha_i|^2}{f_i} \right]$$

[Derevianko 18]
[Foster, Rodd, Safdi 18]
[Center et al 20]; [... ...]

some applications of this approach of ultralight dark matter

- (i) understanding the behavior of **homogeneous wave DM medium***
- (ii) understanding the behavior of the wave DM **in the solar system***
- (iii) construction of the whole wave DM halo*
- (iv) understanding the **adiabatic evolution** of wave DM halo*

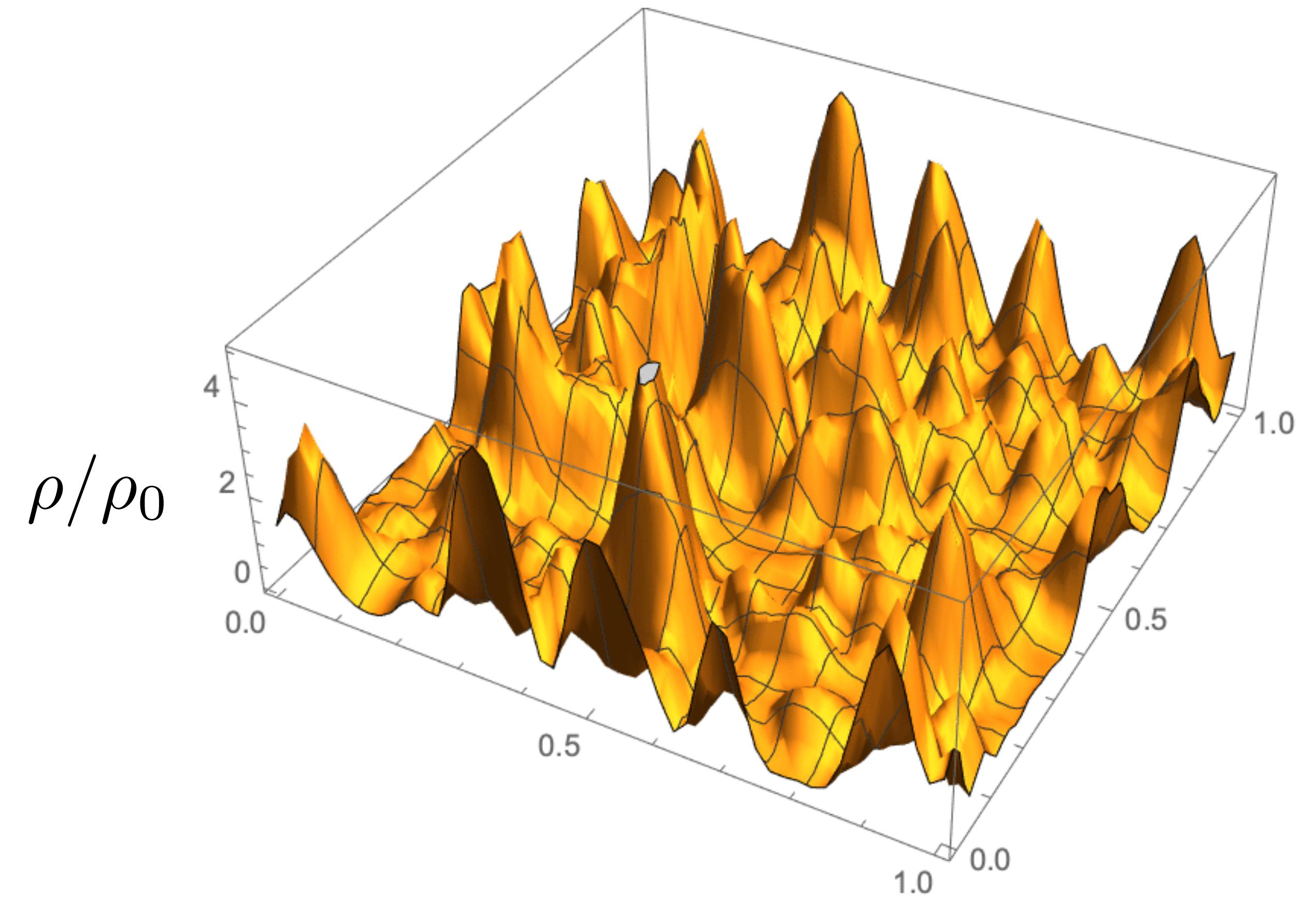
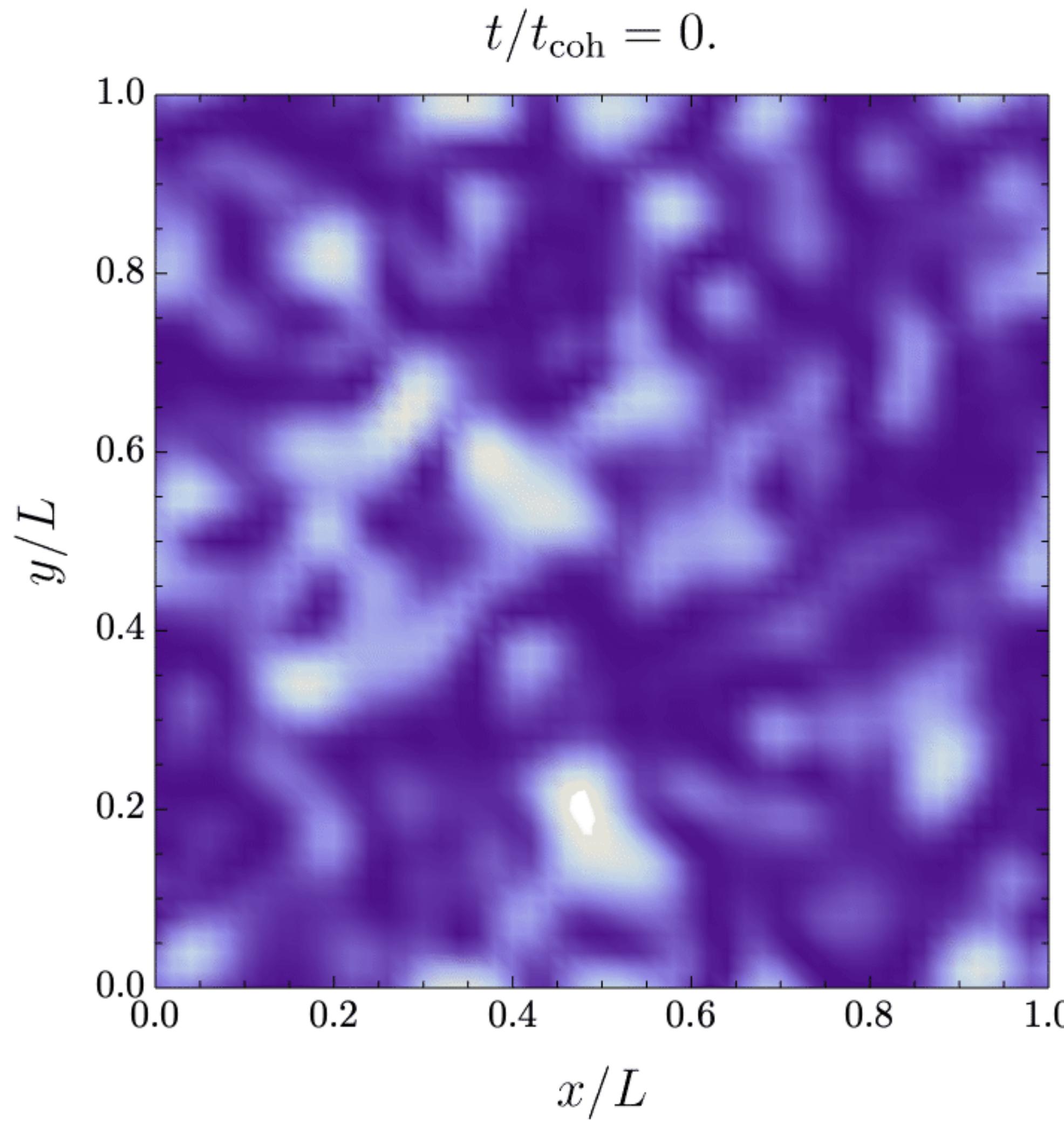
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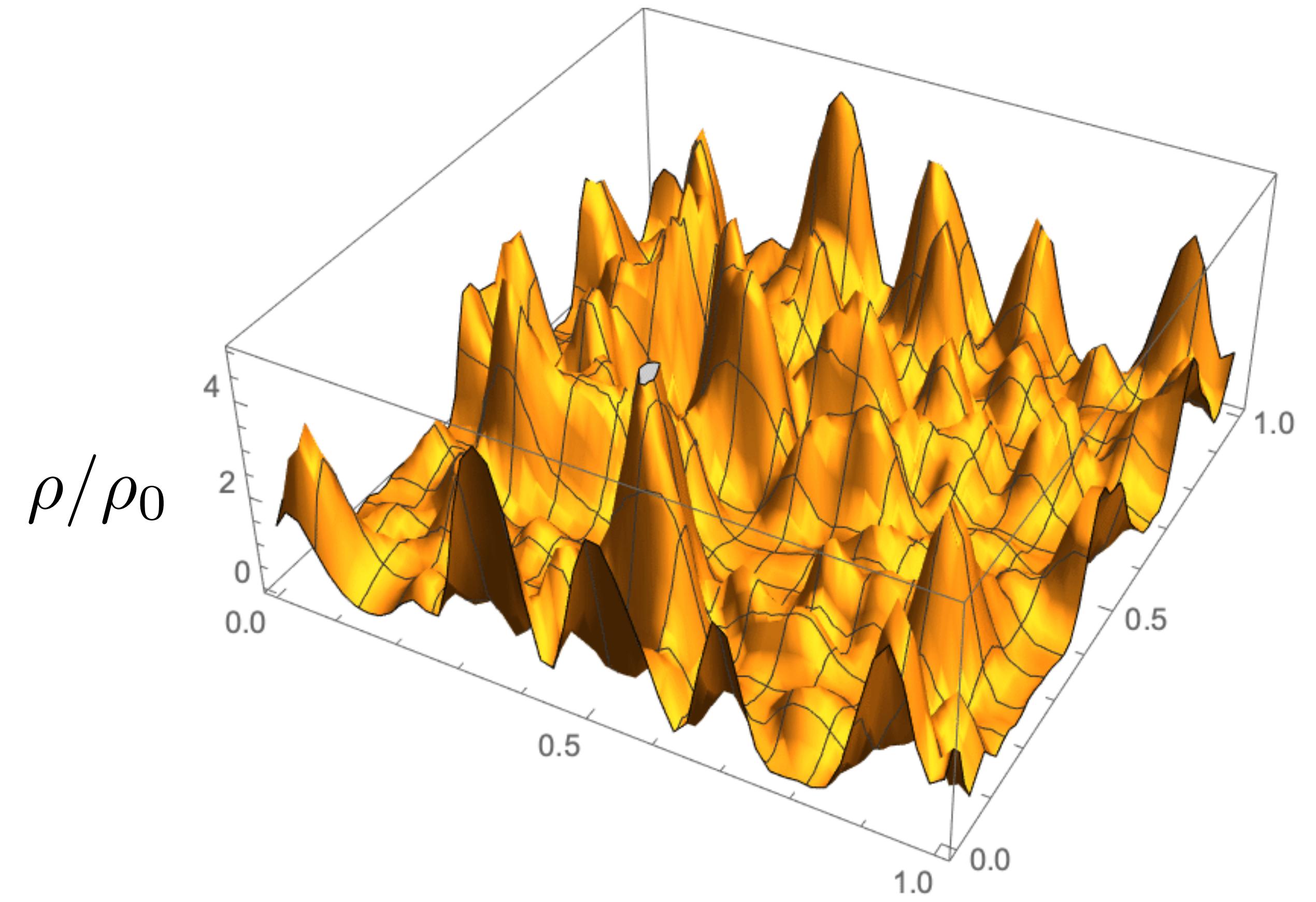
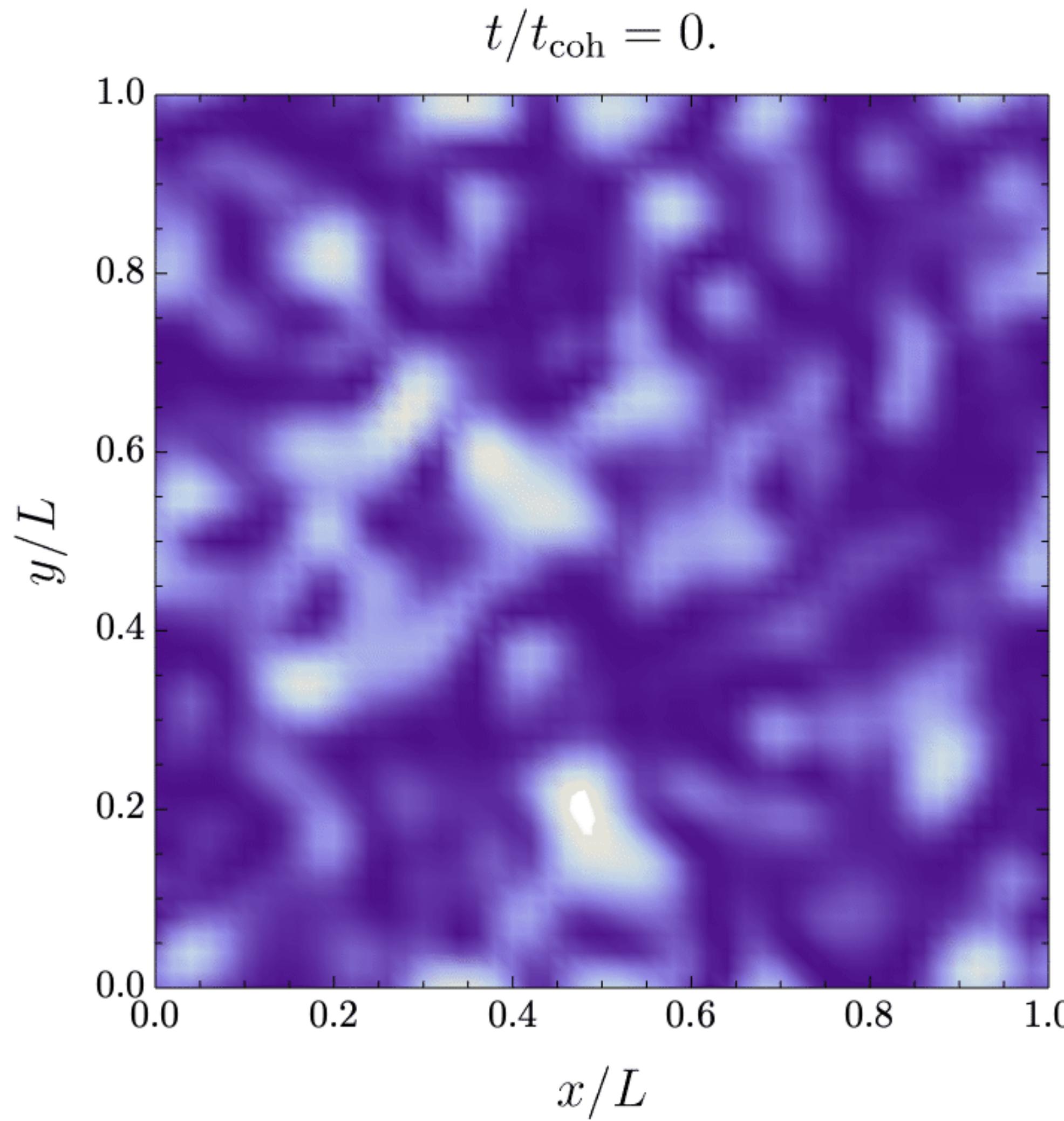
*as a simple example
we may consider a homogeneous system where*

$$\psi_i = \frac{1}{\sqrt{V}} e^{-ik_i x}$$

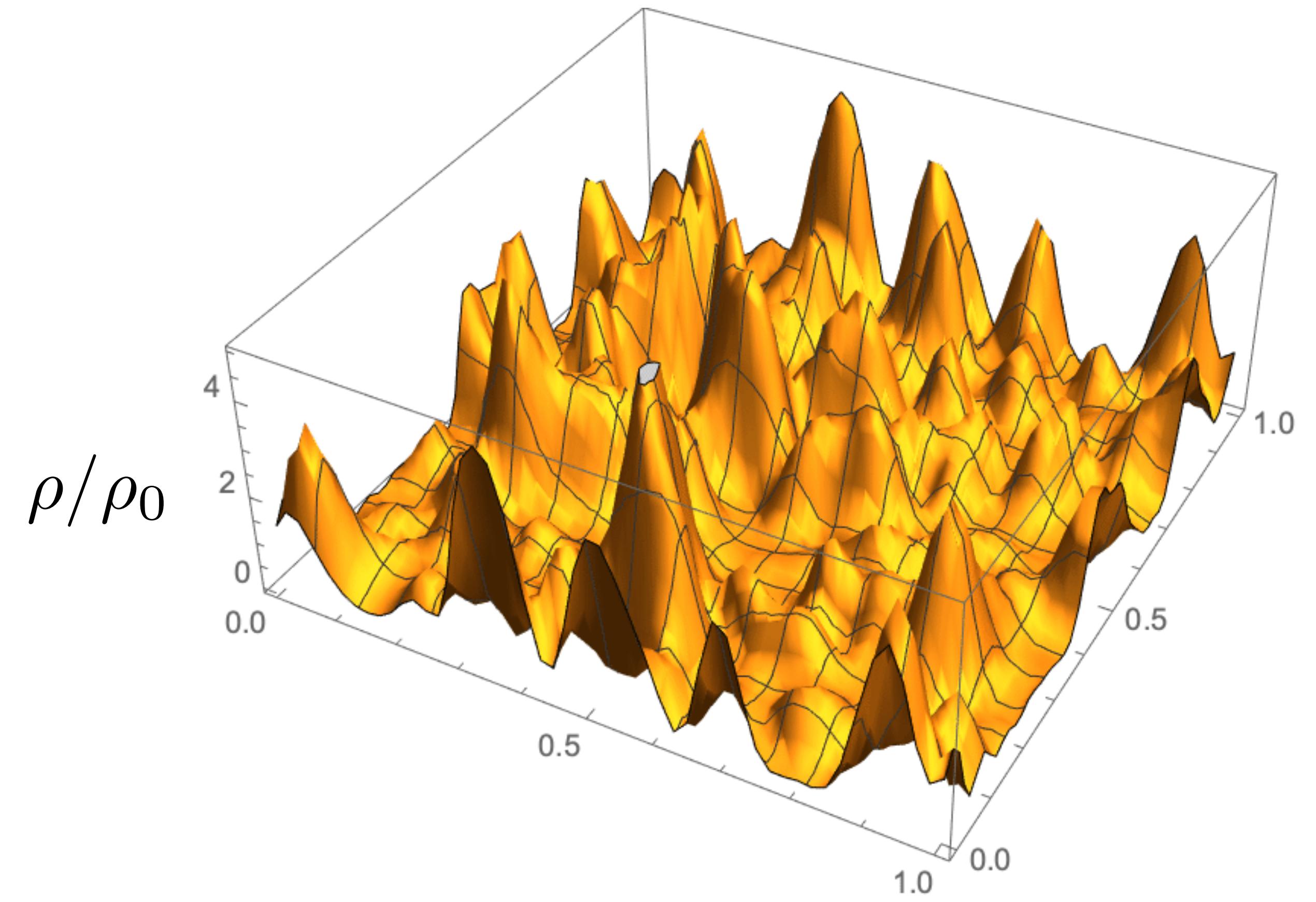
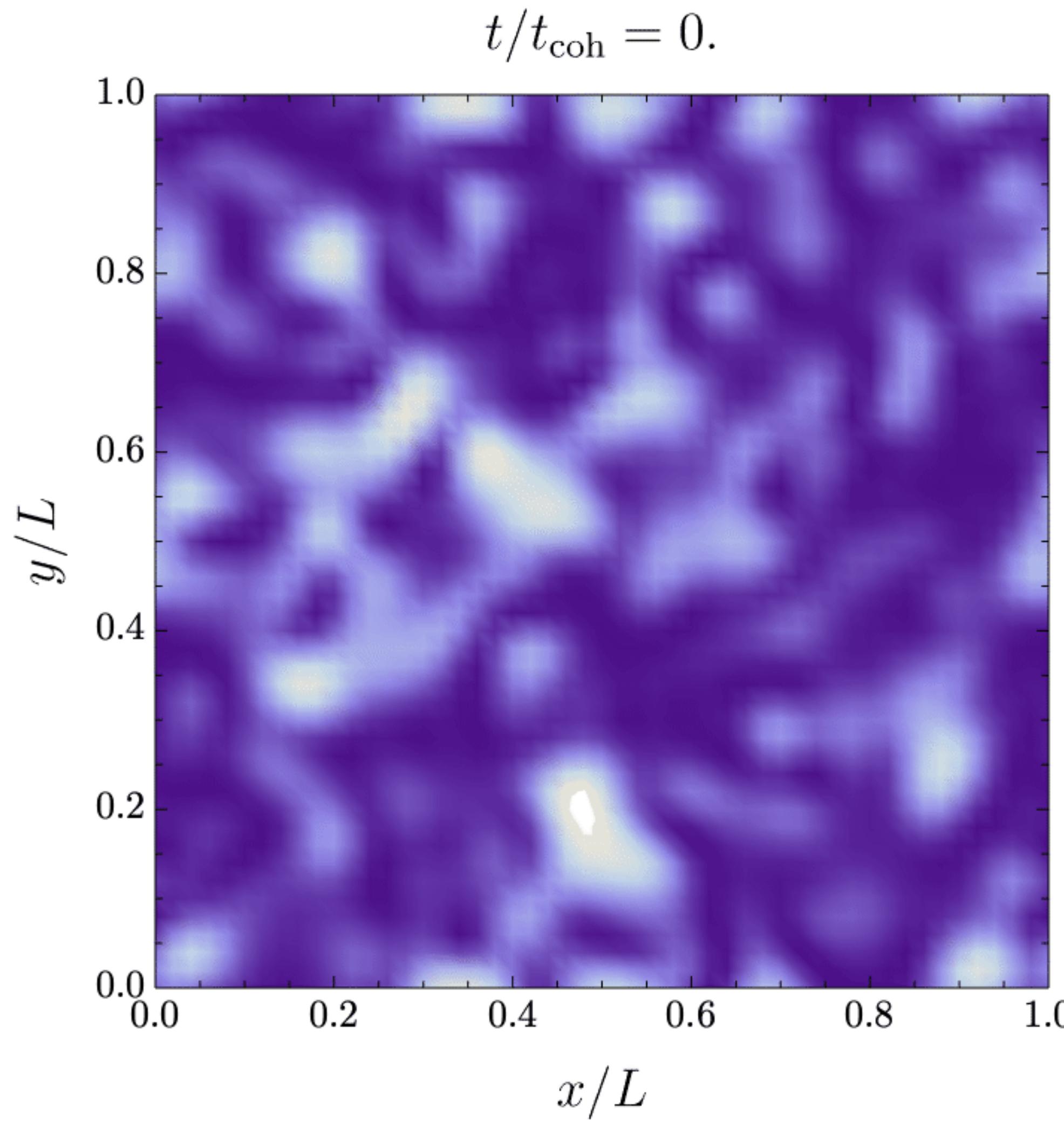
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without performing simulations*



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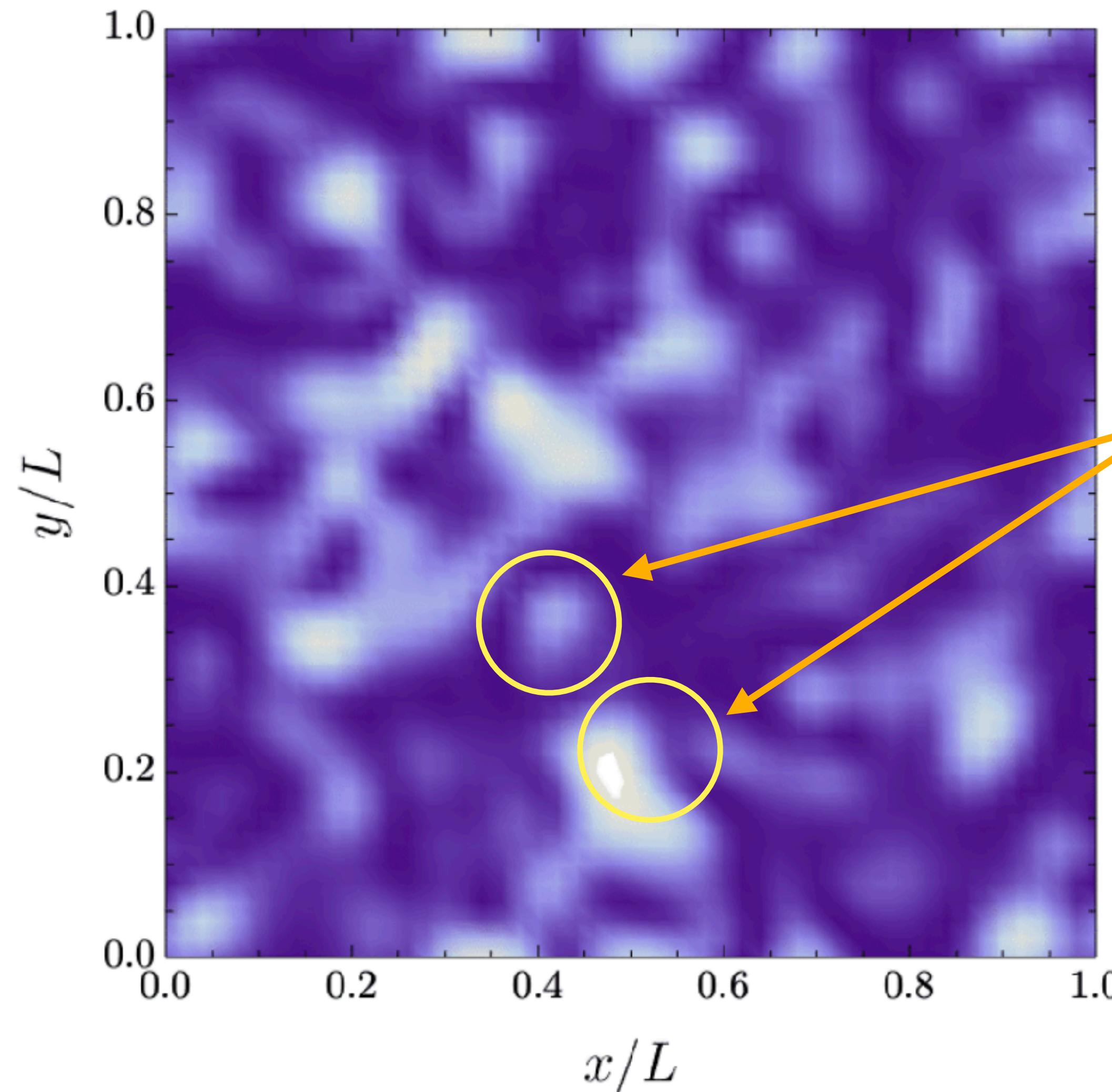


*this description provides a handy tool for analyzing wave DM
without performing simulations*



$$\langle \delta(t, x) \delta(t, y) \rangle \propto \exp[-|\Delta x|^2/\lambda^2]$$

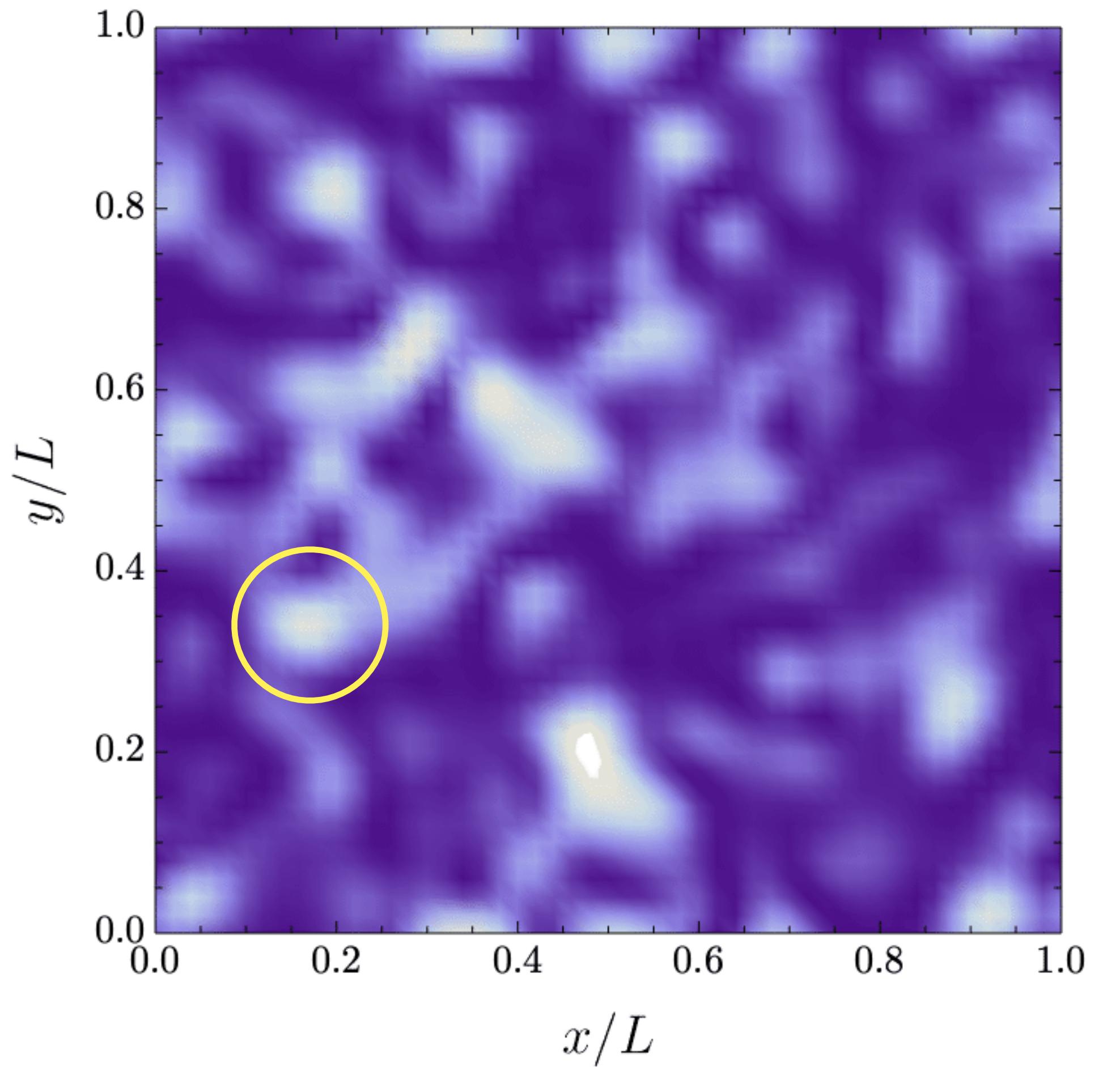
$$t/t_{\text{coh}} = 0.$$



*two patches are
statistically uncorrelated*

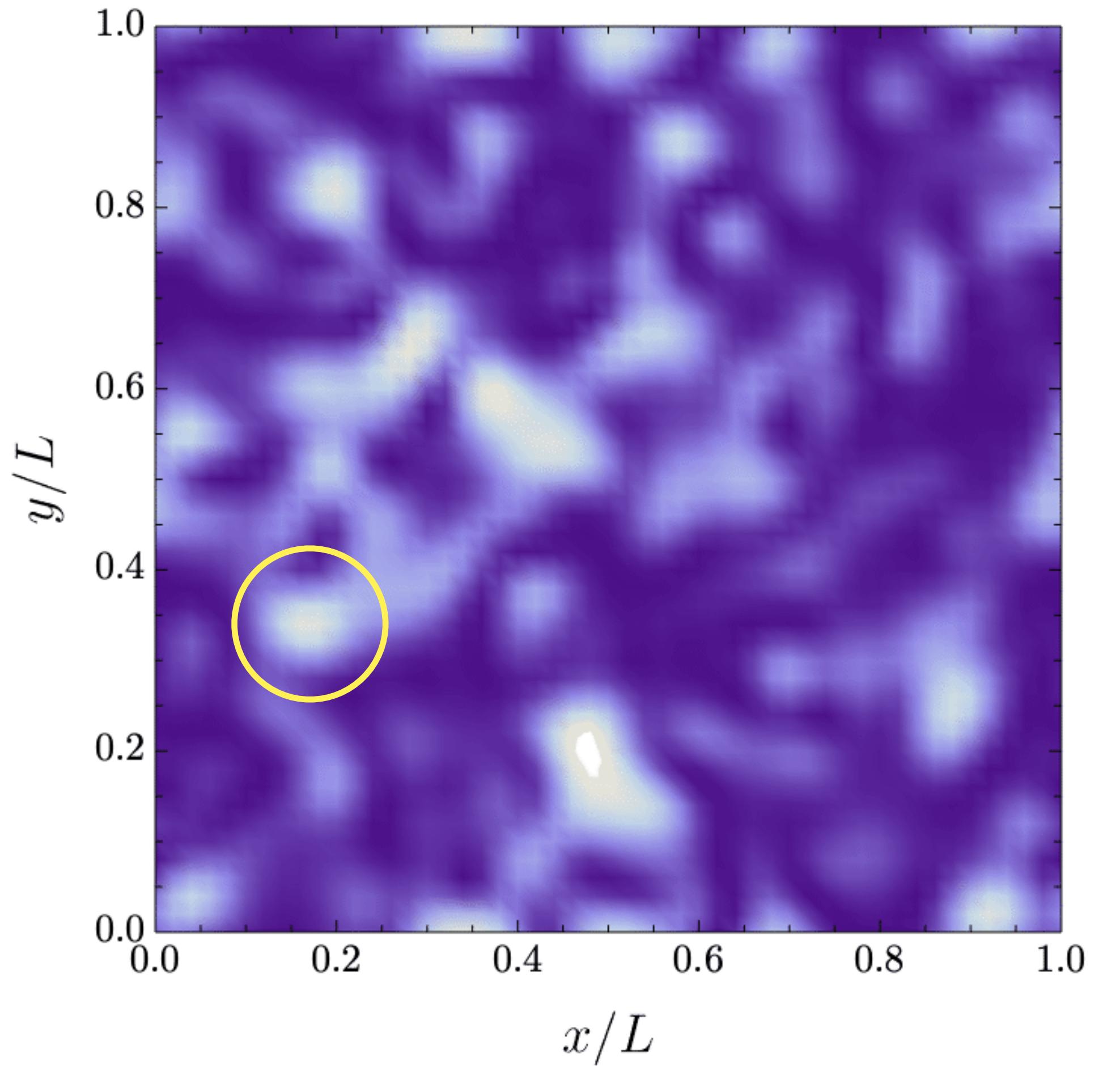
$$\langle \delta(t, x)^2 \rangle$$

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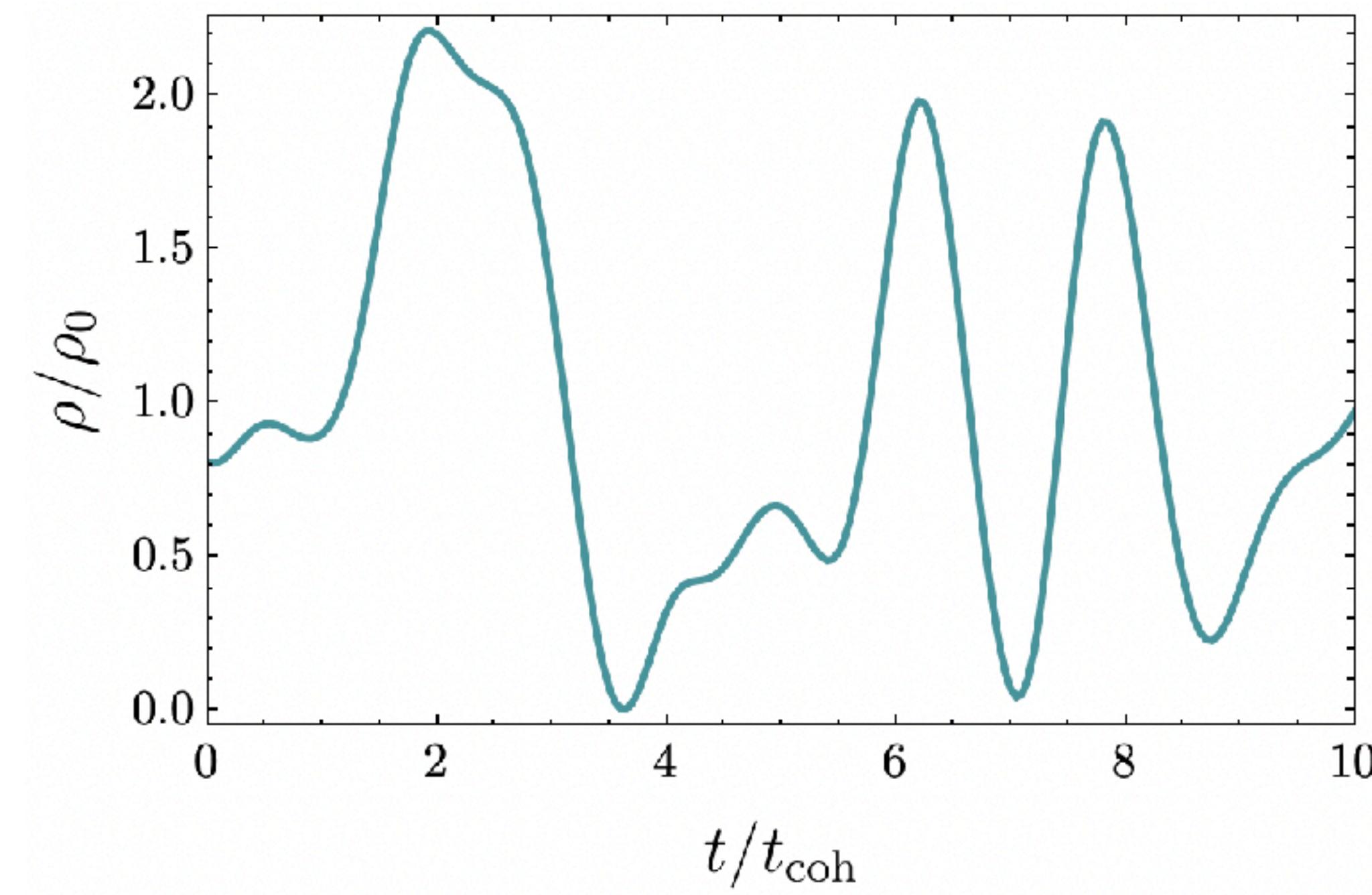
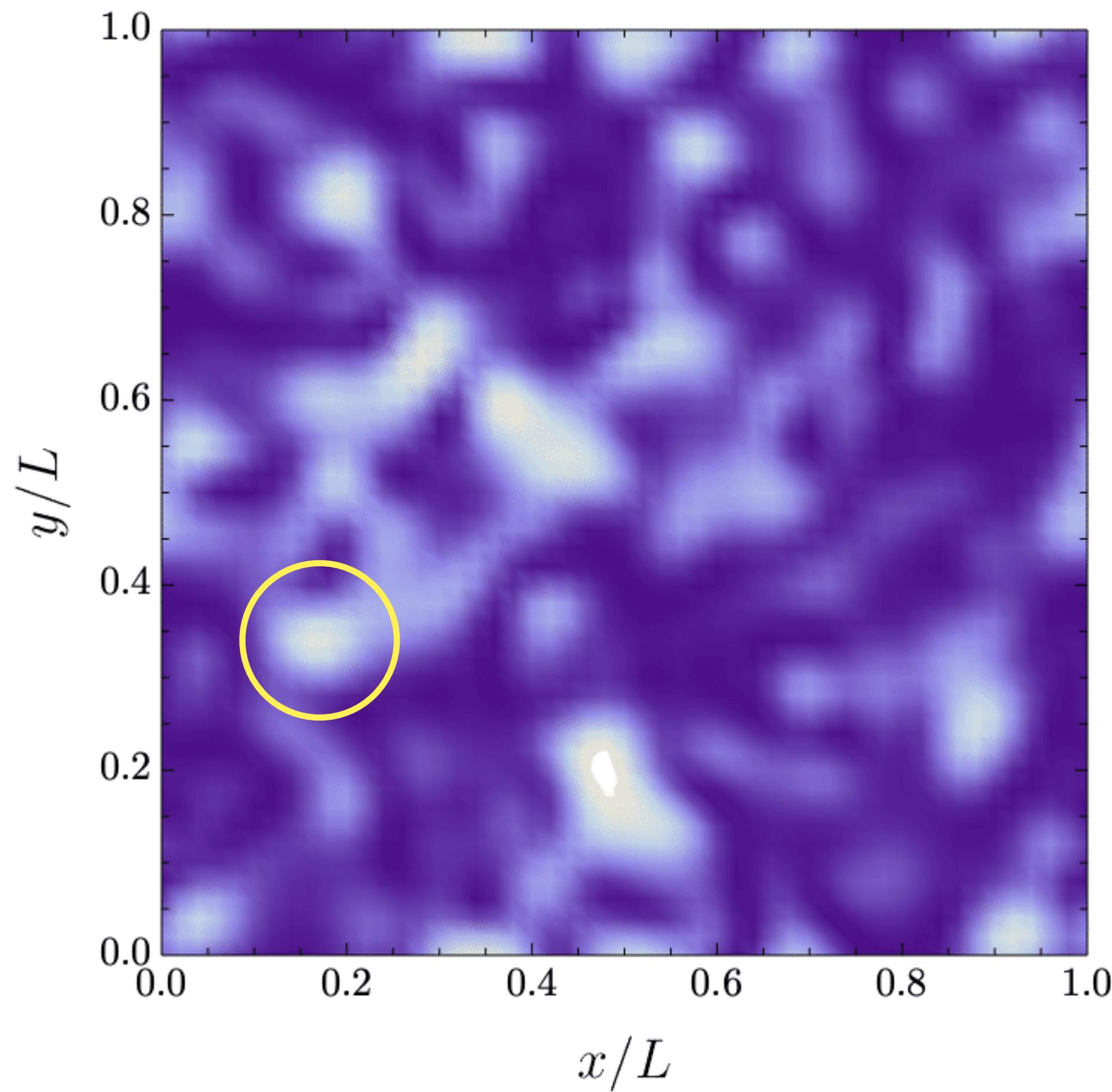
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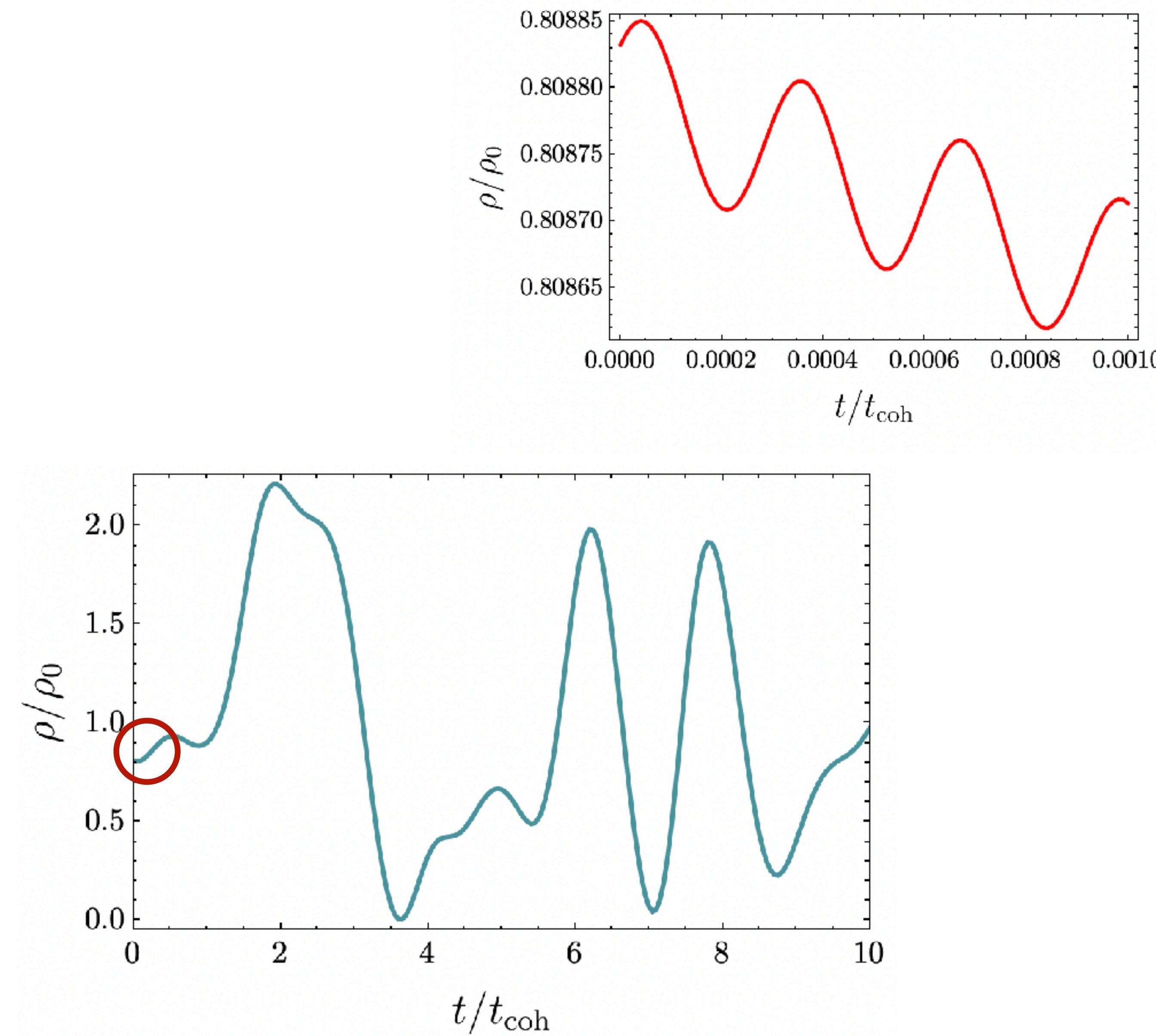
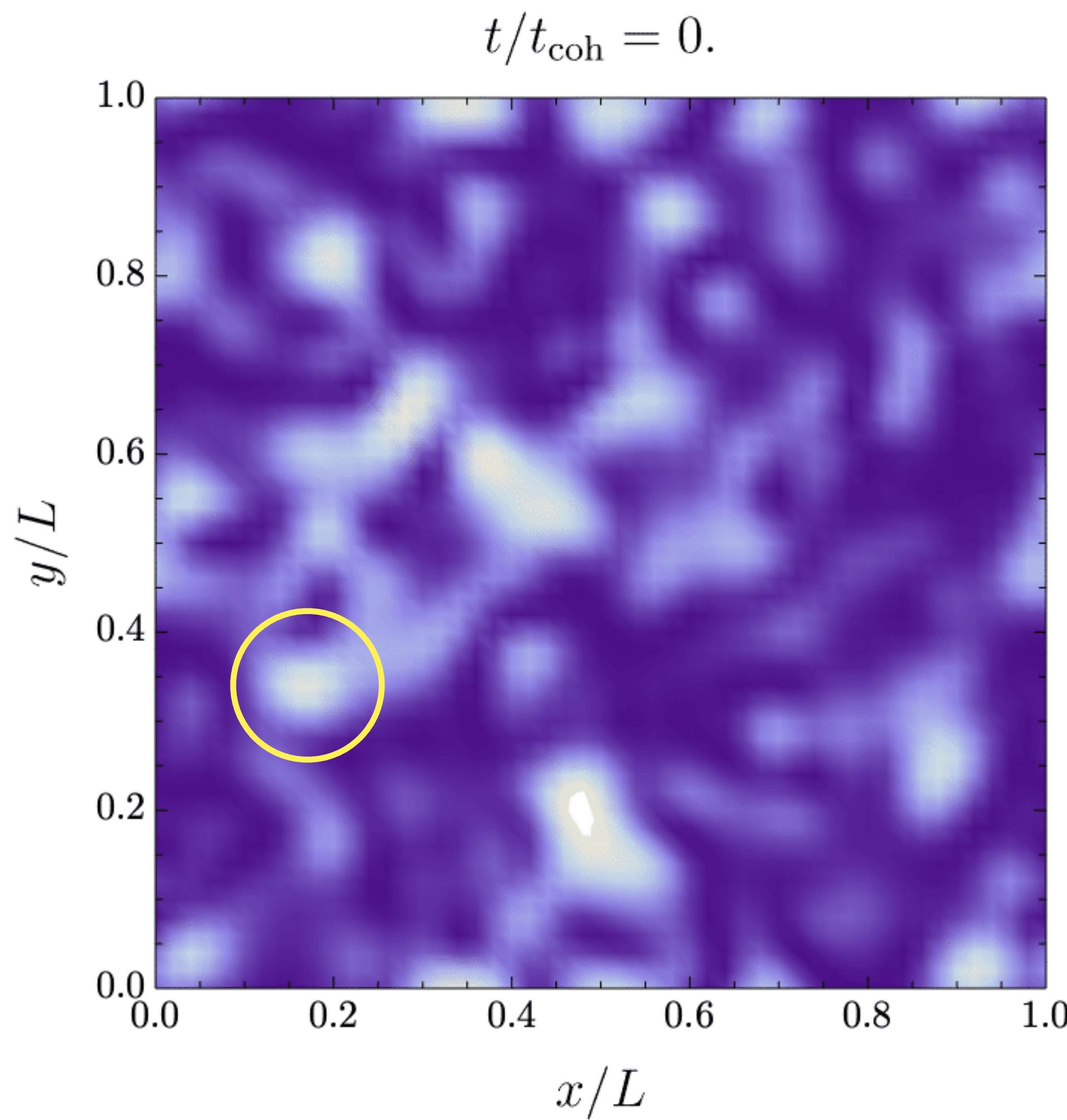


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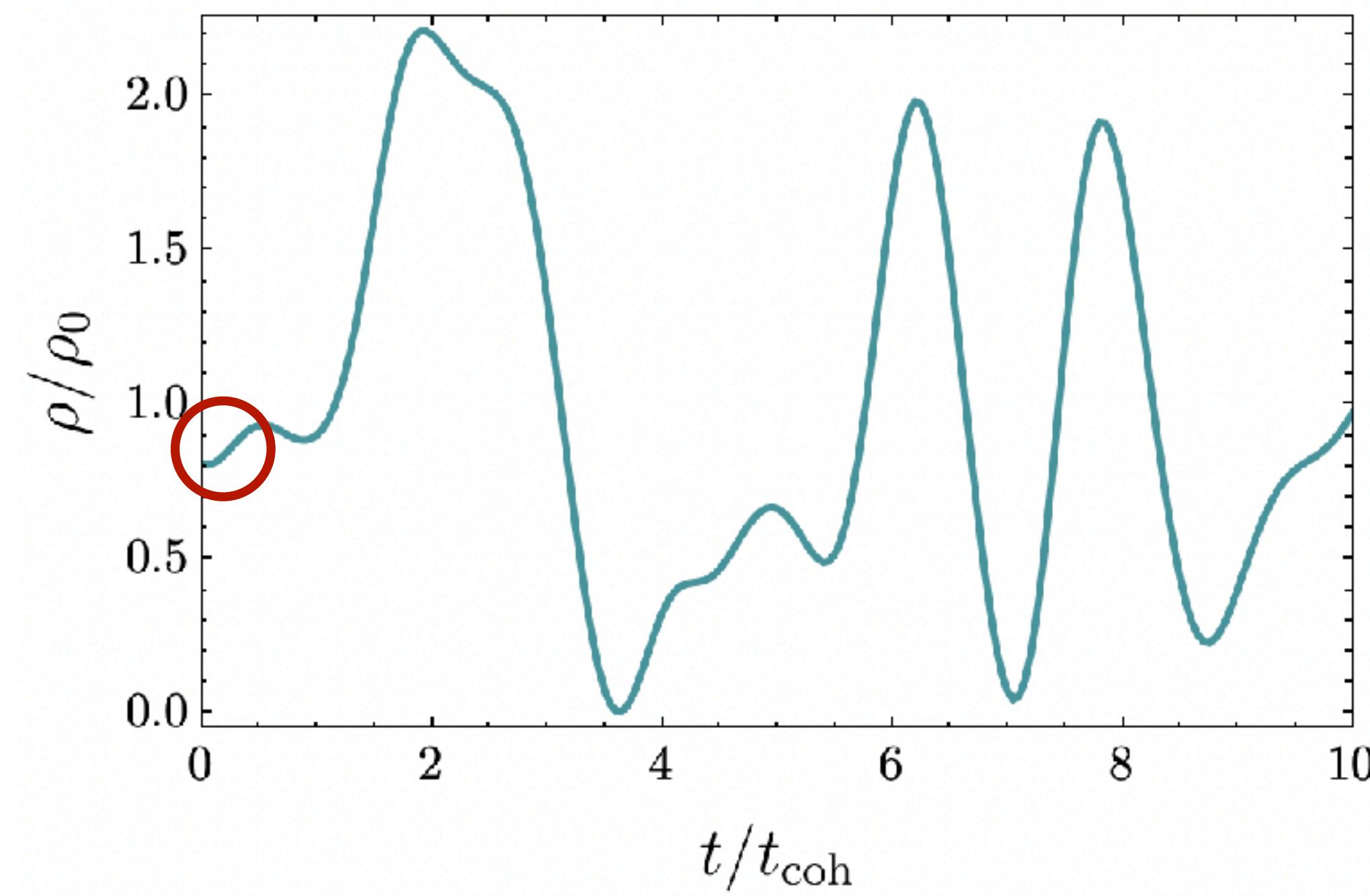
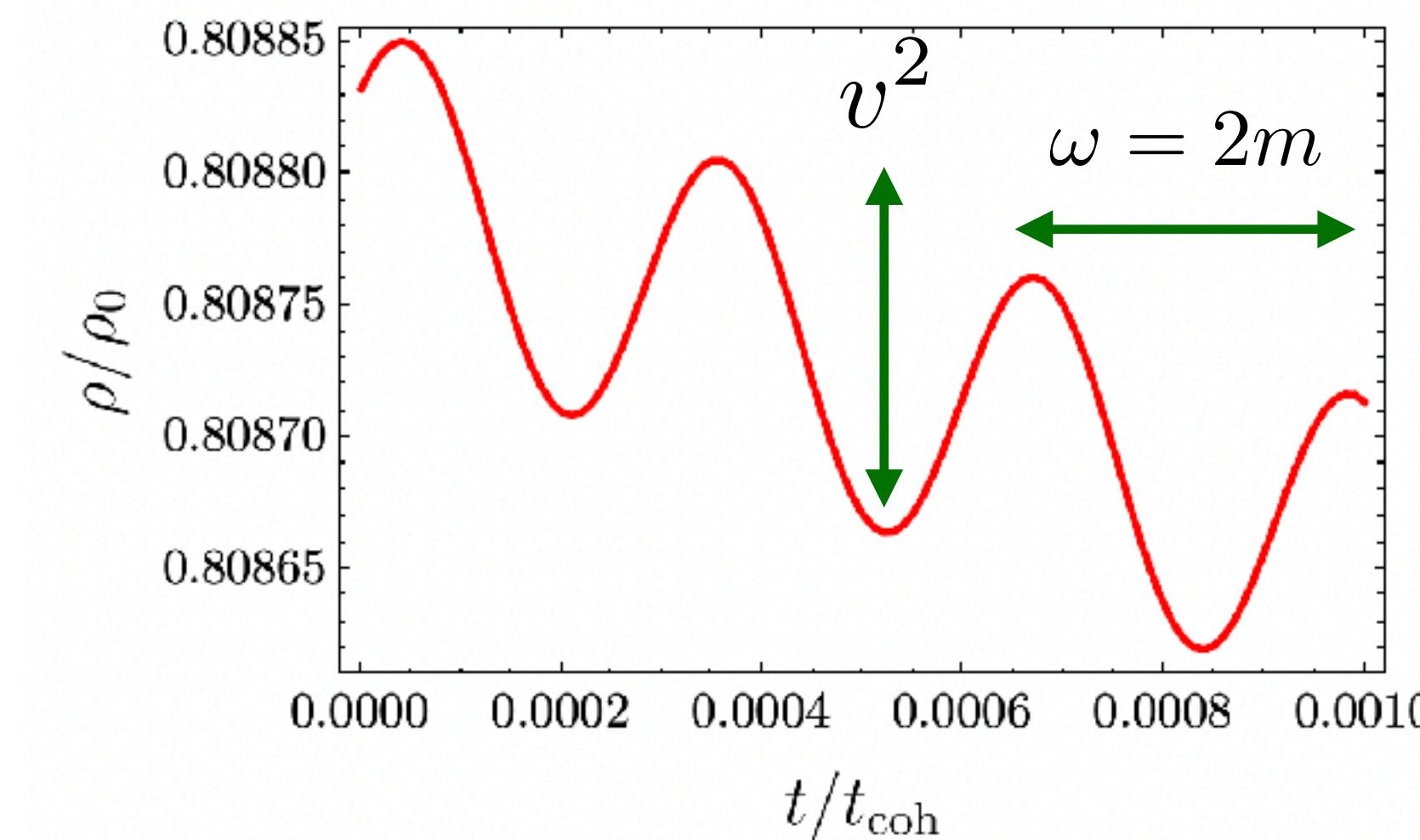
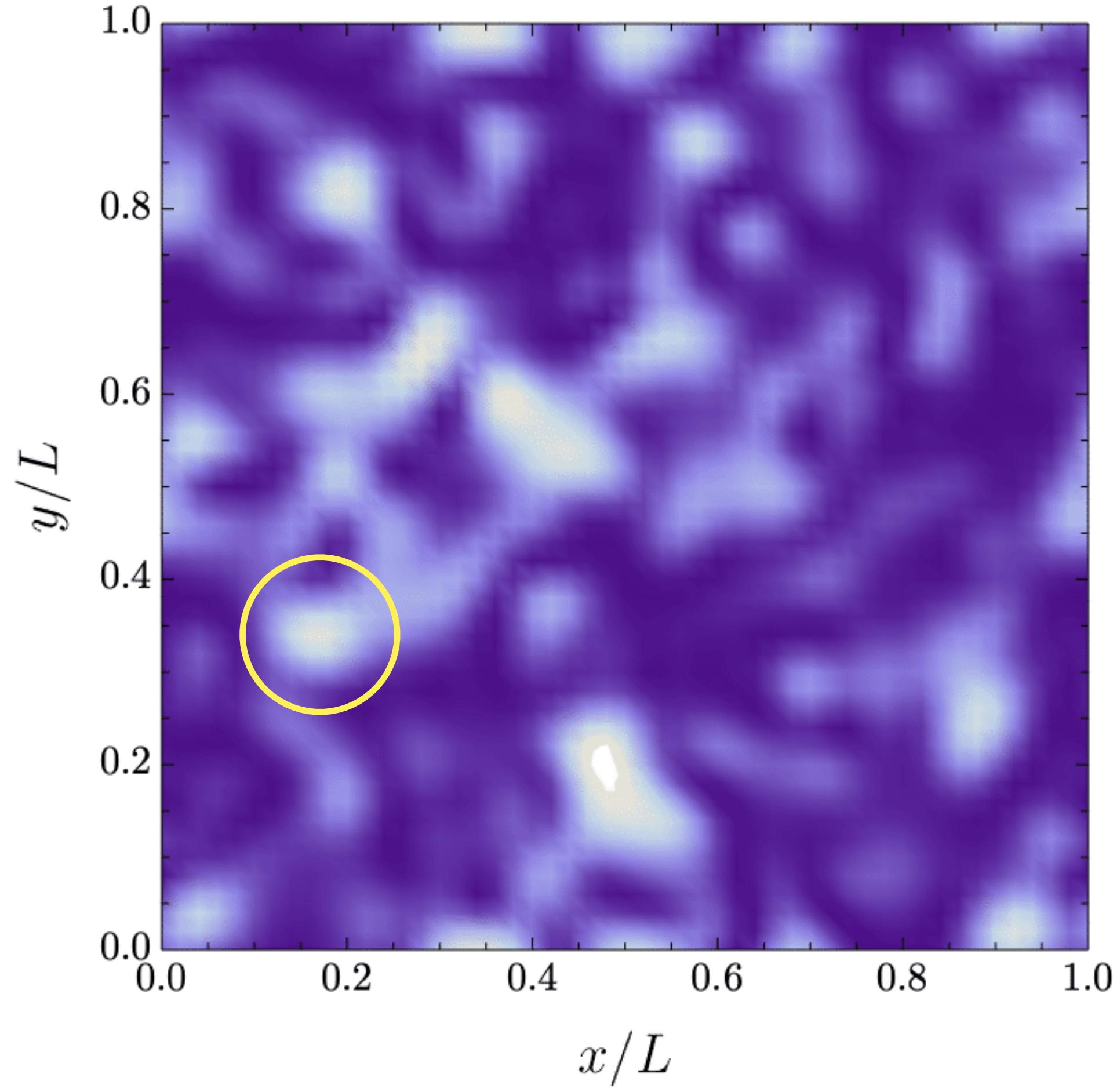


$$\langle \delta(t, x)^2 \rangle$$



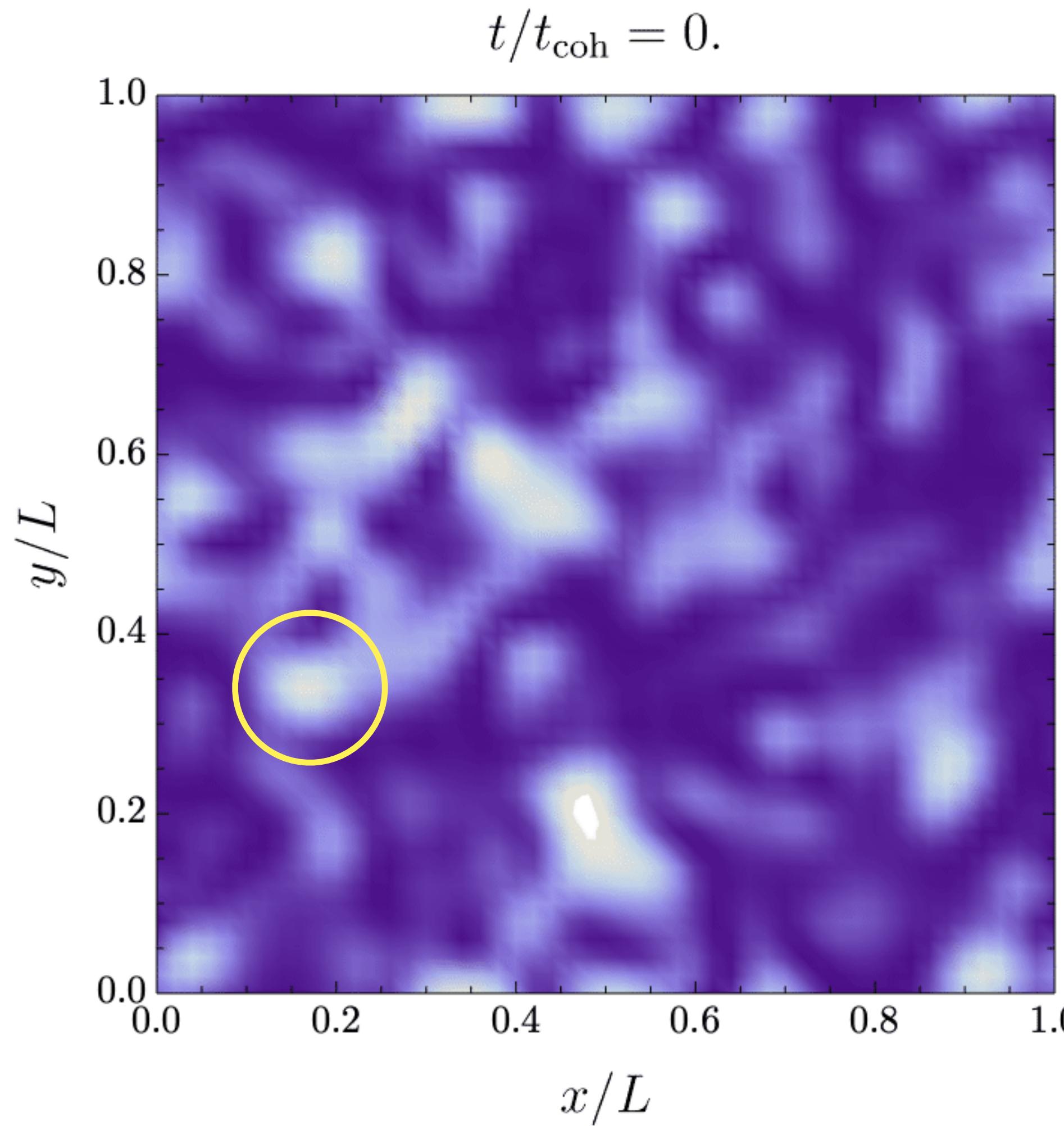
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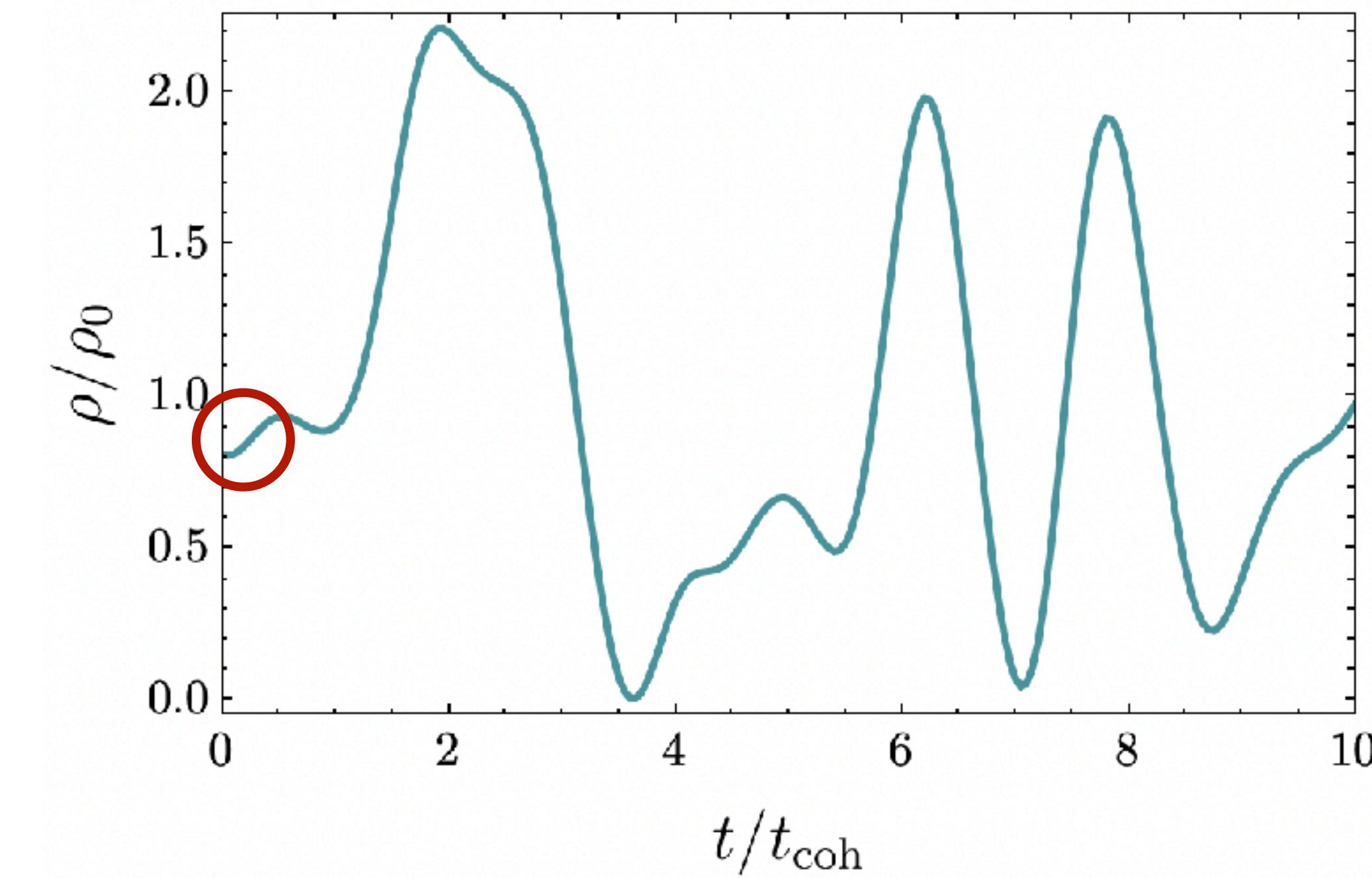
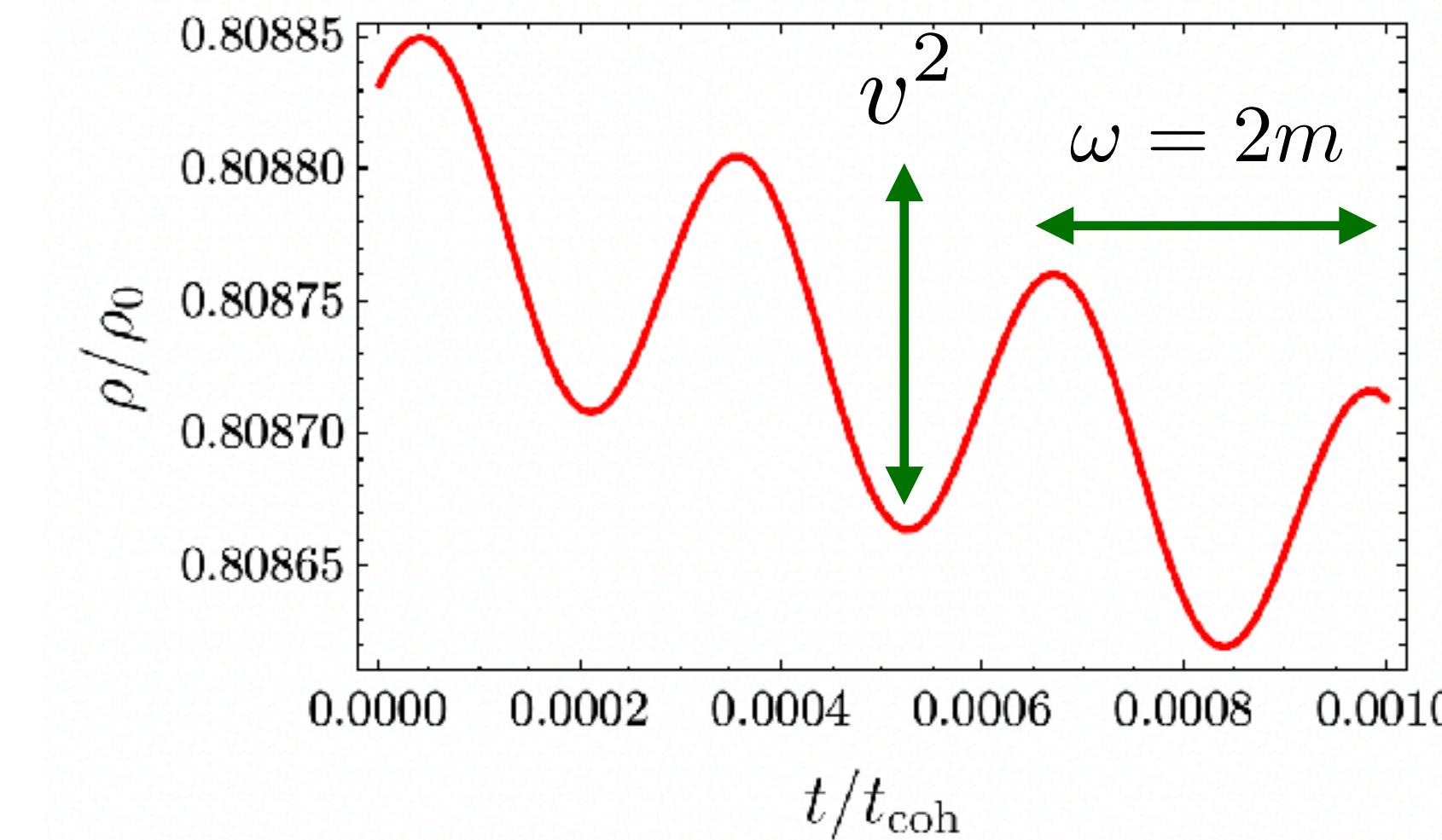


for a single mode

$$\langle \delta(t, x)^2 \rangle$$



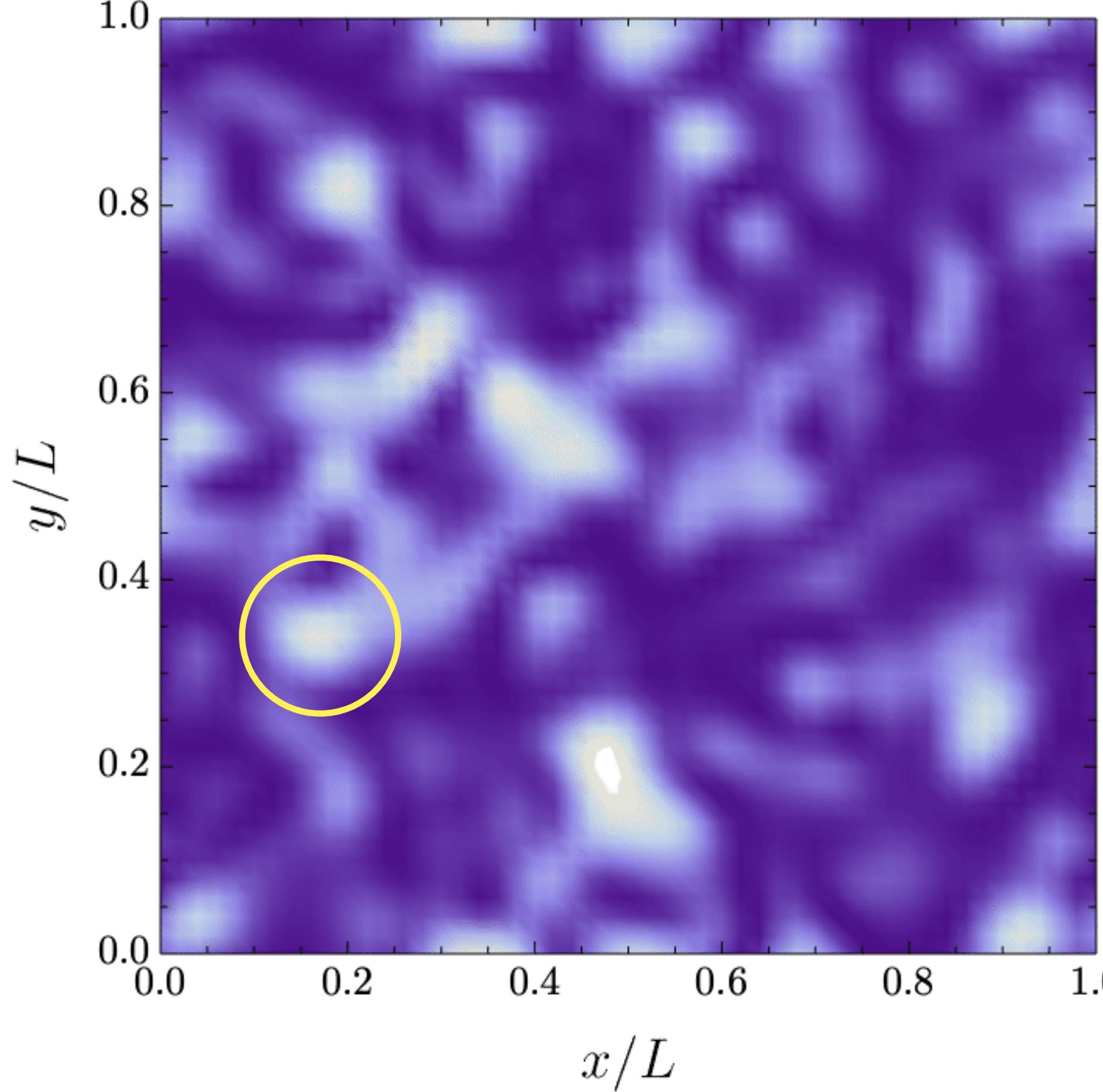
$$\phi = \phi_0 \cos(\omega t - kx)$$



for a single mode

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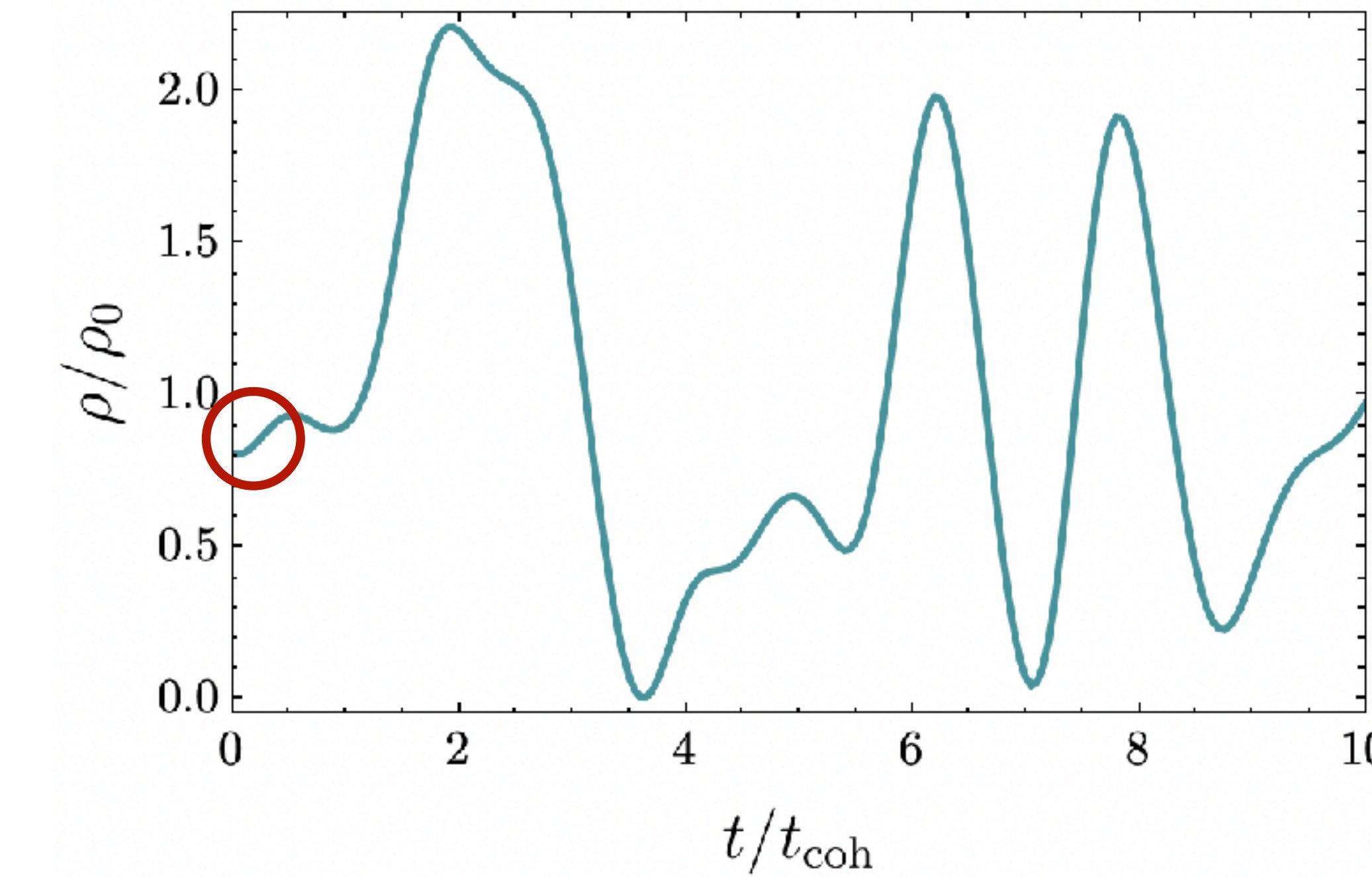
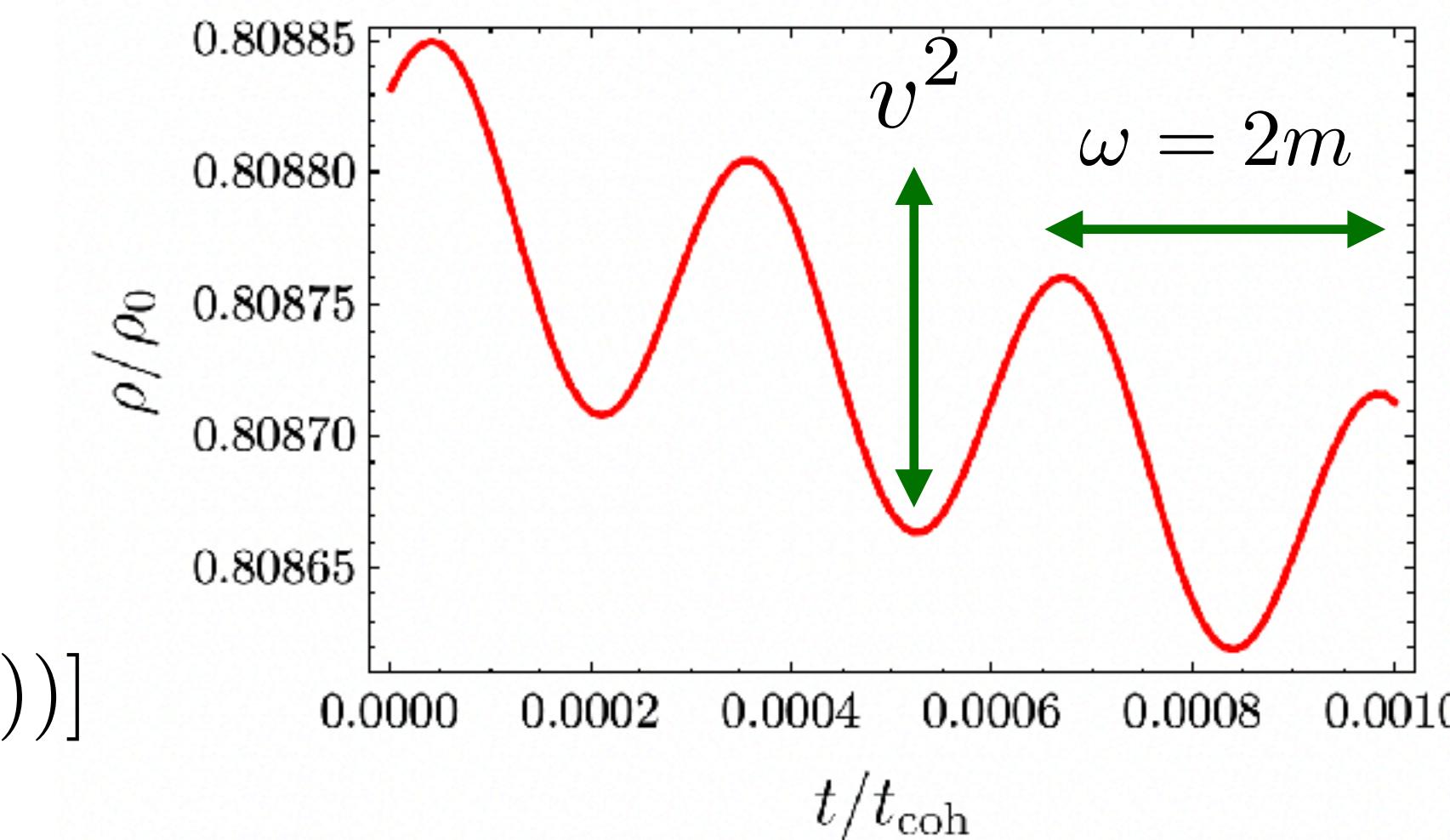
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the energy density is

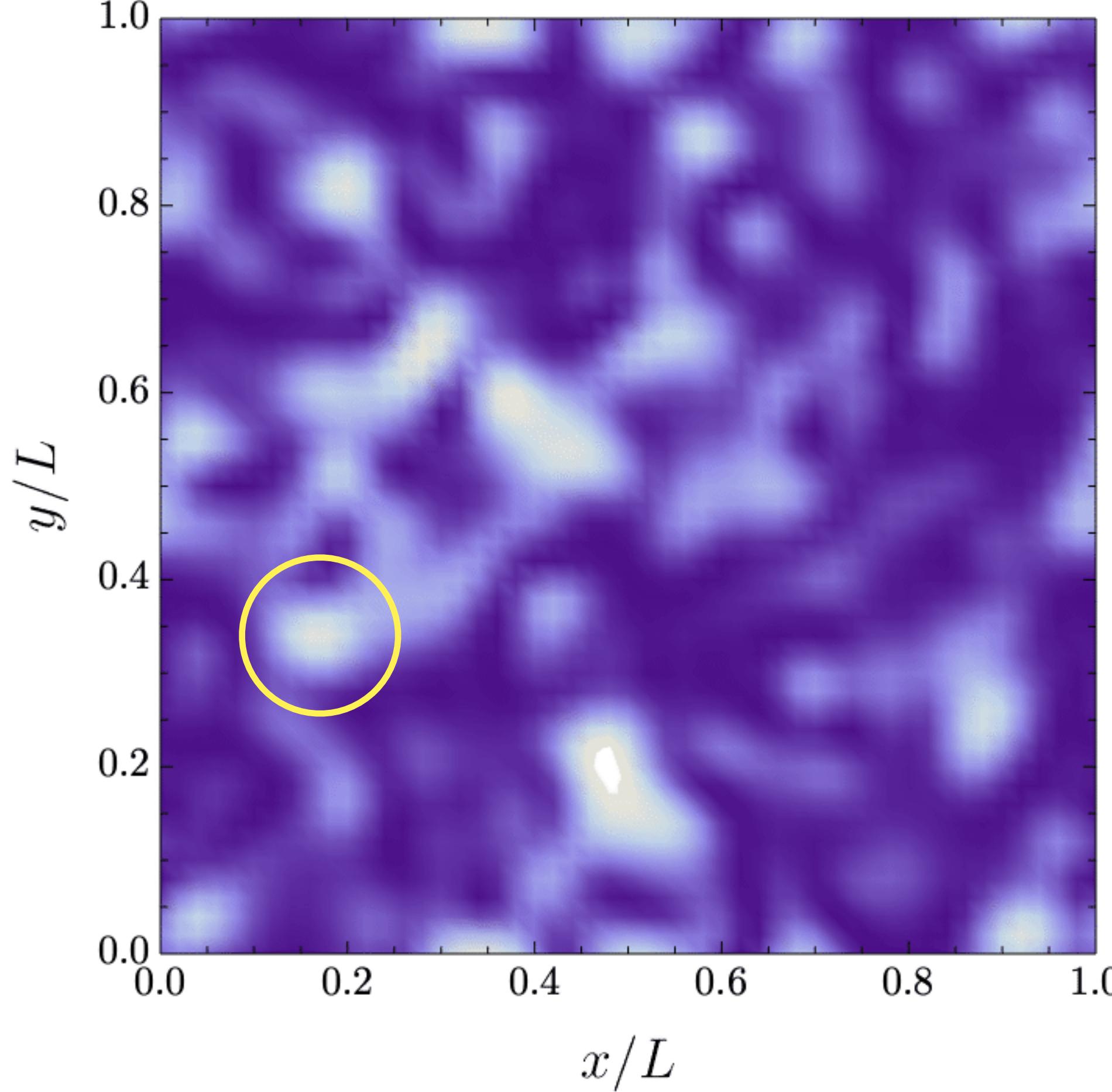
$$\begin{aligned}\rho &= \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 \\ &\simeq \rho_0 [1 - v^2 \cos(2(m t - k x))]\end{aligned}$$



for a single mode

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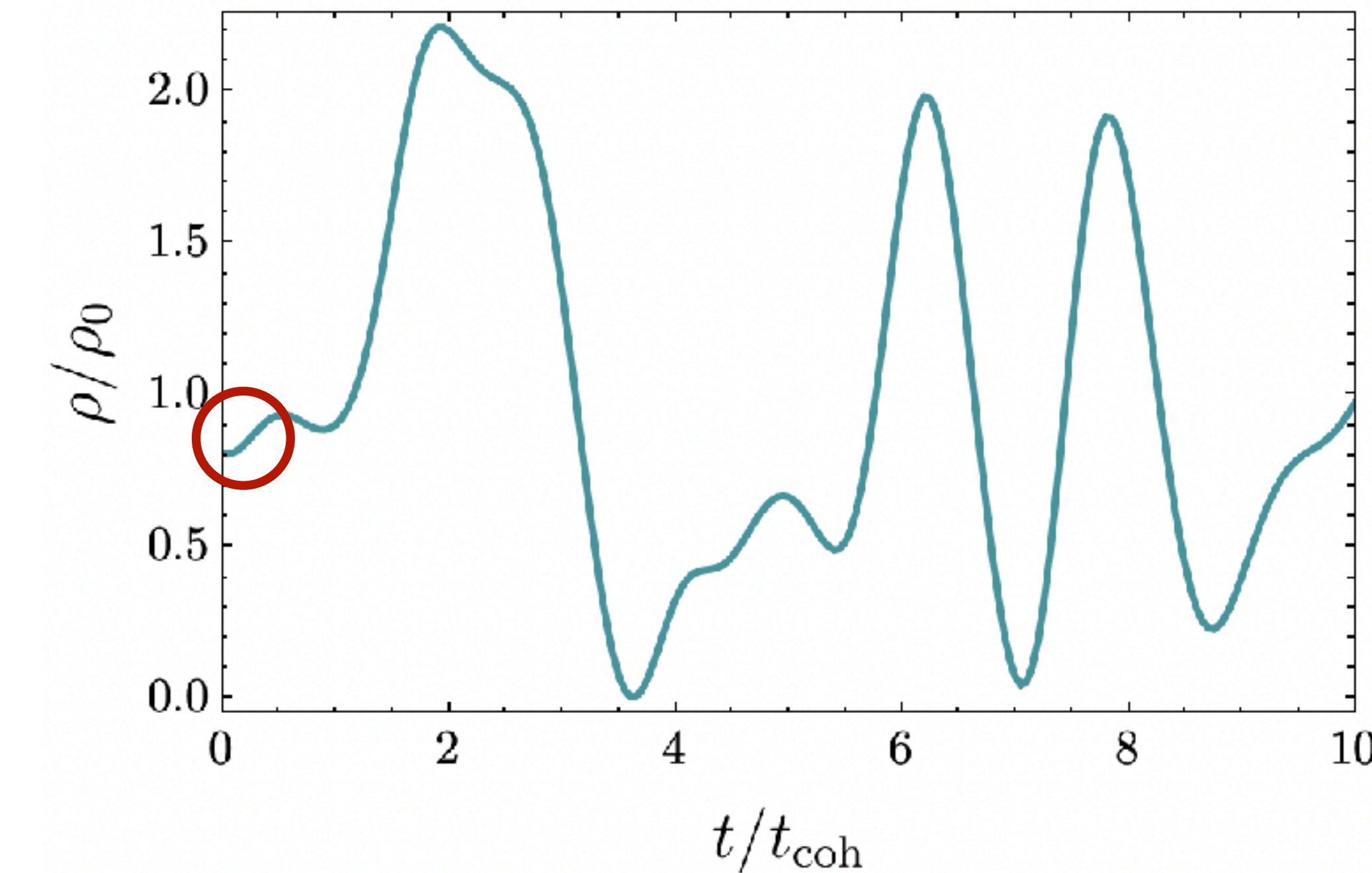
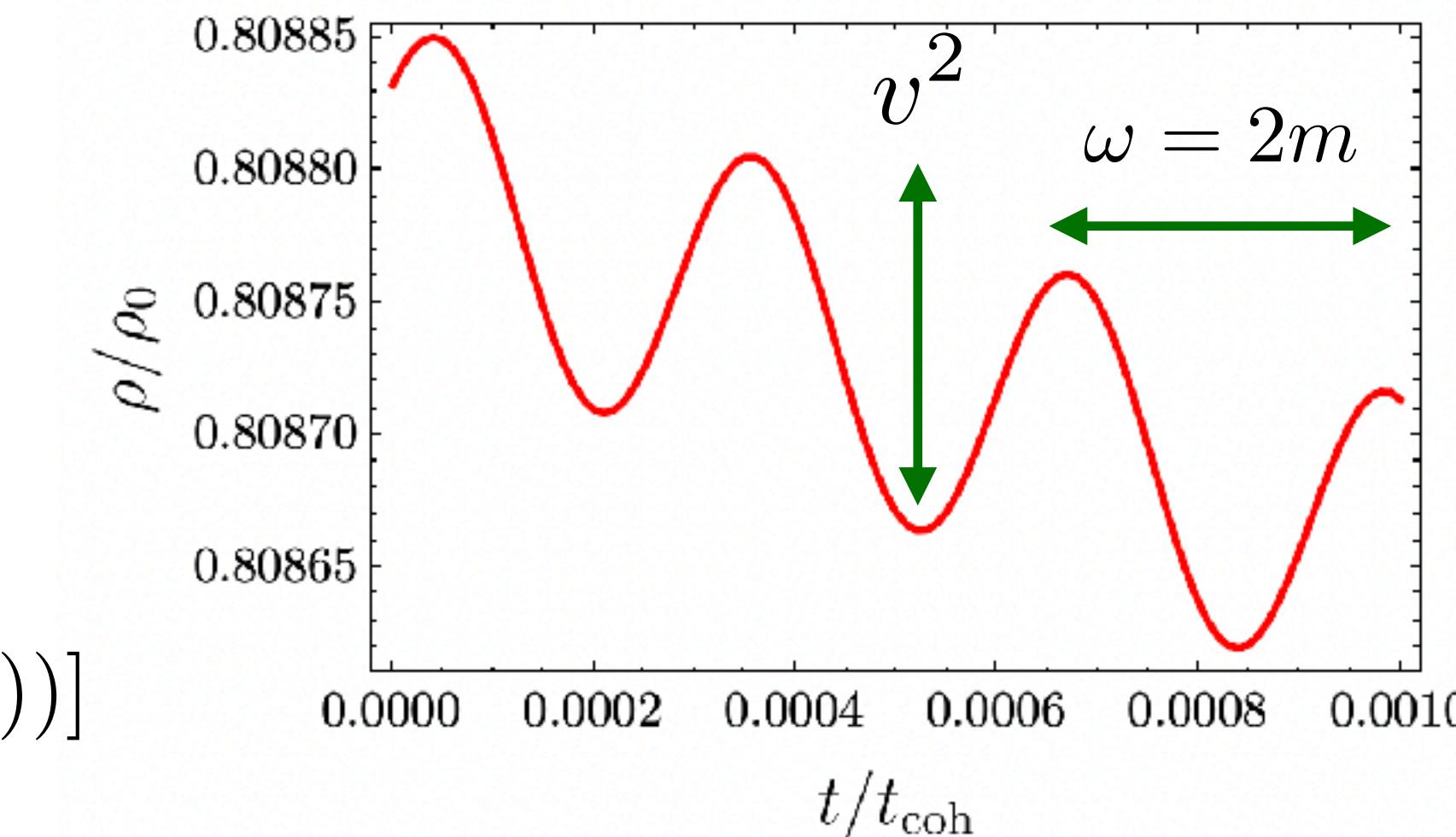


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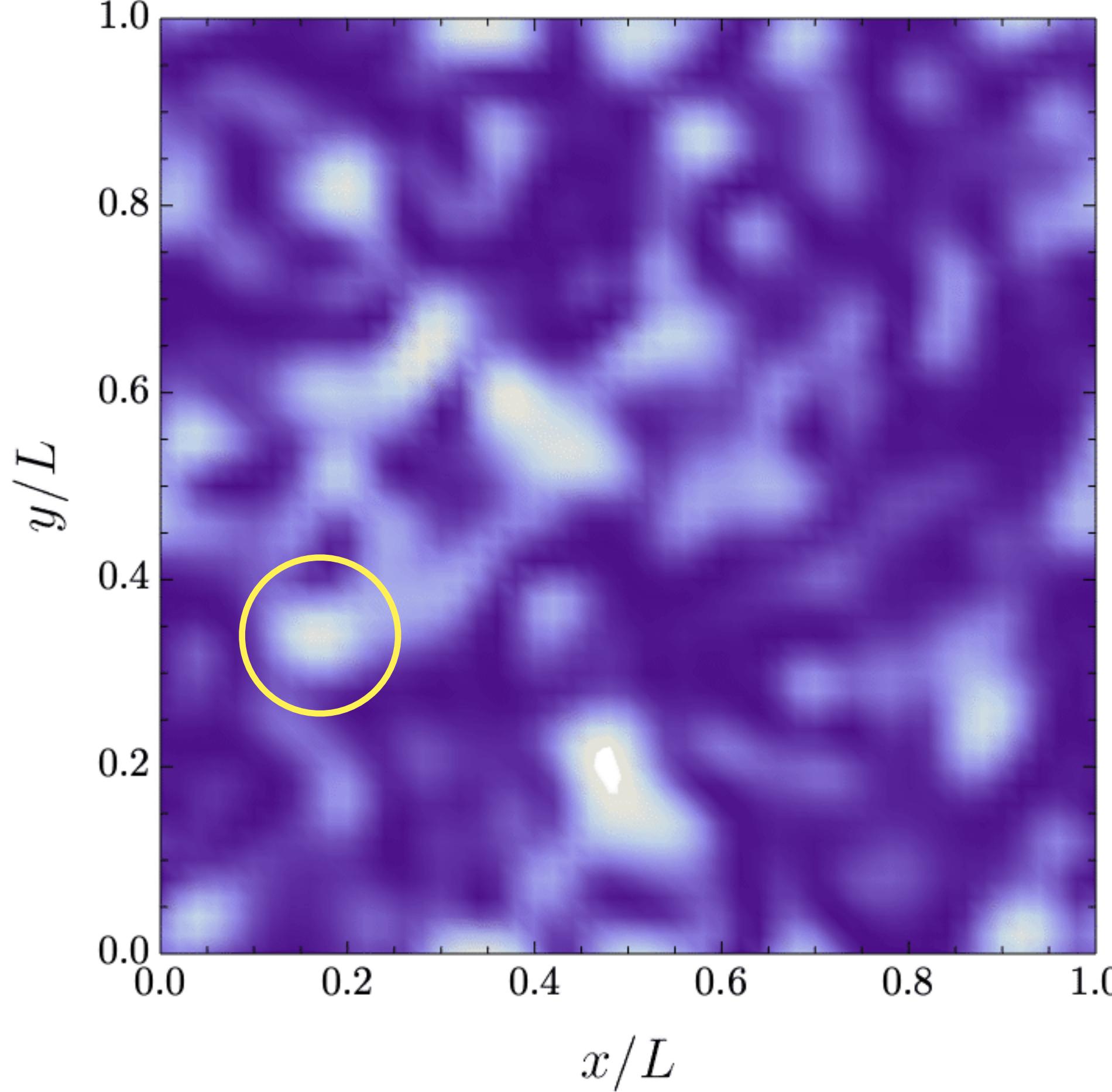
$$\approx \rho_0 [1 - v^2 \cos(2(m t - k x))]$$



for a single mode

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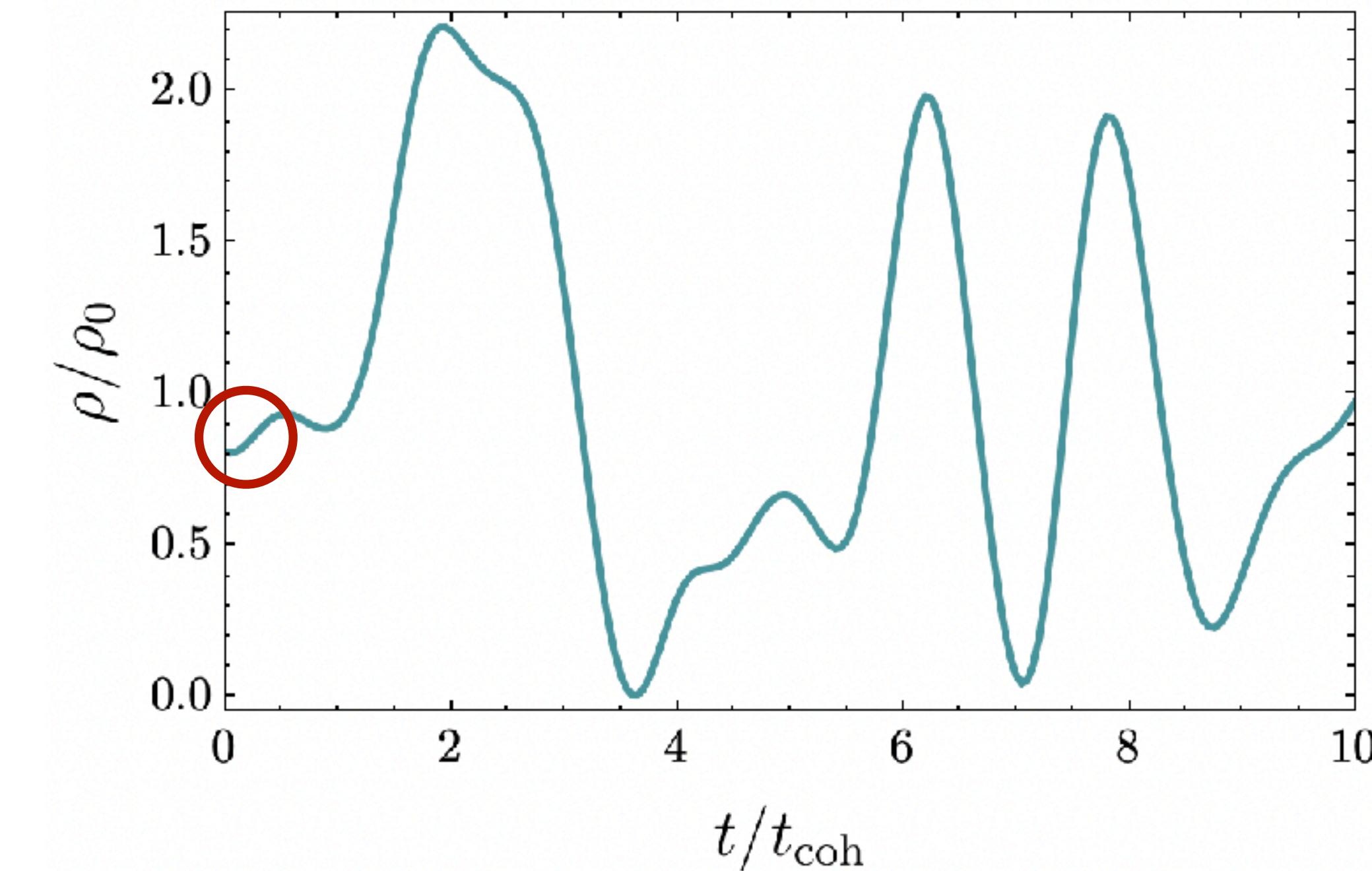
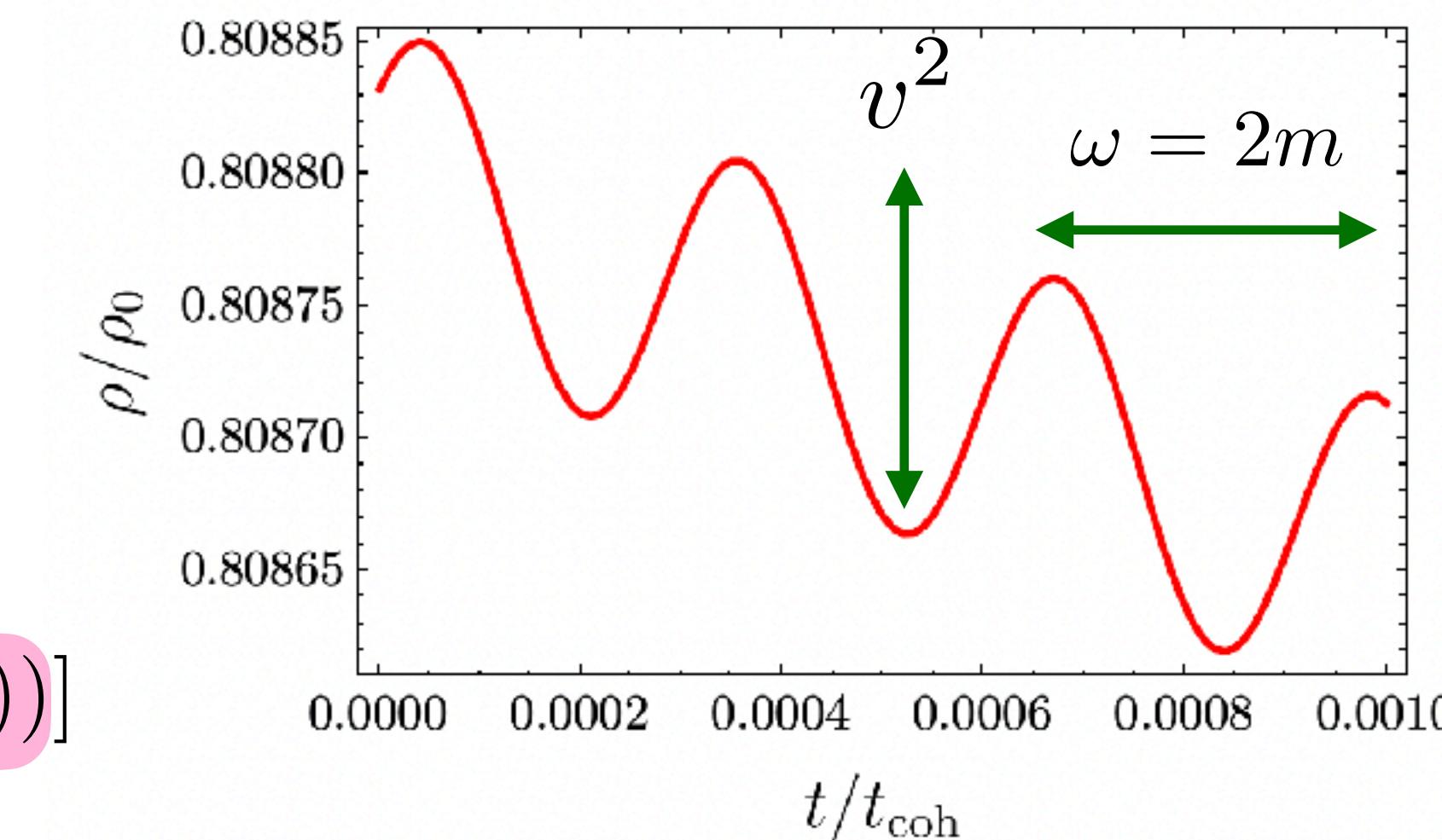


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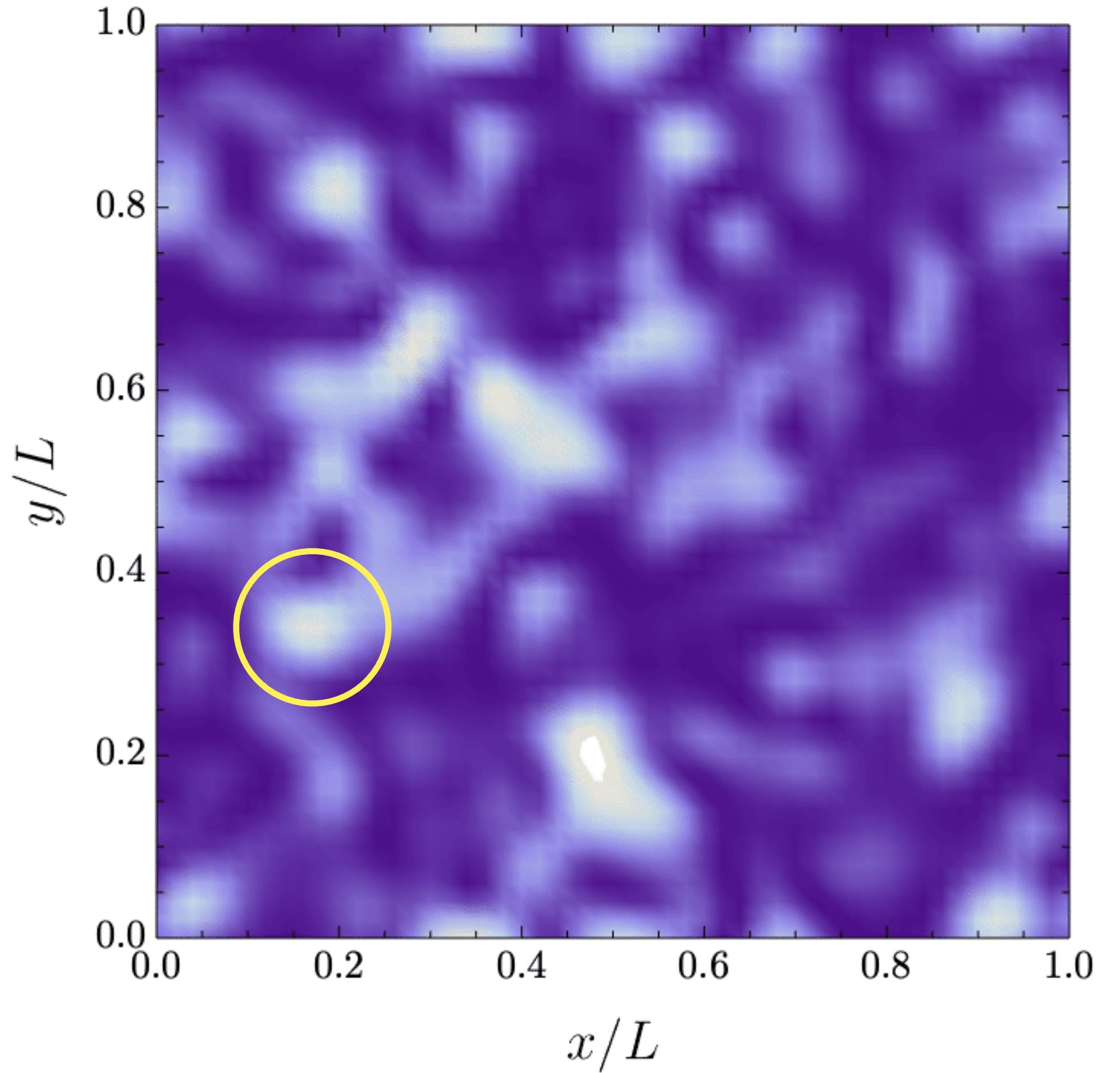
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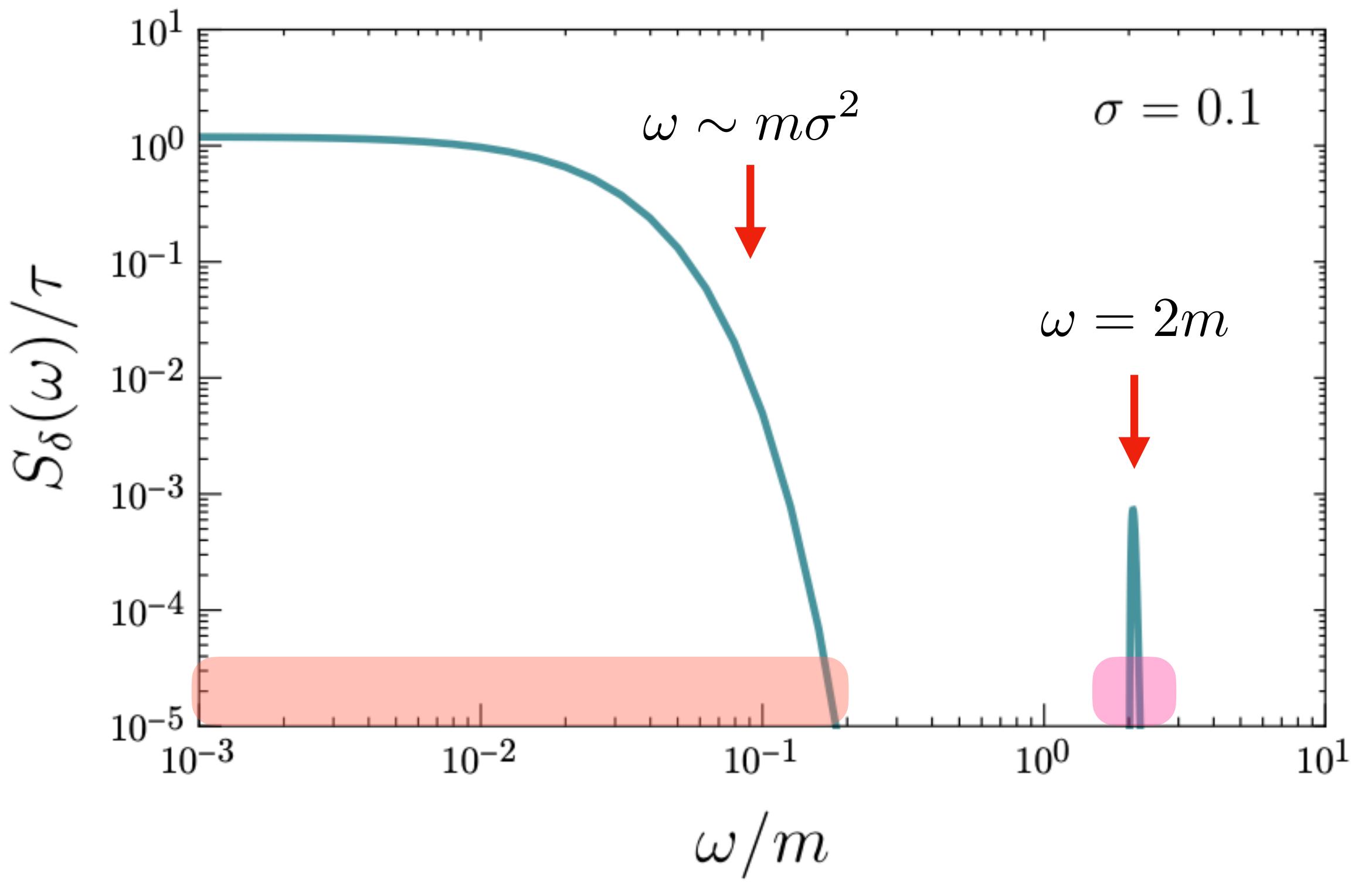


$$\langle \delta^2(x) \rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$

$$t/t_{\text{coh}} = 0.$$

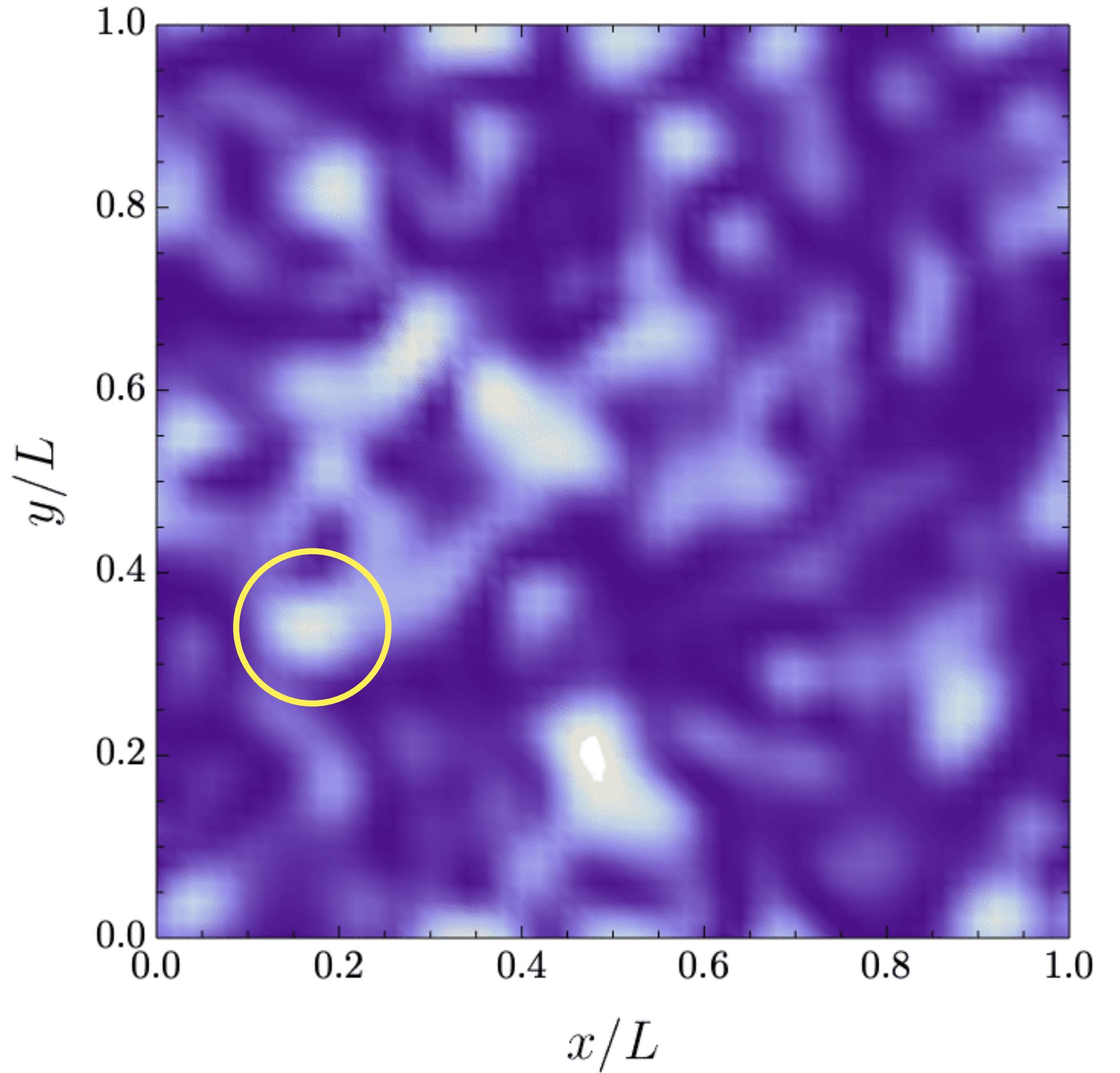


$$S_\delta(\omega) = \tau [\sigma^4 A_\delta(\omega) + B_\delta(\omega)]$$

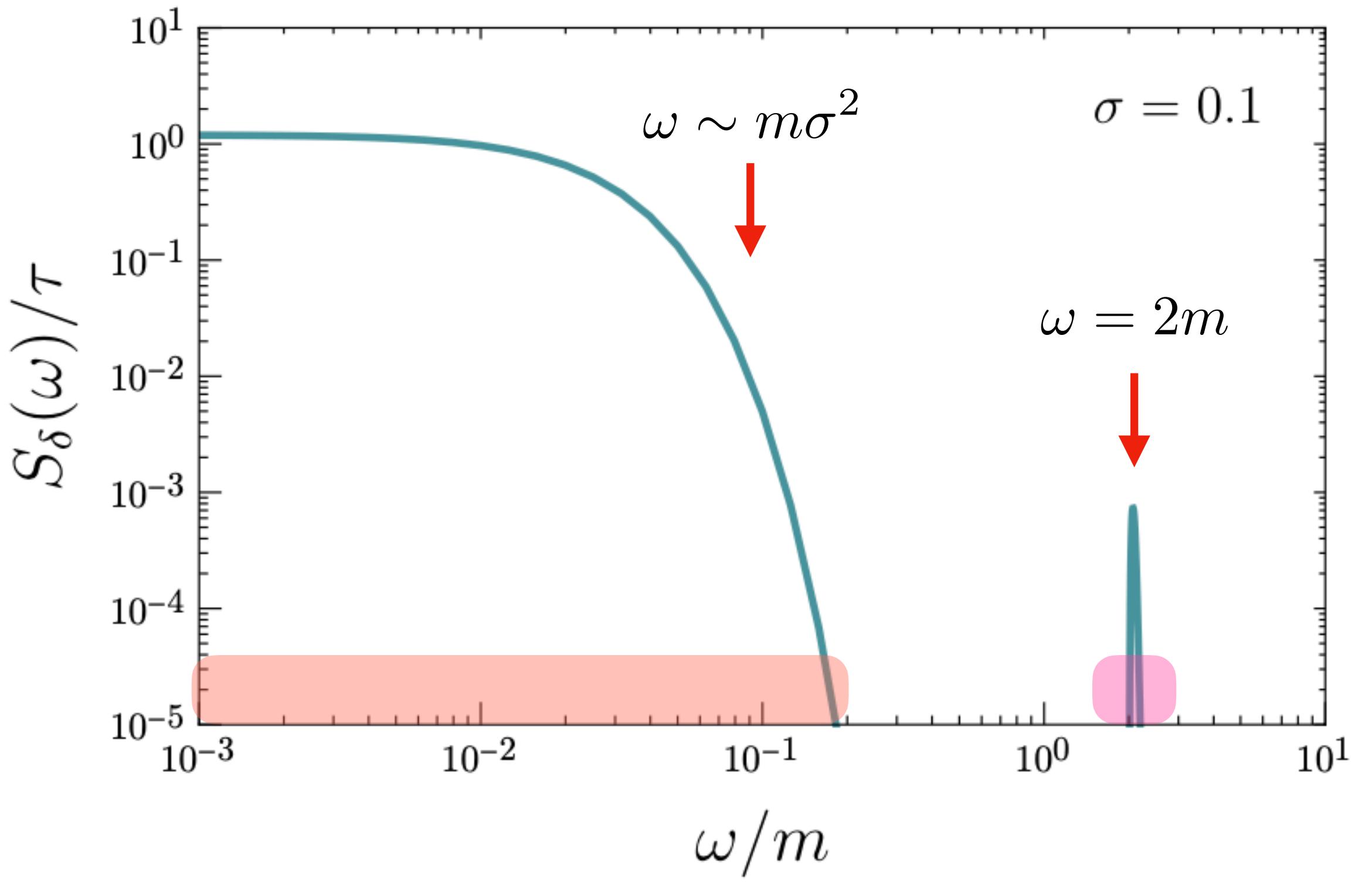


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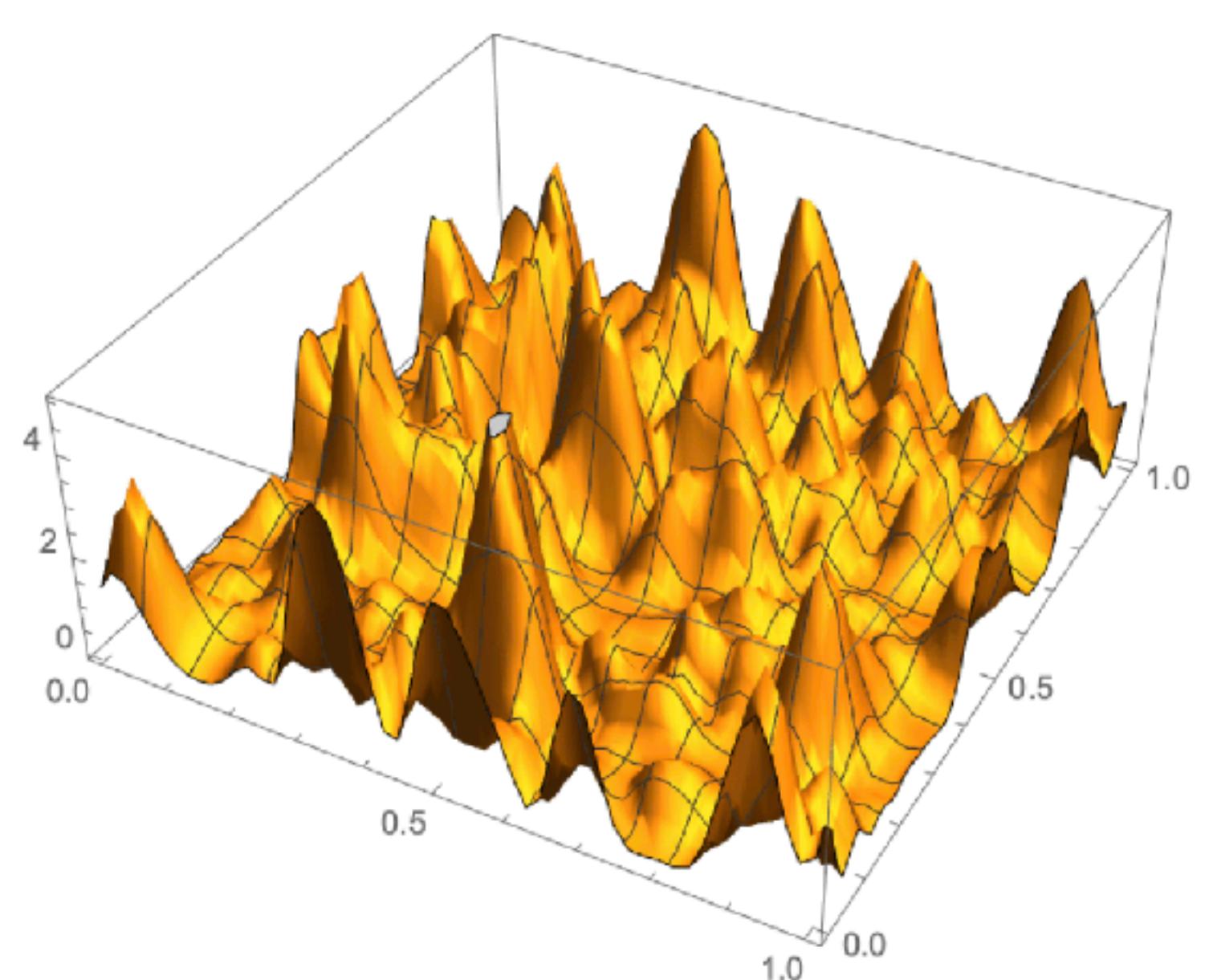
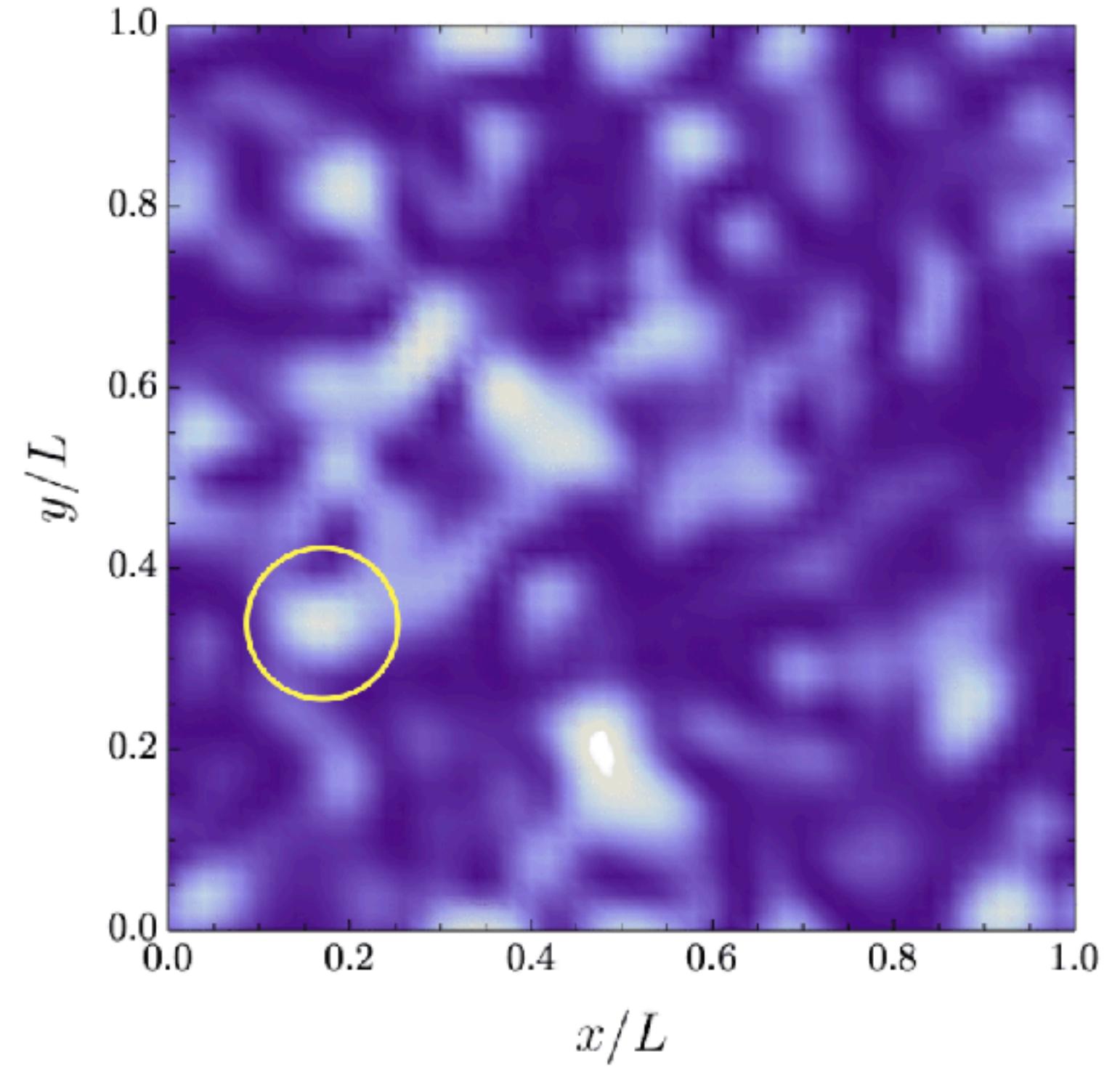
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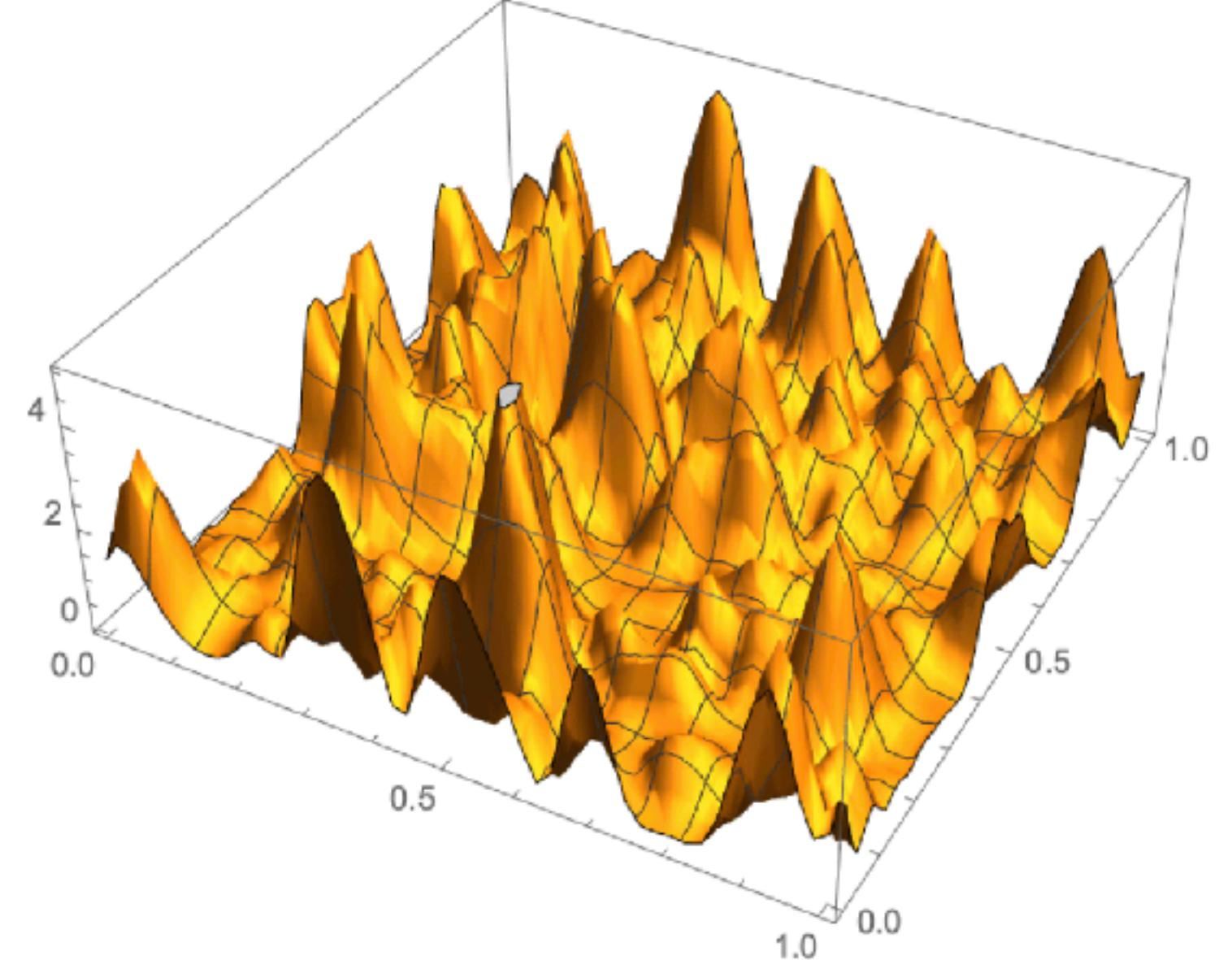
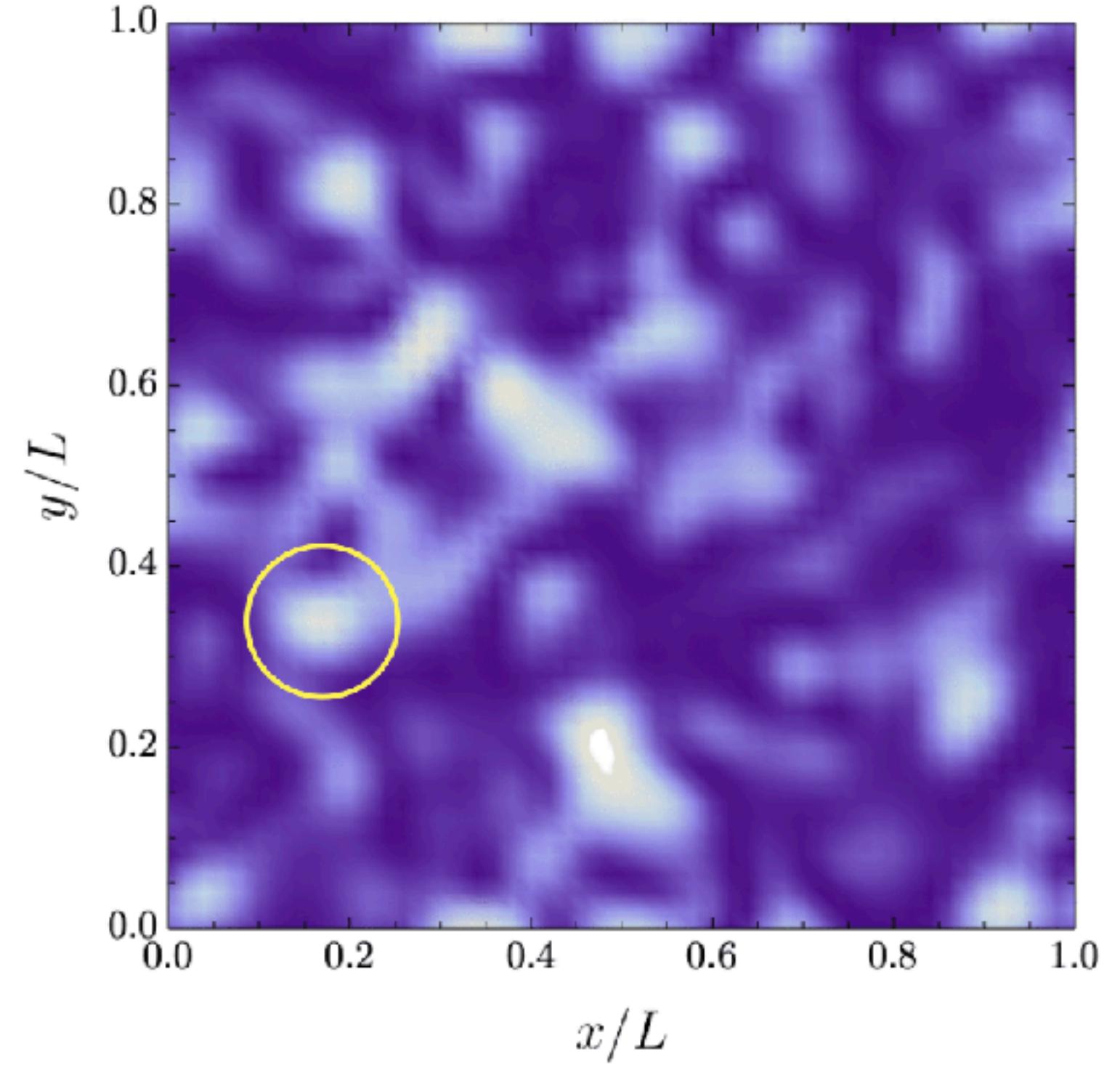
$$t/t_{\text{coh}} = 0.$$



*density fluctuation at each patches
follows the exponential distribution*

$$p(\rho)d\rho = \frac{1}{\rho_0} e^{-\rho/\rho_0} d\rho$$

$$t/t_{\text{coh}} = 0.$$



density fluctuation at each patches follows the exponential distribution

$$p(\rho)d\rho = \frac{1}{\rho_0} e^{-\rho/\rho_0} d\rho$$

a probability to observe density fluctuation $\rho > \rho_c$ is

$$P(\rho > \rho_c) = e^{-\rho_c/\rho_0}$$

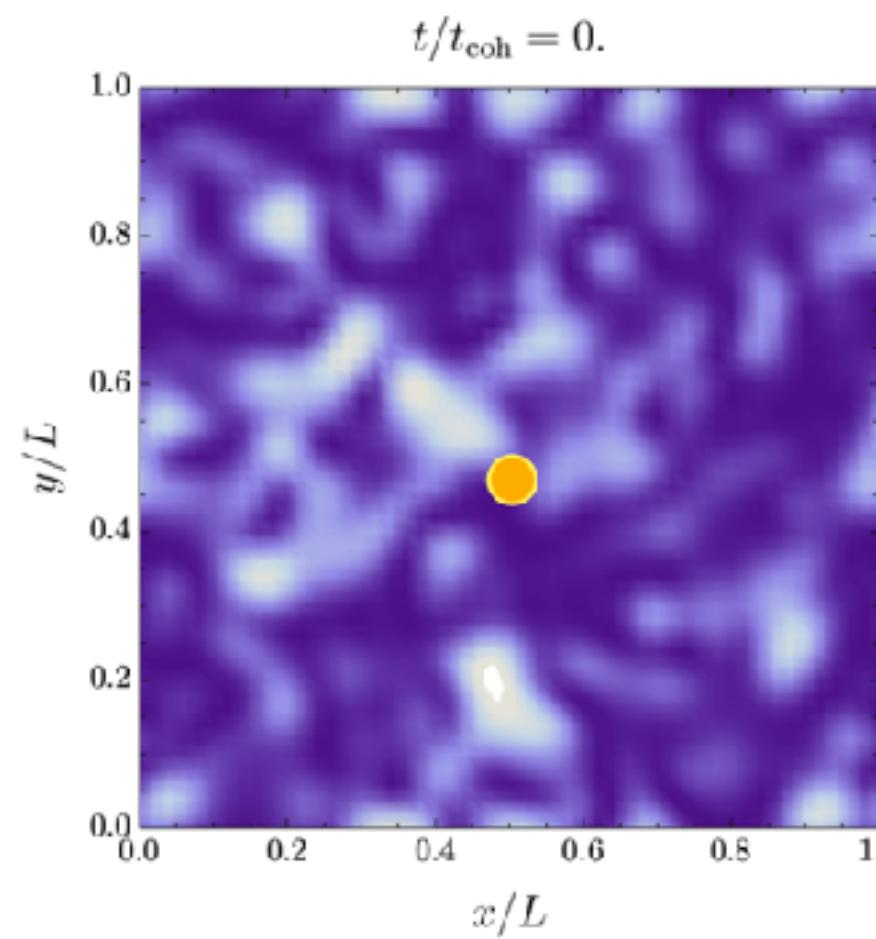
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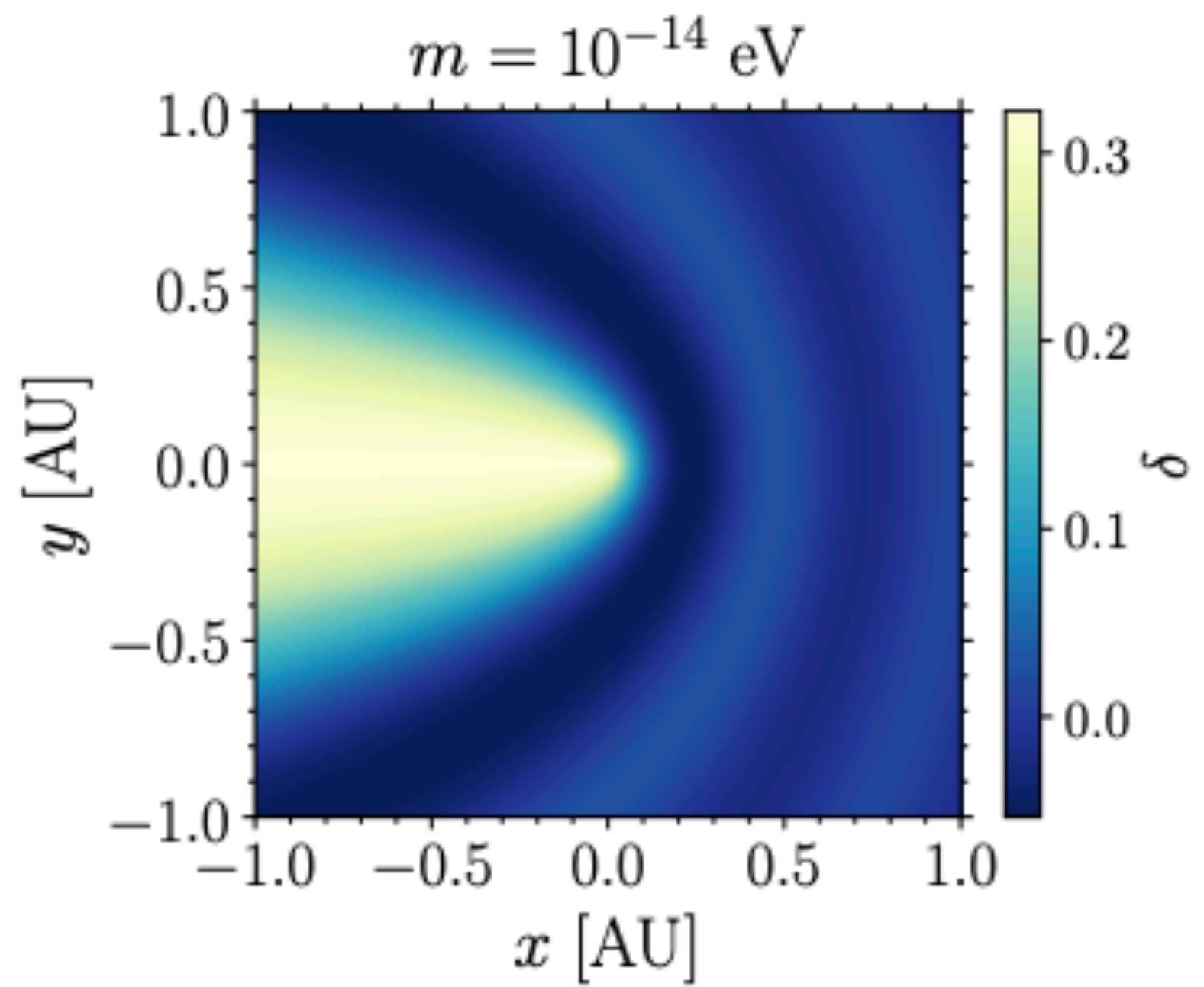
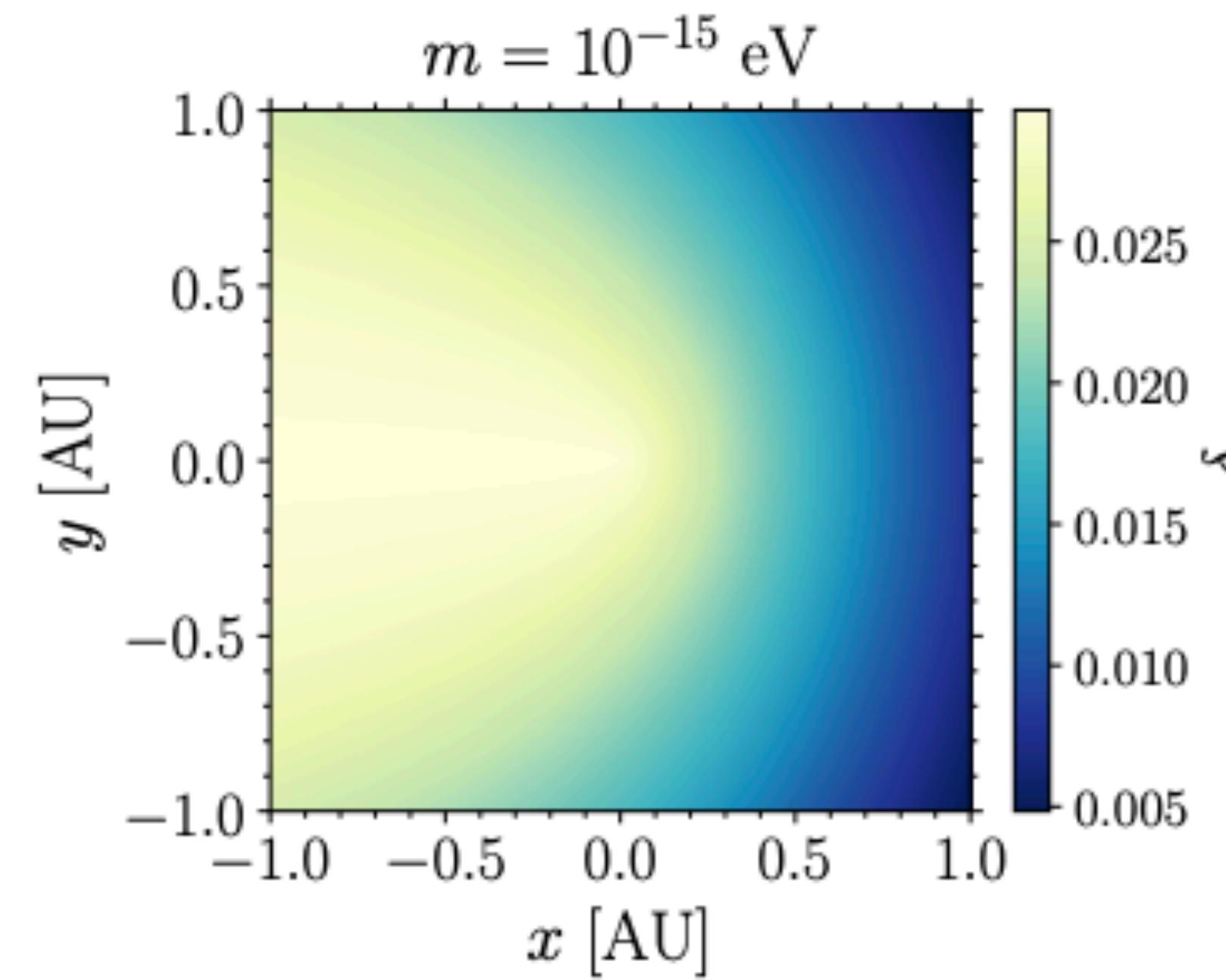


*if there's Sun in the middle of the box
and dark matter has mean velocity (dark matter wind)*

$$\epsilon_i \Psi_i = H \Psi_i = \left[-\frac{\nabla^2}{2m} + \Phi \right] \Psi_i$$

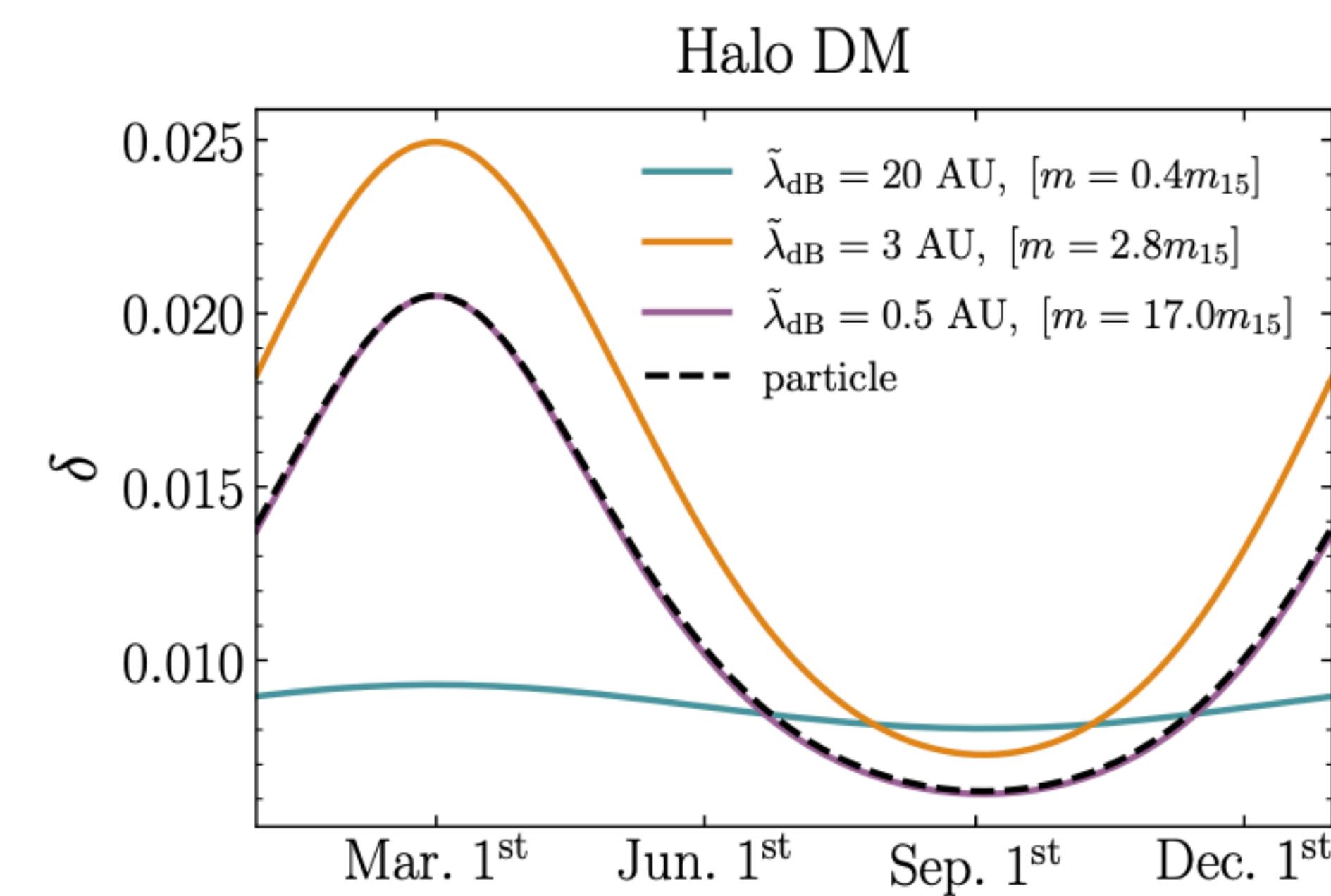
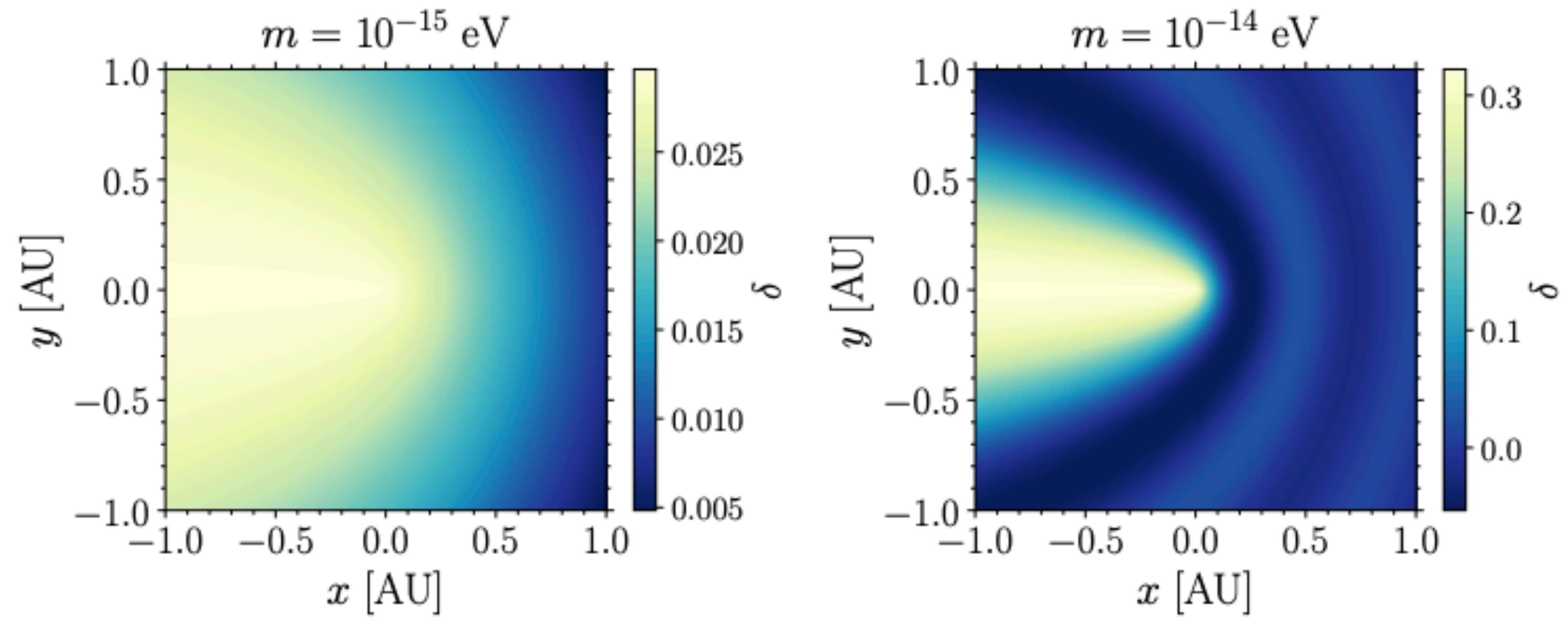
$$\psi_{\mathbf{k}}(\mathbf{x}) = e^{i\mathbf{k}\cdot\mathbf{x}} \Gamma(1 - i\beta) e^{\pi\beta/2} M[i\beta, 1, ikr(1 - \hat{k}\cdot\hat{x})].$$

*mean density contrast $\langle \delta \rangle$
(for monochromatic case)*



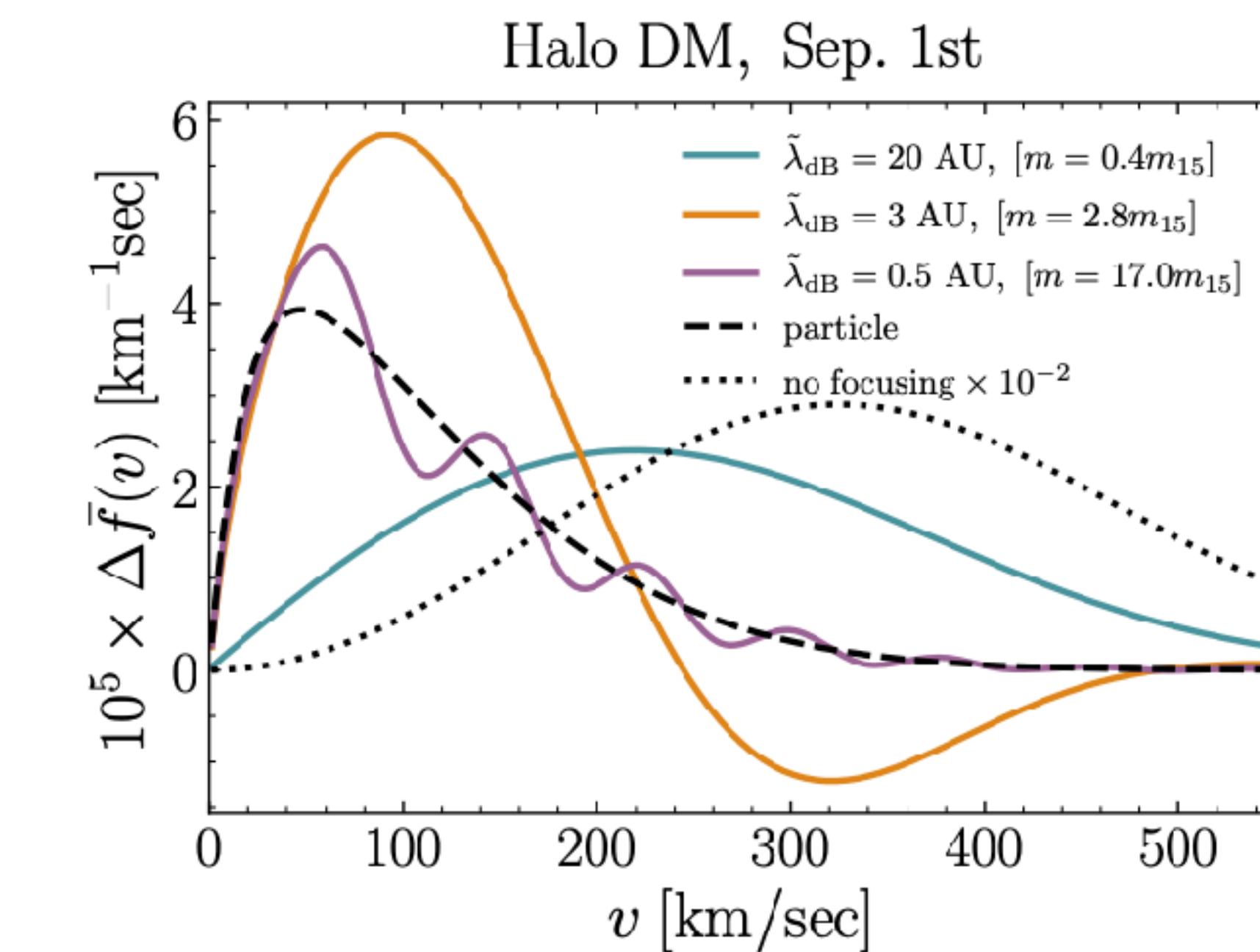
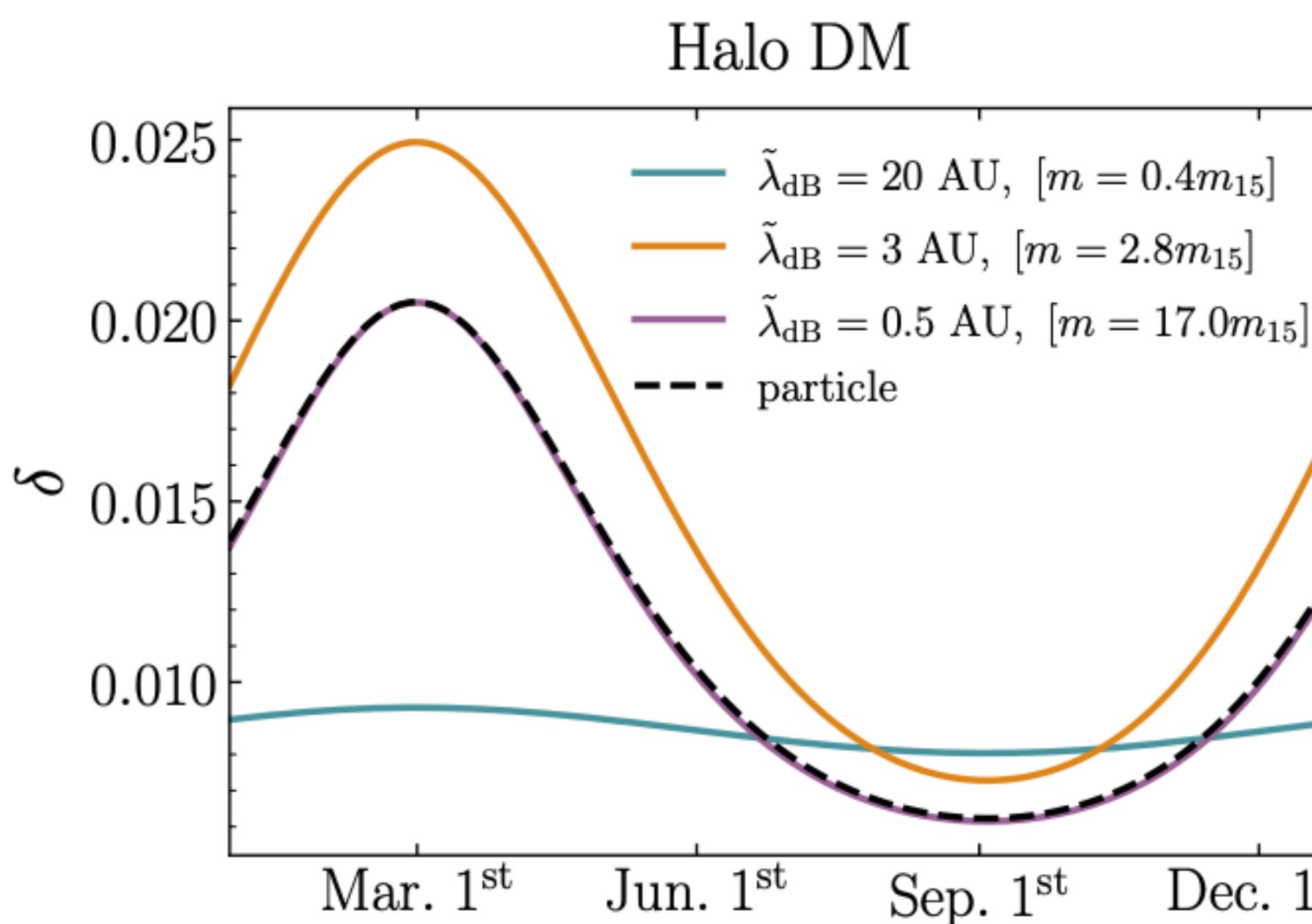
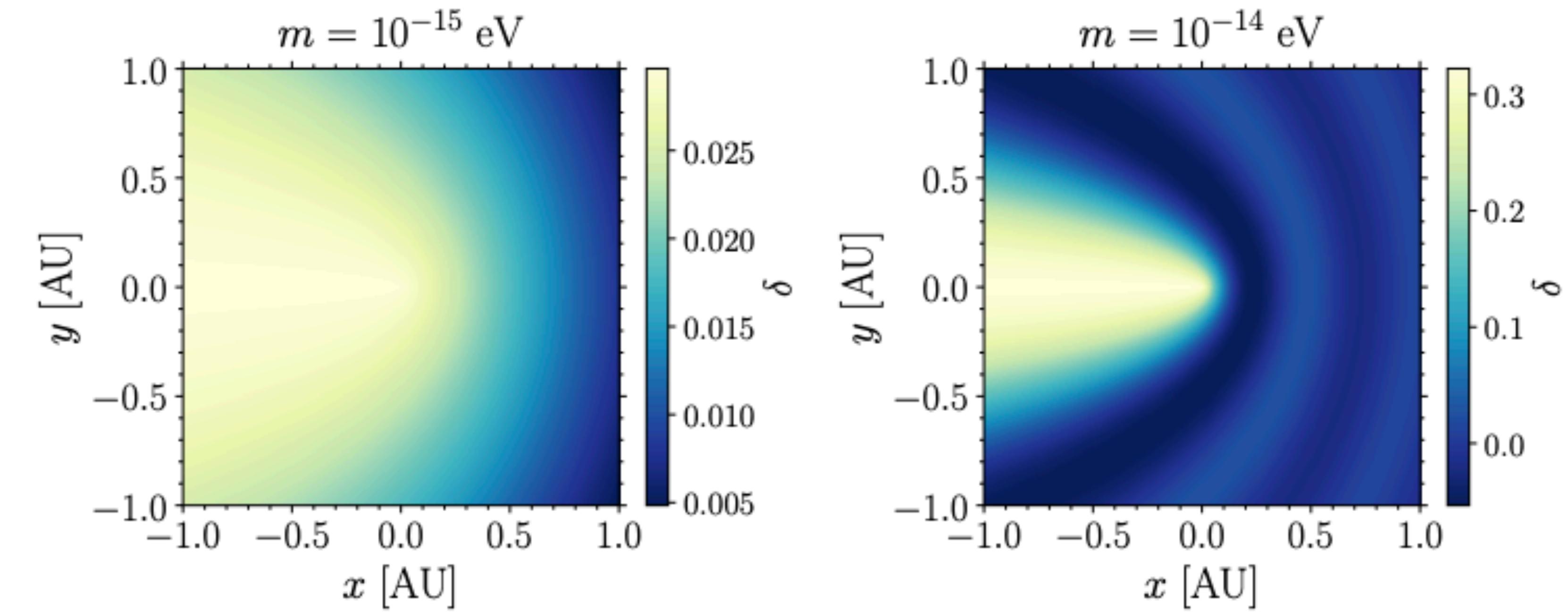
*mean density contrast $\langle \delta \rangle$
(for monochromatic case)*

annual modulation



*mean density contrast $\langle \delta \rangle$
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annual modulation



*deformed
spectral shape*

some applications of this approach of ultralight dark matter

- (i) understanding the behavior of **homogeneous wave DM medium***
- (ii) understanding the behavior of the **wave DM in the solar system***
- (iii) construction of the whole wave DM halo**
- (iv) understanding the **adiabatic evolution** of wave DM halo*

*it is also possible to construct
the whole wave DM halo in this way*

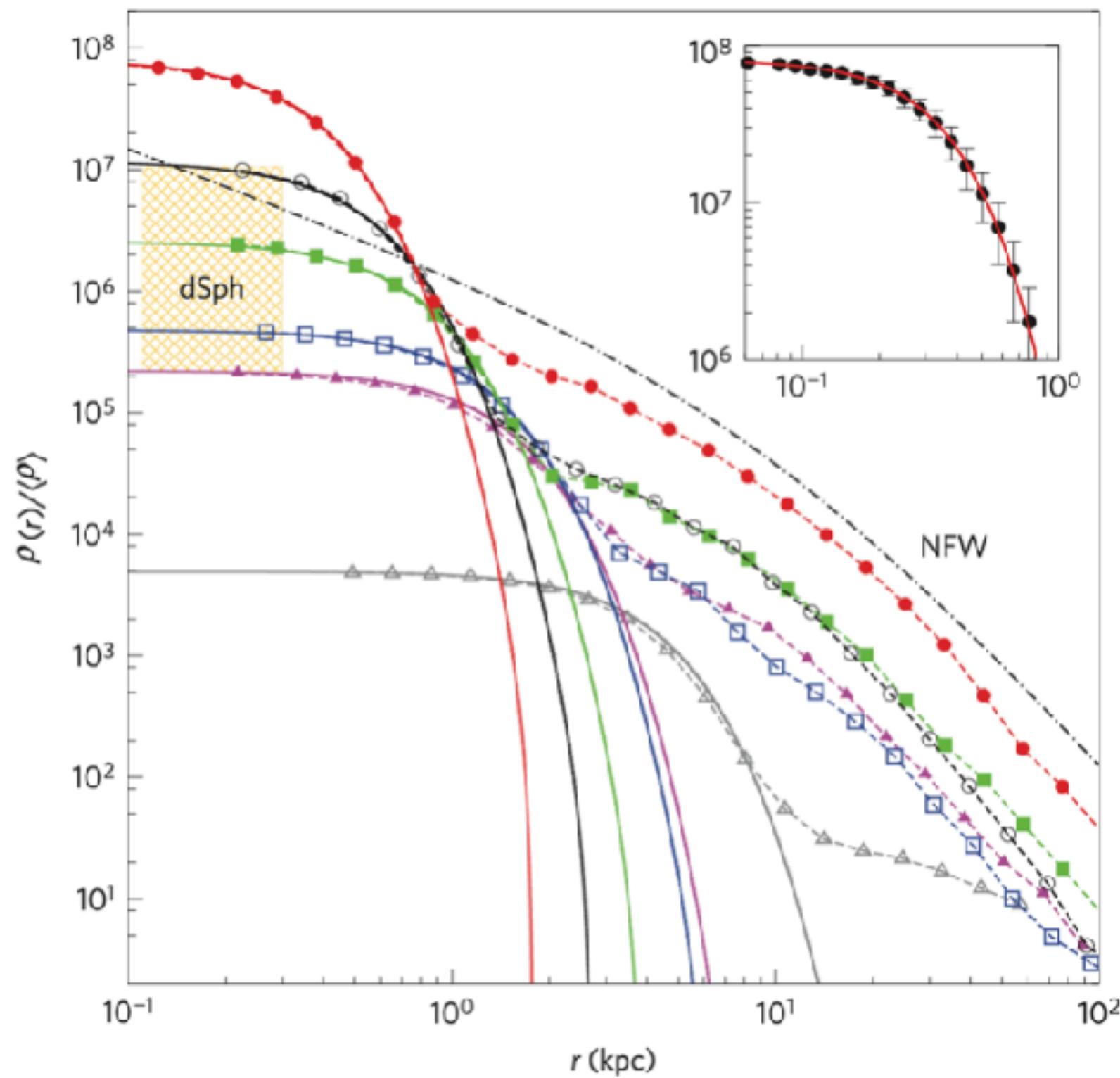
[Widrow, Kaiser (93)]

[Lin, Schive, Wong, Chiueh (18)]

[Yavetz, Li, Hui (22)]

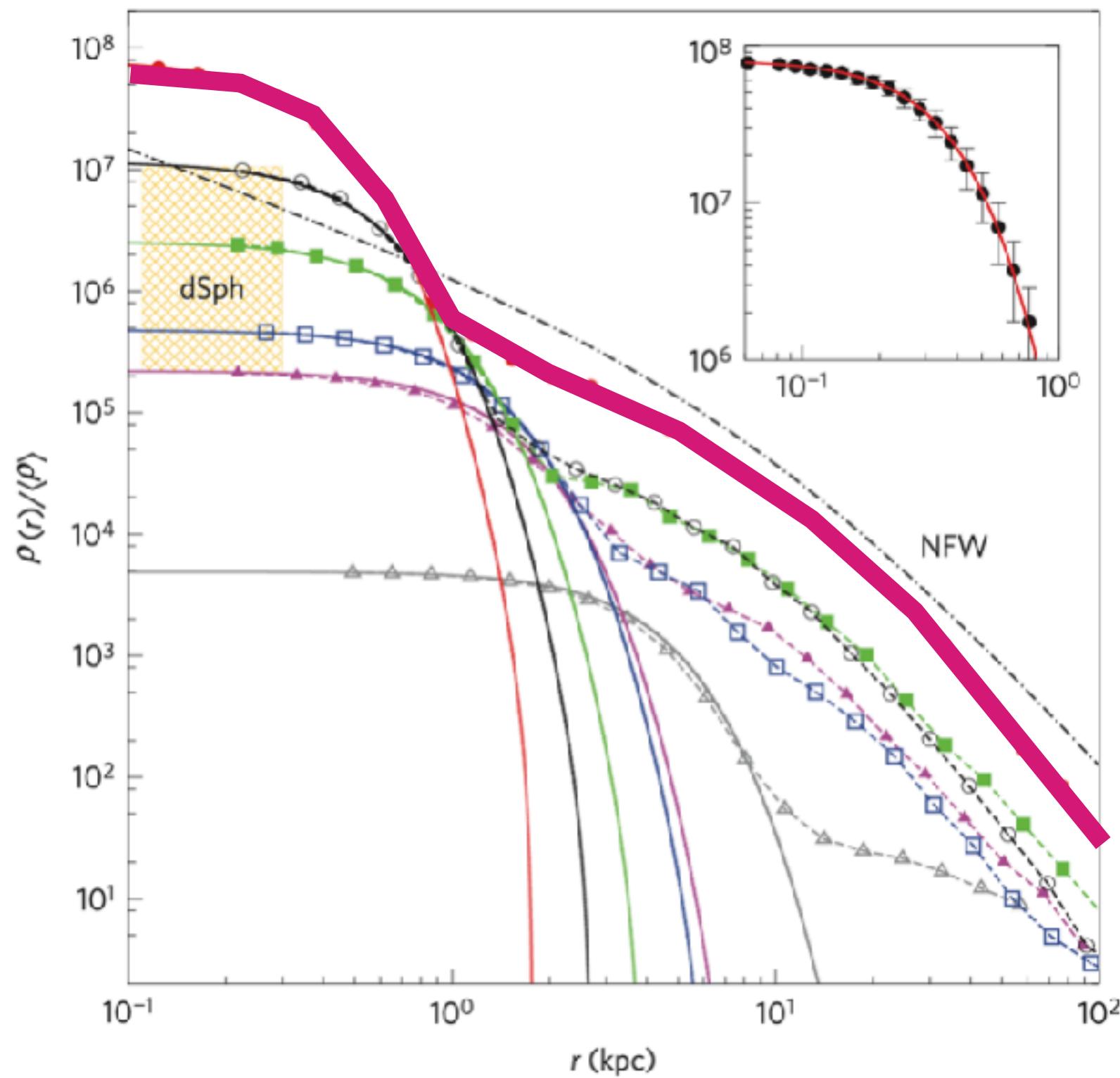
*it is also possible to construct
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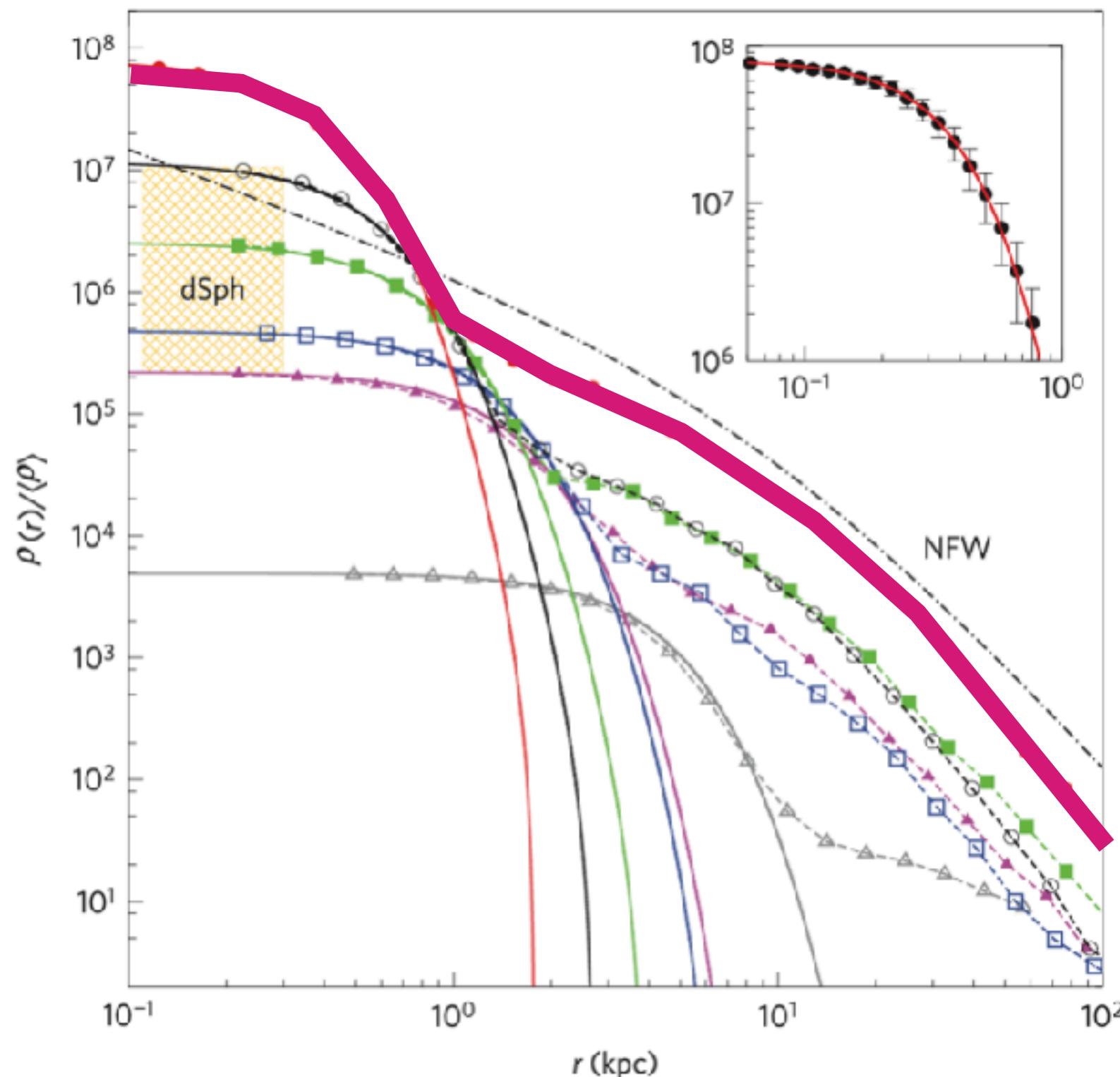


*ansatz:
soliton + NFW profile*

$$\rho_t$$

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*ansatz:
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$$\rho_t$$

this will fix the gravitational potential as well as density profile

*since the gravitational potential is fixed
wave equations to obtain eigenmodes of the system*

$$\psi_{n\ell m} = R_{n\ell}(r) Y_\ell^m(\Omega)$$

$$(rR_{n\ell})'' + [2m^2(E_{n\ell} - \Phi) - \ell(\ell + 1)/r^2](rR_{n\ell}) = 0$$

the field is expanded as

$$\hat{\phi} = \sum_{n\ell m} \frac{1}{\sqrt{2\omega}} \left[\alpha_{n\ell m} \psi_{n\ell m} + \alpha_{n\ell m}^\dagger \psi_{n\ell m}^* \right]$$

quasi-probability distribution

$$P(\alpha_{n\ell m}) = \frac{1}{\pi f_{n\ell m}} \exp \left[-\frac{|\alpha_{n\ell m}|^2}{f_{n\ell m}} \right]$$

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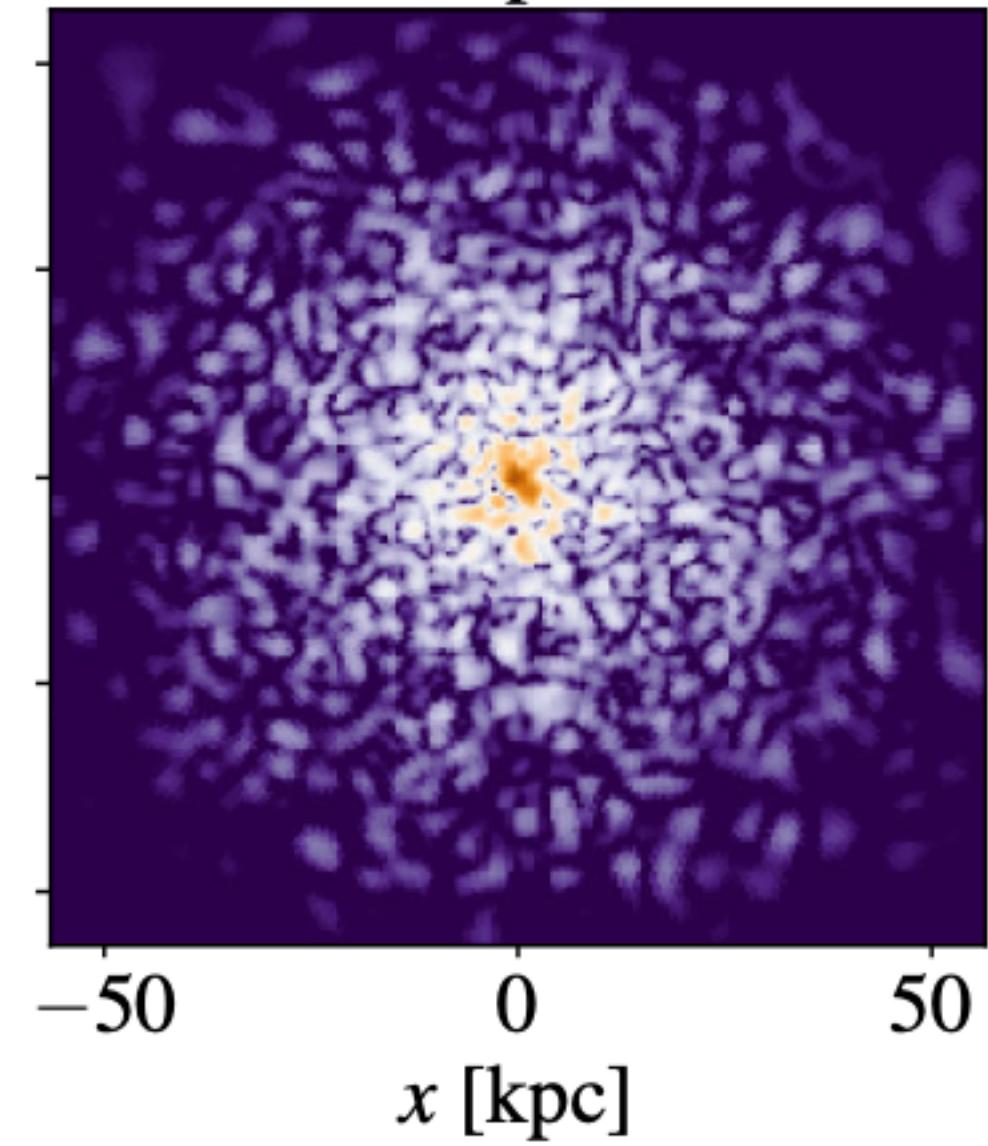
the occupation number f_{nlm} can be determined by comparing

$$\bar{\rho}(r) = \frac{m}{4\pi} \sum_{n\ell} (2\ell + 1) f_{n\ell} |R_{n\ell}(r)|^2 \quad \text{vs} \quad \rho_t$$

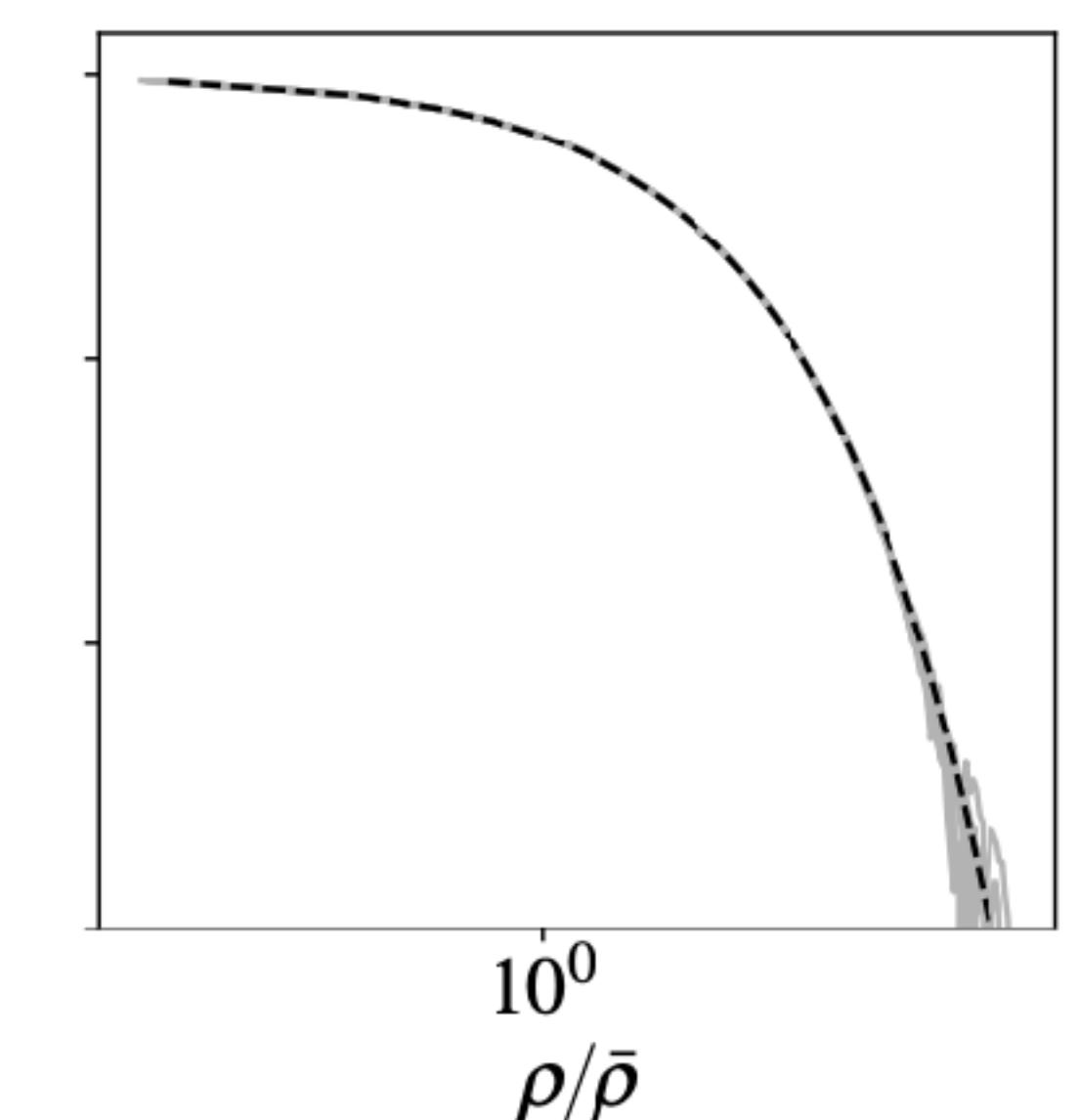
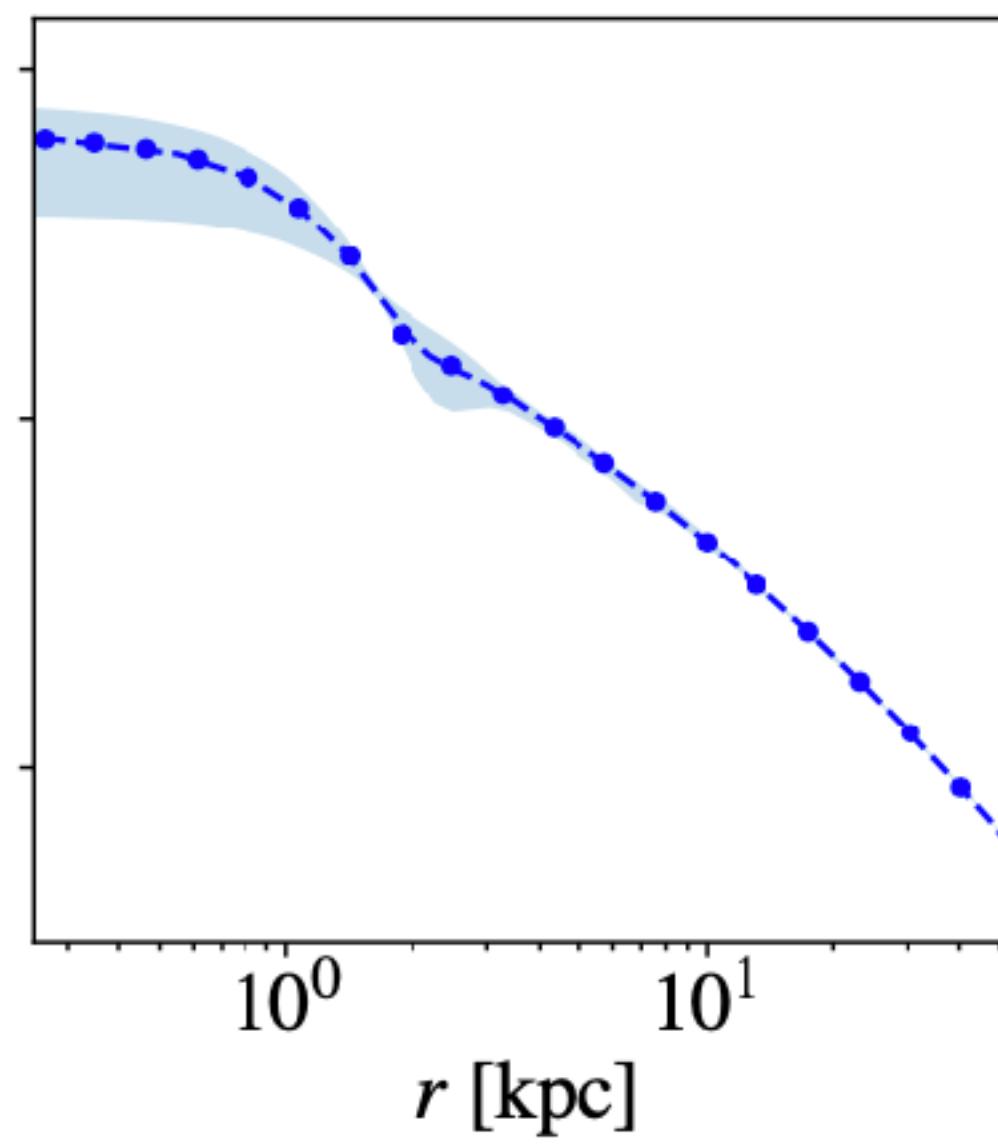
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Isotropic Fit

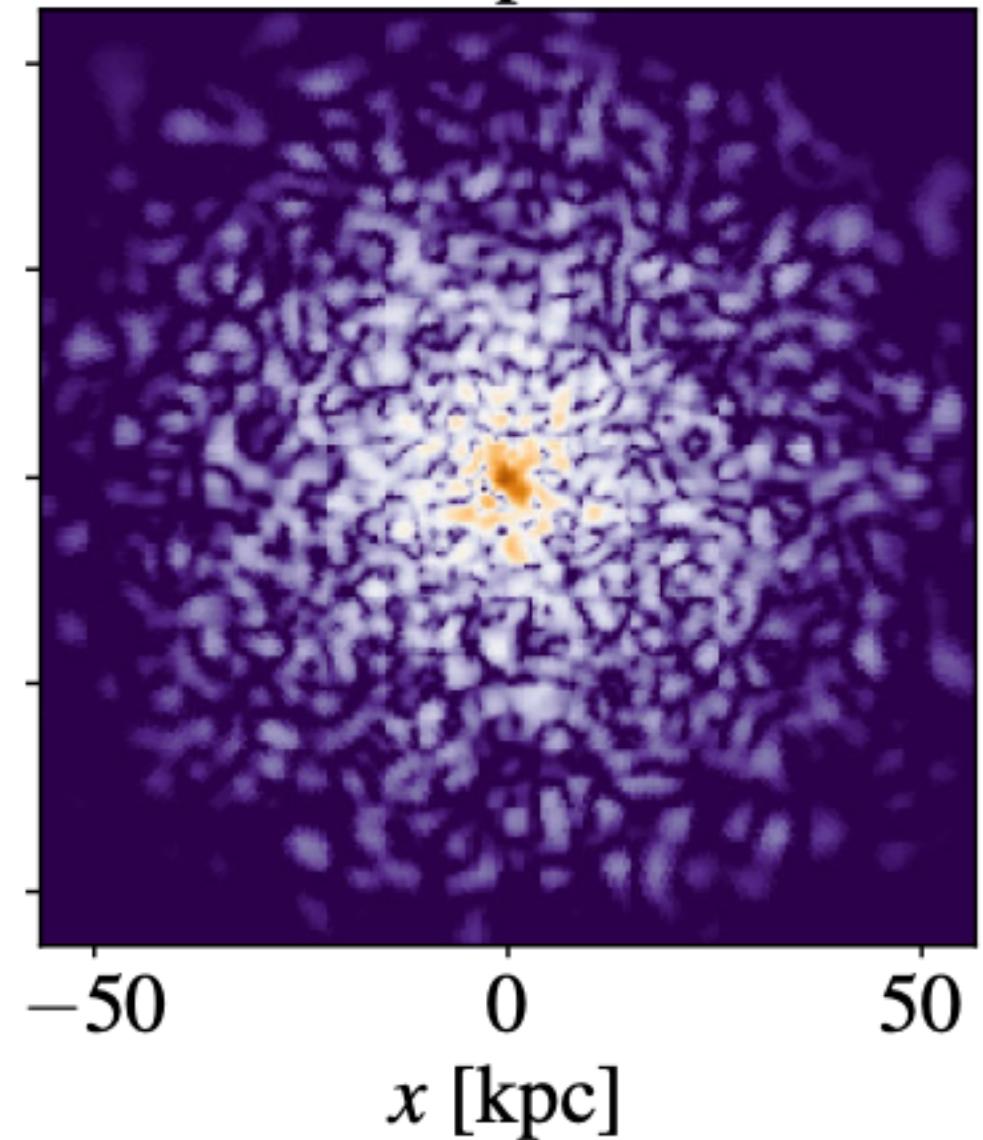


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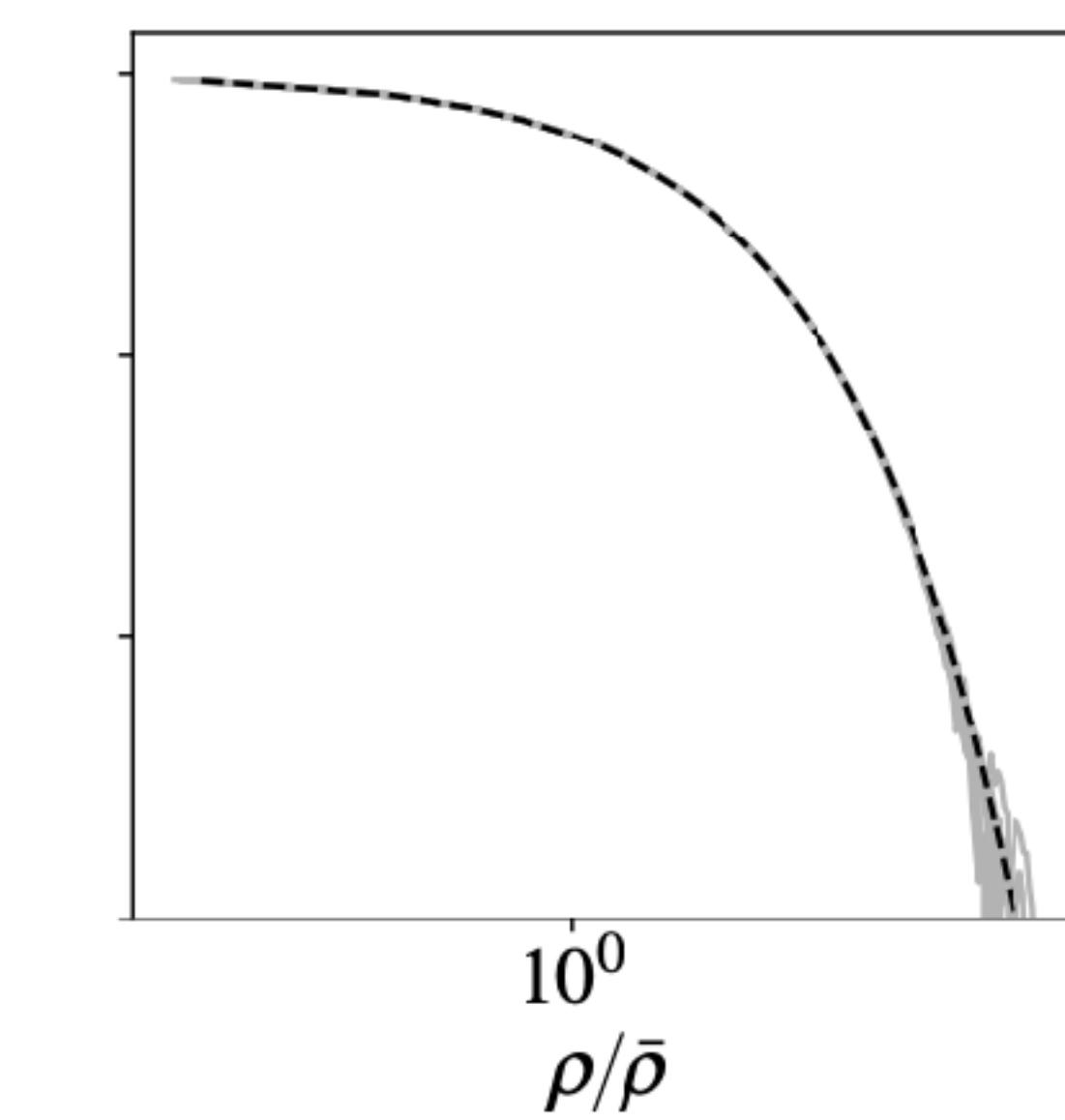
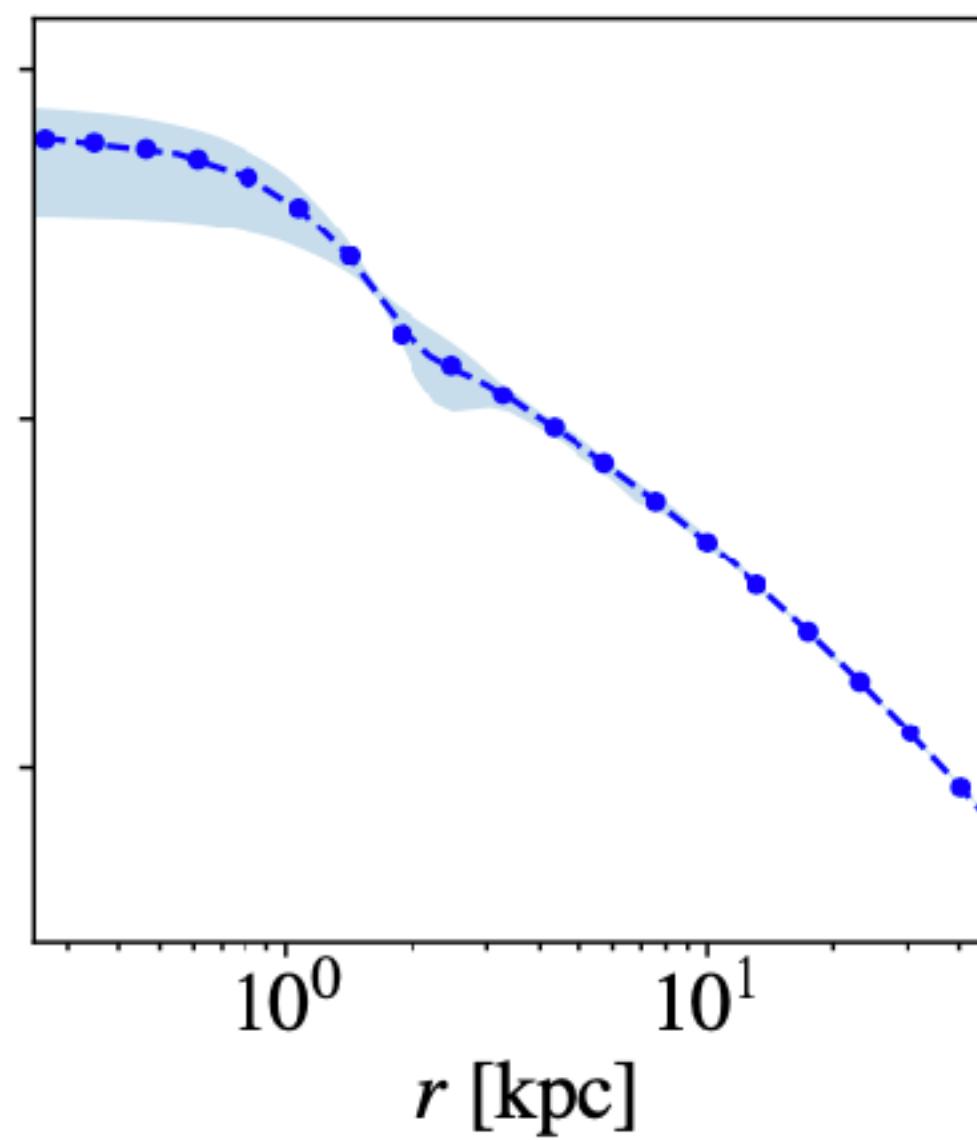


$p(\rho)$

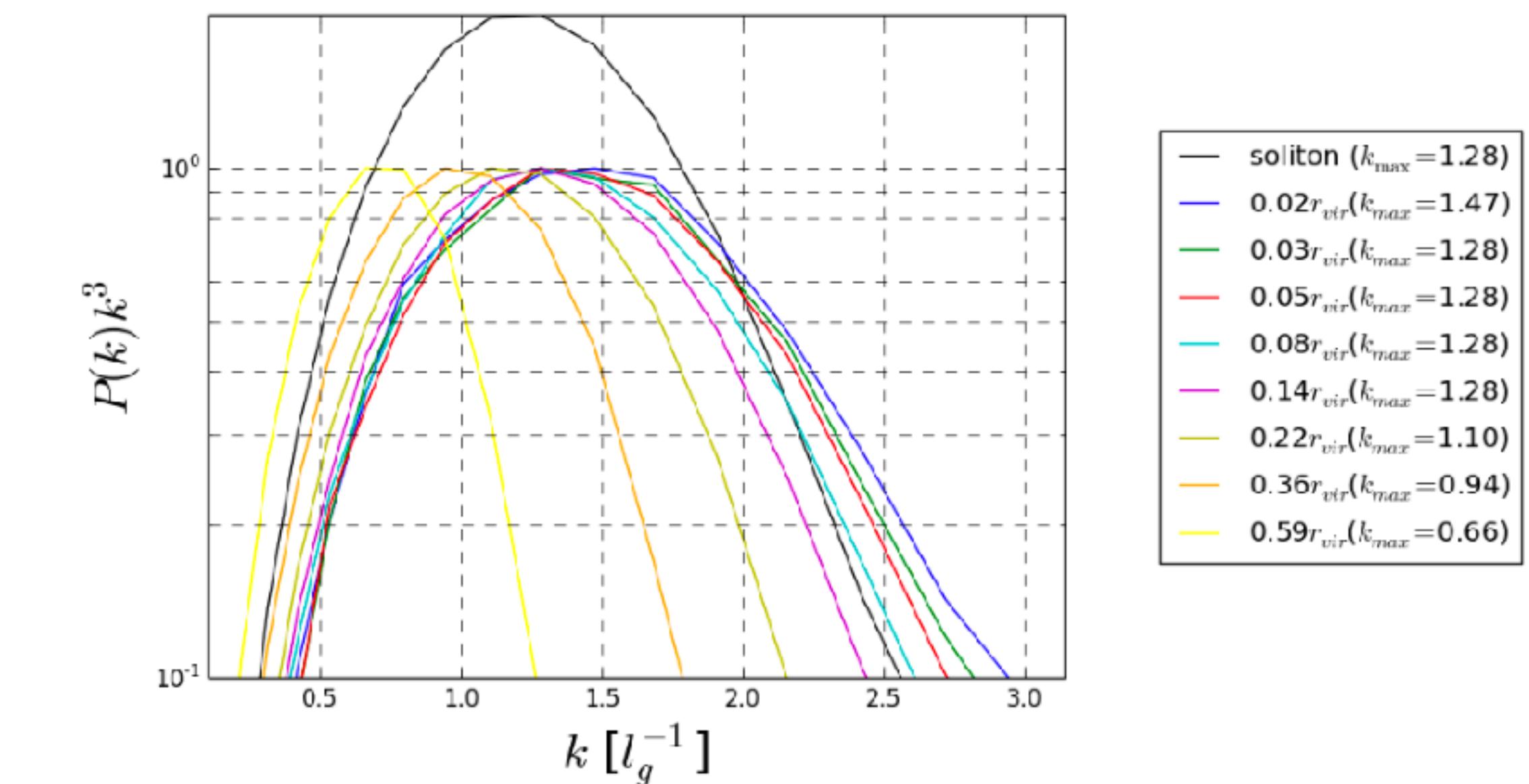
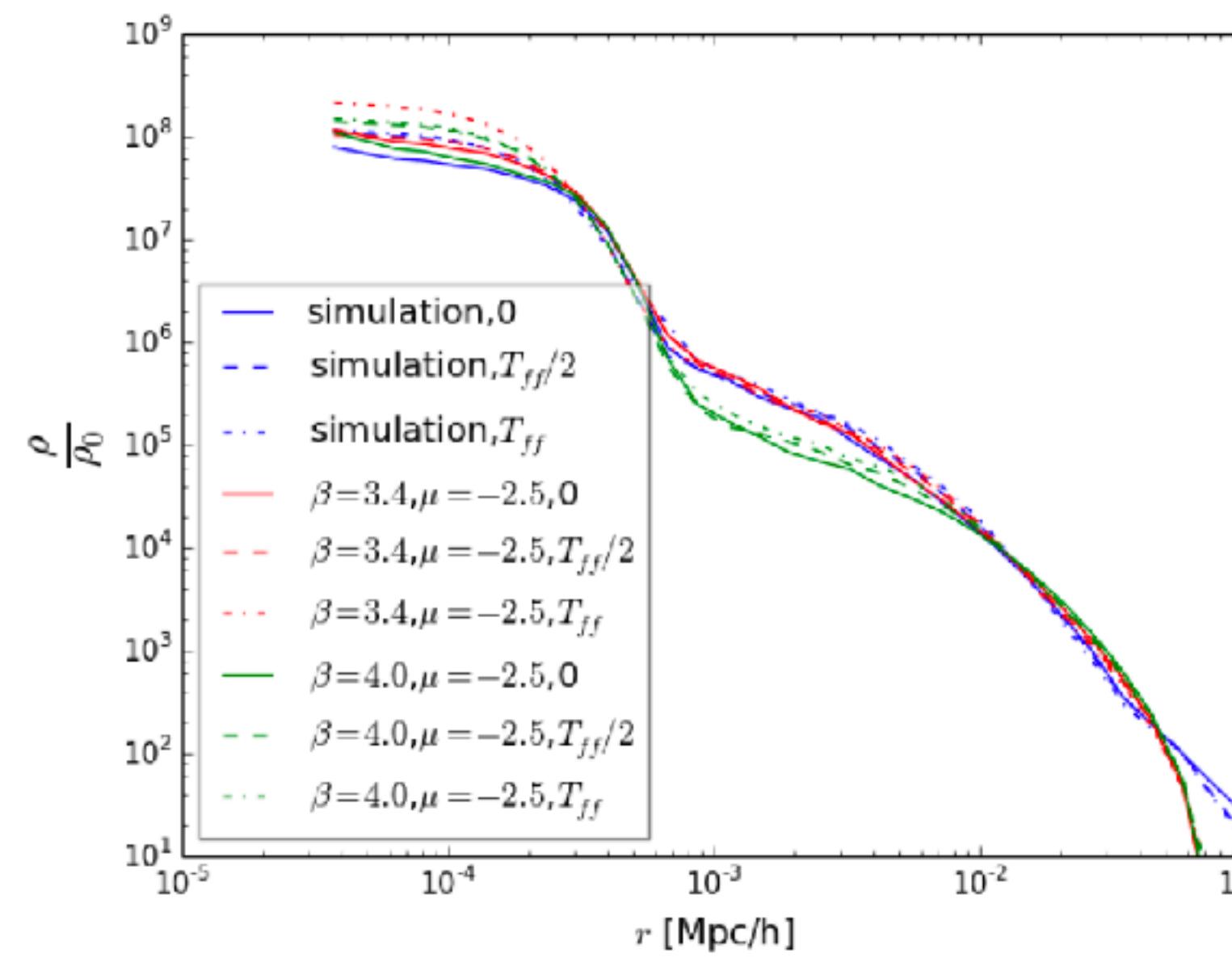
Isotropic Fit



[Yavetz, Li, Hui (22)]



[Lin et al (18)]

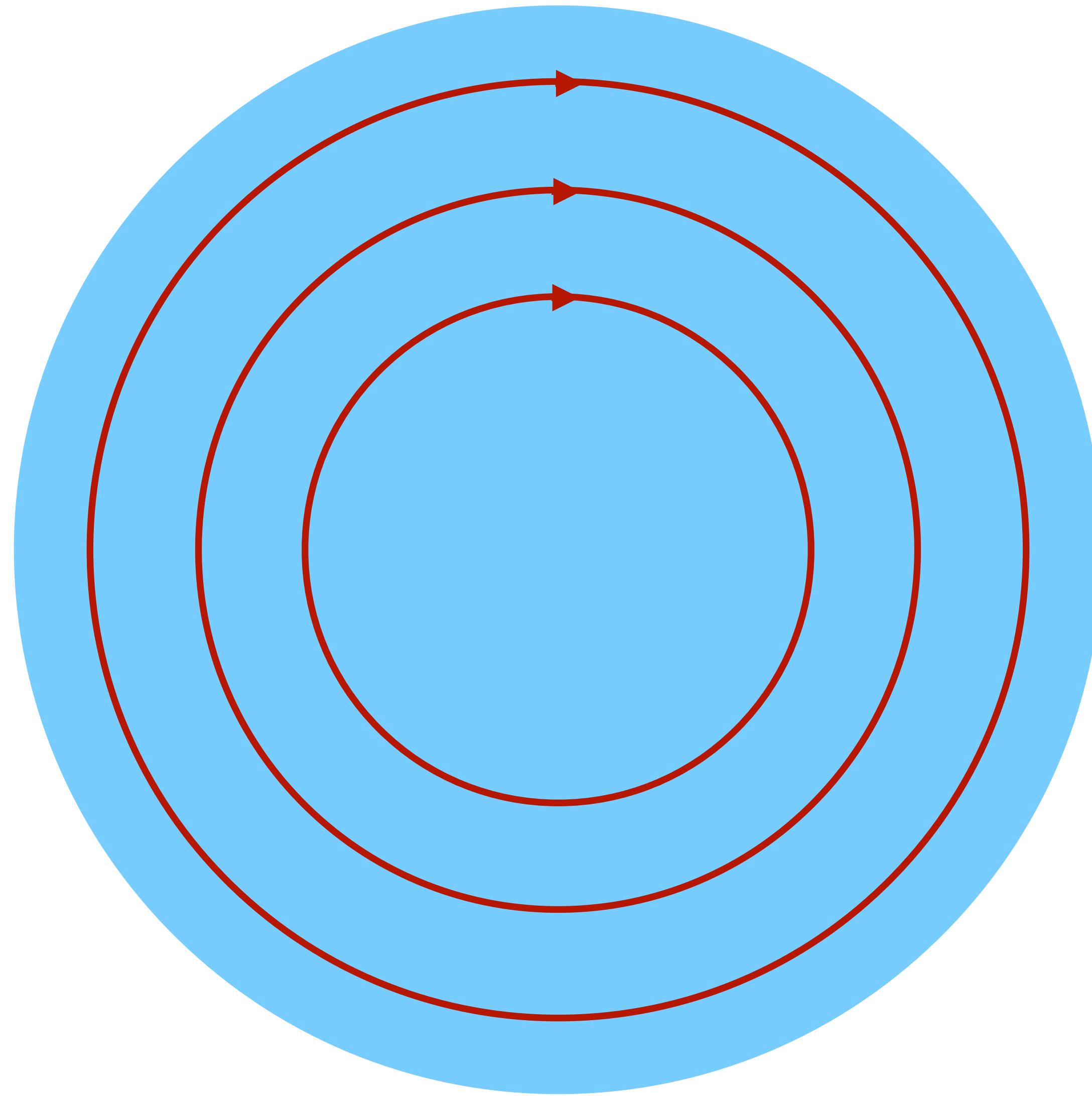


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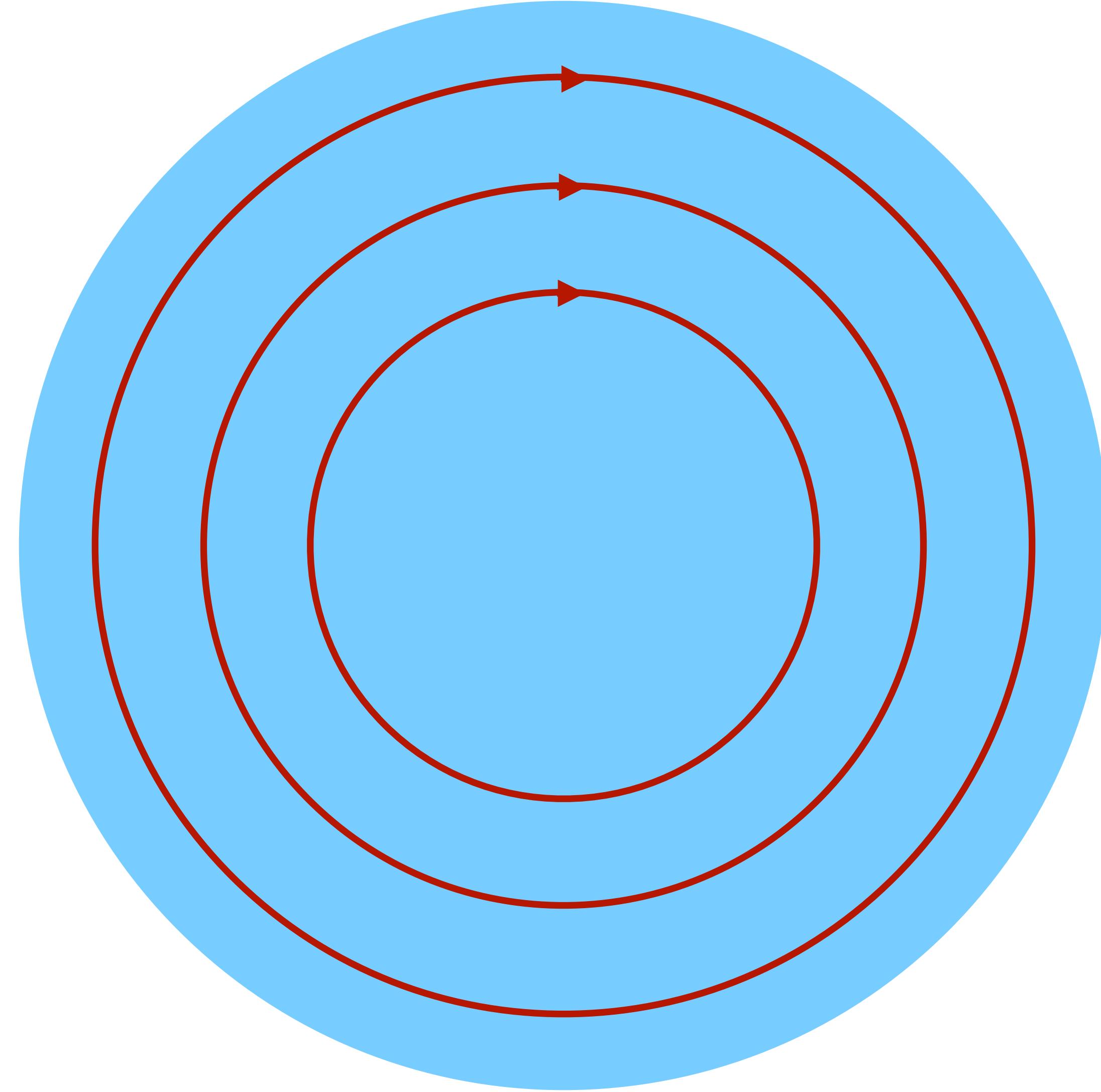
*furthermore, this tool could be useful
for studying the adiabatic evolution of the system*

consider for instance



gas
star (or dark matter)

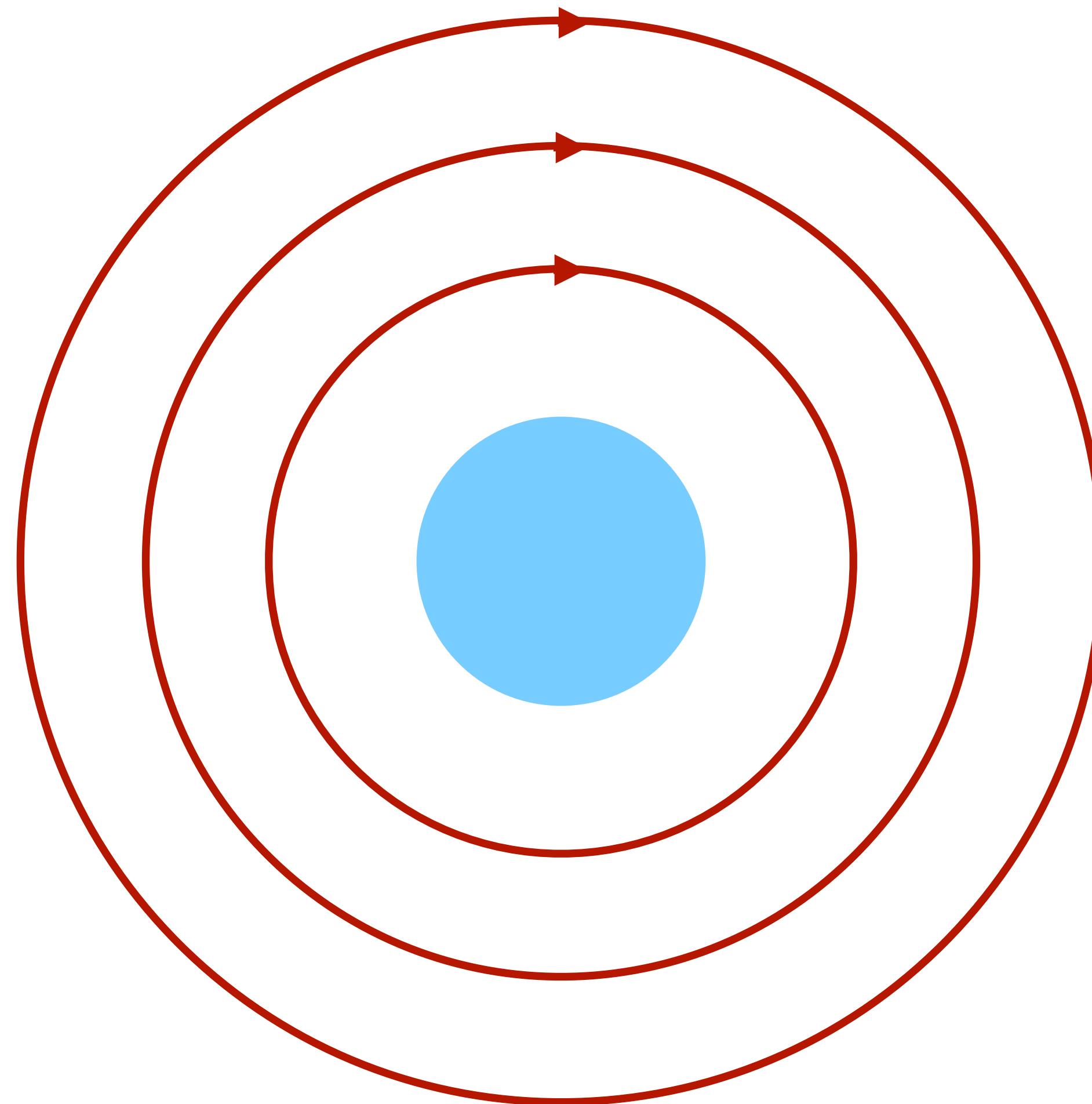
*now imagine gas cloud contracts adiabatically
due to some cooling process*



$$L = rv = \sqrt{GM_{\text{enc}}(r)r}$$

gas
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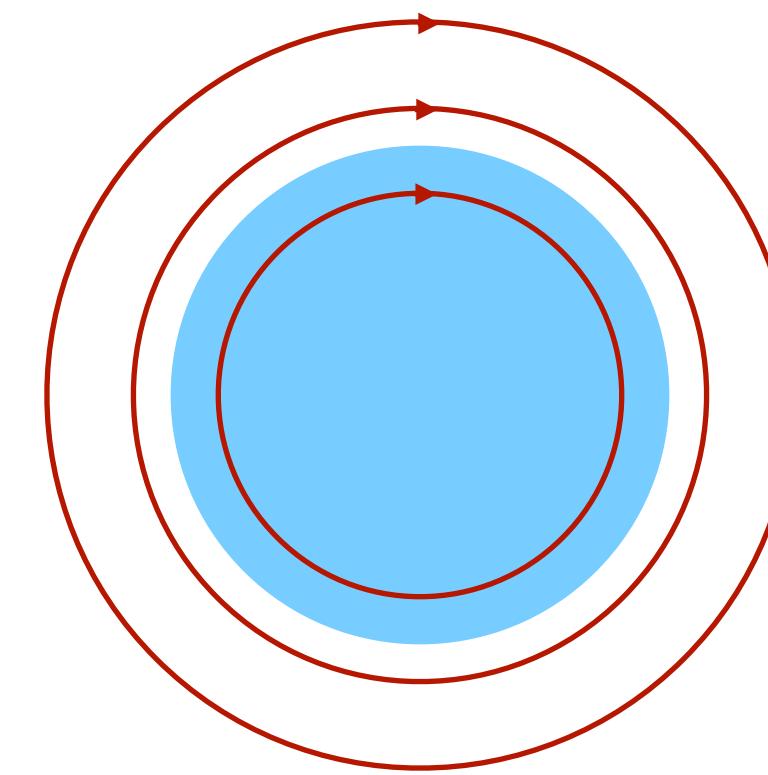
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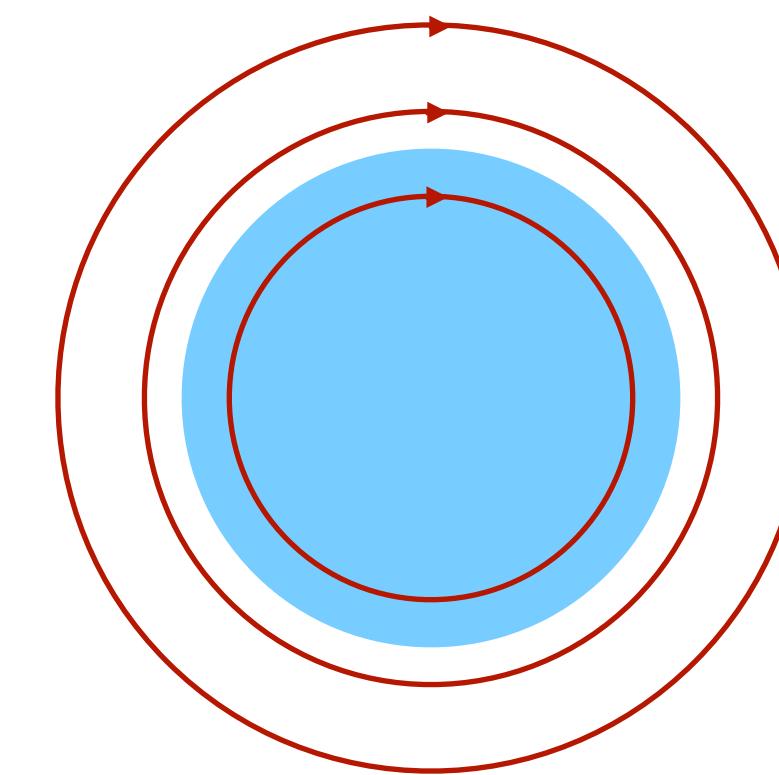
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[Young (90)]
[Blumenthal et al (86)]
[Quinlan et al (95)]
[ven der Marel (99)]

*stellar distribution becomes steeper
as a result of adiabatic evolution of gas (or black hole)!*

$$L = rv = \sqrt{GM_{\text{enc}}(r)r}$$

gas
star (or dark matter)

*dark matter could also contract
in response to the adiabatic evolution of the system*

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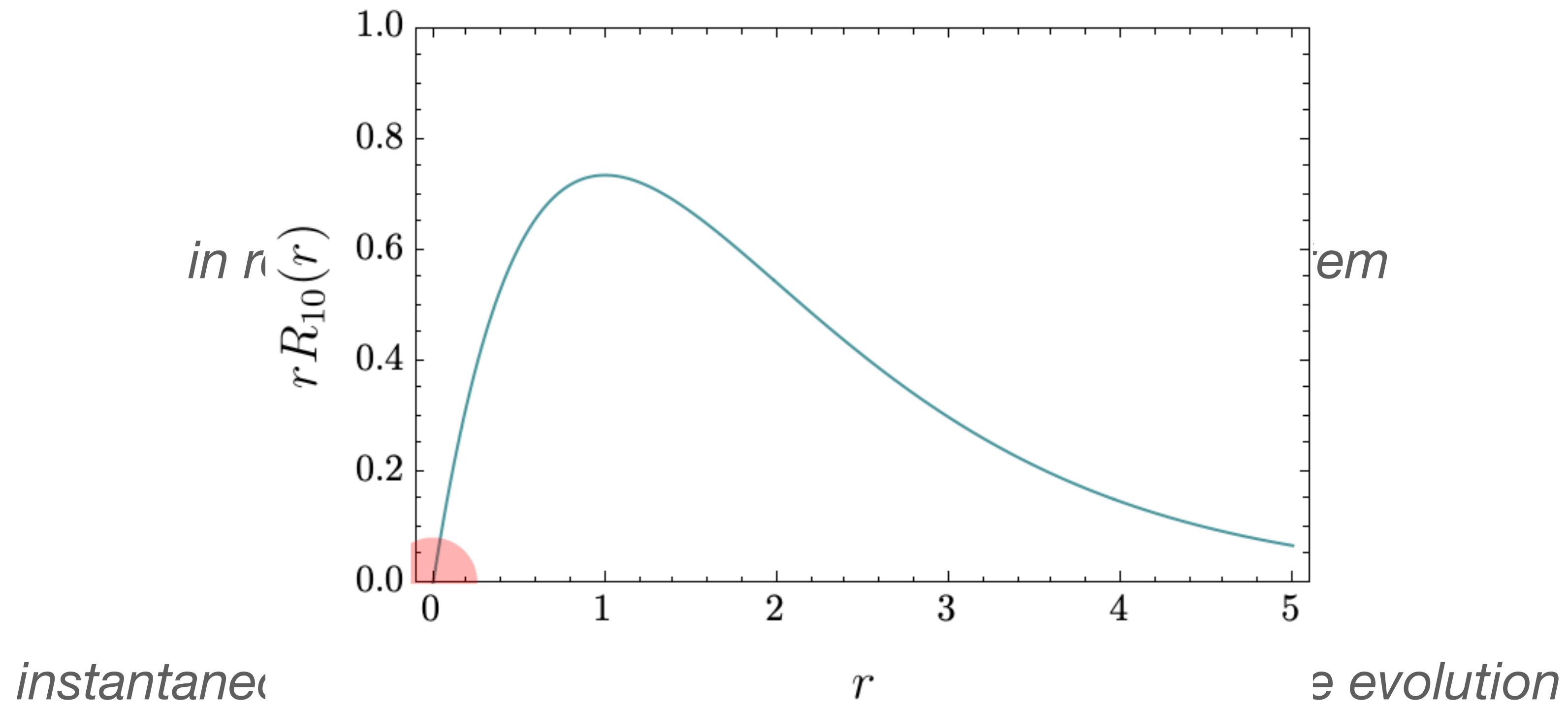
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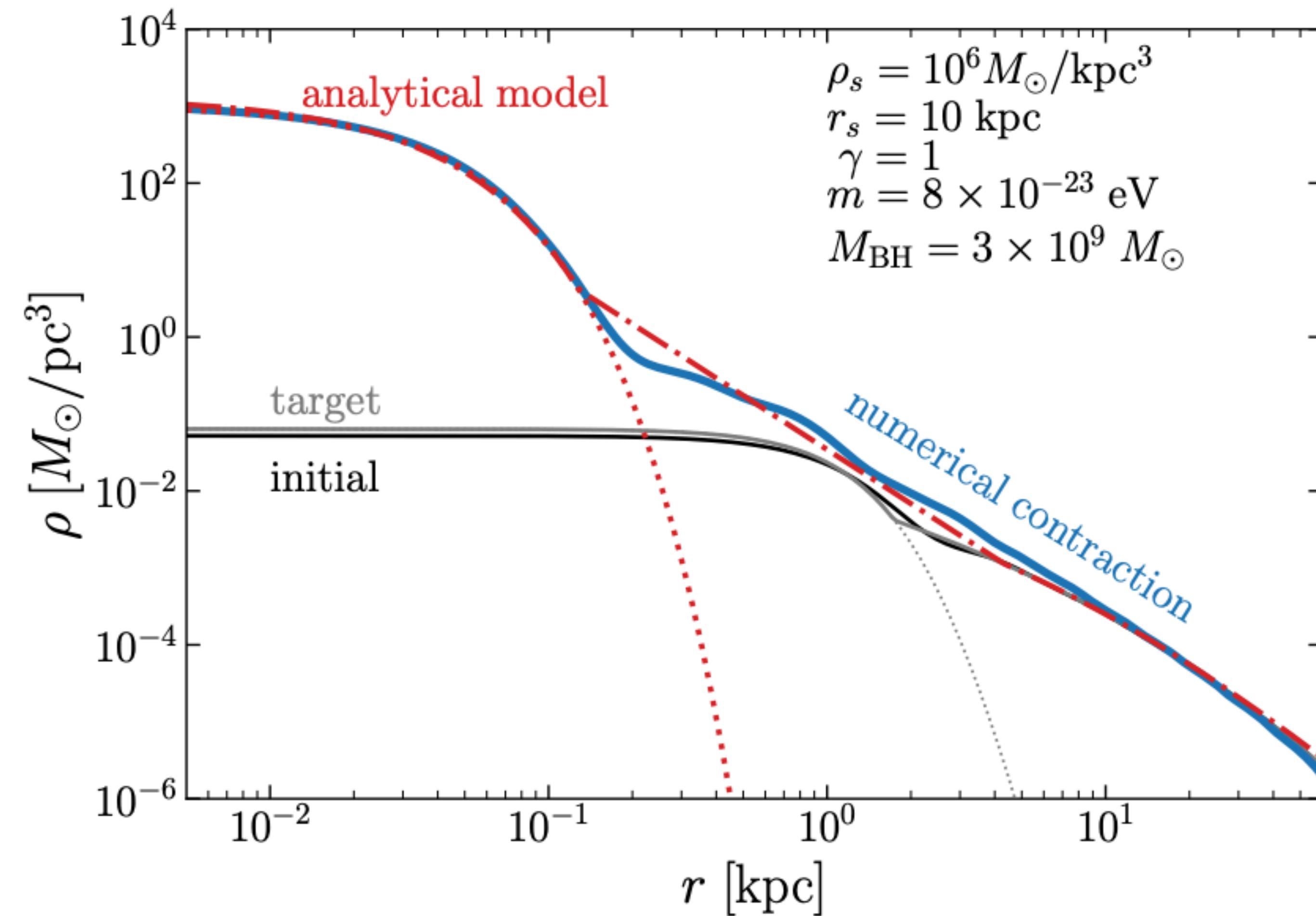
changes adiabatically
stays the same



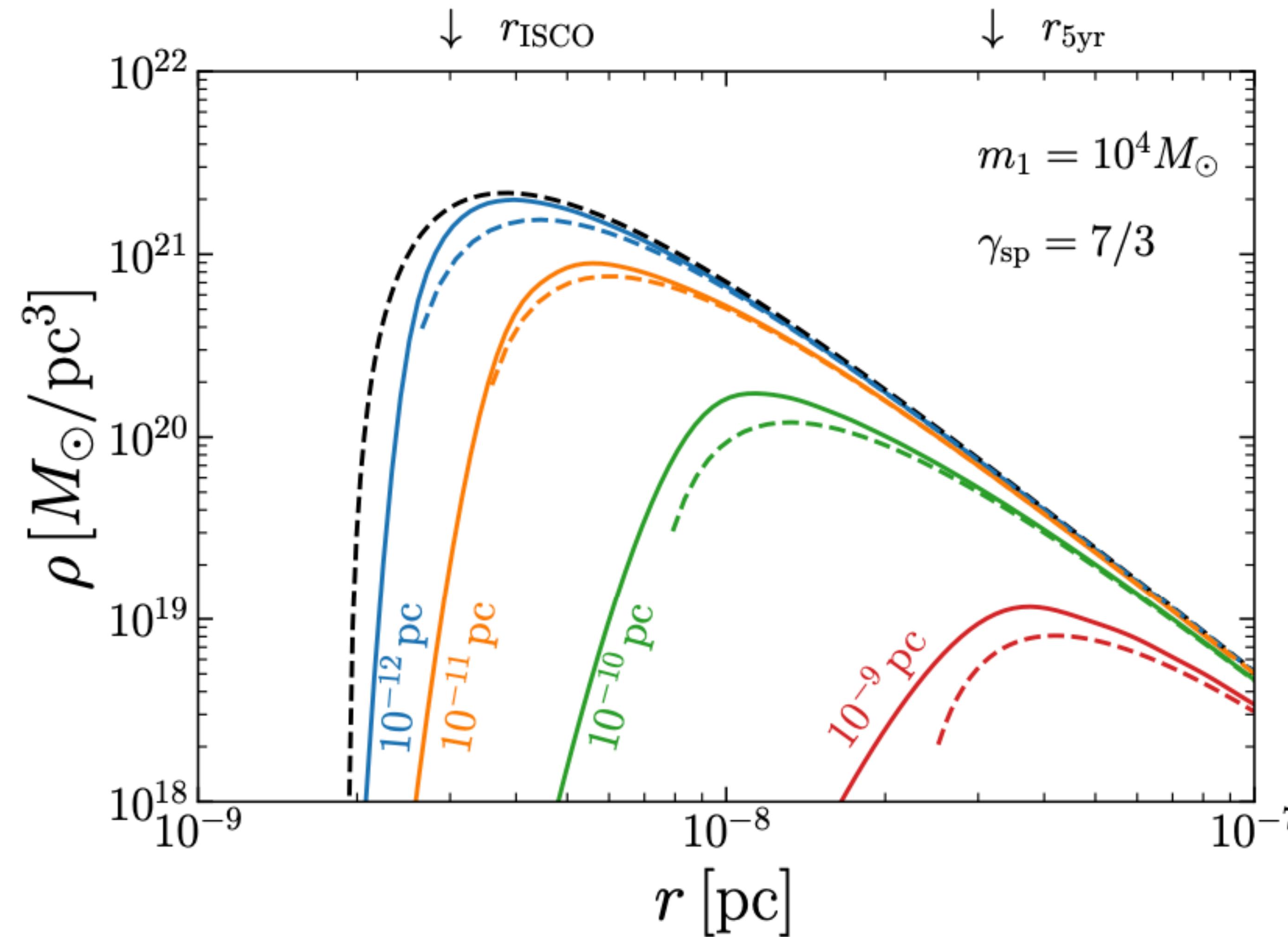
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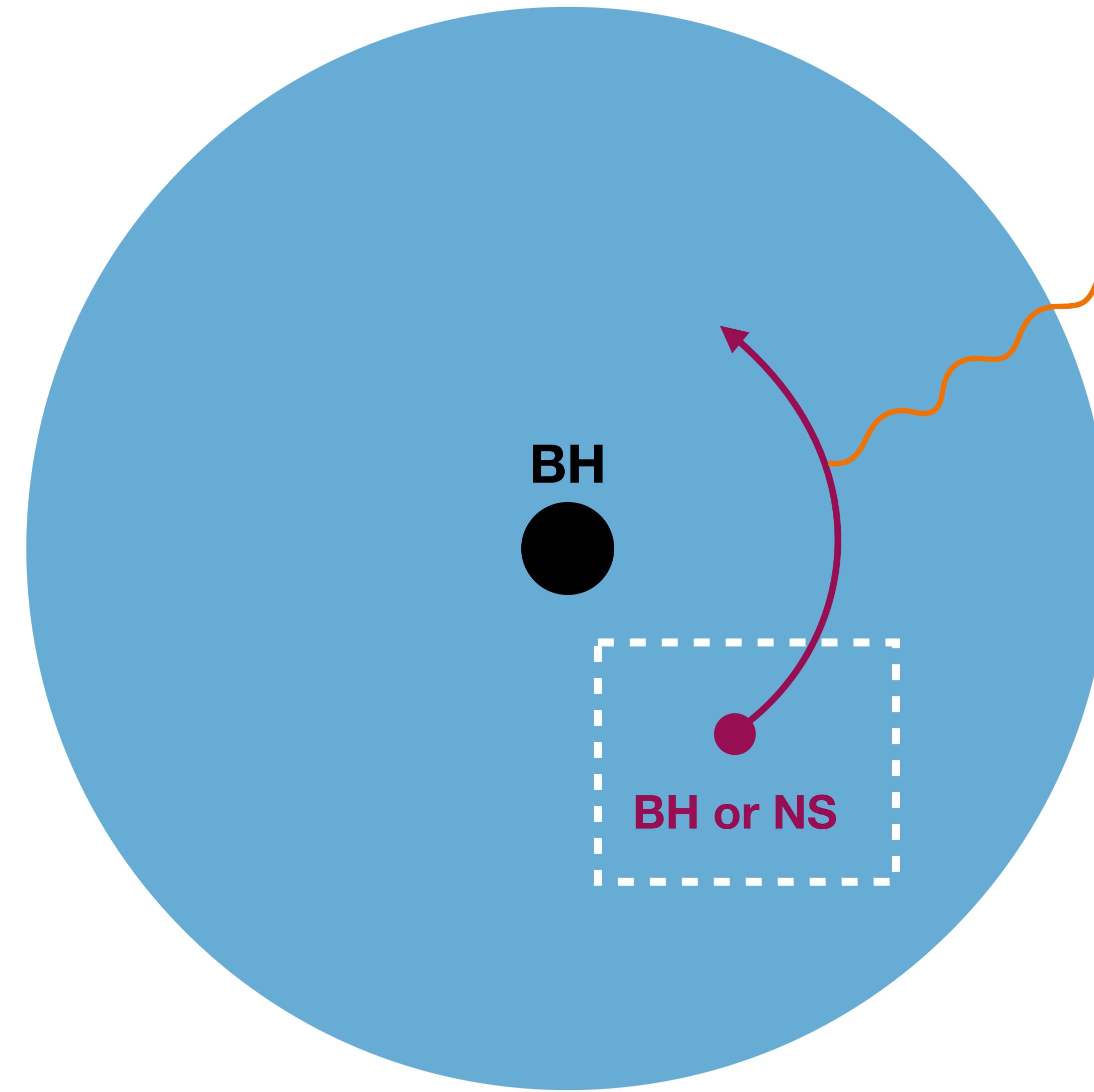
stays the same



or if the central soliton & low angular moments are shallowed
by the black hole over $t \sim 13$ Gyr

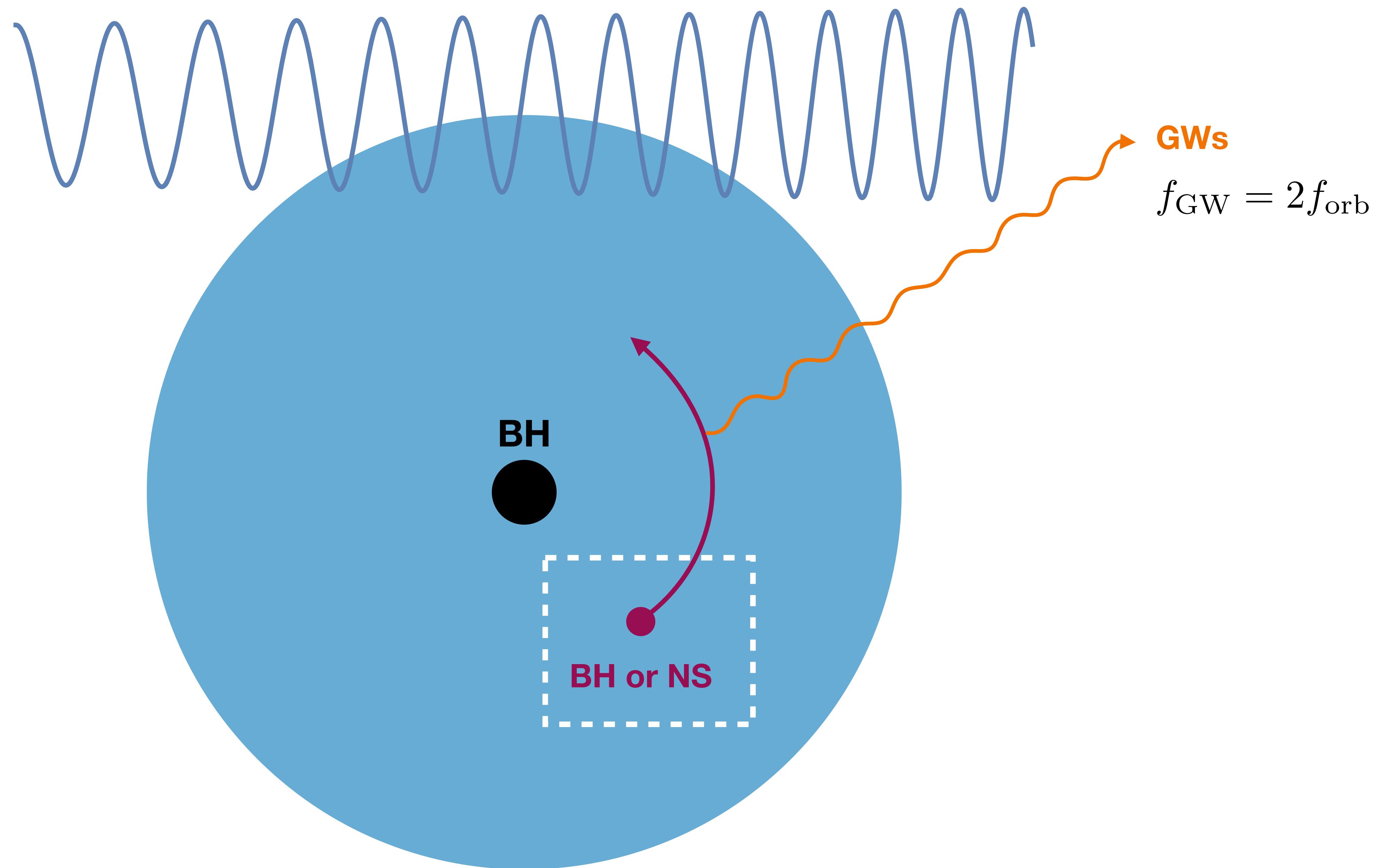


*an interesting way to test this compressed wave DM halo
is through the observation of GWs*

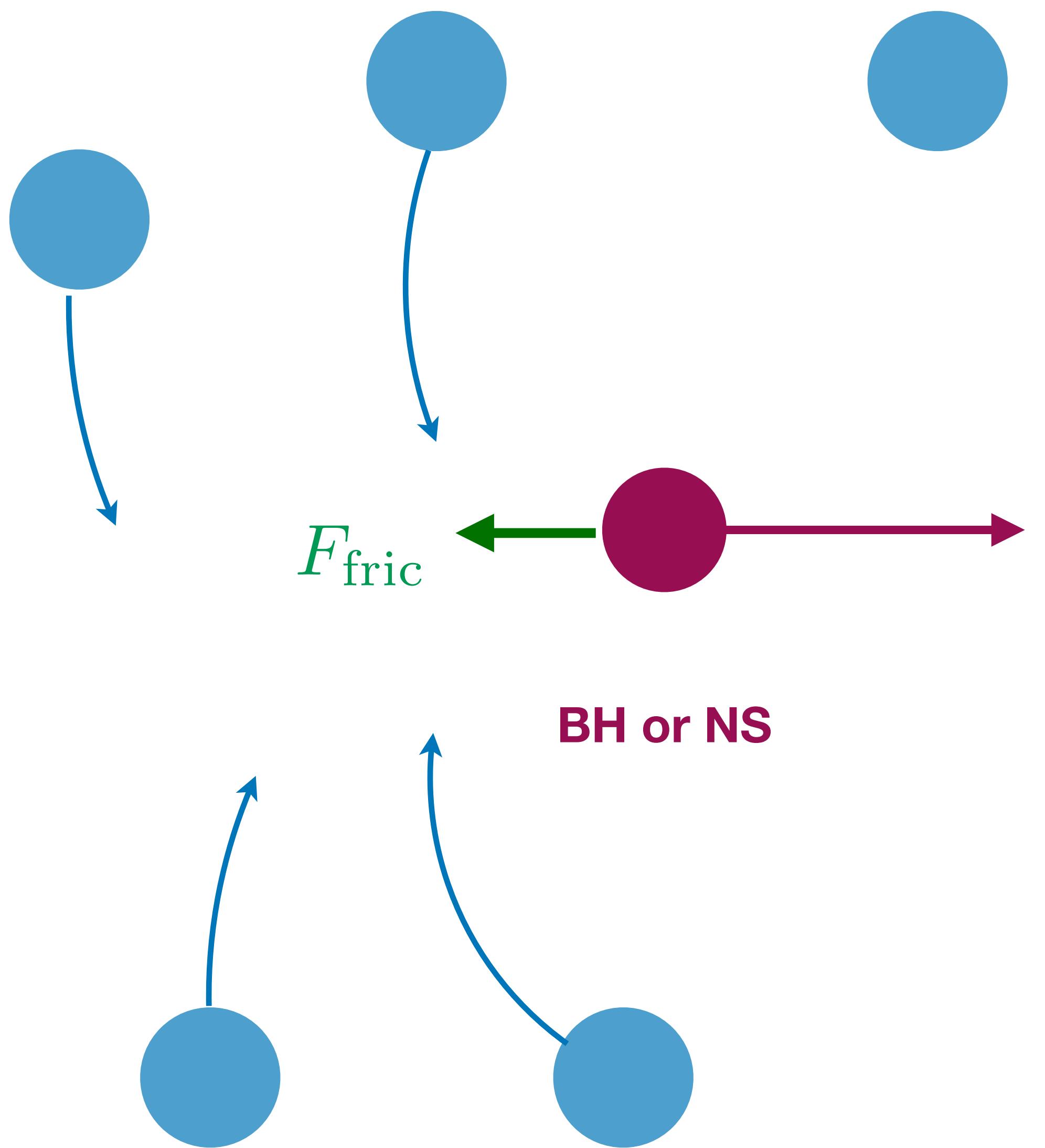


Adiabatically compressed DM halo

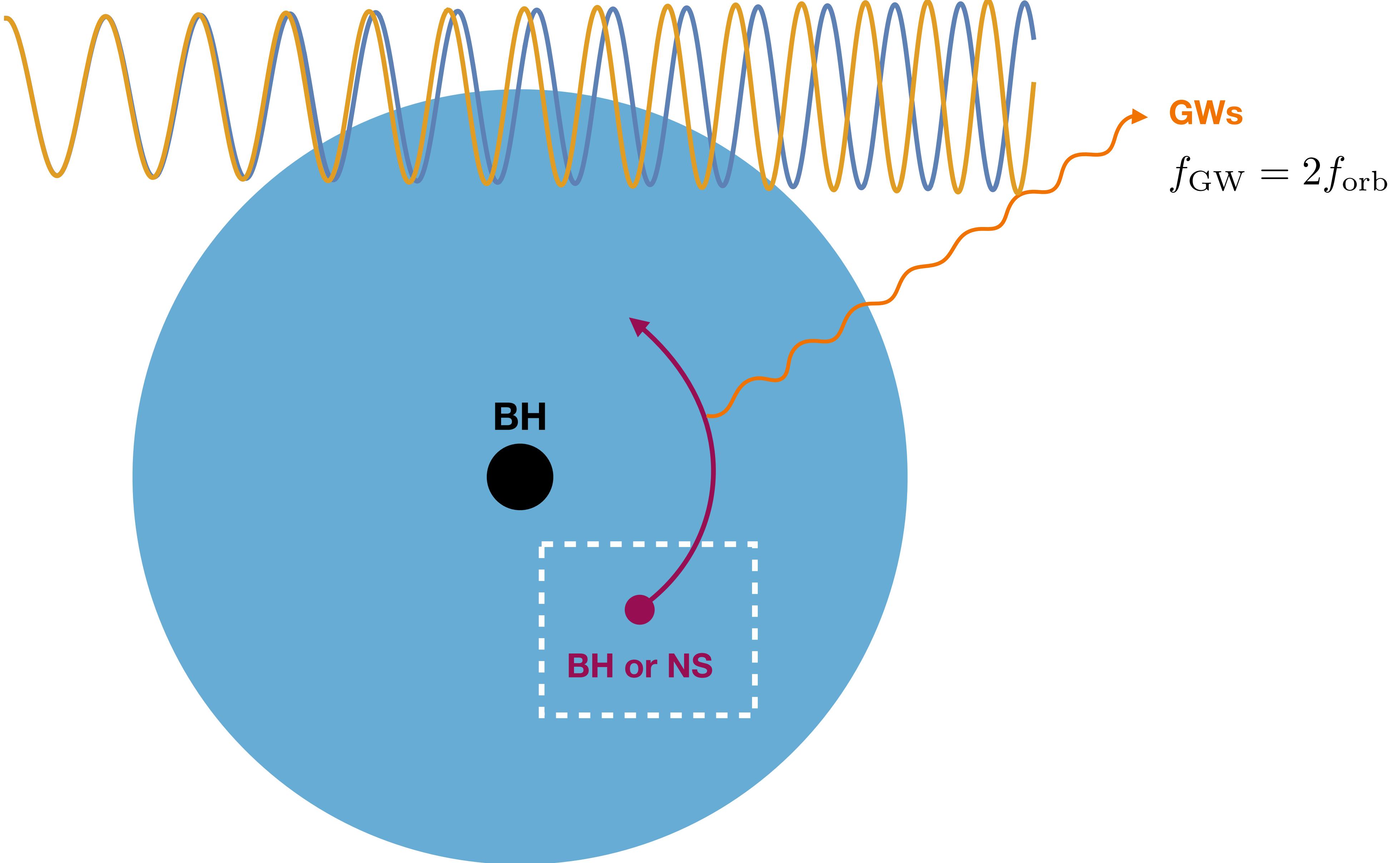
$$f_{\text{GW}} = 2f_{\text{orb}}$$



Adiabatically compressed DM halo



DM overdensity forms
behind the BH or NS
exerting additional frictional force



Adiabatically compressed DM halo

$$m = 10^{-13} \text{ eV}$$

[HK, Lenoci, Stomberg, Xue (22)]

