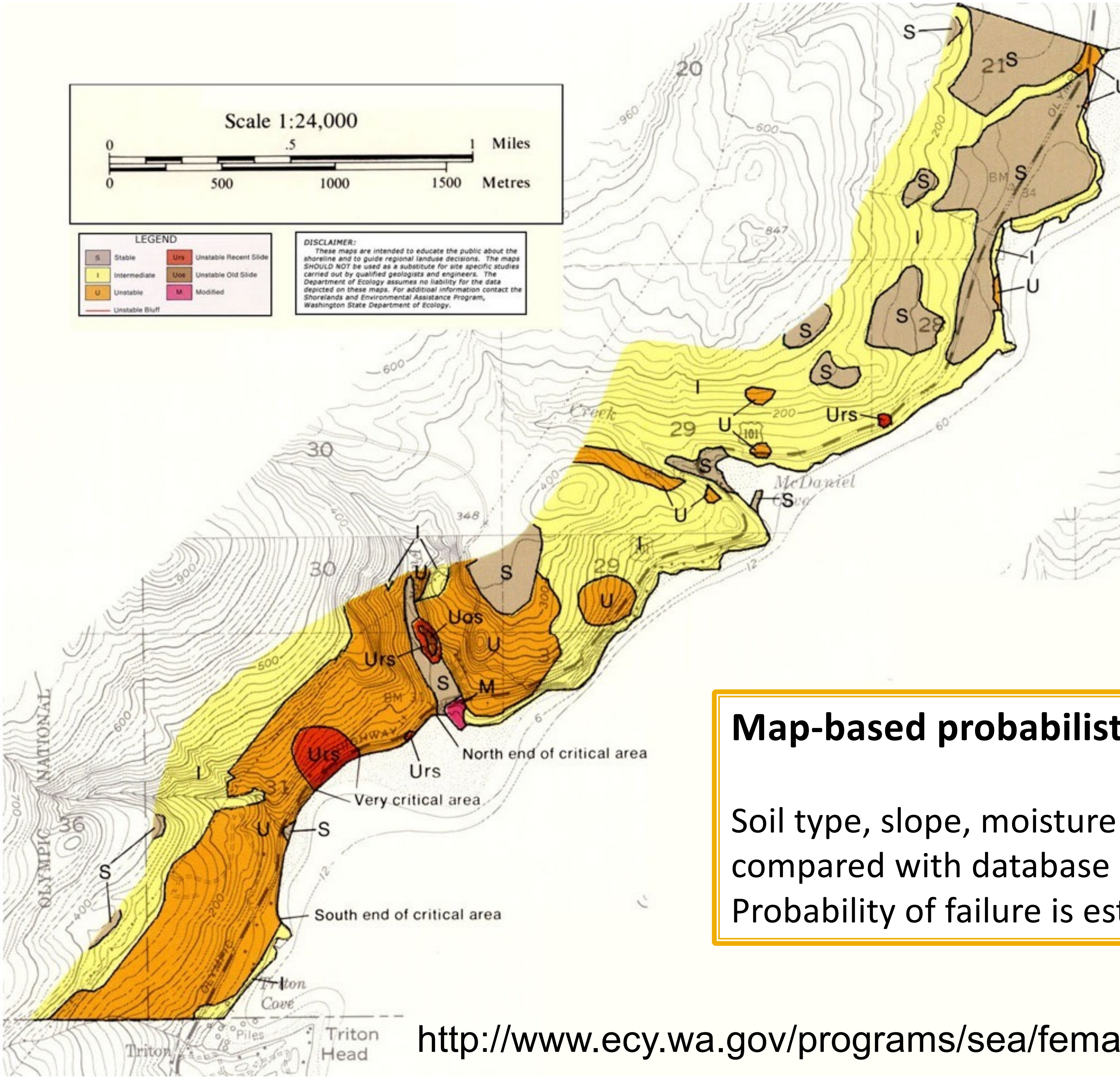
CE 544 – BRIGHAM YOUNG UNIVERSITY

Limit Equilibrium Procedures Part 1

Slope Stability Analysis Methods

- Map-based probabilistic analysis
- Deformation analysis
- Limit equilibrium analysis



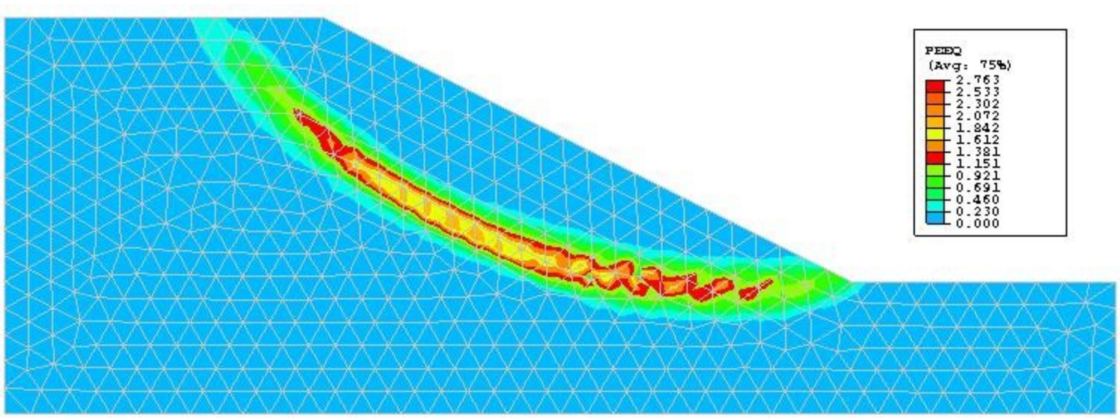
Map-based probabilistic analysis

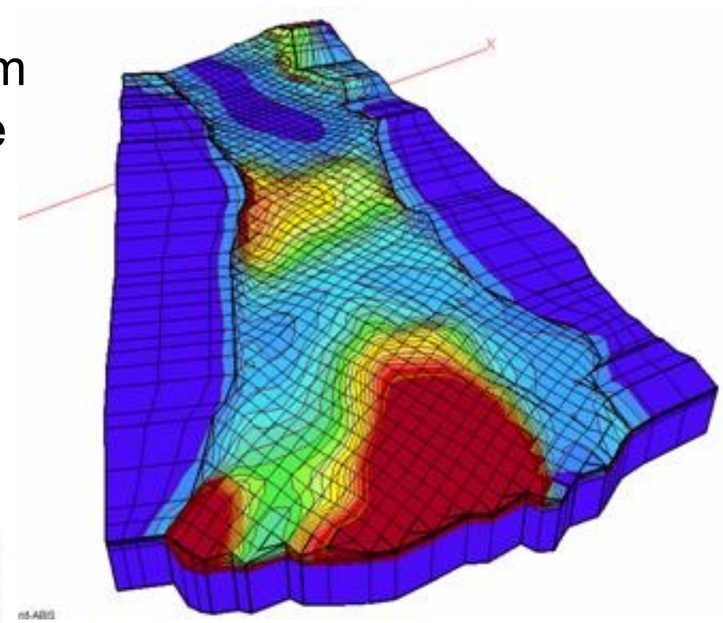
Soil type, slope, moisture content etc. compared with database of past failures. Probability of failure is estimated

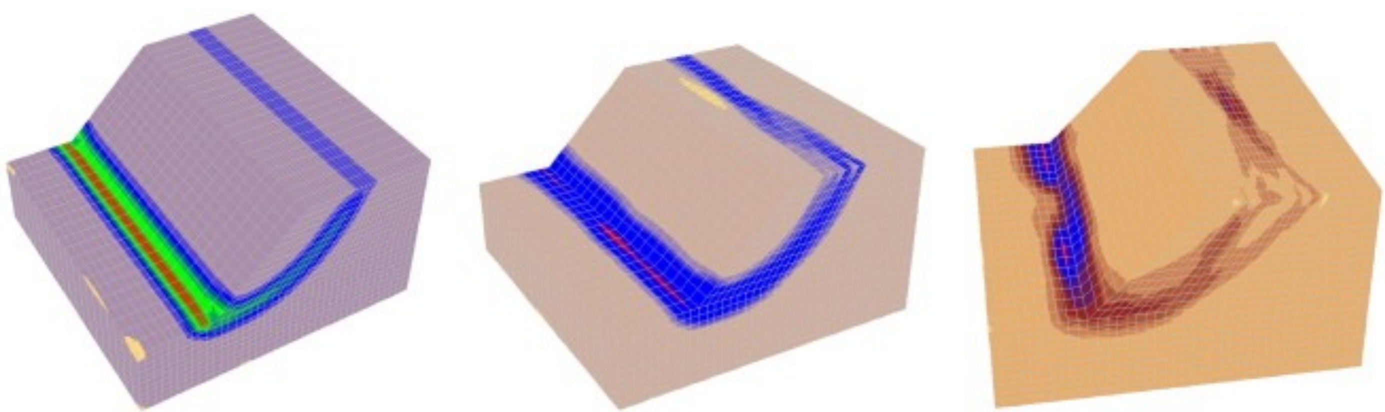
http://www.ecy.wa.gov/programs/sea/femaweb/jefferson.htm

Deformation Analysis

Stress-strain relations, peak-residual strength, anisotropy and other factors are combined to form a model for plastic deformation. Solved with finite element technique







(a) homogeneous, FOS=1.17 (b) m=3, FOS=1.14 (c) m=1, FOS=1.04

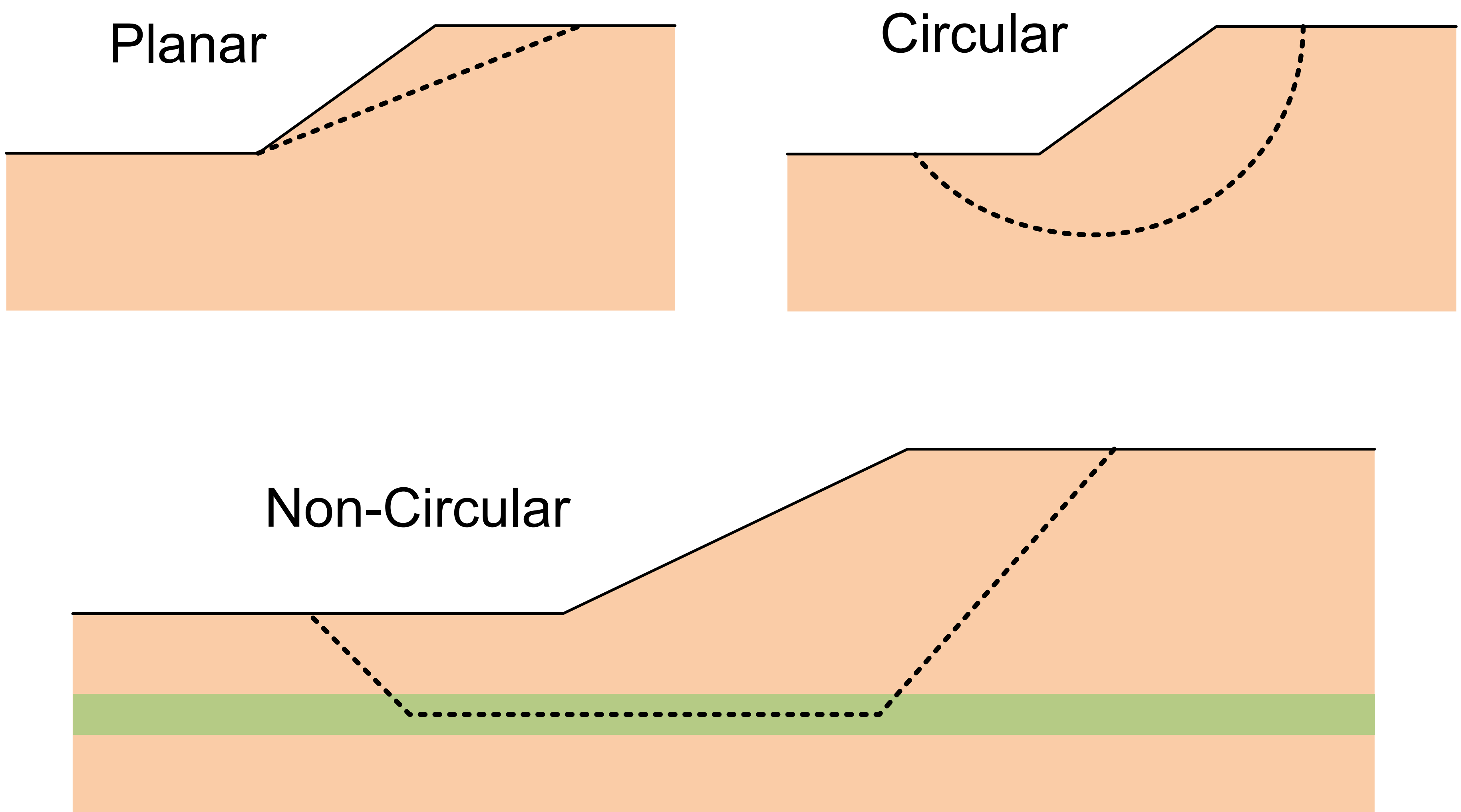
Limit Equilibrium Method

- Candidate failure surface is selected
- Stresses along failure surface are calculated assuming soil is at limit of static equilibrium (τ)
- Total available strength along surface is calculated (s)
- Factor of safety is computed as:

$$\mathrm{FS} = \frac{s}{\tau}$$

- Process is repeated until critical failure surface with minimum factor of safety is found

Failure Surfaces



Factor of Safety

$$FS = \frac{s}{\tau}$$

$$\tau = \frac{s}{FS}$$

$$\tau = \frac{c + \sigma\tan\phi}{FS}$$

$$\tau = \frac{c}{FS} + \frac{\sigma\tan\phi}{FS}$$

$$c_d = \frac{c}{FS}$$

$$\tan\phi_d = \frac{\tan\phi}{FS}$$

c_d = developed or mobilized cohesion

$\tan\phi_d$ = developed or mobilized friction

Equilibrium Conditions

- Static equilibrium must be satisfied
- Three conditions

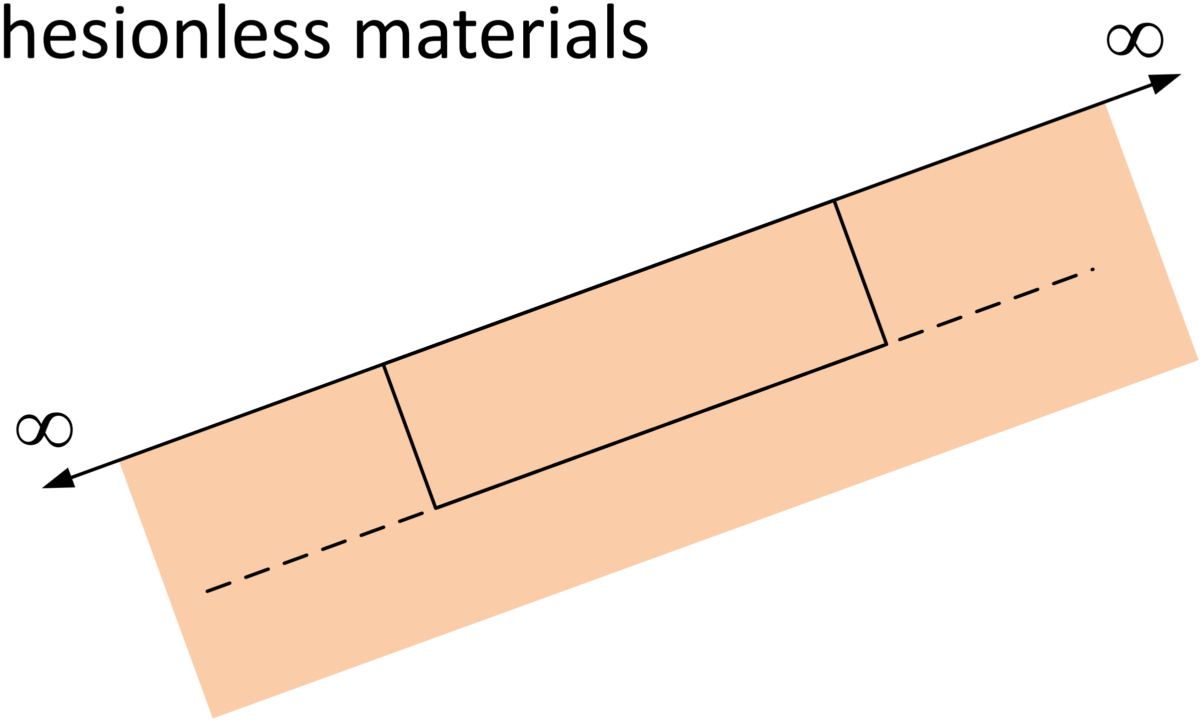
$$\Sigma \mathrm{F_x} = 0 \qquad \Sigma \mathrm{F_y} = 0 \qquad \Sigma \mathrm{M} = 0$$

- Typically, # equations < # unknowns → simplifying assumptions must be used
- Some techniques satisfy all three conditions, some satisfy just a subset

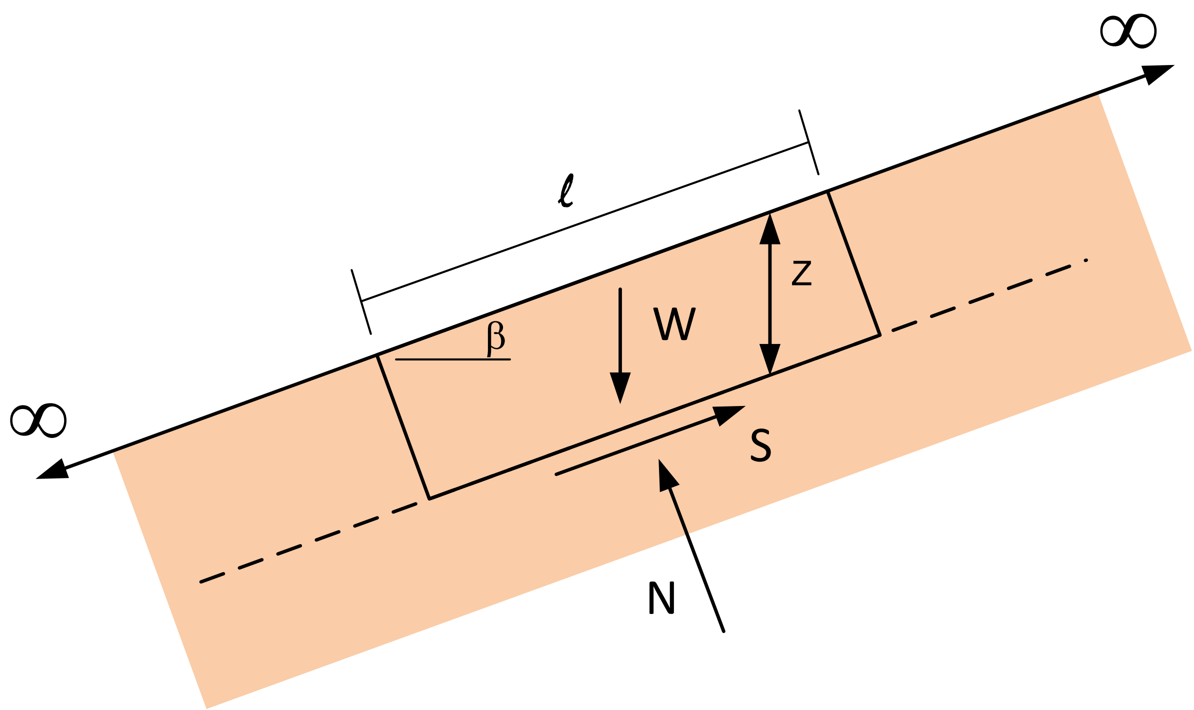
Infinite Slope Analysis

Infinite Slope Analysis

- Can be used as a reasonable approximation for cases with:
  - Shallow firm strata
  - Cohesionless materials



Infinite Slope Analysis



$$S = W\sin\beta$$

$$N = W\cos\beta$$

$$W = \gamma\ell z\cos\beta$$

$$S = \gamma\ell z\cos\beta\sin\beta$$

$$N = \gamma\ell z\cos^2\beta$$

Infinite Slope Analysis

$$\tau = \frac{S}{\ell} = \frac{\gamma \ell z \cos\beta \sin\beta}{\ell} = \gamma z \cos\beta \sin\beta$$

$$\sigma = \frac{N}{\ell} = \frac{\gamma \ell z \cos^2\beta}{\ell} = \gamma z \cos^2\beta$$

$$F = \frac{s}{\tau}$$

$$F = \frac{c + \sigma \tan\phi}{\tau}$$

Infinite Slope Analysis

$$F = \frac{c + \gamma z \cos^2\beta \tan\varphi}{\gamma z \cos\beta \sin\beta}$$

Total Stress Analysis

$$F = \frac{c' + (\gamma z \cos^2\beta - u)\tan\varphi'}{\gamma z \cos\beta \sin\beta}$$

Effective Stress Analysis

Infinite Slope Analysis

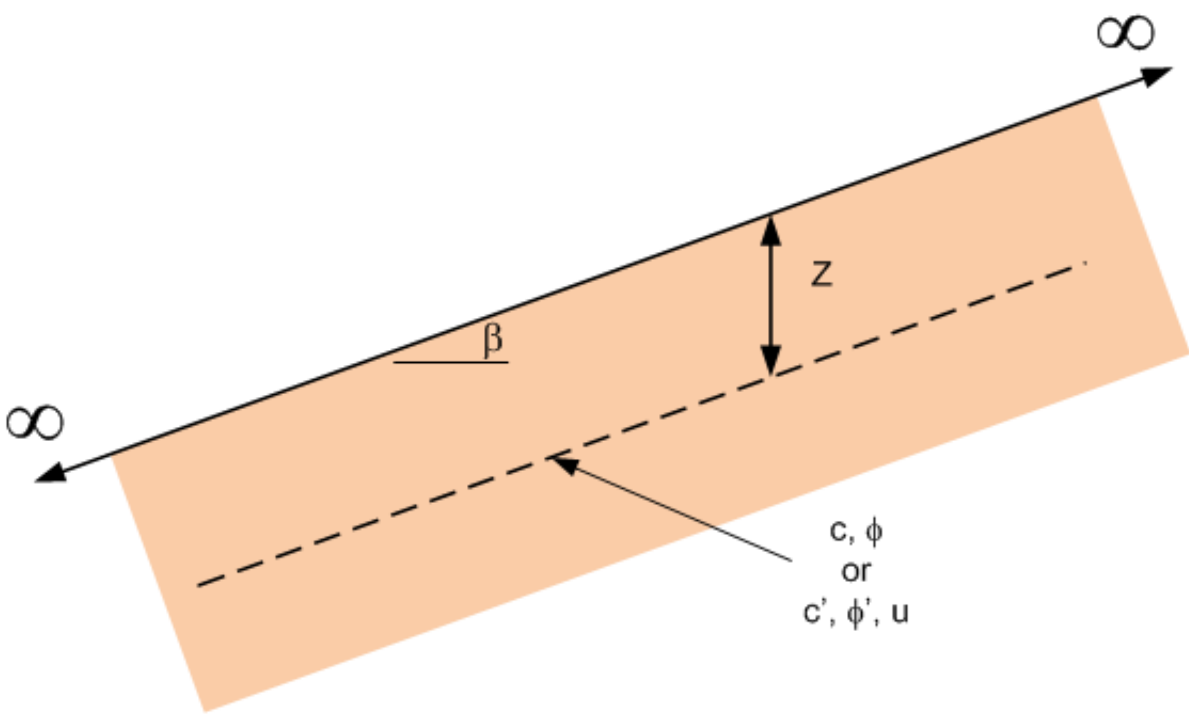
For c=0, u=0:

$$F = \frac{\gamma z \cos^2\beta \tan\varphi}{\gamma z \cos\beta \sin\beta}$$

$$F = \frac{\cos\beta \tan\varphi}{\sin\beta}$$

$$F = \frac{\tan\varphi}{\tan\beta}$$

Infinite Slope Analysis



Total Stress Analysis

- Non-submerged slopes

$$\mathrm{FS} = \frac{c}{\gamma z}\frac{2}{\sin(2\beta)} + \frac{\tan\phi}{\tan\beta}$$

- Submerged slopes

$$\mathrm{FS} = \frac{c}{\gamma' z}\frac{2}{\sin(2\beta)}$$

$c = S_u$, $\phi = 0$

Effective stress analysis

- General case

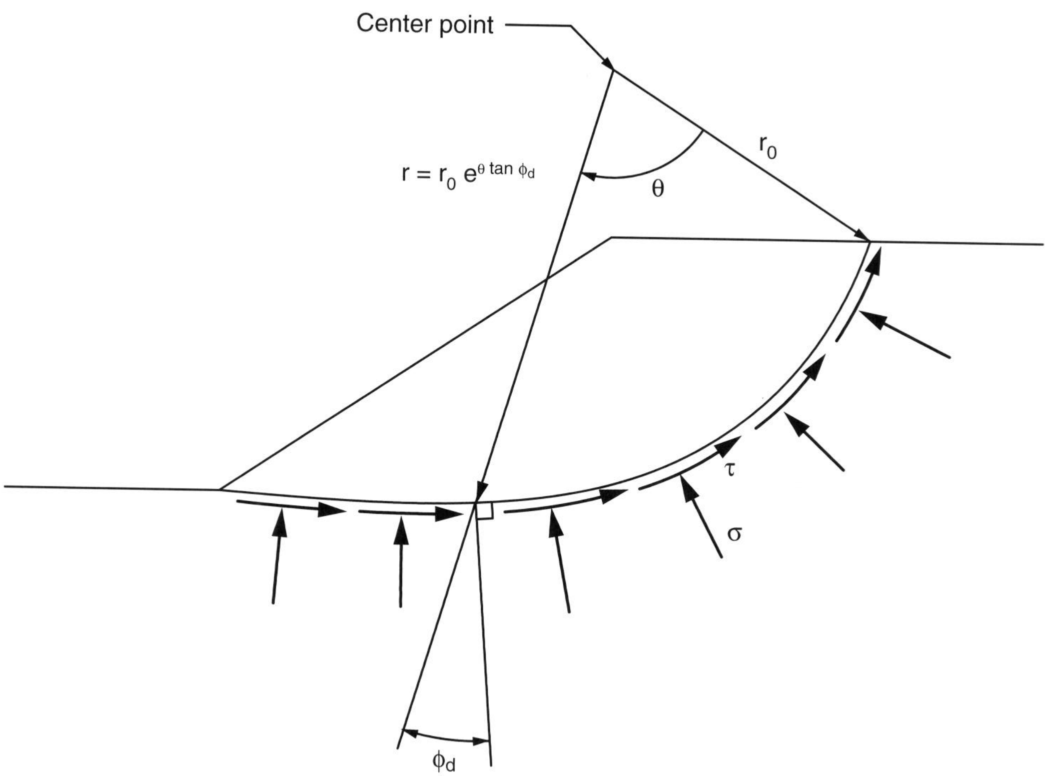
$$FS = \frac{(\bar{c} - u\tan\bar{\phi})}{\gamma z}\frac{2}{\sin(2\beta)} + \frac{\tan\bar{\phi}}{\tan\beta}$$

- Submerged slopes

$$FS = \frac{\bar{c}}{\gamma' z}\frac{2}{\sin(2\beta)} + \frac{\tan\bar{\phi}}{\tan\beta}$$

Log Spiral Technique

Log Spiral Technique



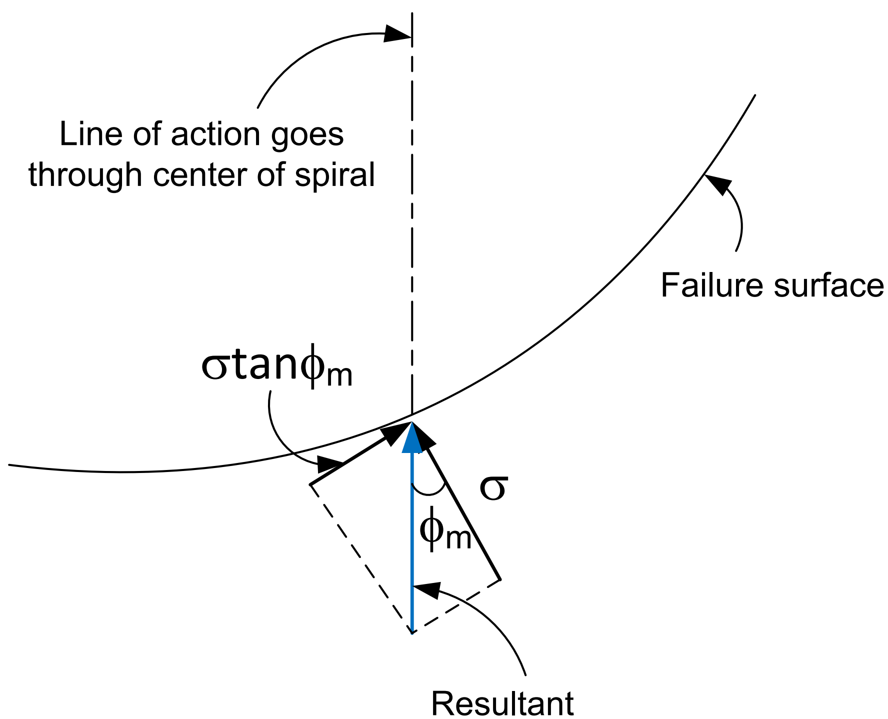
$$\tau = \frac{c}{FS} + \frac{\sigma \tan\phi}{FS}$$

$$c_d = \frac{c}{FS} \quad \tan\phi_d = \frac{\tan\phi}{FS}$$

c_d = developed or mobilized cohesion

$\tan\phi_d$ = developed or mobilized friction angle

Log Spiral Technique



Sum moments about center point:

(1) Moment due to wt of soil mass, W
(2) Moment due to mobilized cohesion Cm
(3) Moments due to mobilized friction and normal forces <u>cancel</u>

(1) and (2) give you one equation with one unknown (F), but geometry depends on F so problem is solved iteratively.

Log Spiral Summary

- Achieves statically determinate solution by assuming log spiral shape for slope
- Explicitly satisfies moment equilibrium; implicitly satisfies force equilibrium
  - Complete equilibrium
  - Relatively accurate

Log Spiral Summary, cont.

- Best procedure for homogenous slopes
- Tough to solve by hand
- Popular method for
  - Charts
  - Simple programs

Swedish Method (ϕ=0)

Swedish Method (ϕ=0)

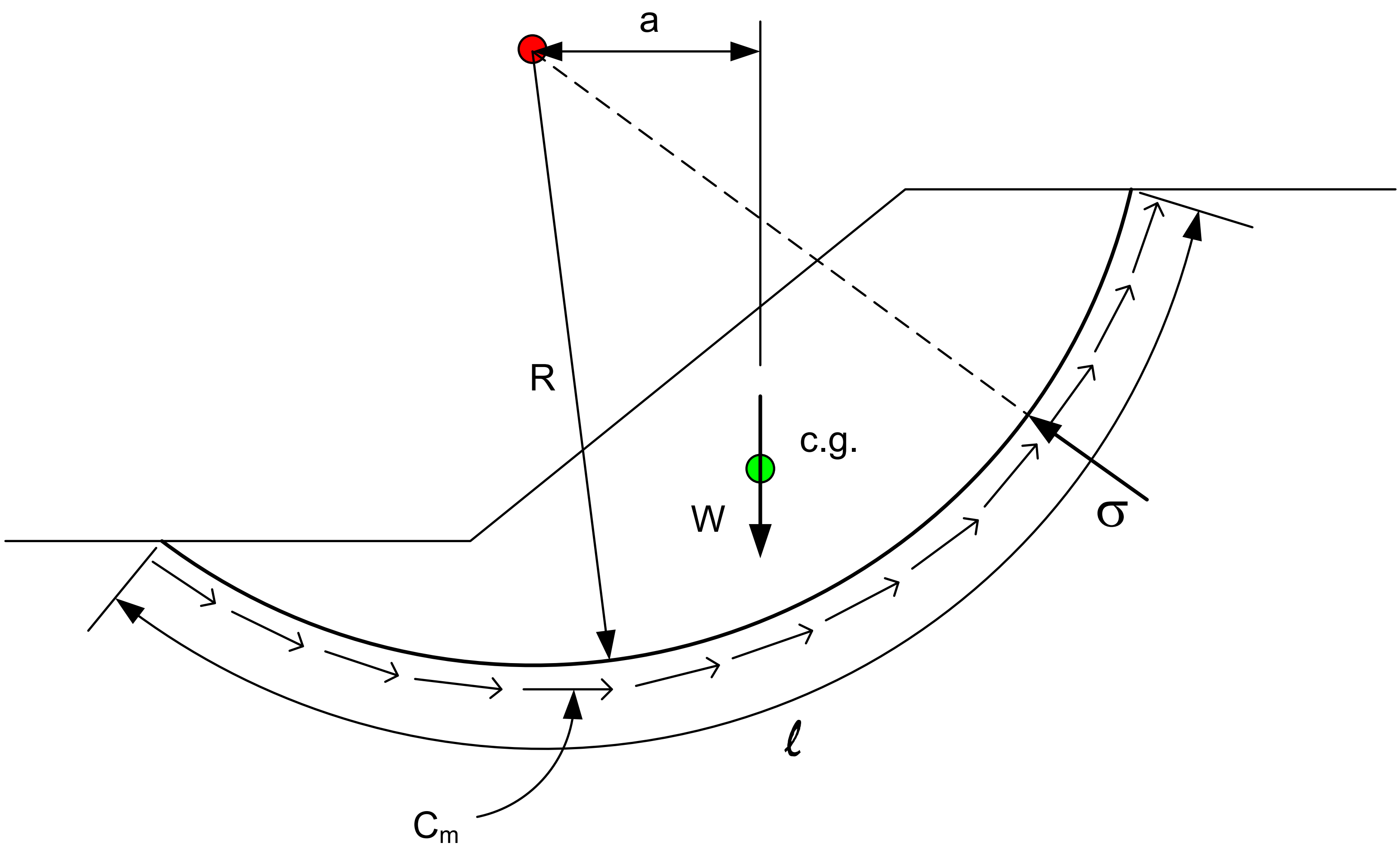
- Applicable to
  - Saturated soil
  - Undrained conditions
- Can be thought of as special case of log spiral:

$$r = r_o e^{\theta \tan \cancelto{0}{\varphi_m}}$$

(i.e. circular surface)

$$r = r_o e^0 = r_0$$

Swedish Method



Swedish Method

Sum moments about the center of the circle:

$$\mathrm{Wa} - \mathrm{c_m}\ell\mathrm{R} = 0$$

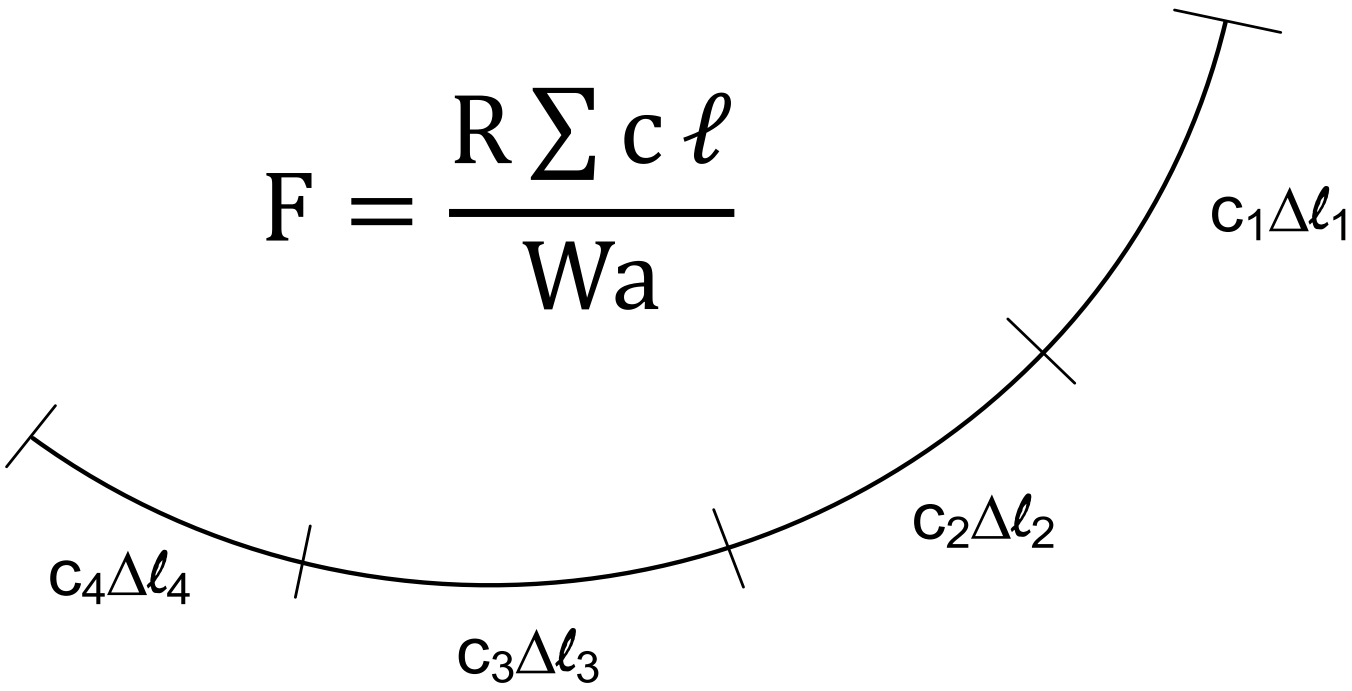
$$\mathrm{Wa} = \mathrm{c_m}\ell\mathrm{R} = \frac{\mathrm{c}}{\mathrm{F}}\ell\mathrm{R}$$

$$\mathrm{F} = \frac{\mathrm{c}\ell\mathrm{R}}{\mathrm{Wa}} = \frac{\text{Available resisting moment}}{\text{Actual driving moment}}$$

Swedish Method

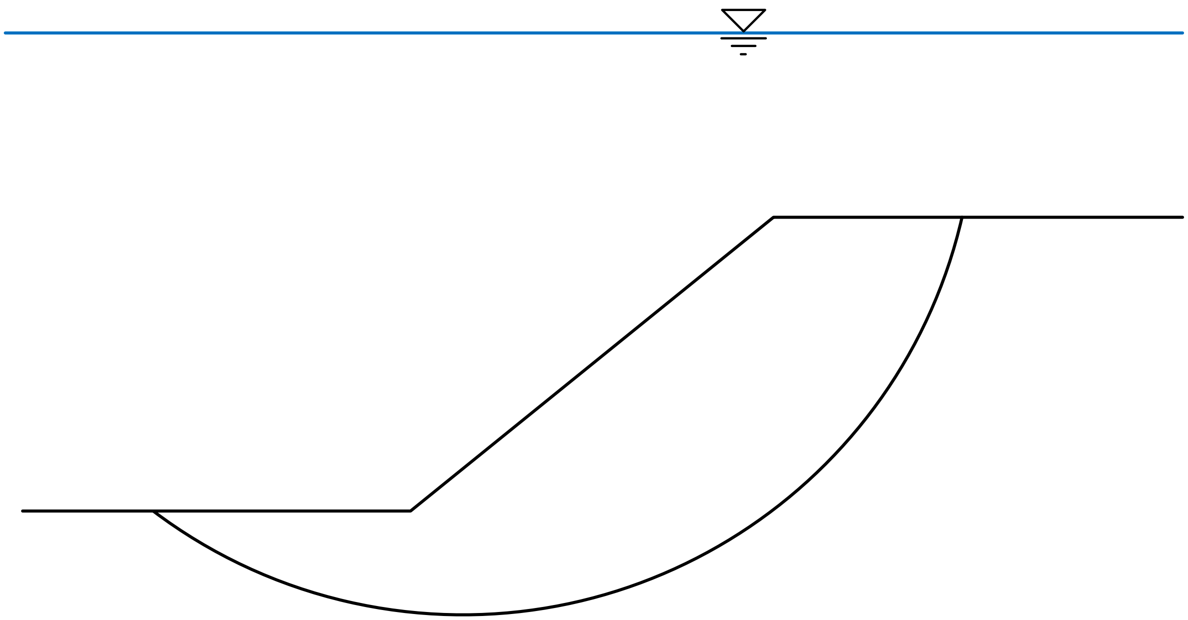
If c varies along the slope, break up slope and sum the individual moments:

$$F = \frac{R \sum c\,\ell}{Wa}$$



Swedish Method

Submerged Slopes



Use submerged unit wt. to account for buoyant force

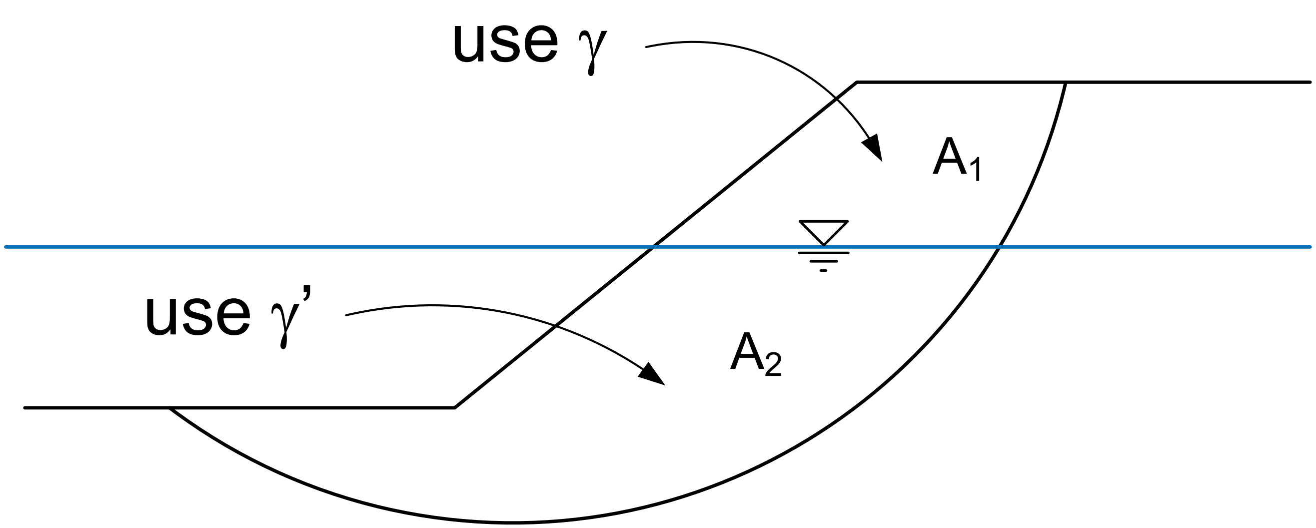
$$Wa = (\gamma-\gamma_w)Aa$$

$$Wa = \gamma' Aa$$

A = Area of sliding mass

Swedish Method

Partially Submerged Slopes



$$F = \frac{c\ell R}{\gamma A_1 a_1 + \gamma' A_2 a_2}$$

Circular Surface, $\phi \neq 0$

ΣMoments about center:

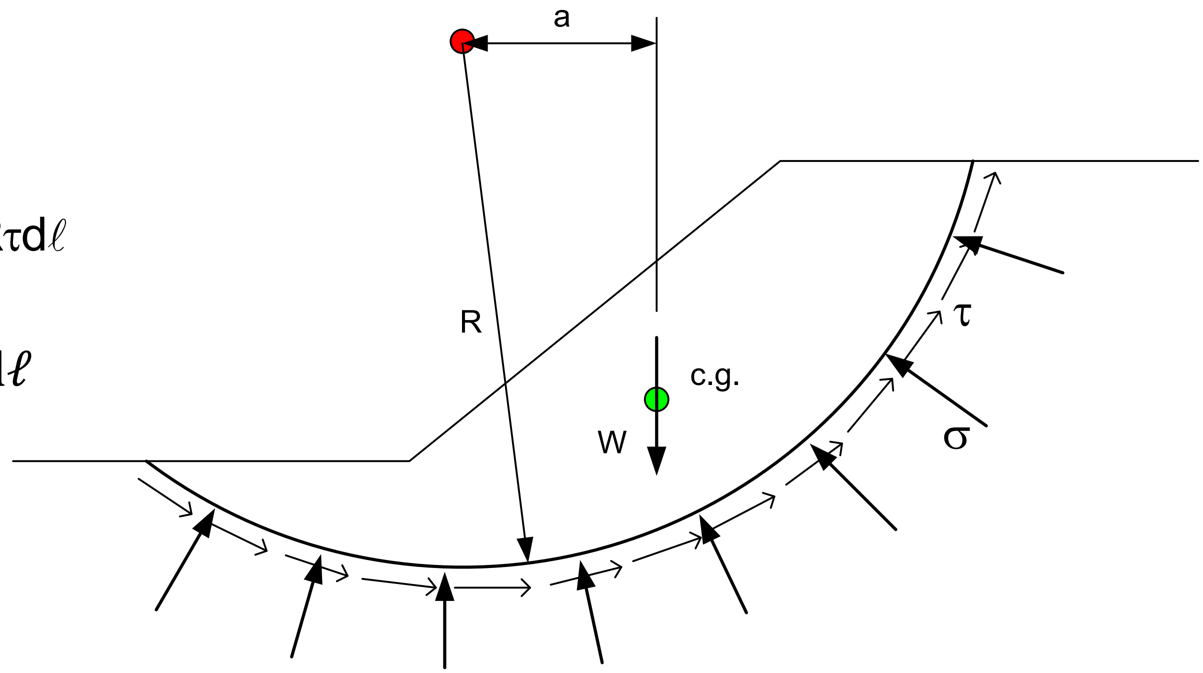
1) Due to weight, M_γ = Wa
2) Due to normal stress, M_σ = 0
3) Due to shear stress, $M_\tau = \int R\tau d\ell$

$$\sum M = M_\gamma - M_\tau = Wa - \int R\tau d\ell$$

$$\tau = \frac{c'}{F} + (\sigma - u)\frac{\tan\phi'}{F}$$

$$Wa - R\int\left[\frac{c'}{F} + (\sigma - u)\frac{\tan\phi}{F}\right]d\ell = 0$$

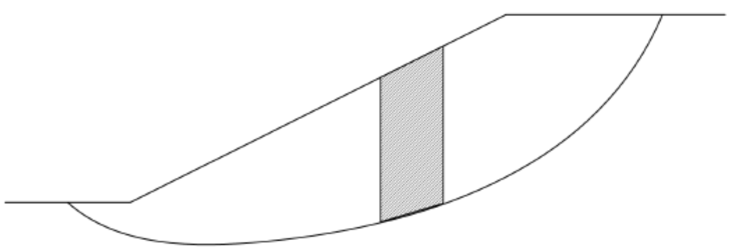
$$F = \frac{R\int[c' + (\sigma - u)\tan\phi]\,d\ell}{Wa}$$

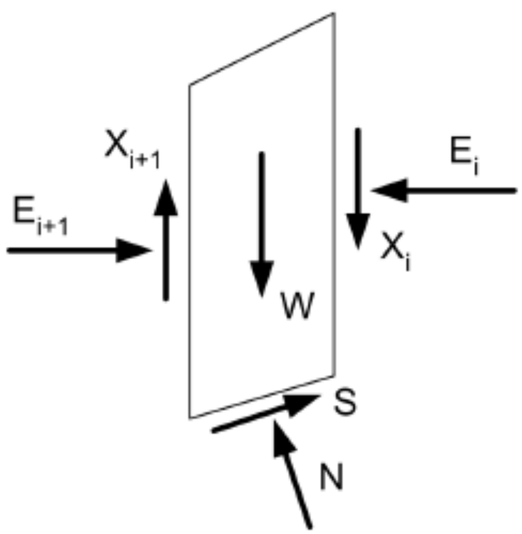


Note: σ varies along the surface and is unknown so you can't solve directly for F

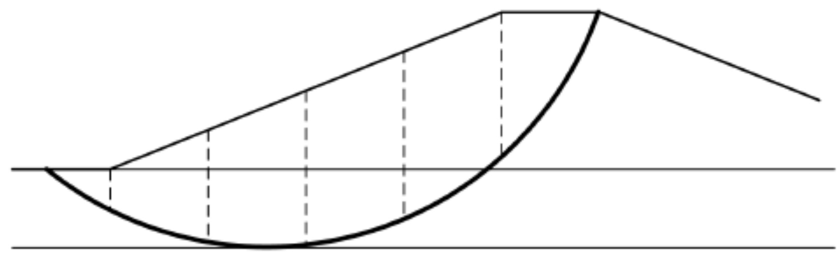
General Method of Slices

Method of Slices

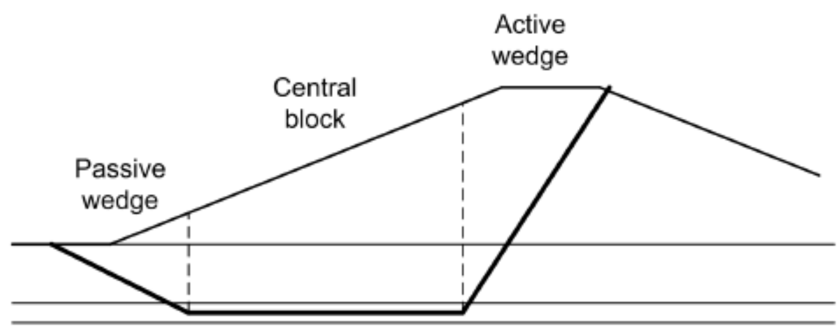




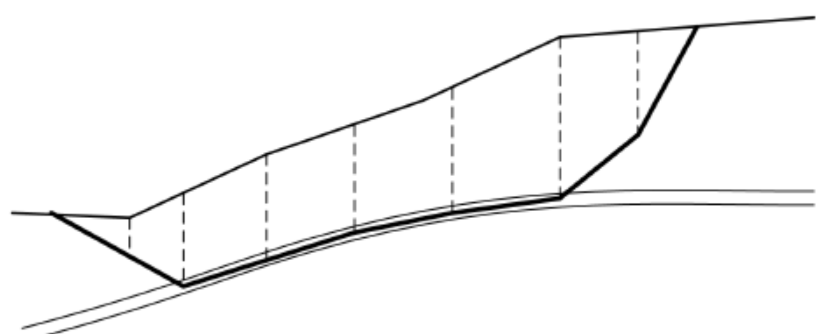
$$F = \frac{\sum\left[c'\Delta\ell + \left(W\cos\alpha - u\Delta\ell\cos^2\alpha\right)\tan\phi'\right]}{\sum W\sin\alpha}$$



a. Circular

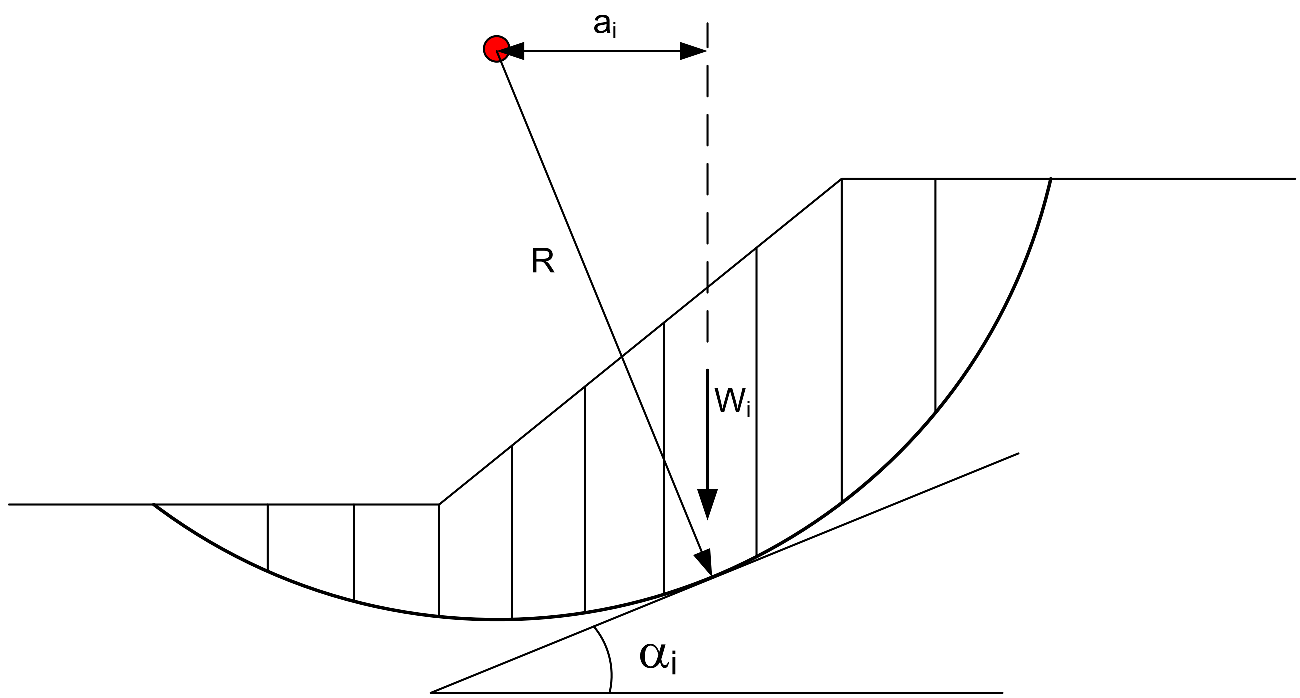


b. Wedge

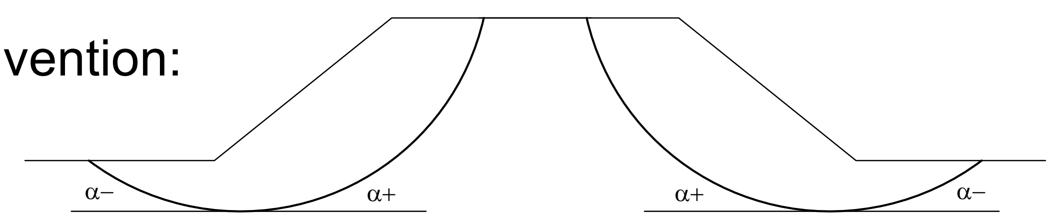


c. General noncircular

General Method of Slices



Sign convention:



General Method of Slices

Moment arm:

$$a_i = R\sin\alpha_i$$

Driving moment:

$$M_d = R\sum W_i \sin\alpha_i$$

Resisting moment:

$$M_r = \sum RS_i = R\sum S_i$$

$$M_r = R\sum \tau_i \Delta\ell_i$$

$$M_r = R\sum \frac{s_i \Delta\ell_i}{F}$$

$$M_d = M_r$$

$$R\sum W_i \sin\alpha_i = R\sum \frac{s_i \Delta\ell_i}{F}$$

$$\sum W_i \sin\alpha_i = \sum \frac{s_i \Delta\ell_i}{F}$$

$$F = \frac{\sum s_i \Delta\ell_i}{\sum W_i \sin\alpha_i}$$

$$s = c + \sigma\tan\phi$$

$$F = \frac{\sum (c + \sigma\tan\phi)\Delta\ell_i}{\sum W_i \sin\alpha_i}$$ ← General eq.

General Method of Slices

If ϕ=0

$$F = \frac{\sum (c + \sigma \tan\cancelto{0}{\phi})\Delta\ell_i}{\sum W_i \sin\alpha_i}$$

$$F = \frac{\sum c\Delta\ell_i}{\sum W_i \sin\alpha_i}$$

This equation is often called the "Swedish Method"

Compare to Swedish Circle equation:

$$F = \frac{c\ell R}{Wa}$$