Theory Questions

Question 1 : Color Theory

- Normalized Chromaticity coordinates of

P1 : $X_{P1C} = \frac{X_1}{X_1+Y_1+Z_1}$; $Y_{P1C} = \frac{Y_1}{X_1+Y_1+Z_1}$; $Z_{P1C} = \frac{Z_1}{X_1+Y_1+Z_1}$ — ①

P2 : $X_{P2C} = \frac{X_2}{X_2+Y_2+Z_2}$; $Y_{P2C} = \frac{Y_2}{X_2+Y_2+Z_2}$; $Z_{P2C} = \frac{Z_2}{X_2+Y_2+Z_2}$ — ②

P3 : $X_{P3C} = \frac{X_3}{X_3+Y_3+Z_3}$; $Y_{P3C} = \frac{Y_3}{X_3+Y_3+Z_3}$; $Z_{P3C} = \frac{Z_3}{X_3+Y_3+Z_3}$ — ③

- Normalized Chromacity coordinates of C in terms of chromacity coordinates of P1, P_2 & P_3

C : $X_c = \frac{X}{X+Y+Z}$, $Y_c = \frac{Y}{X+Y+Z}$, $Z_c = \frac{Z}{X+Y+Z}$

$\because$ C can be expressed as a linear combination of P1, P2 & P3, so

$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$
$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$
$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$

$\therefore \Rightarrow \alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)$

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- Using ①, ② & ③ we get ⇒

$X_1 = (x_1 + y_1 + z_1) \, x_{p1c}$

$X_2 = (x_2 + y_2 + z_2) \, x_{p2c}$

$X_3 = (x_3 + y_3 + z_3) \, x_{p3c}$

$$\Rightarrow x_c = \frac{\left(\alpha_1 (x_1 + y_1 + z_1) x_{p1c} + \alpha_2 (x_2 + y_2 + z_2) x_{p2c} + \alpha_3 (x_3 + y_3 + z_3) x_{p3c}\right)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

$$\beta_1 = \frac{\alpha_1 (x_1 + y_1 + z_1)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

$$\beta_2 = \frac{\alpha_2 (x_2 + y_2 + z_2)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

$$\beta_3 \cancel{\alpha} = \frac{\alpha_3 (x_3 + y_3 + z_3)}{\alpha_1 (x_1 + y_1 + z_1) + \alpha_2 (x_2 + y_2 + z_2) + \alpha_3 (x_3 + y_3 + z_3)}$$

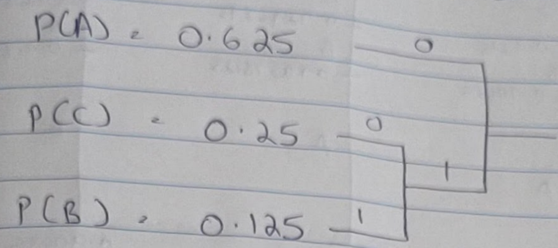
$$\therefore \quad x_c = \beta_1 x_{p1c} + \beta_2 x_{p2c} + \beta_3 x_{p3c}$$

|||ly ⇒ $y_c = \beta_1 y_{p1c} + \beta_2 y_{p2c} + \beta_3 y_{p3c}$

$\Rightarrow z_c = \beta_1 z_{p1c} + \beta_2 z_{p2c} + \beta_3 z_{p3c}$

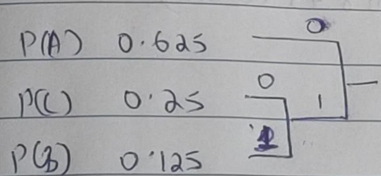
∴ Chromacity coordinates of any color C can be represented as a linear combination of chromacity coord. of resp. primaries

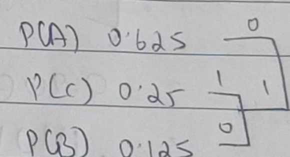
Question 2: General Compression

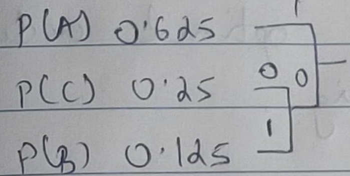


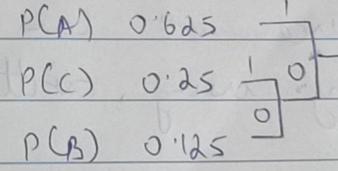
Symbol	Probability	Huffman code	Code length
A	0.625	0	1
B	0.125	10	2
C	0.25	11	2

Average code length = $0.625 \times 1 + 0.125 \times 2 + 0.25 \times 2 = 1.375$

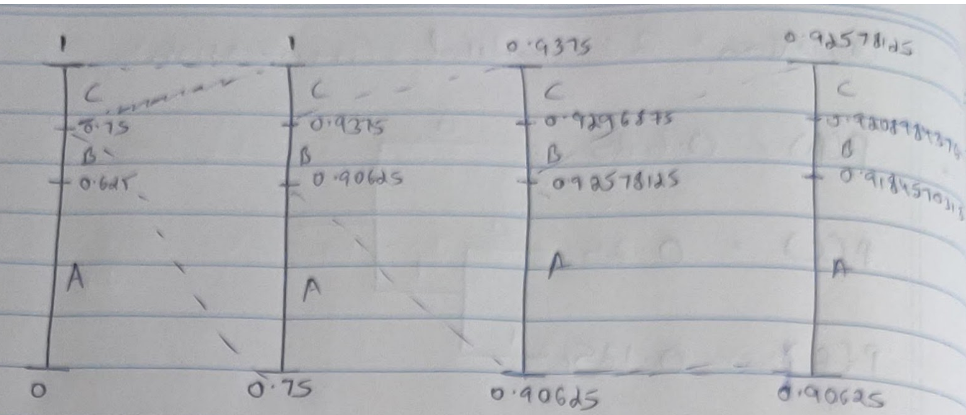








4 possible unique Huffman codes are possible



Consider an example for code : CBAA

Using Huffman code we get 111000
$\Rightarrow$ Code length = 6

Now using arithmetic coding we get

CBAA = [0.90625, 0.918457031)

= 0.90625

= .11101

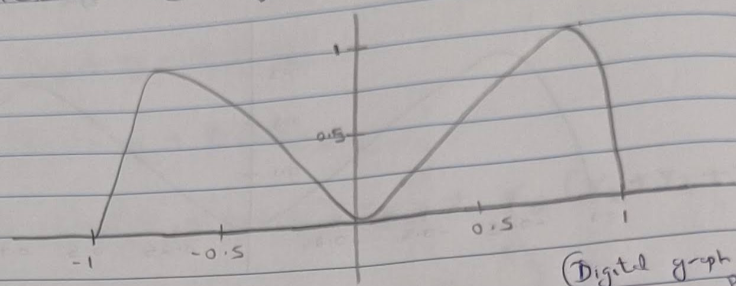
Code length = 5

Hence we can find more optimal codes by using arithmetic coding.

Reason : Huffman coding uses fixed codes while arithmetic coding doesn't.

Question 3 : Entropy Coding

- $H(x) = -(x^2 \log_2 x^2 + (1-x^2)\log_2(1-x^2))$



(Digital graph next page)

- When $x = 0$ or $x = \pm 1$, the Entropy is minimum

- Let $H'(x) = kx \log_2 (1 - x^k) - kx \log_2 x^k = 0$

we have $x = 0$ or $x = \pm \frac{1}{\sqrt{2}}$. Taking these

with boundary condition $x = \pm 1$ back to $H(x)$

we get $x = 0$ or $x = \pm 1$ for minimum.

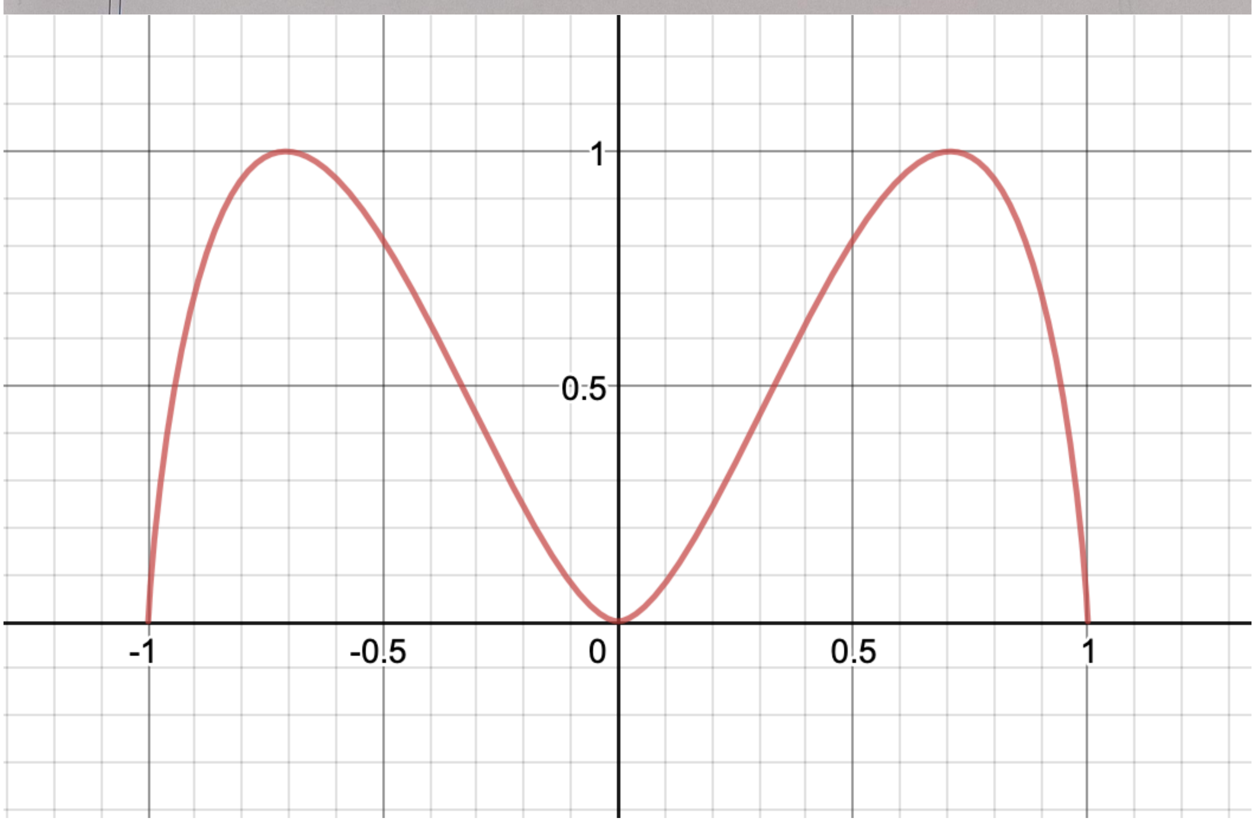
& $x = \pm \frac{1}{\sqrt{2}}$ for maximum.

When $x = \pm \frac{1}{\sqrt{2}}$, the entropy is maximum as found above. From graph, we get ± 0.707 as maximum value.

- As proved in ~~pen~~ previous question

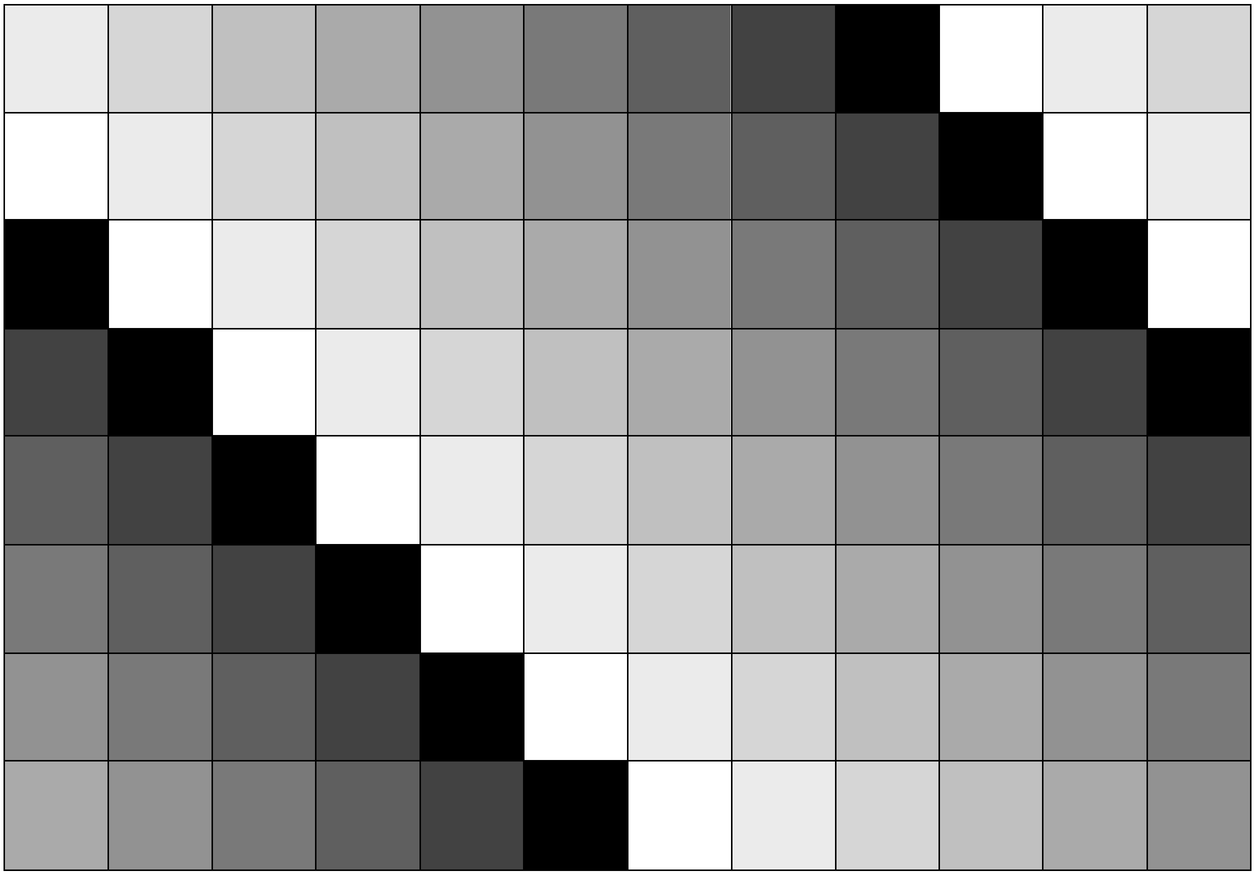
$H'(x) = kx \log_2 (1-x^k) - kx \log_2 x^k = \pm \frac{1}{\sqrt{2}}$

we can see that for $x = \pm \frac{1}{\sqrt{2}}$, $H(x)$ has maximum value.

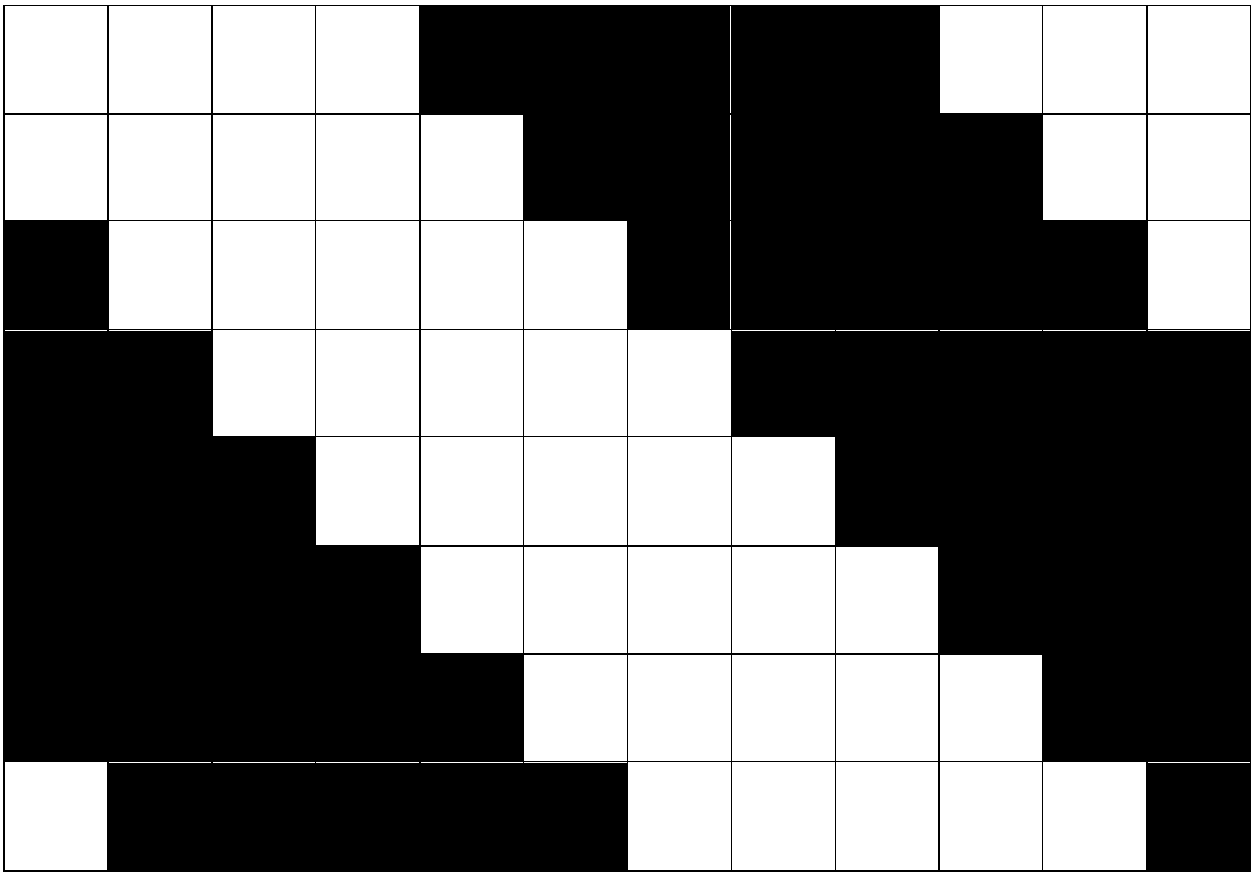


Question 4: Image Dithering

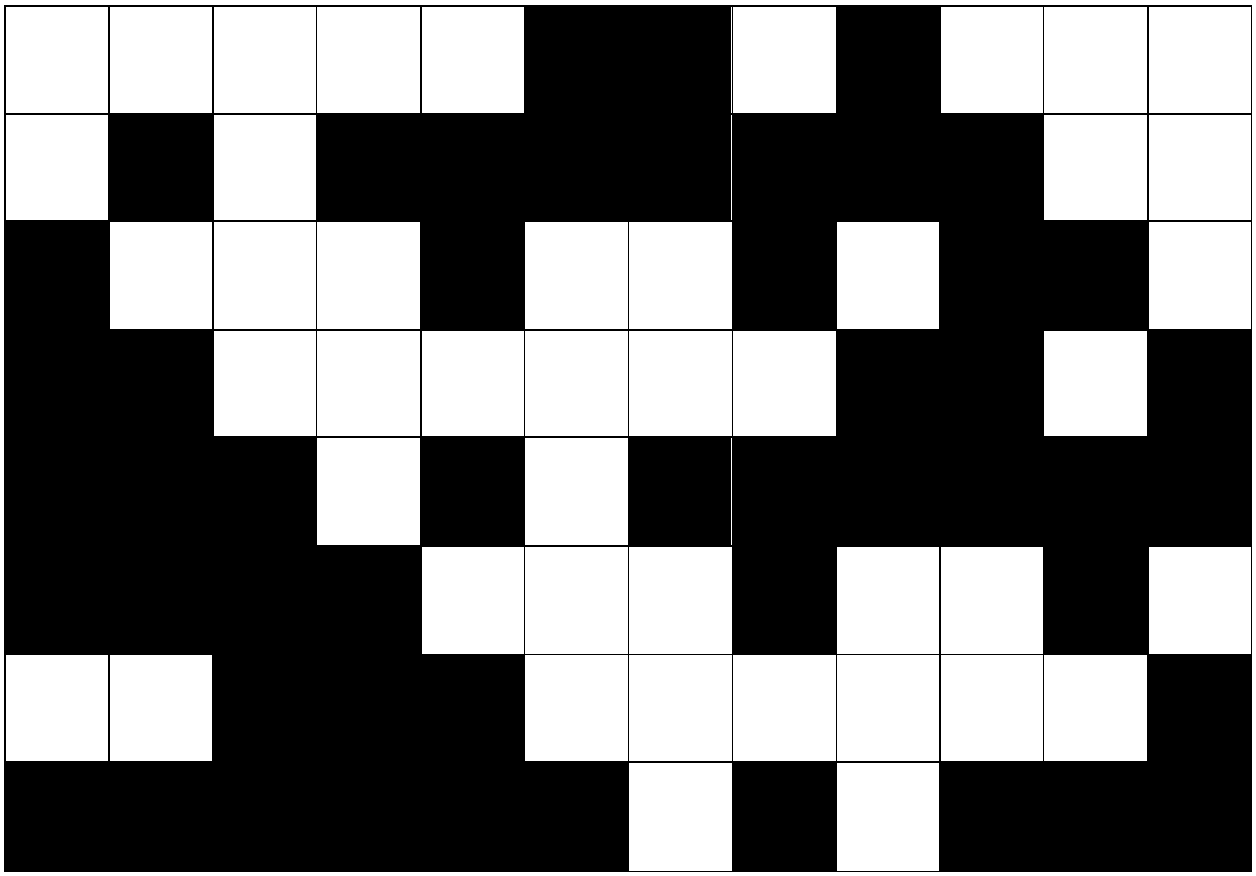
1.



2.



3.



4.