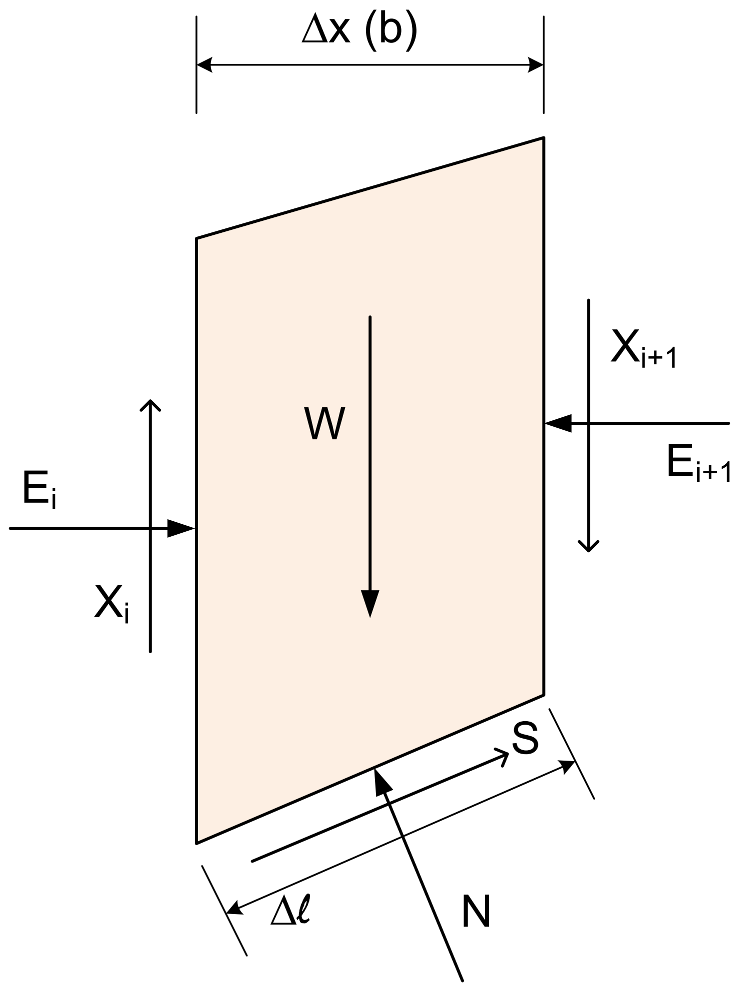
CE 544 – BRIGHAM YOUNG UNIVERSITY

Limit Equilibrium Procedures Part 2

General Method of Slices



$S = f(c', \phi', u, N, F, \Delta\ell)$

Unknowns:

1 factor of safety
n values of N
n locations for N
n-1 values of E
n-1 values of X
n-1 locations for E

Total: 5n-2

Equilibrium Equations:

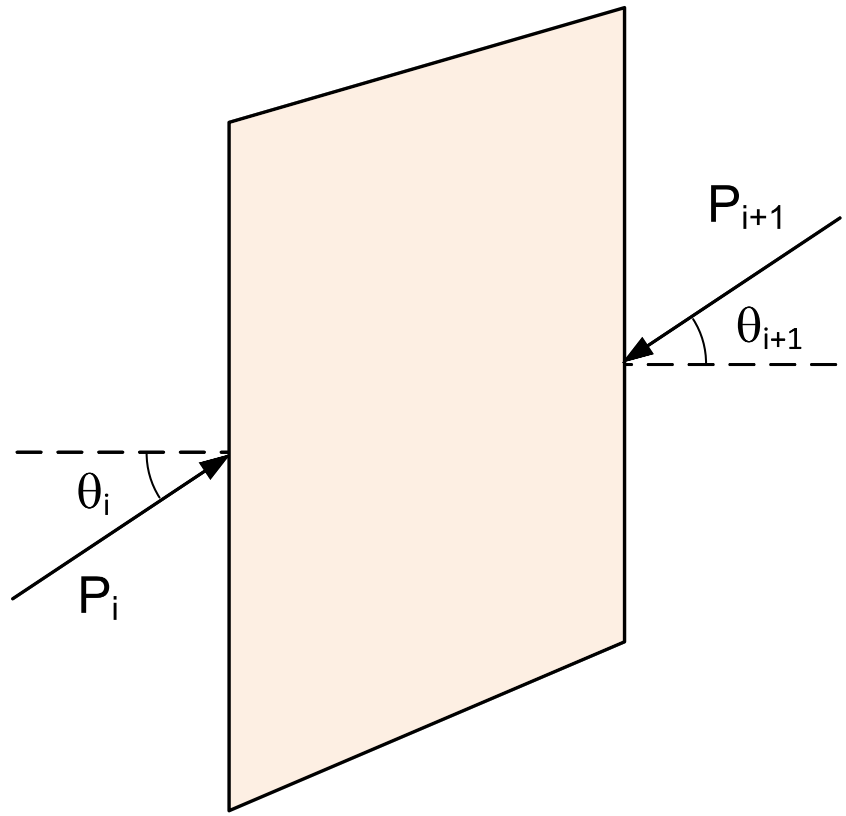
n equations for $\Sigma F_V=0$
n equations for $\Sigma F_H=0$
n equations for $\Sigma M=0$

Total: 3n

5n-2 > 3n, therefore statically indeterminate

General Method of Slices

Note: side forces can be represented by magnitude and direction. Same number of equations and unknowns.



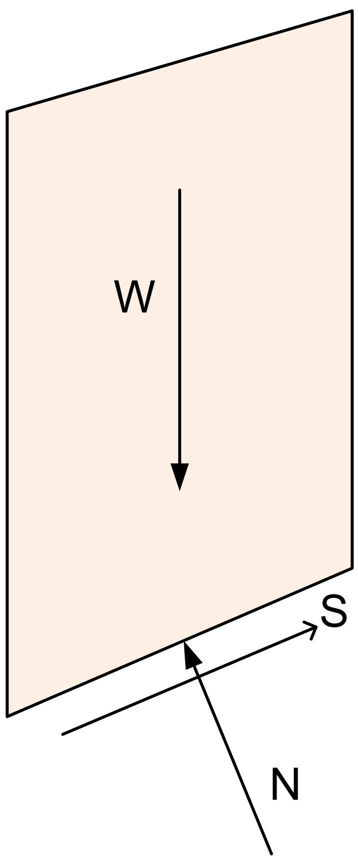
General Method of Slices

- Several techniques based on method of slices
- Each technique uses a different set of assumptions to reduce # of unknowns
- Some techniques satisfy **complete equilibrium**
  - 2n-2 assumptions
  - 3n equations satisfied
- Some don’t satisfy equilibrium
  - >2n-2 assumptions
  - <3n equations satisfied

Ordinary Method of Slices

Ordinary Method of Slices (OMS)

Key assumption → Side forces can be neglected (i.e., sum to zero on each slice)



Σ Forces perpendicular to base of slice:

$$N = W\cos\alpha$$

$$\sigma = \frac{N}{\Delta\ell} = \frac{W\cos\alpha}{\Delta\ell}$$

General equation:

$$F = \frac{\sum(c + \sigma\tan\phi)\Delta\ell}{\sum W\sin\alpha}$$

Substituting σ:

$$F = \frac{\sum c\Delta\ell + W\cos\alpha\tan\phi}{\sum W\sin\alpha}$$

Ordinary Method of Slices

If $\phi = 0$:

$$F = \frac{\sum c\Delta\ell + W\cos\alpha\underbrace{\cancel{\tan\phi}}_{0}}{\sum W\sin\alpha}$$

$$F = \frac{\sum c\Delta\ell}{\sum W\sin\alpha}$$

← This is the same solution we found earlier for the generalized method

Ordinary Method of Slices

For effective stress analysis:

$$F = \frac{\sum (c' + \sigma' \tan \phi')\Delta\ell}{\sum W \sin \alpha}$$

$$\sigma' = \frac{W\cos\alpha}{\Delta\ell} - u$$

$$F = \frac{\sum [c'\Delta\ell + (W\cos\alpha - u\Delta\ell)\tan\phi']}{\sum W\sin\alpha}$$

← This equation can lead to unrealistically low or even negative effective stresses (unconservative)

Ordinary Method of Slices

Alternate Formulation

Define vertical effective weight:

$$W' = W - ub$$

Σ forces perpendicular to base:

$$N' = W'\cos\alpha$$

$$N' = (W - ub)\cos\alpha$$

$$N' = W\cos\alpha - ub\cos\alpha$$

$$b = \Delta\ell\cos\alpha$$

$$N' = W\cos\alpha - u\Delta\ell\cos^2\alpha$$

$$\sigma' = \frac{N'}{\Delta\ell}$$

$$\sigma' = \frac{W\cos\alpha}{\Delta\ell} - u\cos^2\alpha$$

Substituting:

$$F = \frac{\sum[c'\Delta\ell + (W\cos\alpha - u\Delta\ell\cos^2\alpha)\tan\phi']}{\sum W\sin\alpha}$$

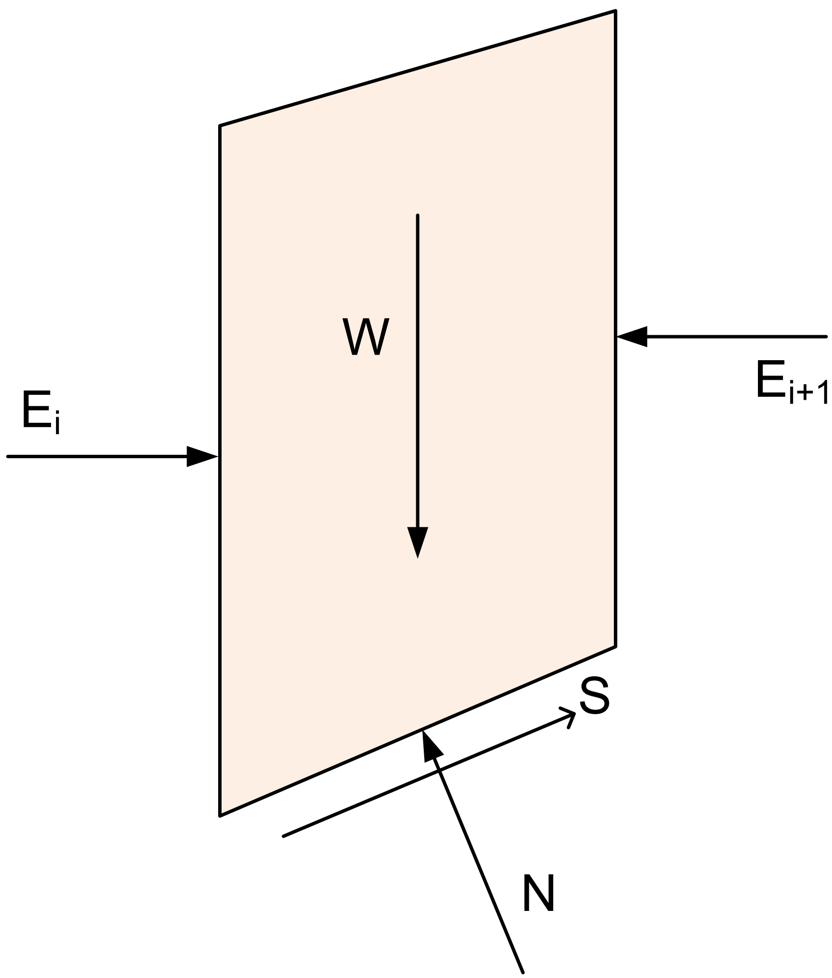
This is the preferred formulation for OMS

Summary - OMS

- Circular shear surface
- Only satisfies moment equilibrium
- For ϕ=0, OMS gives the same solution as the Swedish method
- F can be calculated directly, without iteration
- Less accurate than other procedures of slices

Simplified Bishop's Method

Simplified Bishop's Method



Simplifying assumption:
Side forces are horizontal. I.e., vertical side force components (X_i, X_{i+1}) are equal to zero.

Simplified Bishop's Method

Σ forces in the vertical direction:

$$\mathrm{N}\cos\alpha + \mathrm{S}\sin\alpha - \mathrm{W} = 0$$

$$\mathrm{S} = \tau\Delta\ell$$

$$\mathrm{S} = \frac{\mathrm{s}\Delta\ell}{\mathrm{F}}$$

$$\mathrm{S} = \frac{1}{\mathrm{F}}[\mathrm{c}'\Delta\ell + (\mathrm{N} - \mathrm{u}\Delta\ell)\tan\phi']$$

Substituting:

$$\mathrm{N}\cos\alpha + \left(\frac{1}{\mathrm{F}}[\mathrm{c}'\Delta\ell + (\mathrm{N} - \mathrm{u}\Delta\ell)\tan\phi']\right)\sin\alpha - \mathrm{W} = 0$$

Simplified Bishop's Method

Solving for N:

$$N = \frac{W - (1/F)[c'\Delta\ell - u\Delta\ell\tan\phi']\sin\alpha}{\cos\alpha + (\sin\alpha\tan\phi')/F}$$

$$\sigma' = \frac{N}{\Delta\ell} - u$$

From general equation (based on moment equilibrium):

$$F = \frac{\sum(c' + \sigma'\tan\phi')\Delta\ell}{\sum W\sin\alpha}$$

Simplified Bishop's Method

Combining the three previous equations and solving for F:

$$F = \frac{\sum \left[\frac{c' \Delta \ell \cos\alpha + (W - u \Delta \ell \cos\alpha) \tan\phi'}{\cos\alpha + (\sin\alpha \tan\phi')/F} \right]}{\sum W \sin\alpha}$$

For total stress analysis:

Note that F is on both sides. Must be solved iteratively.

$$F = \frac{\sum \left[\frac{c \Delta \ell \cos\alpha + W \tan\phi}{\cos\alpha + (\sin\alpha \tan\phi)/F} \right]}{\sum W \sin\alpha}$$

Simplified Bishop's Method

For ϕ=0, equation reduces to:

$$F = \frac{\sum c\Delta\ell}{\sum W\sin\alpha}$$

Which is the same equation derived for log spiral, Swedish method, and OMS

Simplified Bishop's Method

Once F is found, N can be computed as:

$$N = \frac{W - (1/F)[c'\Delta\ell - u\Delta\ell\tan\phi']\sin\alpha}{\cos\alpha + (\sin\alpha\tan\phi')/F}$$

For OMS:

$$N = W\cos\alpha$$

Difference in solution is due to differences in N. Both use the same overall equation:

$$\sum SR = \sum WR\sin\alpha$$

Summary – Simplified Bishop's

Unknowns:

n normal forces along base of slice

1 factor of safety

Total: n+1

Equilibrium Equations:

n equations for $\Sigma F_v=0$

1 equation for overall $\Sigma M=0$

Total: n+1

Summary – Simplified Bishop's

- Circular slip surface
- Horizontal side forces
- Satisfies
  - Moment equilibrium
  - Force equilibrium in vertical direction
- More accurate than OMS, especially for effective stress analysis with high pore pressures

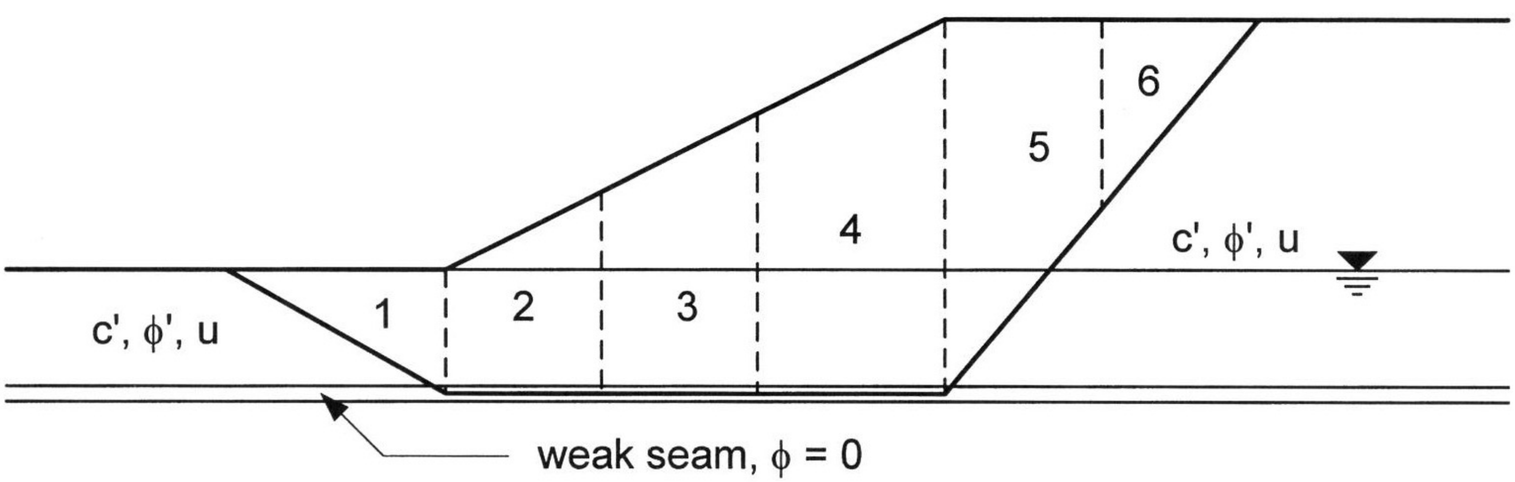
Bishop's Complete Equilibrium Procedure

- Similar to simplified procedure except it is not assumed that all $(X_{i+1} - X_i) = 0$.
- A set of values for vertical side forces is assumed and vertical and horizontal force equilibrium is checked. Process is repeated until equilibrium is satisfied.
- Time-consuming and complicated

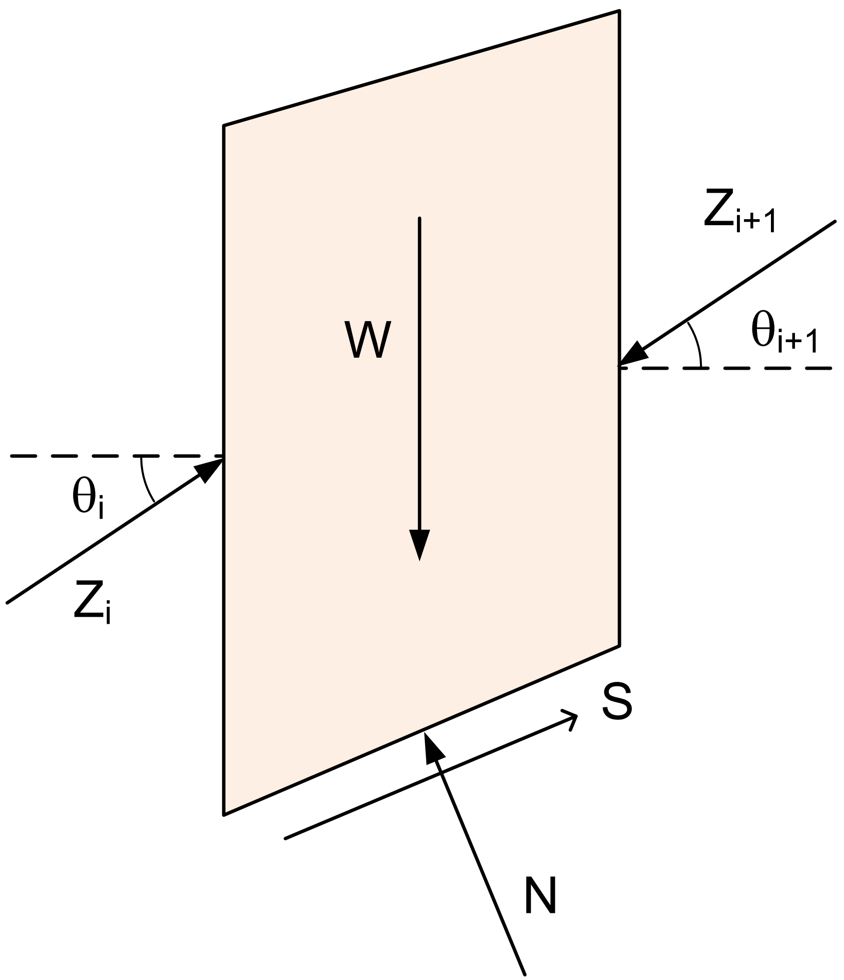
Force Equilibrium Procedures

Force Equilibrium Procedures

- Can be used on non-circular surfaces
- No attempt is made to satisfy moment equilibrium



Force Equilibrium Procedures



Unknowns:

1 factor of safety
n values of N
n-1 values of Z
n-1 values of θ

Total: 3n-1

Since we are not satisfying moment equilibrium, locations no longer matter

Equilibrium Equations:

n equations for $\Sigma F_v=0$
n equations for $\Sigma F_H=0$

Total: 2n

3n-1 > 2n, therefore n-1 assumptions necessary

Best strategy is to assume n-1 values of θ

Force Equilibrium Procedures

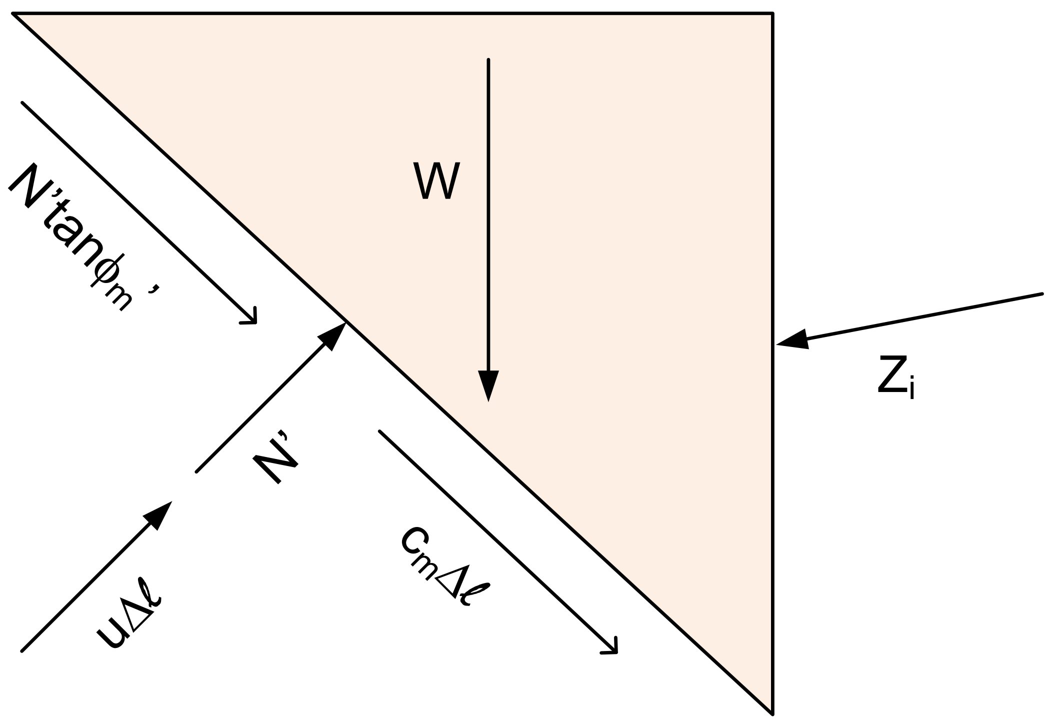
Procedure

(1) Assume F, compute:

$$c_m = \frac{c}{F} \qquad \tan\phi_m = \frac{\tan\phi}{F}$$

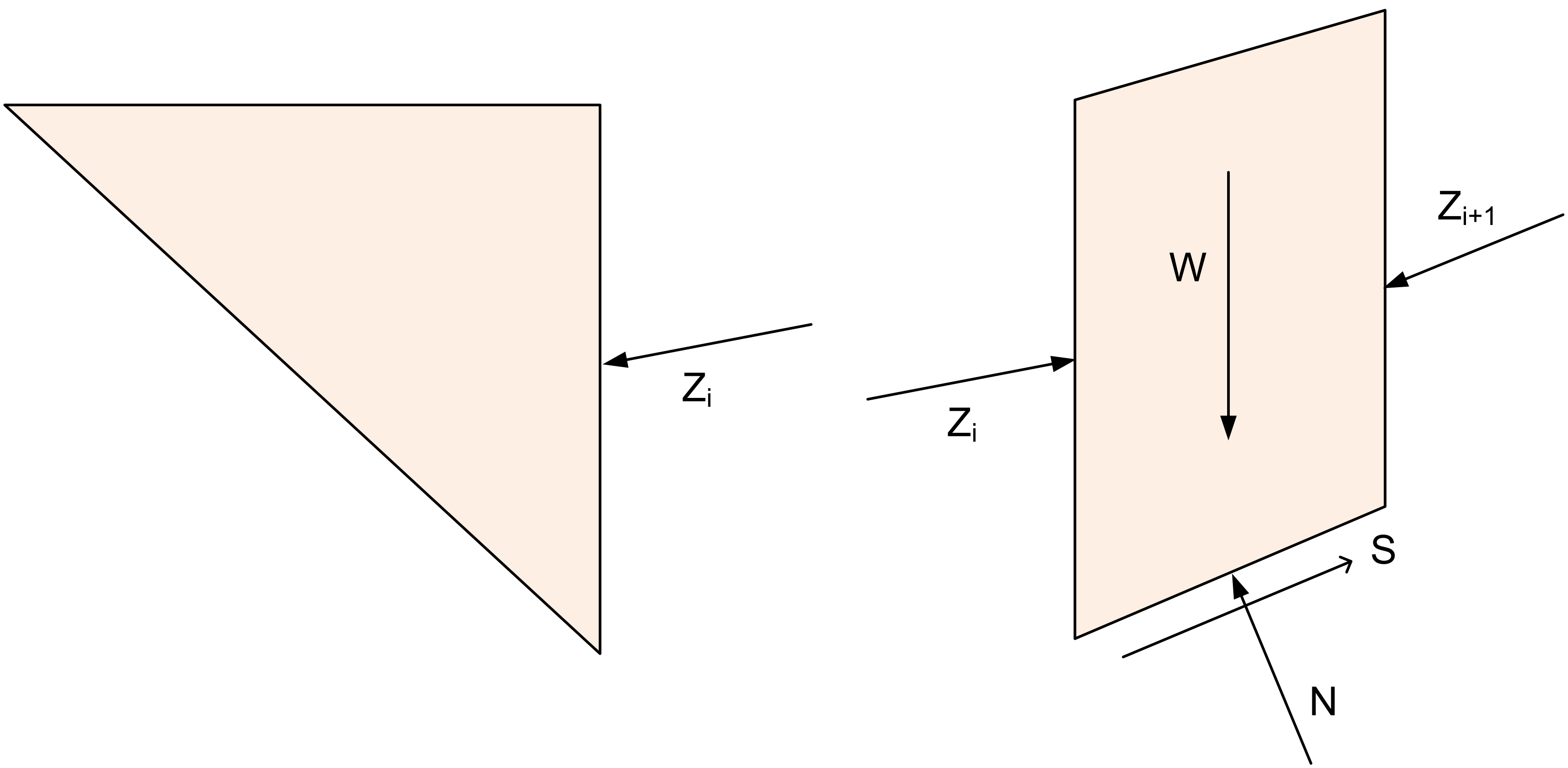
Force Equilibrium Procedures

(2) Start on slice #1. Solve for N and Z by satisfying $\Sigma Fx=0$ and $\Sigma Fy=0$.



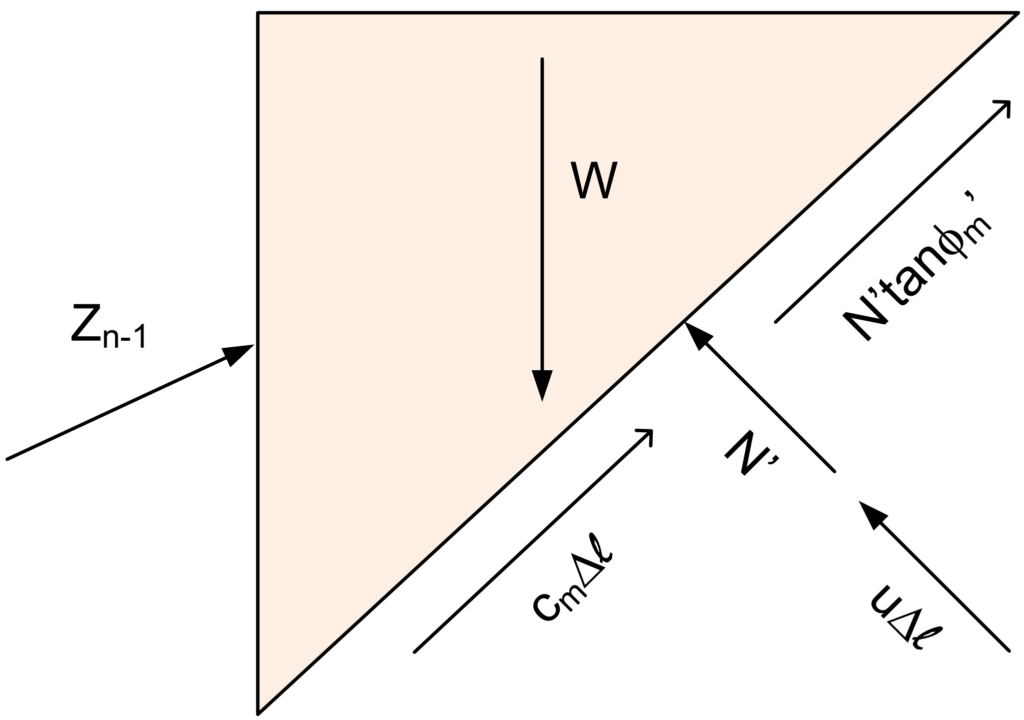
Force Equilibrium Procedures

(3) Repeat for each of the slices in sequence:



Force Equilibrium Procedures

(4) Forces should balance on the last slice:



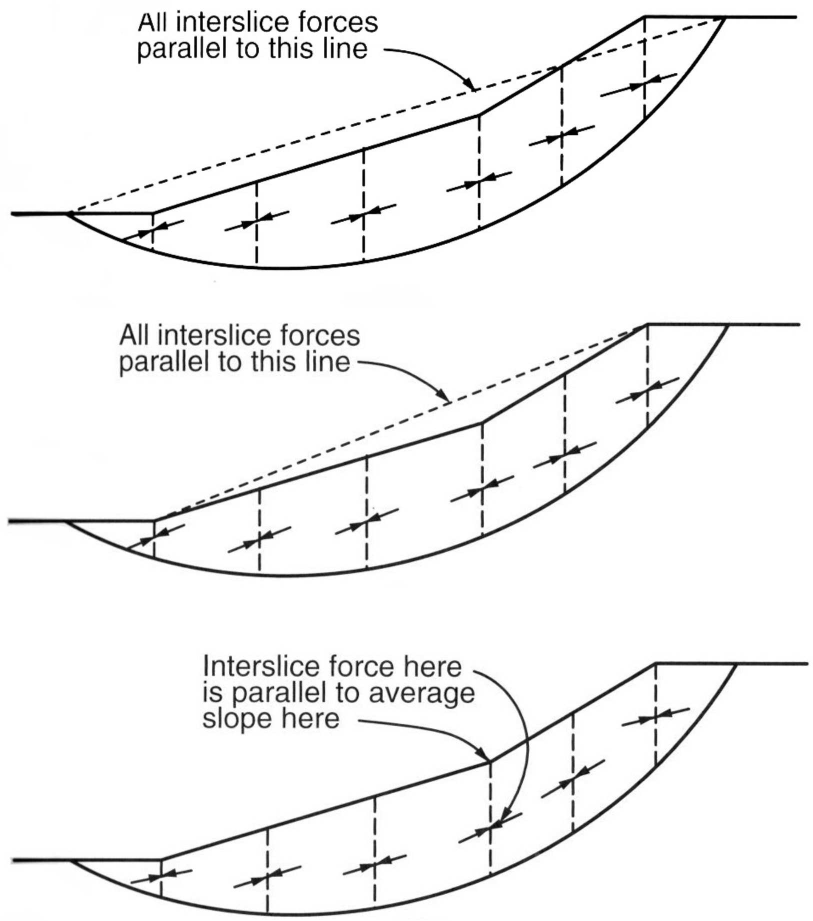
2 equations
1 unknown

If forces do not balance, try a new F and repeat steps 2-4

Side Force Assumptions

- Lowe and Karafaith
  - For each slice, use average slope of ground surface and slip surface
- Simplified Janbu
  - The side forces are assumed to be horizontal
- U.S. Army Corps of Engineers
  - Parallel to slope (see next slide)

Side Force Assumptions, Cont.



U.S. Army Corps of Engineers

"Modified Swedish Procedure"

Summary – Force Equilibrium Methods

- Works on both circular and non-circular surfaces
- Satisfies force equilibrium only, not moment equilibrium
- Requires iteration
- Key assumption = side force inclinations

Complete Equilibrium Procedures

Complete Equilibrium Procedures

Unknowns:

1 factor of safety
n values of N
n-1 values of E
n-1 values of X
n locations for N
n-1 locations for E

Total: 5n-2

Equilibrium Equations:

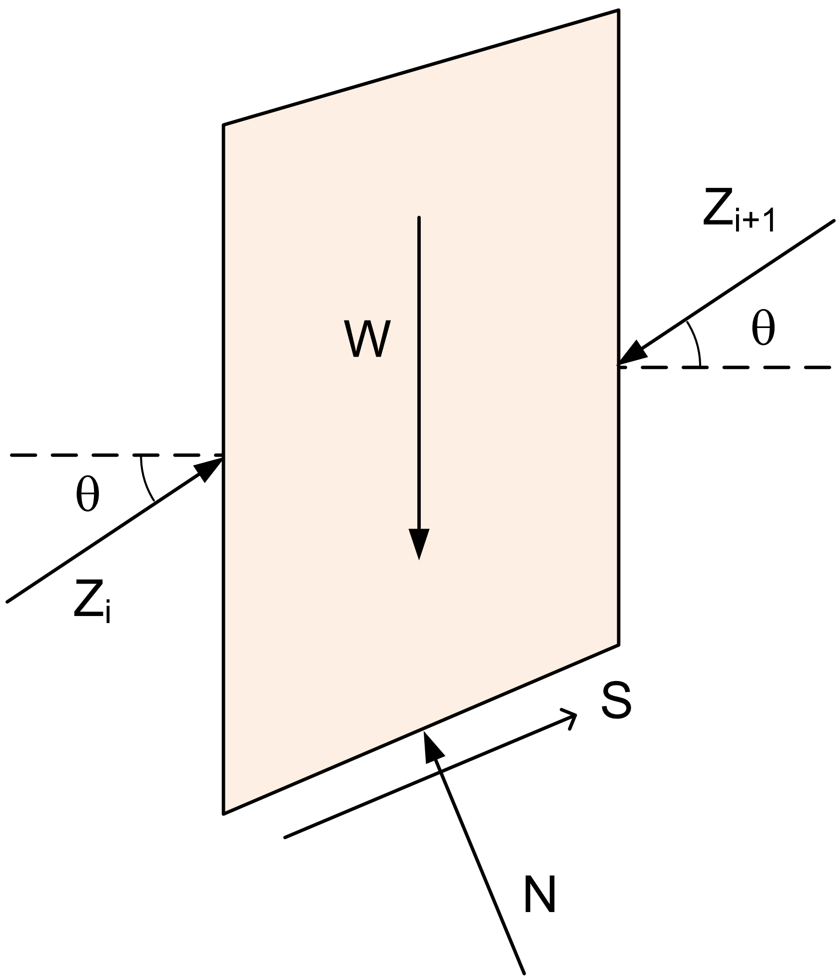
n equations for $\Sigma F_V=0$
n equations for $\Sigma F_H=0$
n equations for $\Sigma M=0$

Total: 3n

We need 2n-2 assumptions to make the problem statically determinate

Assume n locations of normal forces, N (middle of slice). This leaves n-2 assumptions still required

Spencer's Method



Simplifying assumption:

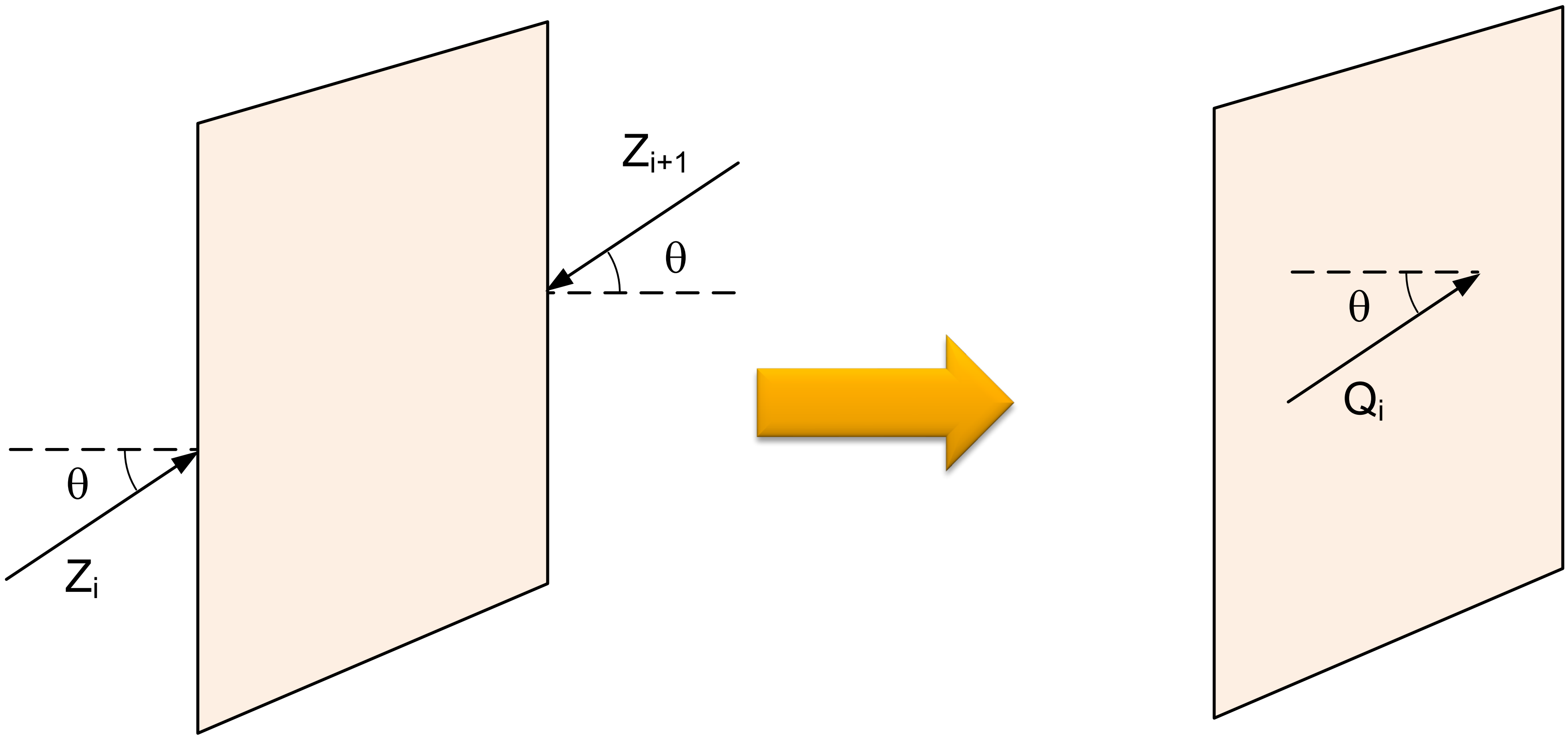
All side forces are parallel.
(θ=constant)

Unknowns:

1 factor of safety
n values of N
n-1 values of Z
1 side force inclination θ
n-1 locations of Z

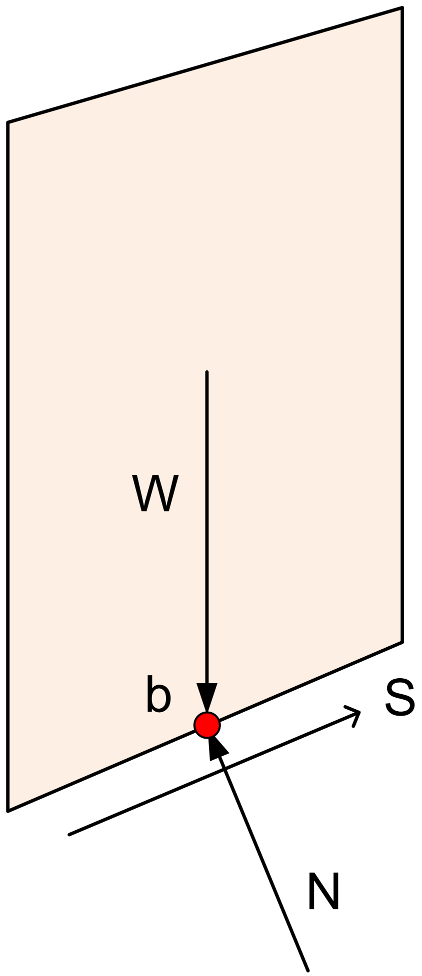
Total: 3n

Spencer’s Method

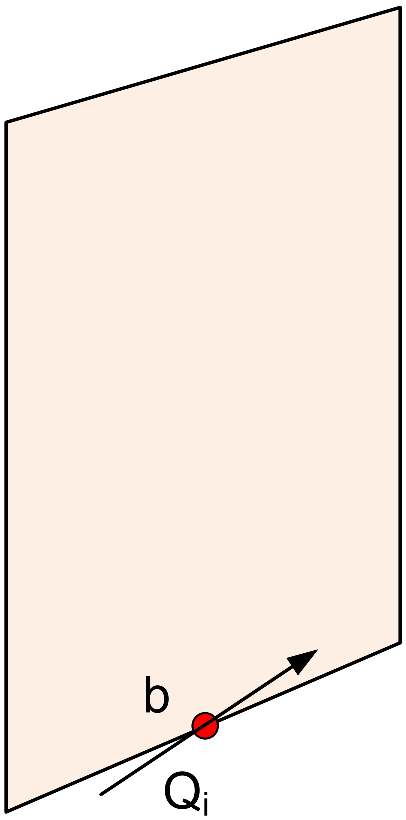


Side forces Z_i and Z_{i+1} are combined into a single resultant side force Q_i, acting at an angle = θ.

Spencer's Method

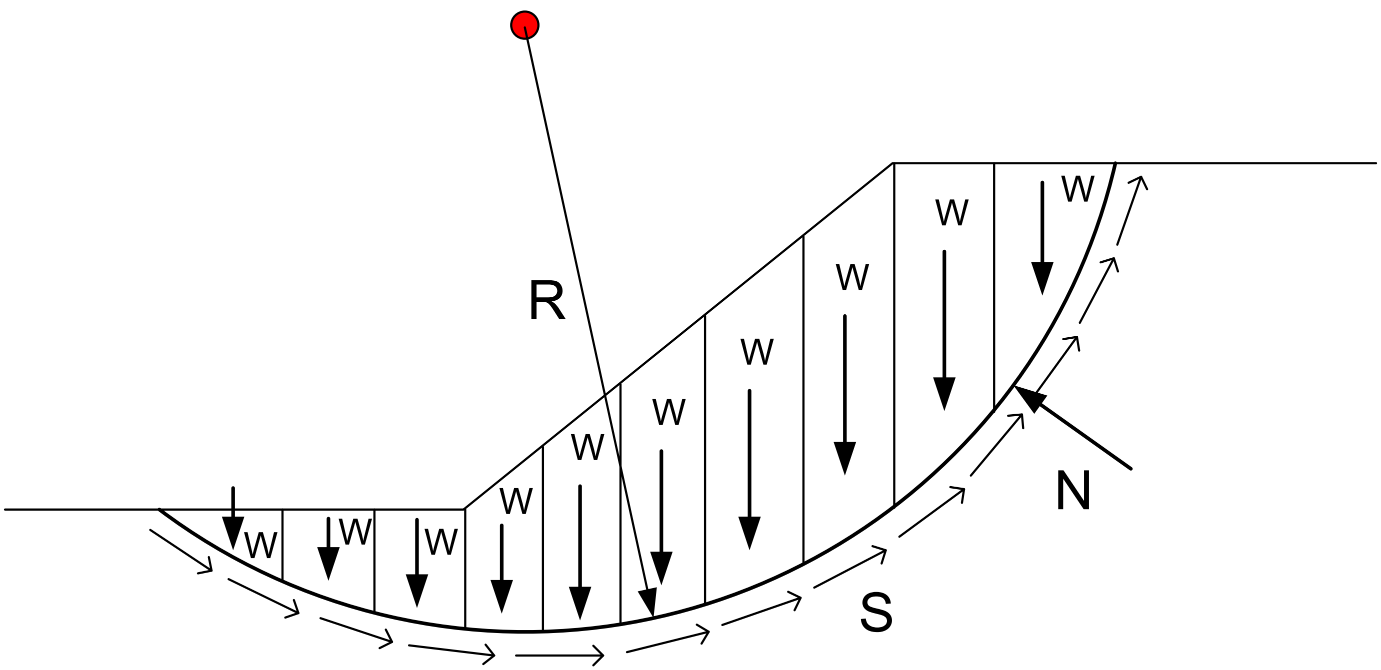


He also assumed that W,S, and N all act through the same point b (center of the bottom of the slice)



Therefore, Q_i must also act through the same point b in order to maintain moment equilibrium

Spencer's Method

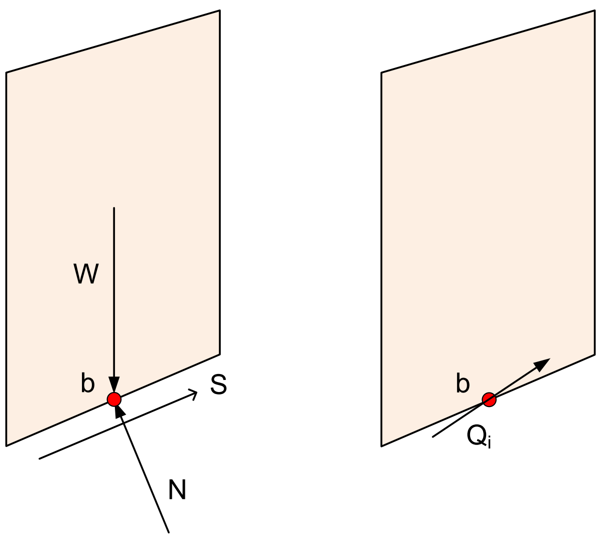


For overall moment equilibrium:

$$\sum M = \sum WR\sin\alpha - \sum SR = 0$$

(same as before)

Spencer's Method



Since Q acts through b, Q must be equal to sum of W, S, and N. Therefore, work in terms of Q.

Two components of Q:

$$Q_{\perp} = Q\sin(\alpha - \theta)$$ (perp. to base)

$$Q_{||} = Q\cos(\alpha - \theta)$$ (parallel to base)

Σ Moments (in terms of Q):

$$\sum M = \sum RQ\cos(\alpha - \theta)$$

Spencer's Method

For overall force equilibrium:

$$\sum F_v = \sum Q\sin\theta = 0$$

$$\sum F_H = \sum Q\cos\theta = 0$$

Since θ = constant, both simplify to:

$$\sum Q = 0$$

Spencer's Method

By summing forces on individual slices, you can derive equation for Q in terms of known quantities:

$$Q = \frac{\left\{W\sin\alpha - \frac{c'}{F}\Delta x\sec\alpha - [W\cos\alpha - u\Delta x\sec\alpha]\,\frac{\tan\phi'}{F}\right\}}{\cos(\alpha-\theta)\left[1 + \frac{\tan(\alpha-\theta)\tan\phi'}{F}\right]}$$

Now we have two equations:

$$\sum Q\cos(\alpha-\theta) = 0$$

$$\sum Q = 0$$

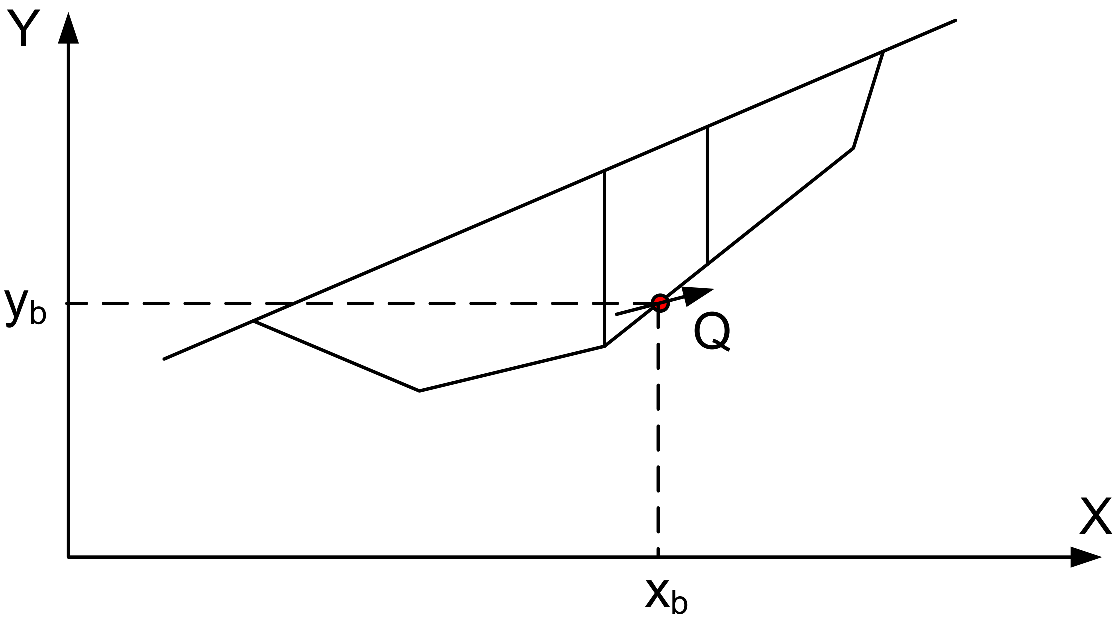
And two unknowns:

F and θ

Once again, equations must be solved iteratively.

Spencer's Method

Non-circular surfaces:



Set up a coordinate system and sum moments about the origin

$$\sum M = \sum \{x_b(-Q\sin\theta) + y_b(Q\cos\theta)\} = 0$$

Once again, we have two equations and two unknowns.

Morgenstern & Price Method

- Interslice shear force is related to interslice normal force by:

$$X = \lambda f(x) E$$

λ = unknown scaling factor
f(x) = assumed function

- The normal force N acts at the base of the slice
- Satisfies complete equilibrium
- More work than Spencer's method, but gives about the same results

Chen & Morgenstern's Method

- Improvement to M&P method.
- Interslice shear force is related to interslice normal force by:

$$X = [\lambda f(x) + f_o(x)]E$$

- Equation is thought to better represent side force relationship at ends
- Satisfies <u>complete equilibrium</u>

Sarma's Procedure

- Similar to M&P and C&M methods
- Interslice shear force is related to shear strength as follows:

  $$X = \lambda f(x) S_v$$

- Developed for applications in seismic stability and includes seismic coefficient
- Satisfies <u>complete equilibrium</u>

Comparison of Methods

- See tables 6.2 and 6.3 in text
- Spencer’s method is often the preferred method
  - Accurate
  - Simplest complete equilibrium method
  - Can be used for circular or non-circular surfaces