MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2020/2021 ACADEMIC YEAR

THIRD YEAR **SECOND**SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, MATHEMATICS & ECONOMICS, MATHEMATICS & COMPUTER SCIENCE AND APPLIED STATISTICS

AMS 313 –TEST OF HYPOTHESIS

DURATION: 2 HOURS

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a. Differentiate between type I and type II errors as used in tests hypothesis (2 marks)

b. State two types of statistical hypothesis (2 marks)

c. Briefly explain the term uniformly most powerful test (2 marks)

d. For a normal distribution with mean μ and variance $\sigma^2 = 5$, an experimenter wishes to test $H_0 : \mu = 7$ versus $H_a : \mu > 7$. What is the smallest sample size n such that $\alpha = 0.05$ level test has power at least 0.80 when alternative $\mu = 8$? (6 marks)

e. The accompanying table gives the scores of a group of 15 students in mathematics and statistics

Student	Maths	Stats
1	22	53
2	27	68
3	36	42
4	38	49
5	42	51
6	58	65
7	58	51
8	60	71
9	62	55
10	65	74
11	66	68
12	56	64
13	66	67
14	67	73
15	62	65

Using Wilcoxon's signed – rank test, determine if the locations of the distribution of scores for these students differ significantly for the two subjects. Give bounds for the p-value and indicate the appropriate conclusion with $\alpha = 0.01$ (6 marks)

f. A political scientist wishes to examine the relationship of the voter image of a conservative's political candidate and the distance in miles between the residence of the voter and the residence of the candidate. Each of 10 voters rated the candidate on a scale of 1 to 20. The resulting data are shown in the following table.

Voter	Rating	Distance
1	15	75
2	7	165
3	5	300
4	19	15
5	17	180
6	12	200
7	9	120
8	18	60
9	3	300
10	8	200

i. Calculate the correlation coefficient, r. (6 marks)

ii. Do these data provide sufficient evidence to indicate a negative correlation between rating and distance at $\alpha = 0.01$? (6 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a. Define the power of hypothesis test (2 marks)

b. A random sample of 100 observations of breaking force is available. Assume that these observations follow a $N(\mu,800)$ distribution of interest is the null hypothesis $H_0 : \mu = 56$ versus the alternative $H_a : \mu > 56$. The rejection rule is being considered as reject H_0 if $\overline{X} = 60$.

i. What is the probability of commuting type I error (4 marks)

ii. If, in fact $\mu = 62$, find the power of the test (4 marks)

c. Let $Y_1, Y_2, ..., Y_{30}$ be a random sample of size $n = 30$ from a normal distribution with unknown mean μ and unknown variance $\sigma^2 = 36$. We wish to test $H_0 : \mu = 12$ versus $H_a : \mu > 12$. Find the uniformity most powerful test with significant level of $\alpha = 0.05$ (10 marks)

QUESTION THREE (20 MARKS)

a. State the Neyman-Pearson Lemma giving precisely what kind of test it is applied on Test Hypotheses (4 marks)

b. Let $f(x_i|\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x_i - \mu)^2}{2\sigma^2}\right\}$ for $i = 1,2,...,n$, where σ^2 is known. Derive the likelihood ratio test for the hypothesis $H_0 : \mu = \mu_0$ vs $H_0 : \mu > \mu_0$ (16 marks)

QUESTION FOUR (20 MARKS)

a. Let X have a Poisson distribution with mean θ.

i. Use the sequential probability ratio test for testing $H_0 : \theta = 0.02$ against $H_1 : \theta = 0.07$ to show that this test can be based upon the statistic $\sum_i^i x_i$ (3 marks)

ii. If $\alpha_a = 0.20$ and $\beta_a = 0.10$, find $C_0(n)$ and $C_1(n)$ (4 marks)

b. For a comparison of the academic effectiveness of two junior high schools A and B, an experiment was designed using ten sets of identical twins, each twin having just completed the sixth grade in each case, the twins in the same set had obtained their previous schooling in the same classrooms at each grade level. One child was selected at end of the ninth grade, an achievement test was given to each child in the experiment. The results are shown in the accompanying table

Twin Pair	A	B
1	67	39
2	80	75
3	65	69
4	70	55
5	86	74
6	50	52
7	63	56
8	81	72
9	86	89
10	60	47

Using the sign test, test the hypothesis that the two schools are the same in academic effectiveness, as measured by scores on the achievement test, against the alternative that the schools are not equally effective. What would you conclude with $\alpha = 0.05$? (5marks)

Given the five data points in the accompanying table.

y	3.0	2.0	1.0	1.0	0.5
x	-2.0	-1.0	0.0	1.0	2.0

Do the data present sufficient evidence to indicate that the slope β_1 differs from zero? (Test at the 5% significance level) (8 marks)