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STABL5...THE SPENCER METHOD OF SLICES: FINAL REPORT

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STABL5...THE SPENCER METHOD OF SLICES

by

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FINAL REPORT

STABL5...THE SPENCER METHOD OF SLICES

TO: H. L. Michael, Director
Joint Highway Research Project

August 28, 1985

File: 6-14-12

FROM: C.W. Lovell, Research Engineer
Joint Highway Research Project

Project: C-36-36L

The attached report is the final one for the JHRP project entitled "Incorporating Spencer's Method of Slices in Program STABL." The work was performed by J. R. Carpenter under the direction of Professor C. W. Lovell.

The Spencer version of STABL will be designated as either STABL5 (mainframe) or PC STABL5 (micro-computer). The Spencer version of STABL is more rigorous than that previously available in STABL4, and accordingly requires more running time.

The IDOH is encouraged to test the new program against STABL4, before adopting it for routine use.

Respectfully submitted,

C. W. Lovell /ncm

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STABL5 AND PCSTABL5

In order to increase the versatility of STABL, Spencer's method of slices has been implemented in the program. Spencer's method was chosen since it satisfies complete equilibrium of the sliding mass; i.e. equilibrium with respect to moment and force equilibrium. In contrast, the Simplified Janbu method satisfies only vertical and horizontal force equilibrium and not moment equilibrium, while the Simplified Bishop method satisfies only vertical force and overall moment equilibrium but not horizontal force equilibrium. These methods are easily solved and typically give conservative values for the FOS when compared to the more accurate methods of slices satisfying complete equilibrium such as the Spencer or Morgenstern and Price methods (Wright et. al., 1973; Sharma and Lovell, 1983).

Spencer's method of slices is especially well suited for handling horizontal or inclined loads such as tieback loads since the method satisfies complete equilibrium. Spencer's method distributes the force from a load such as a tieback between slices through the interaction of the interslice side forces. Therefore, there is no need to use a technique such as the Load Distribution Method to account for the presence of horizontal or inclined loads. The LDM was developed for use in conjunction with the Simplified Bishop or Simplified Janbu methods which do no consider the interaction of the interslice side forces.

Spencer's method is not used as frequently as the simplified methods since it requires more computation time and also since convergence of the solution is also often a problem. However, for problems with horizontal or inclined loads, Spencer's method is more appropriate than the simplified methods. Convergence problems have been avoided using the Linear Approximation Method of solution which will be described later.

The addition of Spencer's method complements the Simplified Janbu and Simplified Bishop methods existing in STABL. Implementation of this method allows the STABL user to search for critical potential failure surfaces using either the Simplified Janbu or Simplified Bishop methods and reanalyze any critical potential failure surface with Spencer's method to obtain a more accurate value of the FOS.

Stability Equations

Spencer (1967) developed a limiting equilibrium method of slices which satisfies complete equilibrium for circular failure surfaces assuming a constant ratio of the interslice normal and shear forces. This assumption leads to the formation of parallel interslice side forces inclined at a constant angle, θ, on each slice. Spencer (1973) found that a reasonably reliable value for the FOS can be obtained by assuming parallel interslice forces. The method was later extended to potential failure surfaces of a general or irregular shape (Wright, 1969; Spencer, 1973).

The slice forces considered in the derivation of Spencer's method of slices are shown in Figure 1. As with other limiting equilibrium methods, the factor of safety on each slice is assumed to be the same such that all slices of the the sliding mass will fail simultaneously. For all slices of a sliding mass to fail simultaneously, the load from one slice must be transmitted to the next slice through the interslice side forces. The interslice forces Z_l and Z_r are inclined from the horizontal at an angle θ. The interslice forces acting on both sides of each slice can be replaced with a single statically equivalent resultant interslice force, QF, acting through the midpoint of the base of the slice and inclined at an angle θ (Figure 2).

Summing the forces normal and tangential to the base of each slice provides two equations of force equilibrium:

$$\Delta N' + \Delta U_\alpha + QF\sin(\alpha-\theta) + \Delta W(k_h\sin\alpha - (1-k_v)\cos\alpha) - \Delta U_\beta\cos(\alpha-\beta) - \Delta Q\cos(\alpha-\delta) - \Delta T\sin(\alpha-i) = 0 \quad . . . \quad (1a)$$

$$\Delta S_r - QF\cos(\alpha-\theta) - \Delta W((1-k_v)\sin\alpha - k_h\cos\alpha) + \Delta U_\beta\sin(\alpha-\beta) + \Delta Q\sin(\alpha-\delta) + \Delta T\cos(\alpha-i) = 0 \quad . . . \quad (1b)$$

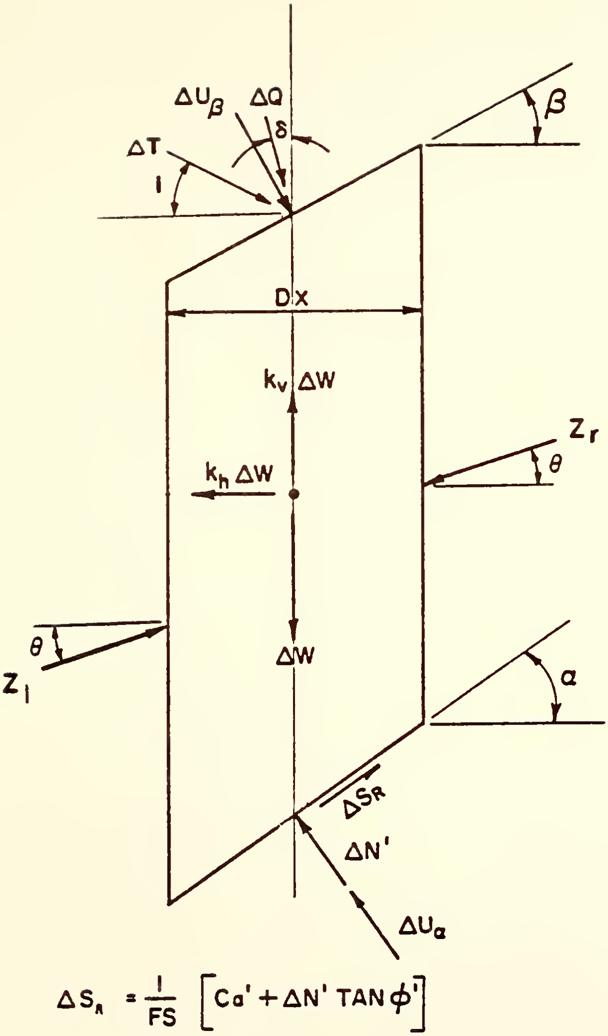


FIGURE 1. Slice Forces Considered for Spencer's Method of Slices

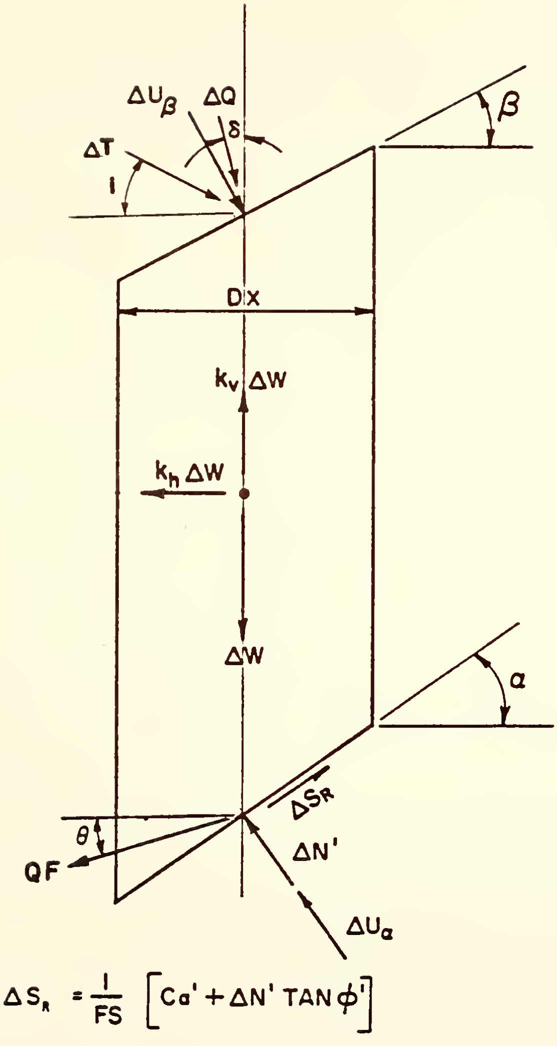


FIGURE 2. Slice Forces Considered in Derivation of Stability Equations for Spencer's Method of Slices

The expression for the effective normal force on the base of each slice may be obtained from equation 1a.

$$\Delta N' = \Delta W((1-k_v)\cos\alpha - k_h\sin\alpha) - \Delta U_\alpha + \Delta U_\beta\cos(\alpha-\beta)$$
$$+ \Delta Q\cos(\alpha-\delta) - QF\sin(\alpha-\theta) + \Delta T\sin(\alpha-i) \quad . \; . \; . \quad (2)$$

The expression for the mobilized resisting shear force at the base of a slice is given by:

$$\Delta S_r = \frac{[C_a' + \Delta N'\tan\phi']}{FS} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where:

FS = Factor of safety: assumed equivalent on all slices

C_a' = Cohesion force = $c' \cdot (dx)/\cos\alpha$

$\Delta N'$ = Effective normal force acting on the base of a slice

ϕ' = Effective angle of shearing resistance

Substituting equation 2 for the effective normal force into the expression for the resisting shear force at the base of each slice (Eqn. 3), and substituting the resulting expression into equation 1b yields the expression for the resultant of the interslice side forces on each slice:

$$QF = \frac{[S_1/FS + S_2]}{\cos(\alpha-\theta)[1 + S_3/FS]} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

where:

$$S_1 = C_a' + \tan\phi'[\Delta W((1-k_v)\cos\alpha - k_h\sin\alpha) - \Delta U_\alpha + \Delta U_\beta\cos(\alpha-\beta) + \Delta Q\cos(\alpha-\delta) + \Delta T\sin(\alpha-i)]$$

$$S_2 = \Delta U_\beta\sin(\alpha-\beta) - \Delta W((1-k_v)\sin\alpha + k_h\cos\alpha) + \Delta Q\sin(\alpha-\delta) + \Delta T\cos(\alpha-i)$$

$$S_3 = \tan\phi'\tan(\alpha-\Theta)$$

FS = Factor of safety: assumed equivalent on all slices

C_a' = Cohesion force = $c' \cdot (dx)/\cos\alpha$

If the overall moment produced about an arbitrary point by all external forces is zero, then the overall moment of the internal forces must also be zero, thus:

$$\sum_{i=1}^{n} QF[R\cos(\alpha-\Theta)] = 0 \quad \text{. . . . . (5)}$$

where R is the distance from the center of rotation about which moments are summed to the center of each slice. For circular potential failure surfaces, the value of R is constant and may be taken out of the summation:

$$\sum_{i=1}^{n} [QF\cos(\alpha-\Theta)] = 0 \quad \text{. . . . . (6)}$$

For surfaces of a general shape where no common axis exists, moments may be taken about a different axis for each slice in turn. It is often convenient to take moments about the center of the base of each slice for irregular surfaces rather than about an arbitrary center of rotation. The approach adopted in obtaining the equilibrium equations does not affect the final solution to a given problem (Spencer, 1970).

If overall force equilibrium is satisfied, then the summation of the internal forces in two mutually exclusive directions must be zero. Hence, for force equilibrium in the horizontal and vertical directions:

$$\sum_{i=1}^{n} [QF\cos\theta] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7a)$$

$$\sum_{i=1}^{n} [QF\sin\theta] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7b)$$

The inclination of the resultant side forces can be expressed as:

$$\theta_i = \theta\, f(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

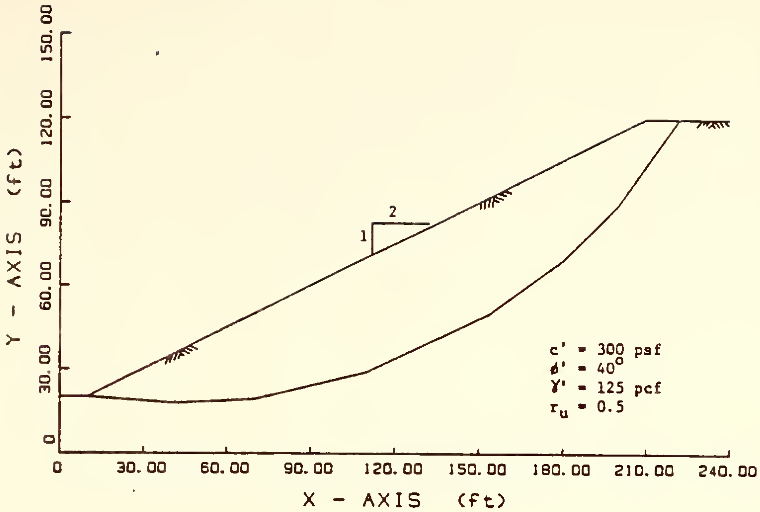
where θ is a scaling angle of inclination and $f(x)$ is an arbitrary function which defines how θ_i varies with the x position of a slice. Parallel side forces occur when $f(x) = 1$ for all values of x. The assumption of parallel resultant side forces is equivalent to the Morgenstern and Price Method (1965) when $f(x) = 1$, thus making Spencer's $\tan\theta$ equivalent to Morgenstern and Price's λ (Spencer, 1973).

If the slope of the resultant interslice side forces is assumed to be parallel; i.e., θ_i = constant, equations 24a and 24b become identical and can be expressed as:

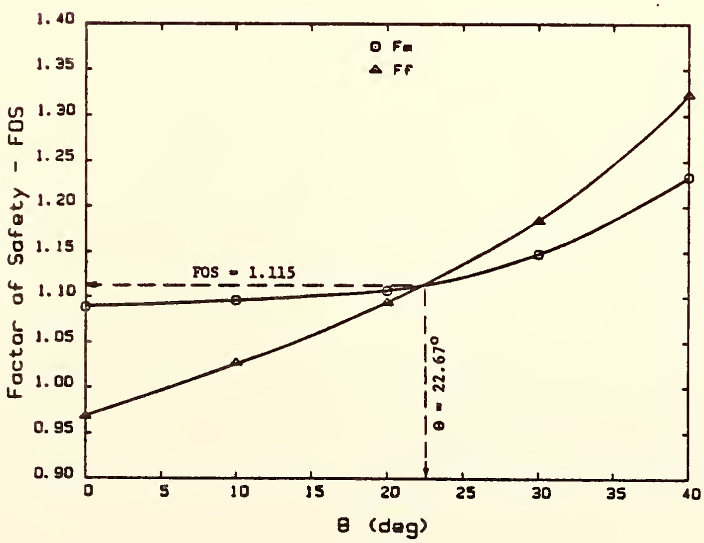
$$\sum_{i=1}^{n} [QF] = \emptyset \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Two factors of safety are obtained when equations 5 and 9 are solved assuming a value of θ. Equation 5 yields a FOS satisfying moment equilibrium (F_m), while equation 9 yields a FOS satisfying force equilibrium (F_f). There is a unique value of the FOS and corresponding value of θ which satisfies both force and moment equilibrium (Figure 3b). Equations 5 and 9 are solved using values of θ until F_m and F_f are equal corresponding to equilibrium of forces and moments for the sliding mass.

It should be noted that at $\theta = \emptyset$, the FOS with respect to moment equilibrium (F_m) corresponds to the Simplified Bishop FOS, while the FOS with respect to force equilibrium (F_f) corresponds to the Simplified Janbu FOS. It can be seen from Figure 3b that the FOS with respect to moment equilibrium (F_m) is much less sensitive to the side force assumption (value of θ), than the FOS with respect to force equilibrium (F_f). From this figure it can also be seen that the Simplified Bishop FOS yields rather accurate values of FOS when compared to complete equilibrium methods. This is



(a)



(b)

FIGURE 3. Variation of F_m and F_f with θ

due to the insensitivity of the F_m curve to the assumption of the slope of the interslice forces.

Reasonableness of the solution can be judged by examining the position of the line of thrust and the magnitude of the interslice shear stresses. Both are obtained from the moment equilibrium equations for the individual slices. This topic will be addressed later in this chapter.

STABL Method of Solution - Linear Approximation Method

Numerous iterative schemes have been used to solve for the FOS satisfying complete equilibrium. It is important to use an iterative scheme that readily converges and also minimizes the number of iterations required to produce a solution. Techniques have included: 1) mathematically sophisticated analyses such as the Newton-Raphson numerical technique (Wright, 1969; Boutrup, 1977); 2) arbitrarily assuming several values of θ, calculating the corresponding values of F_m and F_f, and using a regression analysis to find the intersection of the F_m and F_f curves (Fredlund, 1974); 3) assuming a value of θ, calculating F_m and F_f, and choosing a new value of θ based on the relative magnitude of F_m and F_f for a given θ (Fredlund, 1981); and 4) assuming a value of θ and FOS, calculating F_f, setting F_m equal to F_f, solving for the new value of θ, and substituting that value into the F_m equation (Spencer, 1973; Maksimovic, 1979).

None of the iterative schemes outlined above complemented the routines already present in STABL. Therefore, a new iterative method has been developed which rapidly and accurately determines the FOS satisfying complete equilibrium while avoiding problems of non-convergence. The new method is called the Linear Approximation Method (LAM), and utilizes the INTSCT routines in STABL which calculate the intersection of two straight lines. The method uses values of F_m, F_f and θ to approximate the F_m and F_f curves with straight lines and calculates their intersection. An accurate value of the FOS satisfying complete equilibrium is obtained by successive approximations of the F_m and F_f curves with straight lines for several values of θ. Due to the shape of the F_m and F_f curves, convergence is rapid and often occurs within three iterations. Unlike some of the iterative techniques outlined previously, the LAM is easily comprehended and minimizes the number of iterations required for solution.

Equations 4 and 8 are first solved with initial estimates of θ and FOS. The initial value of θ is taken as one half the approximate slope angle, which is input by the user. Spencer (1967) found that the angle of the resultant interslice side forces satisfying complete equilibrium was less than the slope angle. Therefore, STABL utilizes a user input estimate of the slope angle to begin iteration for the FOS and corresponding angle of the resultant interslice side forces. The solution is not sensitive to the value input for the slope angle; however, a reasonable estimate will minimize iteration time. The initial estimate of the FOS is obtained by first calculating the FOS by either the Simplified Bishop or Simplified Janbu method depending on the type of analysis being performed.

Using the initial estimates of θ and FOS, equations 8 and 4 are solved for the sum of the resultant interslice forces and their corresponding moments. Based on the relative magnitude of the sum of the forces and the sum of the moments, a second value of FOS is chosen and the sum of the forces and moments are recalculated. The factor of safety satisfying force equilibrium for the given value of θ is found by calculating the intersection of the line through the sum of the forces previously calculated with the $\Sigma QF = 0$ axis as shown in Figure 4a. The value of F_f satisfying force equilibrium for the given θ is checked by using that value of FOS to calculate the sum of the forces. The value of F_f is recalculated using a straight line intersection of the two previous values of the sum of the forces with the $\Sigma QF = 0$ axis. Normally three trials are all that are required to find the value of F_f within a tolerance of 0.001. The value of F_m for a given θ is found in the same manner.

The calculation of the sum of the resultant interslice forces and moments for a given value of θ requires little

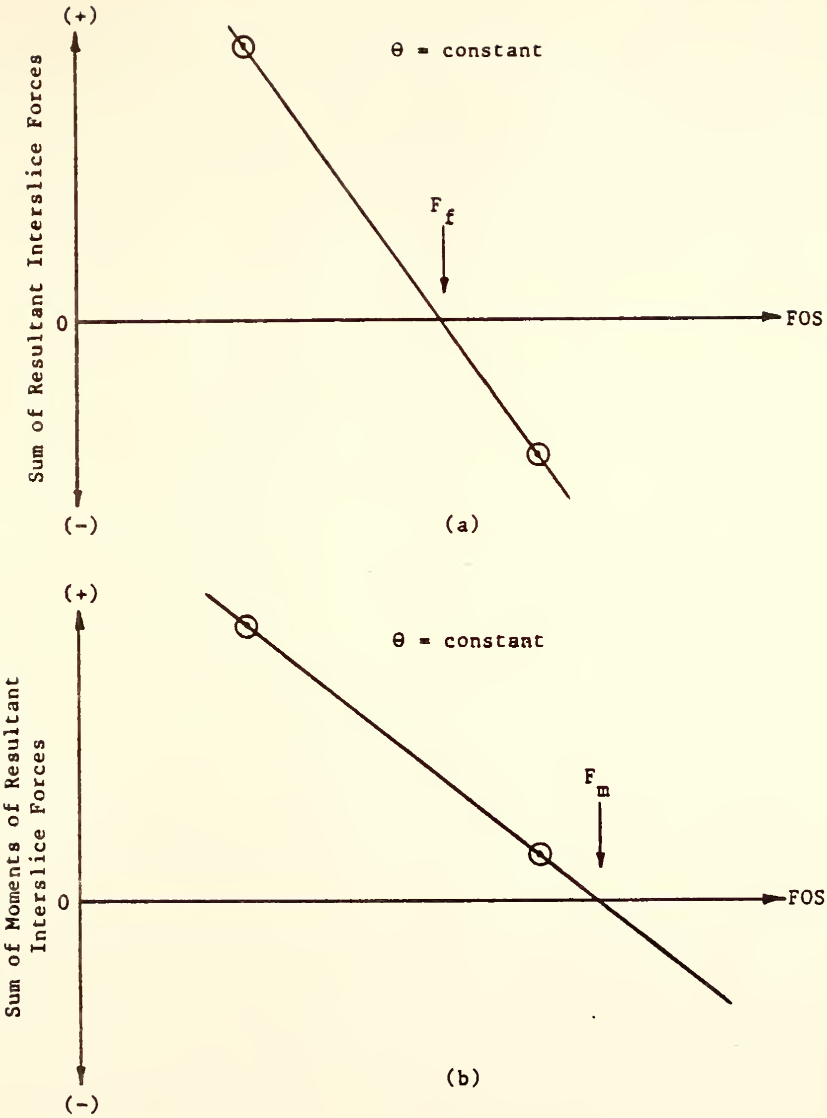


FIGURE 4. Determination of F_f and F_m for a Given Value of Θ by the Linear Approximation Method

calculation time. In addition, calculation of the intersection of two straight lines is simple and also requires very little computation time. The combination of these two facts leads to a very efficient procedure for accurately and rapidly determining the values of F_f and F_m.

A second value of θ is taken as three-fourths the input slope angle. The force and moment equations are again solved for new values of F_f and F_m corresponding to the new value of θ. A second value of θ equal to three-fourths the input slope angle was found to lead to rapid solution of the FOS by the LAM, and was thus chosen.

After two iterations, the F_m and F_f curves are approximated by straight lines and the intersection of the these lines is calculated ((θ_{int}, F_{int}); Figure 5a). It can be seen from Figure 5a that the intersection of the approximation of the F_f and F_m curves by straight lines leads to a very accurate estimate of the value of θ satisfying complete equilibrium and a rather good estimate of the FOS. The difference between θ_{int} and the nearest value of θ used (in this case θ_2) is calculated and another value of θ is selected such that $\theta_3 = \theta_{int} + (\theta_{int} - \theta_2)$. The value of θ_3 is used along with F_{int} to calculate F_m and F_f corresponding to θ_3. Using the new and previous values of F_m, F_f and θ, the intersection of the two curves is again approximated by the intersection of the straight lines representing the F_m and F_f curves (Figure 5b). This

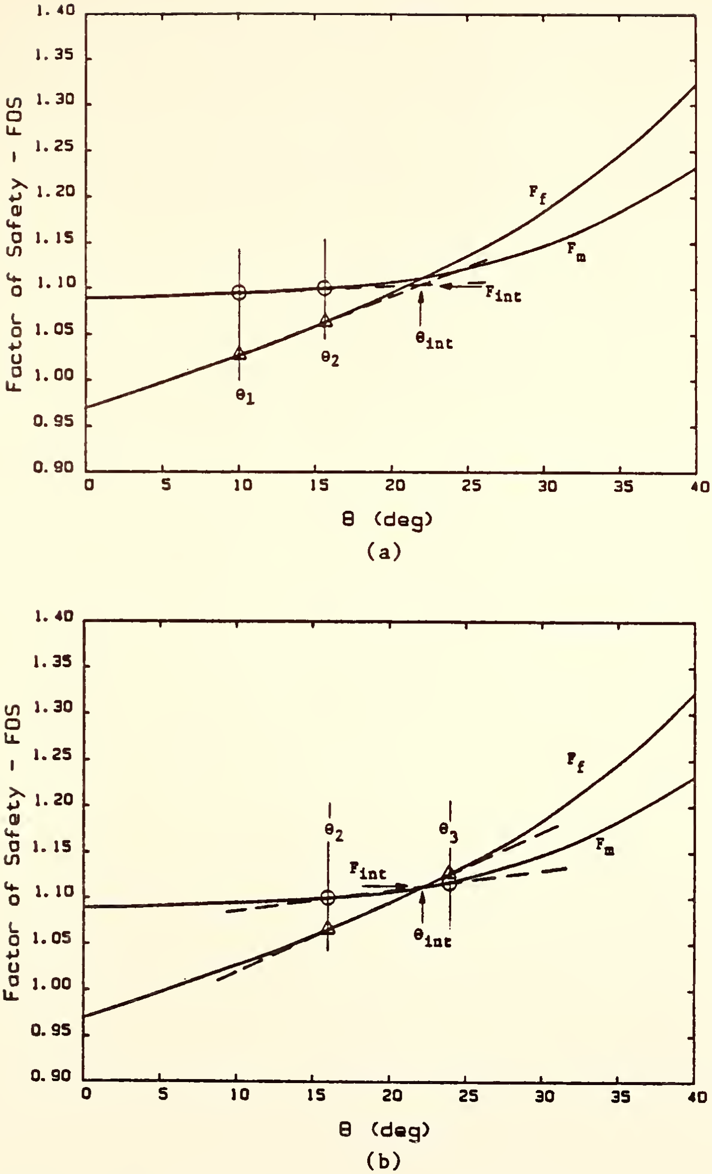


FIGURE 5. Determination of the FOS and θ Satisfying Complete Equilibrium Using the Linear Approximation Method

process is repeated until the difference between the current FOS and the previous FOS is less than 0.001 and the difference between the last two values of θ is less than 0.001 radians, or the difference between F_f and F_m is less than 0.001 for a given θ. This avoids unnecessary calculations when the value of θ being used happens to correspond to the value of θ satisfying complete equilibrium. The program is structured such that a new value of θ will be determined if θ_{int} lies to the left of θ_1 or between θ_1 and θ_2.

Due to the shape of the curves, convergence is rapid and often occurs within three iterations. No problems have been indicated with respect to non-convergence of a solution using the LAM. It is believed that STABL is the only known slope stability program to contain the Linear Approximation Method.

Line of Thrust

As mentioned previously, attention should be paid to the position of the line of thrust (location of the line of action, or points of application, of the interslice side forces on the slices) to check the reasonableness of the solution. A satisfactory solution is one in which the line of thrust passes through the middle third of the slices. Tensile forces are indicated within the slope if the line of thrust lies outside the middle third of the slice. The location of a satisfactory line of thrust is shown in Figure 6 for the example problem shown in Figure 3a.

Once the slope of the interslice forces θ and the FOS satisfying complete equilibrium have been determined, the line of thrust may be calculated. The values of the resultant interslice forces, (Z_l, Z_r), for each slice are calculated by substituting the values of FOS and θ satisfying complete equilibrium into equation 4. Working from the first slice to the last, the points of action of the interslice forces are found by taking moments about the center of the base of each slice in turn.

Spencer (1973) indicated that suitable lines of thrust can be obtained assuming that a tension crack filled with water exists at the upper end of the slip surface. The depth of the tension crack may initially be taken as the depth of zero active effective stress:

$$z_o = \frac{2c'}{\gamma FS(1-r_u)} \sqrt{\frac{1 + \sin\phi'_m}{1 - \sin\phi'_m}} \quad . . . . (10)$$

Spencer demonstrated that reliable factors of safety can be obtained assuming the slope of the interslice forces are parallel. However, he recommended that the slope of the interslice forces should be reduced at the upper end of the slip surface in order to obtain reasonable positions of the line of thrust. Spencer's method as programmed in STABL follows these recommendations.

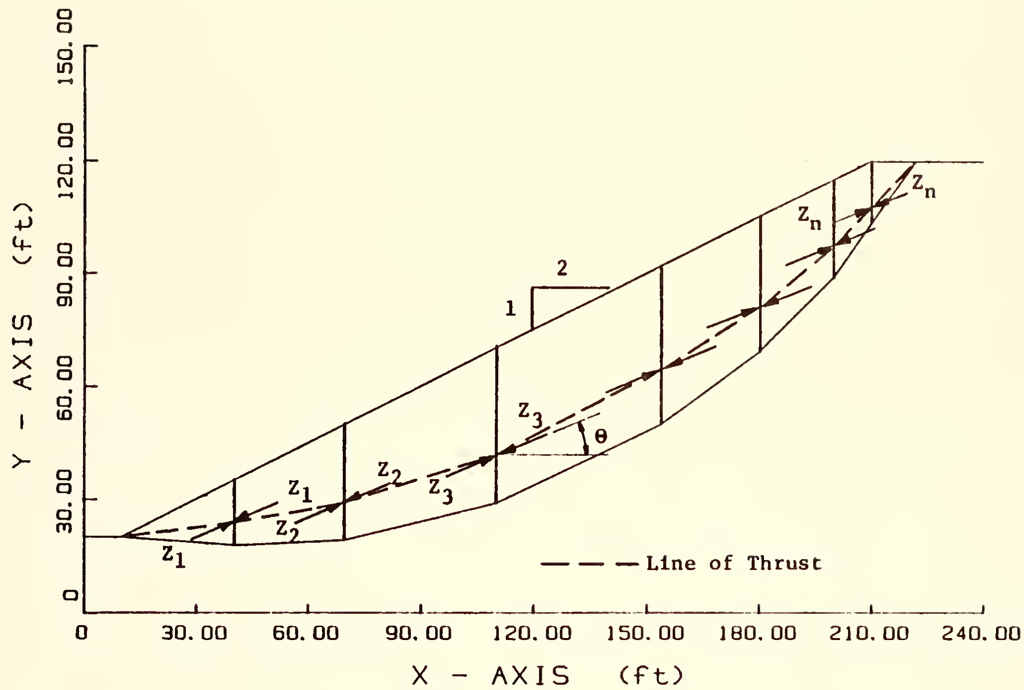


FIGURE 6. Line of Thrust

Spencer Options in STABL

The Spencer option may be invoked by specifying the command "SPENCR" and an estimate of the slope angle. The SPENCR command precedes specification of the surface type and method of solution; i.e., SURFAC, SURBIS, CIRCLE, CIRCL2, RANDOM, BLOCK or BLOCK2.

Spencer's method has been implemented in the STABL program for the primary purpose of obtaining a more accurate value of the FOS and line of thrust for specific surfaces of interest. For critical surfaces, the Spencer method of slices is preferred over the Simplified Janbu or Simplified Bishop methods since it satisfies complete equilibrium of the sliding mass and yields a slightly more accurate FOS. Since determination of the FOS by Spencer's method requires approximately six times more calculation time, it is intended that only specific surfaces of interest will be analyzed utilizing Spencer's method. However, Spencer's method may be used for analysis of either user input specific surfaces, or randomly generated surfaces.

The most efficient use of STABL's capabilities will be realized if the user investigates a number of potential failure surfaces using one of STABL's random surface generation techniques and determines the FOS by either the Simplified Janbu or Simplified Bishop method of slices. Once critical potential failure surfaces have been identified, they may be analyzed using the SPENCR option in conjunction with either the SURFAC or SURBIS option to obtain a more accurate value of the FOS and to gain insight into the reasonableness of the solution through examination of the line of thrust.

When a user input potential failure surface is analyzed, the program will output the values of F_f, F_m and θ calculated during iteration along with the value of FOS and θ satisfying complete equilibrium. The user may use this information to construct a graph similar to that of Figure 50b. When analyzing a user input potential failure surface, the coordinates of the line of thrust, the ratio of the height of the line of thrust above the sliding surface to the slice height for each slice, and the values of the interslice forces are all output. This information allows the user to quickly determine whether or not the line of thrust, and hence the solution, is satisfactory.

The Spencer option may also be used with the STABL options that generate surfaces randomly. However, when the Spencer option is used in conjunction with randomly generated surfaces, only the FOS and angle of the interslice forces satisfying complete equilibrium are output for the ten most critical surfaces. Information regarding the line of thrust, interslice forces or values of F_f, F_m and θ calculated during iteration is not output for randomly generated surfaces; hence the reasonableness of the solution obtained for a randomly generated surface will not be apparent. When the reasonableness of the solution of a randomly generated surface is desired, the surface must be analyzed using the SPENCR option in conjunction with either the SURBIS or SURFAC options.

The STABL5/PCSTABL5 User Manual (Carpenter, 1985b), further describes the Spencer options, input format, restrictions, and error codes. This document also describes some minor program enhancements and provides an example problem using the Spencer option.

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary and Conclusions

Spencer's method of slices was implemented in the programs STABL5 and PCSTABL5 to provide additional versatility to the STABL programs. Spencer's method of slices satisfies complete equilibrium and is capable of transferring the load from one slice to another through the interaction of the interslice shear and normal side forces. Therefore, Spencer's method of slices is particularly well suited for analysis of slopes and retaining walls subjected to tieback loads since it distributes the load from a tieback between slices. The development of Spencer's method was reviewed and its implementation in STABL presented. A new iterative technique was developed by the author for determining the factor of safety and angle of the interslice forces satisfying complete equilibrium of a sliding mass computed by Spencer's method. The new iterative technique is called the Linear Approximation Method.

The Spencer method of slices is preferred for analysis of tiedback slopes and walls over the simplified methods since it satisfies complete equilibrium and accounts for the interaction of the interslice side forces between slices. The FOS obtained by Spencer's method is typically slightly higher than the FOS obtained by the less rigorous Simplified Bishop or Simplified

Janbu methods. Since Spencer's method satisfies complete equilibrium of the sliding mass, it is especially well suited for analysis of tiedback slopes and walls. Spencer's solution is more rigorous than the simplified methods and requires more computation time.

The Linear Approximation Method provides a reliable method for determining the FOS satisfying complete equilibrium. The method not only converges readily, it also searches for the FOS satisfying complete equilibrium, thus minimizing the number of iterations required to obtain a solution. It is believed that the LAM is unique to the STABL programs.

Recommendations

Since Spencer's method of slices satisfies complete equilibrium, this method should be used to analyze the stability of tiedback slopes and retaining structures whenever possible.

Since Spencer's method of slices requires more computer time to arrive at a solution, it is recommended that Spencer's method be used to analyze only those critical potential failure surfaces found by analysis of randomly generated surfaces using the Simplified Bishop or Simplified Janbu method of slices. Following this recommendation will lead to the most efficient utilization of STABL's capabilities and the engineer's time.

It is strongly recommended that the new Spencer routines be thoroughly tested prior to public release of the STABL5 and PCSTABL5 programs.

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