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US Army Corps
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COMPUTER APPLICATIONS IN GEOTECHNICAL
ENGINEERING (CAGE)
AND
GEOTECHNICAL ASPECTS OF THE COMPUTER-AIDED
STRUCTURAL ENGINEERING (G-CASE) PROJECTS

INSTRUCTION REPORT GL-87-1

USER'S GUIDE: UTEXAS2 SLOPE-STABILITY PACKAGE

VOLUME II: THEORY

by

CAGE, G-CASE Task Group on Slope Stability

February 1989

Final Report

Approved For Public Release; Distribution Unlimited

GEOTECHNICAL LABORATORY

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This report describes the theory of a two-dimensional slope-stability analysis and covers the mechanics of limit-equilibrium procedures that utilize the wedge and slices methods. The mechanics of the four different procedures contained in the computer program UTEXAS2 (University of Texas Analysis of Slopes - version 2) are discussed as well as sources of potential errors. The limit-equilibrium equations and the calculation procedures used in the program are described in an appendix.

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PREFACE

This report describes the theory of the two-dimensional slope-stability analysis as performed in UTEXAS2. The mechanics of limit-equilibrium procedures which use the method of slices are covered herein. Bishop's Simplified procedure, force equilibrium procedures that use the Corps of Engineers' Modified Swedish side-force assumption, wedge assumptions described in EM 1110-2-1902, and Spencer's complete equilibrium procedure are all discussed in detail, as well as sources of potential errors. The work is a product of the US Army Corps of Engineers Slope-Stability Task Group, a combination of efforts of the Computer Applications in Geotechnical Engineering (CAGE) and the Geotechnical Aspects of the Computer-Aided Structural Engineering (G-CASE) projects. Both projects are sponsored by the Engineering Division, Engineering and Construction Directorate of the Headquarters, US Army Corps of Engineers (USACE), Department of the Army.

The contents of this report resulted from a combined effort of the task group members:

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This report was assembled by Mr. Earl V. Edris, Jr., Soil Mechanics Division, Geotechnical Laboratory (GL), WES, and revised by the task group. Dr. Wright wrote Appendix A of this report.

The CAGE project was under the general supervision of Dr. William F. Marcuson III, Chief, GL, WES. The G-CASE project was managed and coordinated by Dr. N. Radhakrishnan, Chief, Information Technology Laboratory (ITL), WES, and CASE project manager. The material for this report was edited by Mr. Bobby Odom, ITL, under the Intergovernmental Personnel Act.

COL Dwayne G. Lee, EN, was the Commander and Director of WES. Dr. Robert W. Whalin was Technical Director.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
degrees (angle)	0.01745329	radians
kips (force)	4.448222	kilonewtons

USER'S GUIDE: UTEXAS2 SLOPE-STABILITY PACKAGE

PART I: INTRODUCTION

Purpose of UTEXAS2

1. A large number of slope-stability computer programs have been developed over the past years. A survey of geotechnical computer programs compiled by the US Army Engineer Waterways Experiment Station (WES) in 1982 (Edris and Vanadit-Ellis 1982) listed 37 different programs used throughout the Corps. Being developed for specific purposes, many of the programs are restricted in the range of conditions which can be analyzed. Many of the programs are not well documented, and the specific algorithms implemented are not readily apparent. Systematic evaluation of the various programs is difficult because of their diversity and because each requires a different format for data entry.

2. For these reasons, a joint venture task group of the Computer Applications in Geotechnical Engineering (CAGE) and the Geotechnical Aspects of the Computer-Aided Structural Engineering (G-CASE) projects was tasked by the Headquarters, US Army Corps of Engineers (USACE), to develop a slope-stability package suitable for Corps-wide use. This package of slope-stability programs will offer the following benefits to the Corps of Engineers by:

- a. Providing documented material to the design engineer as a maintained and updated part of the Corps' computer library.
- b. Facilitating review of District work by Divisions and architectural-engineering contract work by both Corps Districts and Divisions.
- c. Enabling different analysis procedures to be conveniently used from a common input data file.

3. The criteria for this limit-equilibrium slope-stability package are contained in Miscellaneous Paper GL-85-8 (CAGE Task Group on Slope Stability 1985) which concluded that:

- a. No program in existence meets all the criteria outlined in the report.
- b. The program UTEXAS (University of Texas Analysis of Slopes), developed by Wright and Roecker (1984) for the Texas Highway Department, most nearly meets all the criteria.

c. Capability and criteria modifications to UTEXAS would be faster and more cost effective than to write a new program.

The version of UTEXAS containing the additional capabilities is called UTEXAS2.

Organization of User's Manual

4. The UTEXAS2 User's Guide is organized into three volumes to avoid a large, cumbersome report. The use of separate volumes also provides for the timely publication of the user-required guidelines of the program. Volume I contains the user guidelines including instructions for input and output, illustrative examples, search procedure recommendations, and error message explanations. Volume II of the series contains the theory and derivations of the equations used in the program. Volume III consists of problems illustrating coding procedures for generic problem types and demonstrating the capabilities and versatility of the program.

Mechanics of Procedures

5. The mechanics of limit-equilibrium procedures are covered in this study of two-dimensional slope-stability analysis. The three force equilibrium procedures discussed in this report include the wedge method described in Engineer Manual (EM) 1110-2-1902 (Headquarters, Department of the Army 1970) and two procedures that use the method of slices. These force equilibrium procedures use the Corps of Engineers Modified Swedish side-force assumption of parallel side forces at a user-specified inclination, EM 1110-2-1902 (Headquarters, Department of the Army 1970), or Lowe and Karafiath's side-force assumption (1960). The mechanics of Bishop's Simplified procedure (Bishop 1955), and Spencer's complete equilibrium procedure (Spencer 1967) are also discussed as are sources of potential error for all methods. The various cases of slope-loading conditions (i.e., steady seepage, sudden drawdown, etc.), characterization of material properties, and internal water-pressure determinations are not covered. These topics, discussed in EM 1110-2-1902 (Headquarters, Department of the Army 1970), are very important but are beyond the scope of this user's guide.

Definition of the Factor of Safety

6. The f~~tor of safety F* in this program is defined with respect to shear stren~~~ as

$$F = \frac{s}{\tau} \tag{1}$$

where

s = available shear strength

τ = shear stress required for just-stable, static equilibrium

The shear strength (s) in Equation 1 is expressed in terms of the Mohr-Coulomb failure criteria. In the case of effective stress analyses, the shear strength is expressed by

$$s = \bar{c} + (\sigma - u) \tan \bar{\phi} \tag{2}$$

where

$\bar{c}, \bar{\phi}$ = shear strength parameters expressed in terms of effective stress

σ = total normal stress

u = pore water pressure

$(\sigma - u)$ = effective normal stress

For total stress analyses, the shear strength parameters are expressed in terms of total stress equivalents, (c, ϕ), and pore water pressures are not considered. Thus, total stress analyses are expressed by the equation

$$s = c + \sigma \tan \phi \tag{3}$$

The only differences between total and effective stress expressions for shear strength and factor of safety are whether or not total stress or effective stress-strength parameters $(c, \phi$ or $\bar{c}, \bar{\phi})$ are used and whether the pore water pressure u appears in the strength equations. Procedures and equations developed on the basis of effective stresses may be applied equally to total stress analyses by using c and ϕ, rather than $\bar{c}$ and $\bar{\phi}$, and by considering the pore water pressure to be zero.

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* For convenience, symbols and abbreviations are listed in the Notation (Appendix B).

7. The factor of safety defined by Equation 1 is computed from requirements of static equilibrium. The factor of safety is introduced into the equations of static equilibrium by first expressing Equation 1 as

$$\tau = \frac{s}{F} \tag{4}$$

where τ represents the shear stress which appears in the equations of static equilibrium. Combining Equation 4 with the expression for the shear strength (Equation 2), the shear stress is expressed as

$$\tau = \frac{\bar{c}}{F} + (\sigma - u) \frac{\tan \bar{\phi}}{F} \tag{5}$$

where the expression on the right-hand side of Equation 5 is used to replace the shear stress in the equations of static equilibrium. The factor of safety applicable to cohesion ($\bar{c}$, c) is assumed to be the same factor applicable to the frictional component ($\tan \bar{\phi}$, $\tan \phi$) of shear strength in Equation 5. A second assumption is that the factor of safety is constant along an assumed shear (or sliding) surface. These assumptions reduce the number of unknowns related to the factor of safety which must be computed from the equilibrium equations and are discussed in detail later in the User's Guide.

8. In limit-equilibrium procedures, the factor of safety is calculated for an assumed shear surface. A number of trial shear surfaces are analyzed until the one producing a minimum factory of safety is found. In comparing the factors of safety calculated by the various procedures, it is appropriate to compare only the minimum factors. The corresponding critical shear surfaces may differ for different procedures and, thus, separate critical shear surfaces must be found for each procedure (Duncan and Wright 1980). If factors of safety for other than the critical shear surfaces are compared, the differences in results for various procedures may be much larger than the differences among minimum factors of safety. The larger differences associated with other-than-critical shear surfaces can be misleading and, in most cases, have no practical meaning.

Subdivision of Soil Mass

9. In order to facilitate limit-equilibrium procedures, the soil mass which is bounded by the assumed shear surface and slope surface is subdivided into a finite number of vertical slices. This allows analysis of relatively inhomogeneous slopes where the properties along the shear surface at the base of each slice may differ from slice to slice.

10. Total forces acting on the free-body diagram of a typical slice employed in the procedures of slices are shown in Figure 1. The total forces

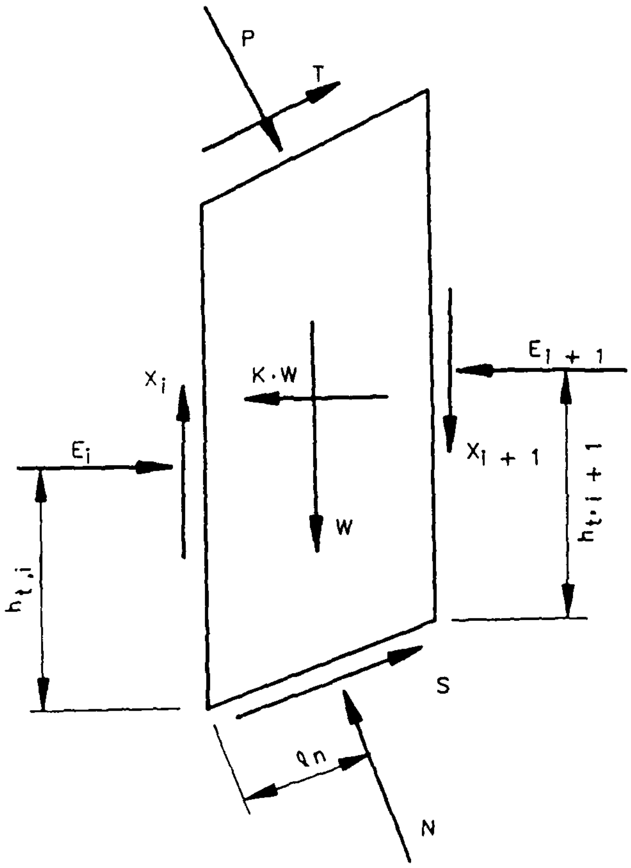

Figure 1. Total forces acting on a typical slice for the procedure of slices

acting on the slice include the weight of the slice W, the shear and normal force on the base of the slice, S and N, respectively, the shear forces on the left and right boundaries of the slice, X_i and X_{i+1}, respectively, and the horizontal forces on the left and right boundaries of the slice, E_i and E_{i+1}, respectively. In the case of effective stress analysis, N, X, and

E may be expressed as $\bar{N}$, $\bar{X}$, $\bar{E}$, and a pore pressure term. In addition, forces such as water forces on the base and sides H_B , H_L , and H_R ; external surface loads P ; external shear forces T ; and "pseudo-static" seismic forces $K \cdot W$ may also be included. These forces are considered as known quantities in terms of their location, direction, and magnitude. The shear force S on the base of the slice is defined in terms of the normal force N and factor of safety F through the Mohr-Coulomb Strength Equation 5. Thus, the shear force can also be considered a known quantity based on given soil parameters, provided that F and N are known or calculated from equilibrium requirements. The unknown quantities which must be assumed or computed in a solution of the equations of static equilibrium include the F , N , and the side forces X and E . In addition, the locations of N on the base of the slice ℓ_N and of the horizontal side forces on the sides of the slice h_t are unknown and must be assumed or computed. The total number of unknowns associated with a soil mass subdivided into n slices is summarized in Table 1 (Wright 1982) with the corresponding number of static-equilibrium equations; 5n - 2 unknowns and 3n equilibrium equations exist.

Table 1

Unknowns and Equilibrium Equations for Procedures of Slices (Wright 1982)

Description	Number*
Unknowns	
Normal force on base of slice N	n
Normal (horizontal) force between slices E	n - 1
Shear (vertical) force between slices X	n - 1
Location of normal force on base of slice ℓ_N	n
Location of side force between slices h_t	n - 1
Factor of safety F	1
Total unknowns	5n - 2
Equilibrium equations	
Summation of forces in the vertical direction	n
Summation of forces in the horizontal direction	n
Summation of moments	n
Total equations	3n

* Number of slices = n.

11. The number of unknown quantities (5n - 2) shown in Table 1 exceeds the corresponding number of equilibrium equations (3n). Accordingly, assumptions must be introduced to obtain a statically determinate solution for the F and for any of the other unknown quantities which are to be calculated. Various assumptions pertaining to the unknown quantities are made in each of the different limit-equilibrium procedures. Differences among the assumptions constitute one of the principal differences among the various methods of slices. In addition, some procedures satisfy all requirements (3n) of planar equilibrium while others only partially satisfy complete equilibrium. Differences among the specific equilibrium conditions satisfied constitute a second principal difference among the various methods of slices procedures.

Simplified Bishop Procedure

12. The Simplified Bishop, also known as Bishop's Modified procedure (Bishop 1955), is one of the most widely used and accepted methods of slices. This procedure, for circular shear surfaces, was proposed by Bishop (1955) as a simplified version of his detailed approach satisfying complete equilibrium. According to this procedure, there are assumed to be no shear forces between slices (X = 0), and forces are resolved in the vertical direction to obtain an equation for N on the base of each slice. The equation for N is incorpcrated into an equation for moment equilibrium about the center of a circular shear surface. The equation of moment equilibrium considers the entire soil mass (all slices) as a single free body and is used to compute the factor of safety. For a slope with n slices, the Simplified Bishop procedure employs and satisfies n + 1 equilibrium equations, consisting of n equations for equilibrium of forces in the vertical direction and one equation for moment equilibrium for the entire soil mass. The corresponding unknowns which are solved from these equations are N on the base of each slice and one factor of safety. Table 2 summarizes the assumptions along with the unknown forces for the Simplified Bishop procedure (Bishop 1955).

13. Although this procedure does not satisfy all requirements for static equilibrium, it has been shown to produce reasonably correct values for the factor of safety (Duncan and Wright 1980). A number of comparisons for relatively homogeneous slope conditions have been made between factors of safety calculated by the Simplified Bishop procedure and by procedures of

Table 2

Assumptions, Unknowns, and Equilibrium Equations for the Simplified Bishop Procedure (Bishop 1955)

Description	Number*
Assumptions	
Vertical shear force on sides of slice X equals 0	
Horizontal side forces are not considered	
Unknowns	
Normal force on base of slice N	n
Factor of safety F	1
Total unknowns	n + 1
Equilibrium equations	
Summation of forces in the vertical direction	n
Summation of moment of forces overall equilibrium equation for total soil mass	1
Total equations	n + 1

* Number of slices = n.

slices which satisfy complete static equilibrium (Whitman and Bailey 1967, Fredlund and Krahn 1977, Duncan and Wright 1980). The factors of safety are usually in agreement within a few percent. The factors calculated by the Simplified Bishop procedure have also been compared with values which were calculated using stresses computed by both linear and nonlinear finite element procedures (Wright, Kulhawy, and Duncan 1973). These comparisons also show that the factors of safety are in close agreement, ±5 percent for homogeneous slope conditions.

14. The principal limitation of the Simplified Bishop procedure results from the assumption of circular shear surfaces. Although several extensions of Bishop's procedure to noncircular shear surfaces have been suggested (Nonveiller 1965, Bell 1969), documentation of the accuracy of such procedures is not available. Thus, the procedure is limited to analyses of slopes where the assumption of circular shear surfaces is reasonable. This procedure does not satisfy horizontal force equilibrium, and caution should be exercised when used for pseudo-static analyses of earthquake conditions.

15. The derivations of the equations used for the Simplified Bishop procedure are presented in Appendix A.

Force-Equilibrium Procedure

16. The force-equilibrium procedures are one type of limit-equilibrium procedure which consider and satisfy the requirements for equilibrium of forces in the vertical and horizontal direction for each slice and the sliding mass, but do not satisfy moment equilibrium. Such force-equilibrium procedures are all suitable for analyses employing shear surfaces of any general shape, and the required calculations can be performed by hand. Accordingly, force-equilibrium procedures have been widely accepted and used by many practicing engineers. The Corps of Engineers uses this procedure as described in EM 1110-2-1902, Engineering and Design Stability of Earth and Rock-Fill Dams (Headquarters, Department of the Army 1970).

17. The unknowns for force-equilibrium procedures are reduced from a total of $5n - 2$ to $3n - 1$ by not considering the locations ℓ_n and h_t associated with the forces N and E. There are $2n$ equations for force equilibrium, and, thus, a total of $n - 1$ assumptions are required to make the problem statically determinate. All force-equilibrium procedures make these $n - 1$ assumptions pertaining to the inclination of the side forces. Side forces, X and E, can be described in terms of components of the resultant Z and its inclination θ from the horizontal. Thus, the side forces shown in Figure 1 can also be expressed by $n - 1$ unknown values of Z and $n - 1$ unknown values of θ. Force-equilibrium procedures assume values for θ to achieve a statically determinate solution, and Z is considered as an unknown. The n values for N, $n - 1$ values for Z, and one value for F are computed by using the $2n$ equations of static equilibrium for all slices. Differences among the many force-equilibrium procedures are a result of the various assumptions which are made concerning the side-force inclination θ. As a result, the factor of safety is directly related to θ as shown in Figure 2.* Table 3 summarizes the unknowns and the assumptions associated with this procedure.

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* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 4.

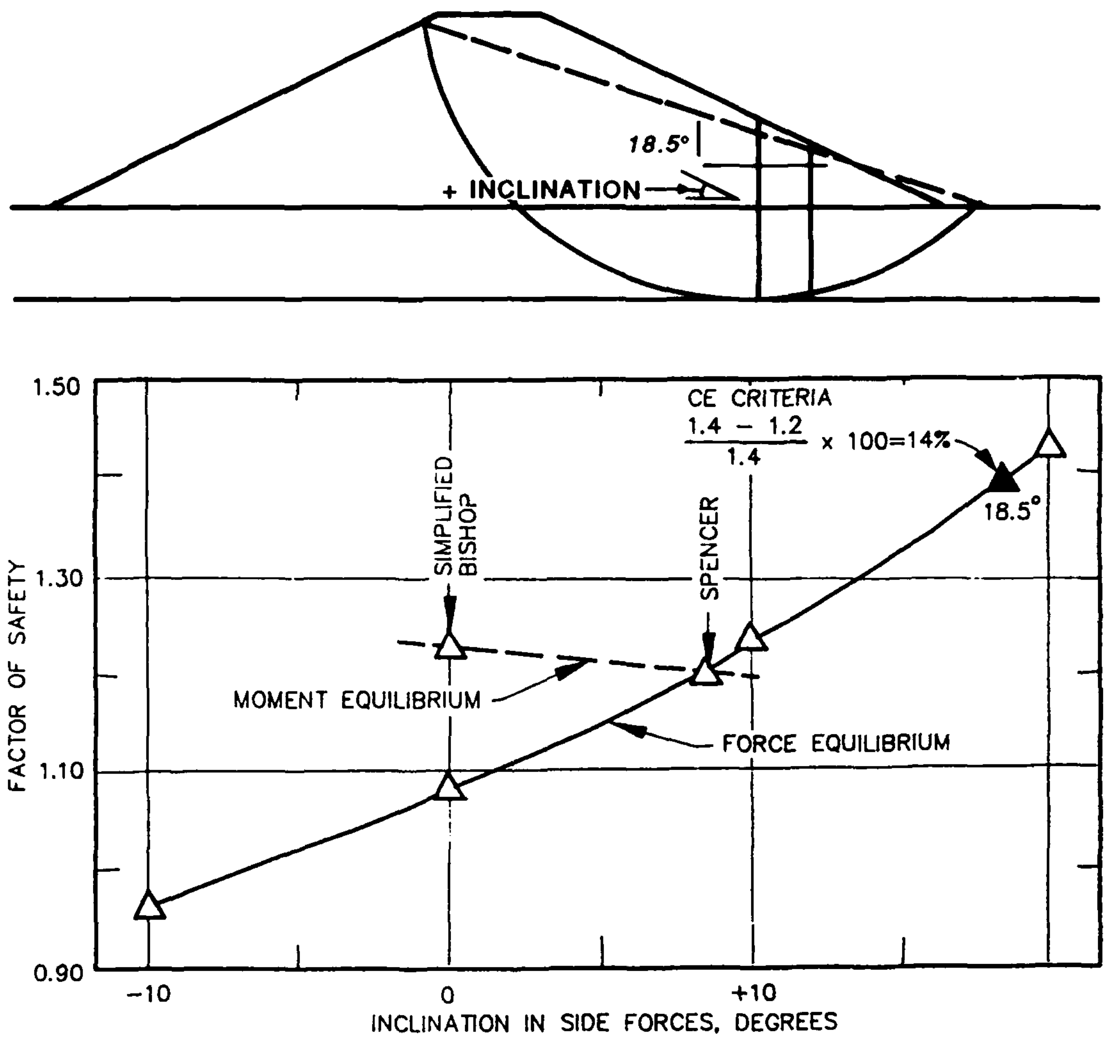

Figure 2. Influence of the parallel side-force inclination on the values of F calculated by force-equilibrium and moment-equilibrium procedures (after Wright 1969)

Table 3

Assumptions, Unknowns, and Equilibrium Equations for Force-Equilibrium Procedures

Description	Number*
Assumptions	
Inclination of resultant interslice force θ is specified and constant	
Unknowns	
Normal force on base of slice N	n
Interslice normal force E or resultant interslice force Z	n - 1
Factor of safety F	1
Total unknowns	2n
Equilibrium equations	
Summation of forces in the vertical direction	n
Summation of forces in the horizontal direction	n
Total equations	2n

* Number of slices = n.

18. Force-equilibrium procedures require an assumption for θ. There are several θ assumptions that could be used. EM 1110-2-1902, Engineering and Design Stability of Earth and Rock-Fill Dams (Headquarters, Department of the Army 1970), specifies that the side-force inclination should be parallel to the average outer slope of the embankment as shown in Figure 2. This assumption has been found to produce factors of safety which are as much as 14 percent higher than values calculated by procedures which satisfy complete equilibrium (Wright 1969). Thus, the assumption is not conservative and may lead to significant overestimates of the factor of safety as shown in Figure 2.

19. There are other θ assumptions that could be utilized, the first being that side forces are horizontal. This assumption is the same as that employed in the Simplified Bishop procedure (Bishop 1955). However, the equilibrium conditions which are satisfied in the force-equilibrium procedures are different from those satisfied in the Simplified Bishop procedure. The

assumption of horizontal side forces in the force-equilibrium procedures often leads to factors of safety which are significantly less than those calculated by the Simplified Bishop procedure or procedures which satisfy all requirements for static equilibrium. Analyses of a variety of both homogeneous and inhomogeneous slopes show that the assumption of horizontal side forces in force-equilibrium procedures can cause the factor of safety to be at least 20 percent less than values calculated by complete equilibrium procedures (Wright 1969). Janbu, Bjerrum, and Kjaernsli (1956) found similar differences of up to 13 percent.

20. Another side-force inclination assumption uses the average of the outer slope and the shear-surface inclination (Lowe and Karafiath 1960). The authors suggested that the side force on each boundary between slices acts at an inclination which is the average of the inclinations of the slope and the shear surface directly above and below the slice boundary, respectively. This side-force assumption has been found to produce values for the factor of safety which are generally within ±10 percent of the values calculated by complete equilibrium procedures (Wright 1969, Duncan and Wright 1980). The differences (±10 percent) are smaller than those associated with the other side-force assumptions discussed above. Thus, the user needs to be aware of the effect of θ assumption input for the Modified Swedish procedure.

21. Using a θ assumption, the Modified Swedish procedure calculates a composite force polygon. The error of closure is determined for each trial factor of safety until the sign of the closure error changes. Then, the trial factors of safety are plotted against the error of closure, and the equilibrium factor of safety at zero error of closure is determined by interpolation. Forces on typical slices and the composite-force polygon are shown in Figure 3. The derivations of the equations used for the Modified Swedish procedure are presented in Appendix A.

Corps of Engineers Wedge Procedure

22. The ability to analyze noncircular shear surfaces is an advantage of the force-equilibrium procedures. The Corps of Engineers wedge procedure discussed in EM 1110-2-1902 (Headquarters, Department of the Army 1970) is a force-equilibrium procedure with specified side-force assumptions that provides the ability to analyze shear surfaces which correspond to zones of

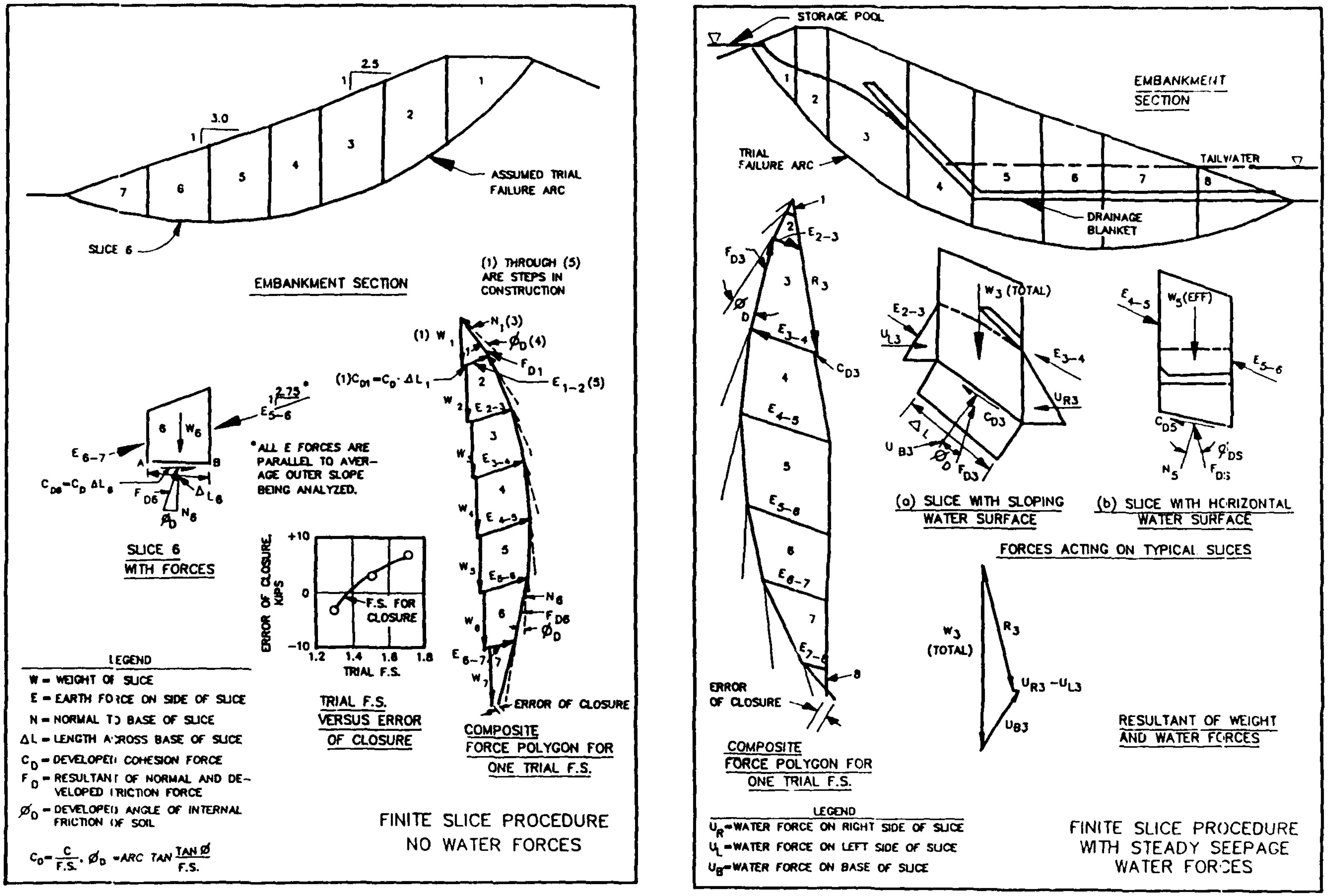

Figure 3. Modified Swedish circular-arc procedure

material weaknesses. Composite interslice or interblock forces are determined for active and passive wedges and central block. Typical force polygons are shown in Figure 4, and the prescribed inclinations of side forces are shown in Figure 5. These side-force inclinations, where the active and passive forces are incl'-ed at different angles, cannot be modeled in the program UTEXAS2.

23. The unknowns, equilibrium equations, and assumptions presented in Table 3 for the force-equilibrium procedures also apply to the wedge procedure. The derivations of the equations used for the wedge procedure are the same as in Appendix A except for the side-force assumptions shown in Figures 4 and 5.

Limitations of the Force-Equilibrium Procedures

24. The main limitation of the force-equilibrium procedures results from the sensitivity of the computed factor of safety to the various side-force assumptions since moment equilibrium is not considered. As shown in Figure 2, the factor of safety is a function of the side-force inclination. Consequently, there is a large degree of uncertainty associated with the correctness of a given side-force assumption for a given problem. For example, when ϕ is equal to zero and a circular shear surface is assumed, the assumption that side forces are parallel to the shear surface produces precisely the same factor of safety as the value which is obtained by complete equilibrium procedures. Other side-force assumptions, in this case ($\phi = 0$), produce incorrect results. However, when ϕ is not equal to zero, the assumption that side forces are parallel to the shear surface may be relatively poor.

Spencer's Procedure

25. Spencer's procedure (Spencer 1967) is a complete equilibrium procedure and is defined as a stability-analysis procedure that fully satisfies the force and moment requirements of static equilibrium for each slice. Factors of safety calculated by several of the available complete equilibrium procedures and compared for a variety of different slopes (Wright 1969, Sarma 1973, Fredlund and Krahn 1977, Duncan and Wright 1980) have shown that the results of these comparisons are in agreement within ±5 percent of a mean value, and no procedure appears to produce consistently high or low values for the factor of safety (Wright 1982). This procedure is suitable for analysis of both

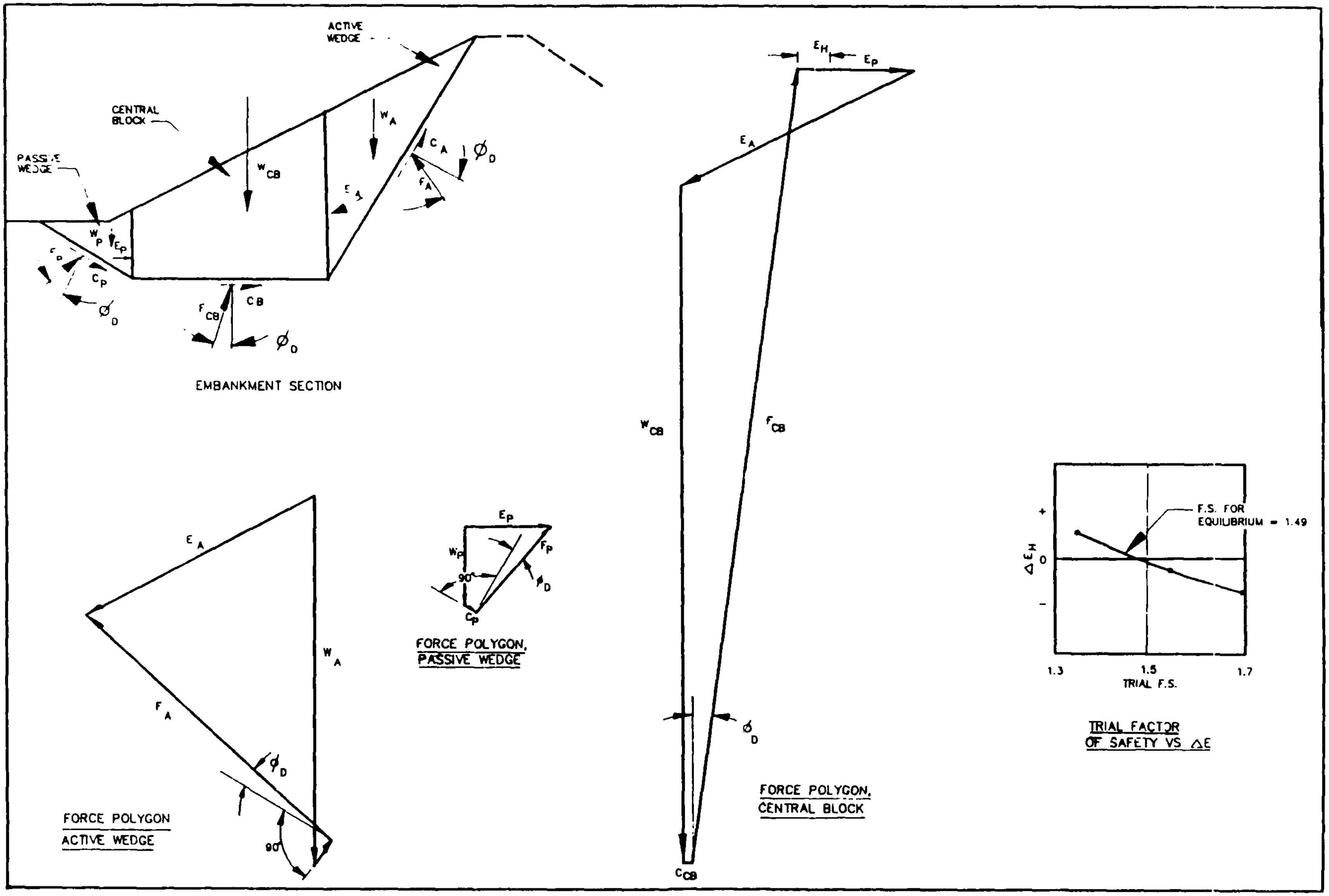

Figure 4. Corps of Engineers wedge procedure

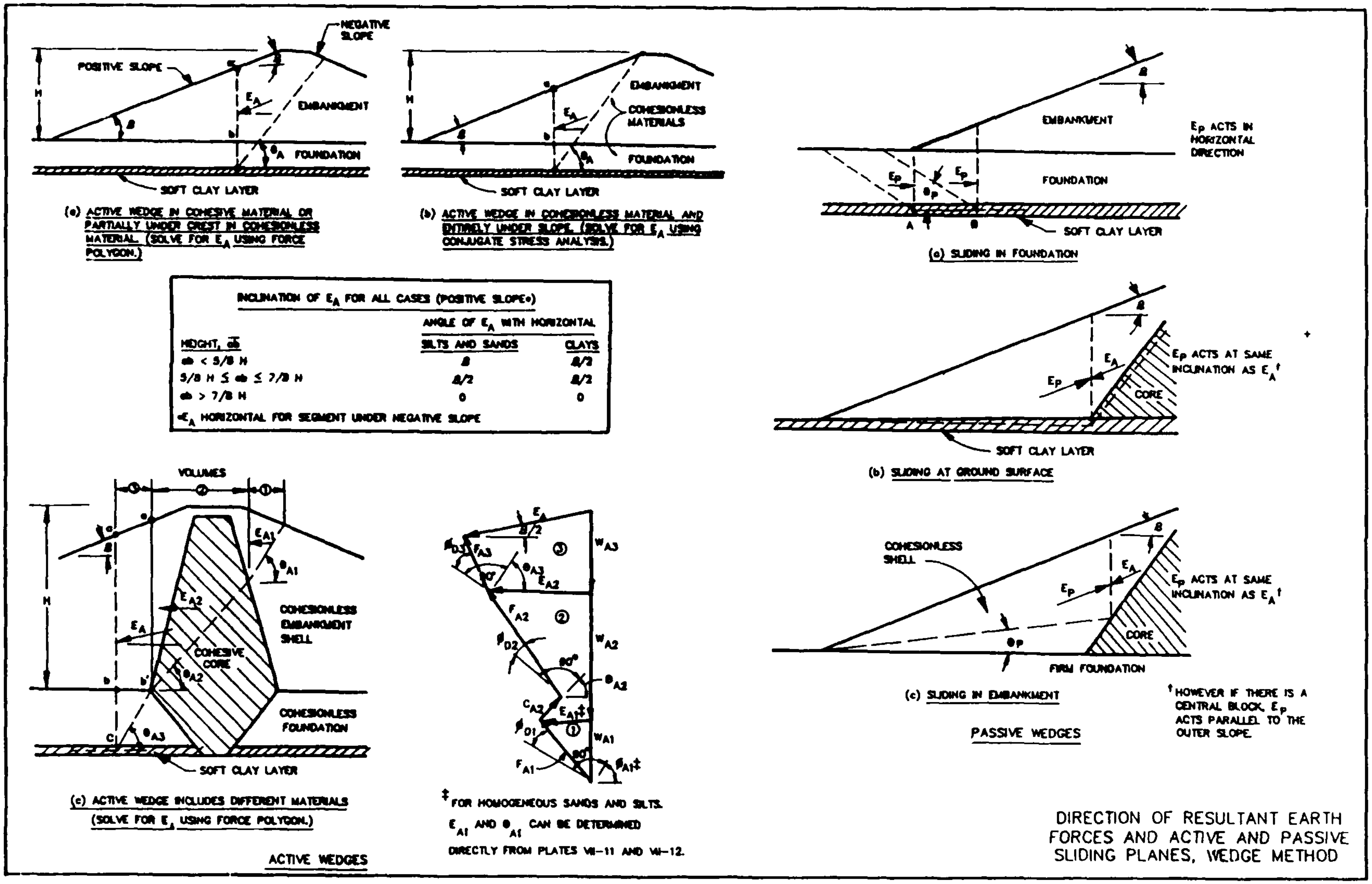

Figure 5. Corps of Engineers wedge method, prescribed inclination of side forces

circular and noncircular or wedge shear surfaces. However, the computations require the use of a computer and are impractical for hand calculations.

26. Spencer (1967) developed this procedure which satisfies both force and moment equilibrium. The θ is not assumed, but determined, so that complete equilibrium is satisfied. The side forces are assumed to be parallel. Also, the assumption is made that the normal forces are located at the center of each slice base. These assumptions result in the reduction by $2n - 2$ of the unknowns, leaving $3n$ unknowns. Thus, the problem becomes statically determinate. Table 4 summarizes the unknowns and equilibrium equations for this procedure. The derivation of the Spencer's procedure is given in Appendix A.

27. It is impractical to solve the equilibrium equation in Spencer's procedure by hand. However, one can check the method by drawing a composite-force polygon in the manner of the force-equilibrium procedure with the Corps of Engineers Modified Swedish side-force assumption (EM 1110-2-1902) (Headquarters, Department of the Army 1970), using the angle of side-force inclination computed by Spencer's procedure (Spencer 1967). The force polygon must close if the solution is correct. The location, magnitude, and sign of the side force for each slice can be checked for correctness. Also, comparison of results can be made with the values obtained by using one of the other procedures.

Table 4

Unknowns, Assumptions, and Equilibrium Equations for the Complete Equilibrium Procedure (Spencer 1967)

Description	Number*
Assumptions	
Side forces assumed parallel: $X = E \cdot \tan \theta$; θ is the same for all slices	
Location of normal forces on base of slice assumed at midpoint on base	
Unknowns	
Normal force on base of slice N	n
Normal (horizontal) force between slices E	n - 1
Side force inclination θ	1
Location of side force between slices h_t	n - 1
Factor of safety F	1
Total unknowns	3n
Equilibrium euations	
Summation of forces in the vertical direction	n
Summation of forces in the horizontal direction	n
Summation of moments	n
Total equations	3n

* Number of slices = n.

PART III: SOURCES OF POTENTIAL ERROR (WRIGHT 1982)

28. Two cases can be readily identified for any of the stability procedures where even the most statically correct limit-equilibrium slope-analysis procedures will produce incorrect or unreasonable values for the factor of safety and for the various other calculated values. The two cases where incorrect or unreasonable values occur are related directly to specific slope and shear surface conditions. The first case where incorrect or unreasonable results may occur corresponds to slopes where material with a relatively high cohesive component (c, $\bar{c}$) of strength exists along the upper crest portion of the shear surface. The second case corresponds to slopes where shear surfaces exit steeply upward through cohesionless material near the toe of the slope. These two cases and the resultant problems they introduce require separate consideration, and different solutions are used to eliminate errors associated with each.

High Cohesive-Strength Components

29. When a relatively high cohesive component of strength exists along the shear surface near the crest of a slope, limit-equilibrium solutions often indicate relatively high tensile stresses; in such cases, N acting on the shear surface and E near the crest of the slope become negative (tensile). Also, near the crest of the slope, the locations of the side forces ("line of thrust") diverge and fall well outside the boundaries of the slope and shear surface, approaching an infinite distance from the slope in the extreme. Wright (1975) has shown that such a pattern for the side-force locations is closely related to existing zones of tensile stress. The tensile stresses implied in the observed solutions are fully consistent with the use of a cohesive component of strength in the Mohr-Coulomb equation and the associated stress state required for a limit state of equilibrium. The zone in which the tensile stresses are observed is analogous to an active earth-pressure zone, and such tensile stresses are well recognized in classical active earth-pressure calculations for cohesive materials. Although tensile stresses are consistent with the assumed conditions, their existence in limit-equilibrium slope-stability solutions may lead to complications and errors. Spencer's limit-equilibrium procedure (Spencer 1967) encounters various degrees of

numerical instability in which the iterative, trial and error procedures used to calculate the factor of safety may not produce a convergent solution when relatively high tensile stresses exist.

30. Tensile stresses resulting from high cohesive-strength components are in many cases unrealistic and may easily exceed any tensile-strength capacity which the soil can provide for stability. Often, the cohesive-strength component assigned to describe the strength of a soil simply represents an intercept obtained by extrapolating a straight-line failure envelope to zero normal stress (σ, $\bar{\sigma}$) on a Mohr-Coulomb diagram. While such a cohesion intercept implies a certain tensile strength, tensile strengths are seldom measured or considered to be realistic for soils. Accordingly, tensile stresses calculated in stability analyses employing Mohr-Coulomb parameters with a significant cohesion should, in many instances, be considered erroneous and eliminated. A simple, practical solution for eliminating tensile stresses from analyses by any limit-equilibrium procedure is to introduce a vertical "crack" and to terminate the upper portion of the shear surface at a vertical slice boundary with an appropriate depth below the crest of the slope. The depth of the crack d_c can be estimated from the Rankine active earth-pressure theory as

$$d_c = \frac{2c_m}{\gamma \tan\left(45 - \frac{\phi_m}{2}\right)} \qquad (7)$$

where

c_m, ϕ_m = mobilized strength parameters ($c_m = c/F$, $\tan \phi_m = [\tan \phi]/F$)

γ = unit weight of soil

Usually, the factor of safety (c_m and ϕ_m) can be estimated with a reasonable degree of accuracy for purposes of establishing a crack depth from Equation 7 prior to stability calculations; an estimated value of unity for F is often adequate. Use of a crack depth substantially greater than the depth given by Equation 7 should be avoided because it will not only eliminate zones of tensile stress but also compressive stress; thus, the factor of safety may be overestimated. Introduction of a crack with an appropriate depth eliminates both unrealistic tensile stresses and virtually all numerical instabilities which are associated with tensile stresses in various procedures.

Shear-Surface Inclinations

31. A second potential source of error in limit-equilibrium analyses results from conditions where shear surfaces exit steeply upward through cohesionless soil. In such cases, N and E near the exit portion of the shear surface may become very large, approaching infinite values in the extreme, or may become negative. In addition, the iterative numerical solution used in the Simplified Bishop (Bishop 1955), force-equilibrium, and complete equilibrium procedures to calculate the factor of safety may oscillate or diverge. These problems are often associated with analyses employing circular shear surfaces for embankments on foundations where a relatively thin layer of cohesionless soil is underlain by a thicker stratum of weaker soil, often soft clay. In these cases, the critical shear surface is often relatively deep and passes steeply upward at its exit. The problem of very large or negative stresses near the toe can also occur for relatively shallow, noncircular shear surfaces, which have a relatively flat, horizontal portion at a shallow depth exiting abruptly at the ground surface. When the assumed inclination of the exiting portion of the surface is too steep, the solution for the factor of safety will diverge, or unrealistic values for the factor of safety and stresses at the toe of the shear surface will result. Whitman and Bailey (1967) recognized the problem associated with steeply exiting, circular shear surfaces and discussed the problem as it pertains to the Simplified Bishop procedure (Bishop 1955) although the problem occurred with force-equilibrium and complete equilibrium slice procedures as well.

32. Wright (1969) has examined a number of cases where unreasonably large or negative values have been calculated for the stresses near the toe of the shear surfaces. In each of these cases, the stresses were consistent with those calculated by trial wedge or Coulomb passive earth-pressure procedures. This consistency was dependent on the fact that the shear-plane and interslice (earth-pressure)-force inclinations which were used for the earth-pressure calculations were the same as those from the corresponding limit-equilibrium slope analyses. However, the inclinations of the shear planes which were being considered did not represent reasonable critical shear-plane inclinations and would not have been expected to form based on reasonable passive earth-pressure considerations; instead, the inclinations of the shear planes were much steeper than passive earth-pressure theories would indicate.

33. There are some adjustments, in such cases, that can be made in the side-force assumptions employed in the various limit-equilibrium procedures. These adjustments can improve the reasonableness of stresses near the toe of steeply inclined shear surfaces. However, such adjustments may require tedious effort on the part of the user and may only partially solve the problem. A preferred alternative solution is to adjust the shape (inclination) of the shear surface in the cohesionless material to conform with a more reasonable state of passive shear resistance. An appropriate inclination for the shear surface can be estimated from Coulomb passive earth-pressure theory. The inclination determined in this manner is a function of the θ (earth-pressure resultant) and the mobilized friction angle $\bar{\phi}_m$ of the cohesionless soil. The shear-surface inclinations α are plotted versus θ for various values of ϕ_m in Figure 6. An appropriate α can be estimated from this figure and the shear surface adjusted to this inclination. Such adjustments to α will entirely eliminate unreasonable stresses in the passive zone and provide a distinctly improved solution.

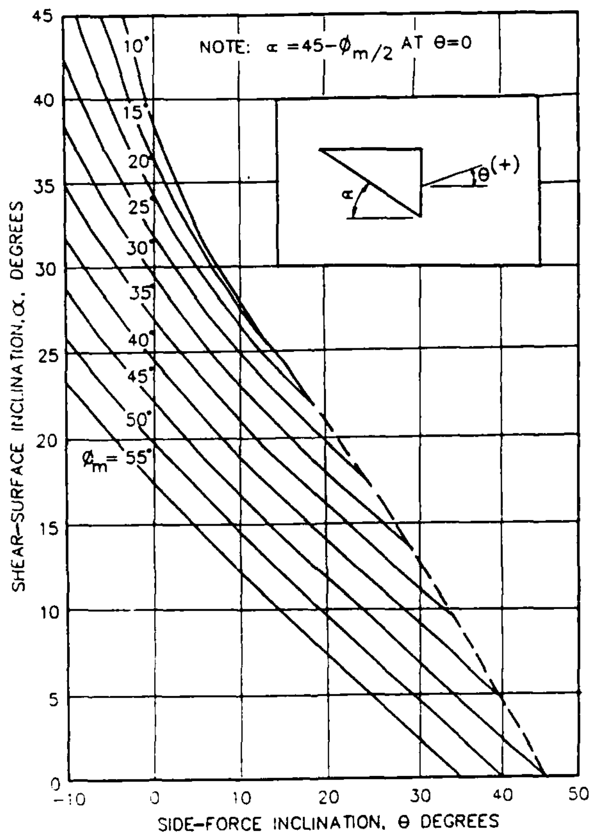

Figure 6. Critical α's from Coulomb passive earth-pressure theory for various values of θ and ϕ_m (after Wright 1982)

PART IV: CONCLUSIONS

34. In conclusion, the relative advantages and limitations of the various analysis procedures are presented in Table 5. These procedures differ principally with respect to the static-equilibrium conditions which they satisfy and the assumptions which are made to achieve a statically determinate solution for the factor of safety. The Spencer procedure (Spencer 1967) satisfies all static-equilibrium requirements.

Table 5

Relative Advantages and Limitations of Limit-Equilibrium Procedures

Procedure	Advantages	Limitations
Simplified Bishop	Simple; value for F is reasonably correct calculations can be performed by hand	Restricted to circular shear surfaces; could give incorrect results for pseudo-static analyses
Force equilibrium (Corps of Engineers Modified Swedish side-force assumption, Lowe and Karafiath's side-force assumption and Corps wedge side-force assumption	Circular and noncircular shear surfaces; calculations can be performed by hand	Factor of safety is sensitive to side-force assumptions, Lowe and Karafiath's side-force assumption appears to work best; solution does not satisfy moment equilibrium
Spencer	Statically correct; circular and noncircular shear surfaces; complete equilibrium	Requires use of computer for most practical applications

35. The Simplified Bishop (Bishop 1955) and all force-equilibrium procedures do not fully satisfy the requirements of static equilibrium and, thus, are not statically complete procedures. The force-equilibrium procedures are relatively inaccurate, and the factors of safety calculated by these procedures may differ by 50 percent or more for some situations from the calculated values using complete equilibrium procedures. Accordingly, the usefulness of

a force-equilibrium procedure rests primarily in its suitability for hand calculations employing noncircular shear surfaces. The Simplified Bishop procedure is probably more correct than either of the force-equilibrium procedures. Factors of safety calculated by the Simplified Bishop procedure have been found to agree within approximately ±5 percent with those calculated by complete equilibrium slice procedures for relatively simple slope and loading conditions. Thus, for at least some cases, the Simplified Bishop procedure appears to be relatively correct. However, this procedure does not consider equilibrium of forces in the horizontal direction. Accordingly, use of the procedure for complex slope and loading conditions where significant horizontal external loads and internal body forces are applied may lead to large inaccuracies.

36. Although procedures which satisfy all requirements for static equilibrium are statically correct procedures and most appear to produce essentially the same value for the factor of safety, the procedures may also produce incorrect or unreasonable values for the factor of safety. For a variety of slopes, the geometry assumed for the shear surface in the "active" or "passive" zone can be important and may influence the computed results. Introduction of a vertical crack in the active zone or adjustment of the inclination of the shear surface in the passive zone is often required to obtain a reasonable solution. The assumption of a constant factor of safety along the shear surface implies that the shear strength is developed simultaneously along the entire shear surface. For cases where this is not true due to dissimilar or brittle materials, this assumption may lead to incorrect or unreasonable results.

37. All limit-equilibrium analysis procedures are subject to the basic limitations imposed by the definition of the factor of safety; however, all procedures which satisfy complete static equilibrium produce almost the same value for the factor of safety. These complete equilibrium procedures provide a useful means for estimating the stability of earth slopes and can be used with considerable confidence for their intended purposes.

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APPENDIX A: LIMIT-EQUILIBRIUM SLOPE-STABILITY EQUATIONS USED IN THE COMPUTER PROGRAM UTEXAS2 (Wright 1986)*

Introduction

1. The computer program UTEXAS2 has the capability of using four different limit-equilibrium slope-stability analysis procedures for computing the factor of safety F (Wright 1986). The four procedures are Spencer's (1967) procedure; Simplified Bishop (1955) procedure; force-equilibrium procedure with the Corps of Engineers Modified Swedish side-force assumption, Engineer Manual (EM) 1110-2-1902 (Headquarters, Department of the Army 1970); and force-equilibrium procedure with Lowe and Karafiath's (1960) side-force assumption. Each of these procedures is a "procedure of slices" in which the soil mass, bounded by the surface of the slope and an assumed shear (sliding) surface, is divided into a number of vertical slices. The equilibrium equations used to compute the factor of safety, as well as other unknown quantities computed in the solution by each of these limit-equilibrium procedures, are presented in this appendix. In addition, the numerical procedures used to solve these equations in the computer program UTEXAS2 are presented.

2. The Modified Swedish and Lowe and Karafiath procedures each satisfy all requirements for equilibrium of forces on individual slices, but do not satisfy moment equilibrium. Except for differences in the assumptions made regarding the inclination of the side forces in these two procedures, the two procedures are otherwise identical. Accordingly, presentation of the equations and numerical solutions is combined for the Modified Swedish and Lowe and Karafiath (1960) procedures and is covered under the single heading of "Force-Equilibrium Procedures."

3. The general nomenclature and sign conventions used are presented in this appendix along with general equations and derivations common to all procedures. The remaining paragraphs of this appendix cover the specific equations, derivations, and numerical solution procedures for Spencer's, the force equilibrium, and Simplified Bishop procedures.

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* References cited in this appendix are included in the References at the end of the main text.

Nomenclature and Sign Convention

4. A typical slice and the system of forces acting on the slice are shown in Figure A1. Coordinates are expressed in a right-hand Cartesian coordinate system with the x axis being horizontal and directed toward the right, and the y axis being vertical and directed upward for positive values. Forces are considered to be positive when they act in the positive x or y direction, unless noted otherwise. Angles are measured from the horizontal plane (x axis) and are considered to be positive in the counterclockwise direction. The forces acting on a slice consist of the weight of the slice W, a normal and a shear force on the top of the slice, P and T, respectively, a force R acting on the base of the slice to represent internal reinforcement, and the normal and shear forces on the base of the slice N and S, respectively. Excluding any force carried by reinforcement, a horizontal force KW which represents the body force for seismic loading in a "pseudo-static" analysis, and the normal and shear forces, E and X, respectively on the sides of the slice, these forces represent the forces in the soil. The normal and shear forces on the top of the slice represent any loads due to concentrated forces or distributed pressures on the surface of the slope.

5. The forces shown in Figure A1 represent total forces including any forces due to water pressures. All forces are shown acting in the direction in which they are assumed to act when their value is positive. Negative values for any of the forces act in the opposite directions to the ones shown in Figure A1. The weight of the slice is assumed to act through the midpoint of the slice, and the normal force is assumed to act at the center of the base of the slice for convenience. Although other locations might be assumed for these forces (W and N), the locations have been found to have a minor effect on the factor of safety. The coordinates of the center of the base of the slice are designated as x_b, y_b; the inclination of the base of the slice is by the angle α; the location of the normal force P on the top of the slice by the coordinates x_p, y_p; the inclination of the top of the slice by the angle β; the location of R by the coordinates x_r, y_r on the base of the slice, and its inclination by the angle ψ. The seismic force acts horizontally along a line with the y coordinate y_k.

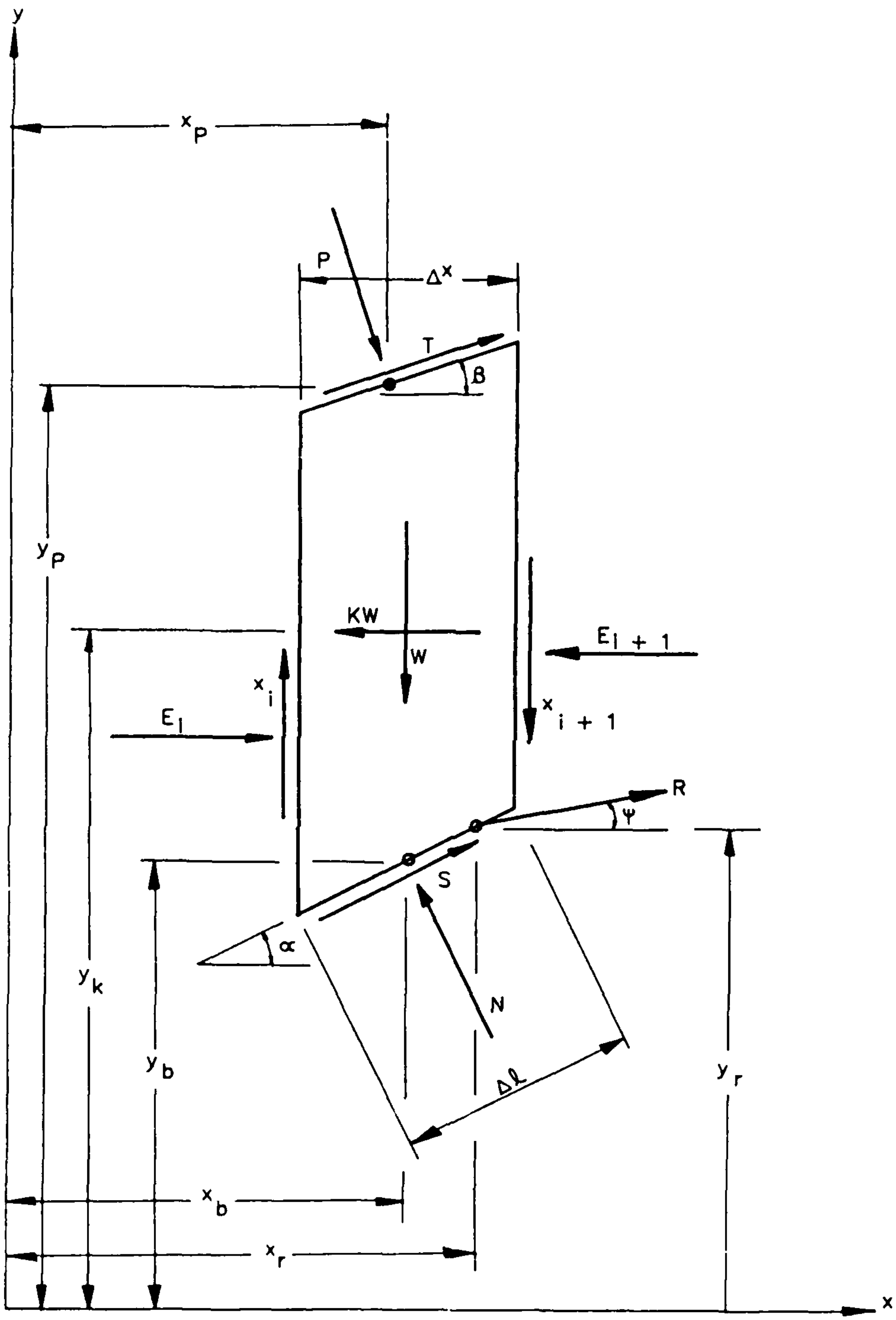

Figure A1. Total forces and coordinate nomenclature for a typical slice

General Equations

6. It is convenient to express the components of the known forces acting in the horizontal direction on a slice by the resultant F_h expressed as

$$F_h = -KW + P \sin \beta + T \cos \beta + R \cos \psi \tag{A1}$$

where the forces acting to the right are considered positive. Similarly, the components of the unknown forces acting in the vertical direction are expressed by the resultant F_v as

$$F_v = -W - P \cos \beta + T \sin \beta + R \sin \psi \tag{A2}$$

where forces acting in the upward direction are considered positive. The moment produced about the center of the base of the slice by the known forces is expressed by M_o as

$$\begin{aligned} M_o = {} & -P \sin \beta(y_p - y_b) - P \cos \beta(x_p - x_b) - T \cos \beta(y_p - y_b) \\ & + T \sin \beta(x_p - x_b) + KW(y_k - y_b) - R \cos \psi(y_r - y_b) \\ & + R \sin \psi(x_r - x_b) \end{aligned} \tag{A3}$$

where a counterclockwise moment is considered to be positive.

7. The shear force on the base of the slice can be expressed as

$$S = \tau \Delta \ell \tag{A4}$$

where

τ = average shear stress on the base of the slice

$\Delta \ell$ = length of the base of the slice

The average shear stress is related to the shear strength and factor of safety by the definition of the factor of safety, which is expressed as

$$F = \frac{s}{\tau} \tag{A5}$$

where s is the available shear strength.

8. The shear strength can be expressed in terms of effective stress shear-strength parameters $\bar{c}$ and $\bar{\phi}$ or total stress shear-strength parameters c and ϕ. In the case where effective stresses are used, the available shear strength is expressed by the Mohr-Coulomb equation as

$$s = \bar{c} + (\sigma - u) \tan \bar{\phi} \tag{A6}$$

which, when substituted into Equation A5 and rearranged, gives the following expression for the shear stress

$$\tau = \frac{\bar{c} + (\sigma - u) \tan \bar{\phi}}{F} \tag{A7}$$

Finally, combining Equations A7 and A4, the following expression is written for the shear force on the base of the slice

$$S = \frac{1}{F} \left[\bar{c}\Delta\ell + (N - u\Delta\ell) \tan \bar{\phi}\right] \tag{A8}$$

or, since $\Delta\ell = \Delta x \sec \alpha$, where Δx = width of slice

$$S = \frac{1}{F} \left[\bar{c}\Delta x \sec \alpha + (N - u\Delta x \sec \alpha) \tan \bar{\phi}\right] \tag{A9}$$

9. In the case where the shear strength is expressed using total stresses,

$$s = c + \sigma \tan \phi \tag{A10}$$

Equations A8 and A9 become

$$S = \frac{1}{F} \left[c\Delta\ell + N \tan \phi\right] \tag{A11}$$

and

$$S = \frac{1}{F} [c\Delta x \sec \alpha + N \tan \phi] \tag{A12}$$

respectively. Comparison of Equations A11 and A8 and Equations A12 and A9 shows that the equations for total stresses may be arrived at from the equations based on effective stresses by simply setting the pore water pressure to zero and replacing $\bar{c}$ and $\bar{\phi}$ in the effective stress equations by c and ϕ. In general, all of the equations for limit-equilibrium slope-stability analyses using total stresses may be derived from the corresponding equations for effective stresses by setting the pore water pressure u to zero and replacing $\bar{c}$ and $\bar{\phi}$ in the effective stress equations by c and ϕ. Accordingly, in the following paragraphs of this report, all equations are presented in terms of effective stress shear strength parameters with the equations for total stresses being an implicit, special case, i.e., $u \to 0$; $\bar{c} \to c$; $\bar{\phi} \to \phi$.

Spencer's Procedure

10. Spencer's procedure (Spencer 1967) satisfies all requirements of static equilibrium. The side forces are assumed to be parallel, i.e., they act at the same inclination. The value of the inclination θ is considered to be an unknown which is solved for along with the factor of safety.

Derivation for the equilibrium equations

11. In deriving the equilibrium equations for Spencer's procedure, it is convenient to represent the side forces by their total resultant force Q on the slice as shown in Figure A2. The requirements for overall equilibrium of forces in the horizontal and vertical directions are then expressed by the following equations, respectively:

$$\Sigma Q \cos \theta = 0 \tag{A13}$$

and

$$\Sigma Q \sin \theta = 0 \tag{A14}$$

where the summation is carried out for all slices. However, since the side-force inclination θ is assumed to be constant in Spencer's procedure,

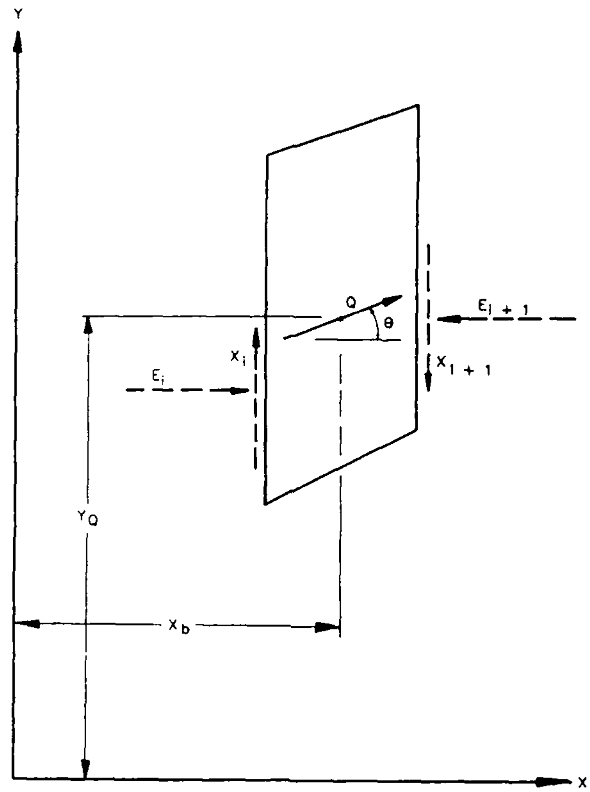

Figure A2. Total resultant side force and line of action coordinate definition for a typical slice

Equations A13 and A14 are reduced to the following single equation for force equilibrium:

$$\Sigma Q = 0 \tag{A15}$$

12. The requirement for overall equilibrium of moments is conveniently expressed by summing moments about the origin of the coordinate system which gives

$$\Sigma Q(x_b \sin \theta - y_Q \cos \theta) = 0 \tag{A16}$$

where, again, the summation is performed for all slices. Expressions for Q and y_Q in Equations A15 and A16 are obtained by considering the equilibrium requirements for individual slices.

13. Expression for Q. The expression for Q is derived by first summing forces in a direction perpendicular to the base of the slice. Forces acting in an upward direction toward the base of the slice are considered

positive. The summation for forces gives the following equation for equilibrium:

$$N + F_v \cos \alpha - F_h \sin \alpha - Q \sin (\alpha - \theta) = 0 \qquad \text{(A17)}$$

which can be solved for the normal force N to give

$$N = -F_v \cos \alpha + F_h \sin \alpha + Q \sin (\alpha - \theta) \qquad \text{(A18)}$$

Substitution of Equation A18 into Equation A9 derived for the shear force S from the Mohr-Coulomb condition gives the following equation for the shear force on the base of the slice:

$$S = \frac{1}{F} \left\{ \bar{c}\Delta x \sec \alpha + [-F_v \cos \alpha + F_h \sin \alpha + Q \sin (\alpha - \theta) - u\Delta x \sec \alpha] \tan \bar{\phi} \right\} \qquad \text{(A19)}$$

14. An expression for the shear force on the base of the slice is also obtained by resolving forces parallel to the base of the slice. By resolving forces parallel to the base of the slice, the following equilibrium equation can be written:

$$S + F_v \sin \alpha + F_h \cos \alpha + Q \cos (\alpha - \theta) = 0 \qquad \text{(A20)}$$

which can be rearranged to give the following expression for the shear force:

$$S = -F_v \sin \alpha - F_h \cos \alpha - Q \cos (\alpha - \theta) \qquad \text{(A21)}$$

The expressions for the shear force given by Equations A19 and A21 can be equated to give

$$\frac{1}{F} \left\{ \bar{c}\Delta x \sec \alpha + [-F_v \cos \alpha + F_h \sin \alpha + Q \sin (\alpha - \theta) - u\Delta x \sec \alpha] \tan \bar{\phi} \right\} = -F_v \sin \alpha - F_h \cos \alpha - Q \cos (\alpha - \theta) \qquad \text{(A22)}$$

Finally, Equation A22 can be solved for the resultant force Q to give

$$Q = \left[-F_v \sin\alpha - F_h \cos\alpha - \frac{\bar{c}}{F} \Delta x \sec\alpha + (F_v \cos\alpha - F_h \sin\alpha + u\Delta x \sec\alpha) \frac{\tan\bar{\phi}}{F} \right] m_\alpha \qquad \text{(A23)}$$

where

$$m_\alpha = \frac{1}{\cos(\alpha - \theta) + \sin(\alpha - \theta) \dfrac{\tan\bar{\phi}}{F}} \qquad \text{(A24)}$$

15. Expression for y_Q. The line of action of the resultant force Q is expressed by the coordinate y_Q located on the line of action at a point directly above the center of the base of the slice. The coordinate y_Q is shown in Figure A2. Summing moments about the center of the base of the slice and noting that the sum must be equal to zero for equilibrium gives

$$-Q \cos\theta(y_Q - y_b) + M_o = 0 \qquad \text{(A25)}$$

where, as shown earlier by Equation A3, M_o represents the moment about a point on the center of the base of the slice due to all of the known forces (KW, P, R, etc.). Equation A25 is solved for y_Q to give

$$y_Q = y_b + \frac{M_o}{Q \cos\theta} \qquad \text{(A26)}$$

Solution of equilibrium equations

16. The expressions for Q (Equation A23) and y_Q (Equation A26) are substituted into the equations of equilibrium (Equations A15 and A16) to produce two equations in two unknowns (F and θ) which must be satisfied to satisfy static equilibrium. The solution to Equations A15 and A16 for the factor of safety and side-force inclination is accomplished using an iterative procedure based on Newton's method for solving two equations in two unknowns.

For assumed values of the factor of safety F_0 and side-force inclination θ_0, it is convenient to write the two equilibrium equations in the form

$$R_1 = \Sigma Q_0 \tag{A27}$$

and

$$R_2 = \Sigma Q_0 \left(x_b \sin \theta_0 - y_{Q_0} \cos \theta_0 \right) \tag{A28}$$

where

Q_0 = value of Q based on the assumed values

F_0, θ_0, R_1, R_2 = force and moment imbalances, respectively, based on the assumed values F_0 and θ_0

Application of Newton's method to find the roots to Equations A27 and A28 corresponding to $R_1 = R_2 = 0$, gives the following for the new estimates for F and θ based on the assumed values:

$$F_1 = F_0 + \Delta F \tag{A29}$$

and

$$\theta_1 = \theta_0 + \Delta\theta \tag{A30}$$

where ΔF and $\Delta\theta$ represent adjustments to the assumed values of F and θ, respectively, to be used for the next iteration. The expressions for ΔF and $\Delta\theta$ are as follows:

$$\Delta F = \frac{R_1 \dfrac{\partial R_2}{\partial \theta} - R_2 \dfrac{\partial R_1}{\partial \theta}}{\dfrac{\partial R_1}{\partial \theta} \dfrac{\partial R_2}{\partial F} - \dfrac{\partial R_1}{\partial F} \dfrac{\partial R_2}{\partial \theta}} \tag{A31}$$

$$\Delta \theta = \frac{R_2 \dfrac{\partial R_1}{\partial F} - R_1 \dfrac{\partial R_2}{\partial F}}{\dfrac{\partial R_1}{\partial \theta} \dfrac{\partial R_2}{\partial F} - \dfrac{\partial R_1}{\partial F} \dfrac{\partial R_2}{\partial \theta}} \tag{A32}$$

In the computer program UTEXAS2, Equations A31 and A32 are used to compute the values of ΔF and $\Delta\theta$, respectively, up to the point in the iterative solution where the respective values become less than 0.5 and 0.15 radians. Once the values become less than these limits ($\Delta F = 0.5$, $\Delta\theta = 0.15$ radians), an "extended" form of Newton's method is used based on the following two equations:

$$R_1 + \Delta F \frac{\partial R_1}{\partial F} + \Delta\theta \frac{\partial R_1}{\partial \theta} + \frac{1}{2} \Delta F^2 \frac{\partial^2 R_1}{\partial F^2} + \Delta F \Delta\theta \frac{\partial^2 R_1}{\partial F \partial \theta} + \frac{1}{2} \Delta\theta^2 \frac{\partial^2 R_1}{\partial \theta^2} = 0 \qquad \text{(A33)}$$

$$R_2 + \Delta F \frac{\partial R_2}{\partial F} + \Delta\theta \frac{\partial R_2}{\partial \theta} + \frac{1}{2} \Delta F^2 \frac{\partial^2 R_2}{\partial F^2} + \Delta F \Delta\theta \frac{\partial^2 R_2}{\partial F \partial \theta} + \frac{1}{2} \Delta\theta^2 \frac{\partial^2 R_2}{\partial \theta^2} = 0 \qquad \text{(A34)}$$

Equations A33 and A34 are derived from Taylor (1937) series expansions including the second-order terms. Estimates of new trial values are obtained by solving these two equations simultaneously for ΔF and $\Delta\theta$.

17. The partial derivatives of R_1 and R_2 in Equations A31 through A34 are obtained from Equations A27 and A28 and are as follows:

$$\frac{\partial R_1}{\partial F} = \Sigma \frac{\partial Q}{\partial F} \qquad \text{(A35)}$$

$$\frac{\partial R_1}{\partial \theta} = \Sigma \frac{\partial Q}{\partial \theta} \qquad \text{(A36)}$$

$$\frac{\partial^2 R_1}{\partial F^2} = \Sigma \frac{\partial^2 Q}{\partial F^2} \qquad \text{(A37)}$$

$$\frac{\partial^2 R_1}{\partial F \partial \theta} = \Sigma \frac{\partial^2 Q}{\partial F \partial \theta} \qquad \text{(A38)}$$

$$\frac{\partial^2 R_1}{\partial \theta^2} = \Sigma \frac{\partial^2 Q}{\partial \theta^2} \qquad \text{(A39)}$$

$$\frac{\partial R_2}{\partial F} = \Sigma \frac{\partial Q}{\partial F} \left(x_b \sin \theta_0 - y_{Q_0} \cos \theta_0 \right) - \Sigma Q_0 \left(\frac{\partial y_Q}{\partial F} \cos \theta_0 \right) \qquad \text{(A40)}$$

$$\frac{\partial R_2}{\partial \theta} = \Sigma \frac{\partial Q}{\partial \theta} x_b \left(\sin \theta_0 - y_{Q_0} \cos \theta_0\right) + \Sigma Q_0 \left(x_b \cos \theta_0 + y_{Q_0} \sin \theta_0 - \frac{\partial y_Q}{\partial \theta} \cos \theta_0\right) \quad \text{(A41)}$$

$$\frac{\partial^2 R_2}{\partial F^2} = \Sigma \frac{\partial^2 Q}{\partial F^2} \left(x_b \sin \theta_0 - y_{Q_0} \cos \theta_0\right) - 2\Sigma \frac{\partial Q}{\partial F}\left(\frac{\partial y_Q}{\partial F} \cos \theta_0\right) - \Sigma Q_0 \left(\frac{\partial^2 y_Q}{\partial F^2} \cos \theta_0\right) \quad \text{(A42)}$$

$$\frac{\partial^2 R_2}{\partial F \partial \theta} = \Sigma \frac{\partial^2 Q}{\partial F \partial \theta} \left(x_b \sin \theta_0 - y_{Q_0} \cos \theta_0\right) + \Sigma \frac{\partial Q}{\partial F} \left(x_b \cos \theta_0 + y_Q \sin \theta_0 - \frac{\partial y_Q}{\partial \theta} \cos \theta_0\right) - \Sigma \frac{\partial Q}{\partial \theta} \left(\frac{\partial y_Q}{\partial F} \cos \theta_0\right) - \Sigma Q_0 \left(\frac{\partial^2 y_Q}{\partial F \partial \theta} \cos \theta_0 - \frac{\partial y_Q}{\partial F} \sin \theta_0\right) \quad \text{(A43)}$$

$$\frac{\partial^2 R_2}{\partial \theta^2} = \Sigma \frac{\partial^2 Q}{\partial \theta^2} (x_b \sin \theta_0 - y_Q \cos \theta_0) + 2\Sigma \frac{\partial Q}{\partial \theta} \left(x_b \cos \theta_0 + y_{Q_0} \sin \theta_0 - \frac{\partial y_Q}{\partial \theta} \cos \theta_0\right) - \Sigma Q \left(x_b \sin \theta_0 - y_{Q_0} \cos \theta_0 - 2 \frac{\partial y_Q}{\partial \theta_0} \sin \theta_0 + \frac{\partial^2 y_Q}{\partial \theta^2} \cos \theta\right) \quad \text{(A44)}$$

18. In evaluating the various partial derivatives of Q in Equations A35 through A44, it is convenient to write the expression for Q (Equation A23) as

$$Q = \frac{C_1 + \frac{C_2}{F}}{C_3 + \frac{C_4}{F}} \tag{A45}$$

where

$$C_1 = -F_v \sin \alpha - F_h \cos \alpha \tag{A46}$$

$$C_2 = -\bar{c}\Delta x \sec \alpha + (F_v \cos \alpha - F_h \sin \alpha + u\Delta x \sec \alpha) \tan \bar{\phi} \tag{A47}$$

$$C_3 = \cos (\alpha - \theta) \tag{A48}$$

$$C_4 = \sin (\alpha - \theta) \tan \phi \tag{A49}$$

Then

$$\frac{\partial Q}{\partial F} = \frac{-1}{\left(C_3 + \frac{C_4}{F}\right)^2} \left[\left(C_3 + \frac{C_4}{F}\right) \frac{C_2}{F^2} - \left(C_1 + \frac{C_2}{F}\right) \frac{C_4}{F^2}\right] \tag{A50}$$

$$\frac{\partial Q}{\partial \theta} = \frac{-1}{\left(C_3 + \frac{C_4}{F}\right)^2} \left(C_1 + \frac{C_2}{F}\right) \left(\frac{\partial C_3}{\partial \theta} + \frac{1}{F} \frac{\partial C_4}{\partial \theta}\right) \tag{A51}$$

$$\begin{aligned} \frac{\partial^2 Q}{\partial F^2} = \frac{1}{\left(C_3 + \frac{C_4}{F}\right)^3} & \left\{\left(C_3 + \frac{C_4}{F}\right) \left[2\left(C_3 + \frac{C_4}{F}\right) \frac{C_2}{F^3} - 2\left(C_1 + \frac{C_2}{F}\right) \frac{C_4}{F^3}\right]\right. \\ & \left. - 2 \frac{C_4}{F^2} \left[\left(C_3 + \frac{C_4}{F}\right) \frac{C_2}{F^2} - \left(C_1 + \frac{C_2}{F}\right) \frac{C_4}{F^2}\right]\right\} \end{aligned} \tag{A52}$$

$$\begin{aligned} \frac{\partial^2 Q}{\partial F \partial \theta} = \frac{-1}{\left(C_3 + \frac{C_4}{F}\right)^3} & \left\{\left(C_3 + \frac{C_4}{F}\right) \left[\frac{C_2}{F^2}\left(\frac{\partial C_3}{\partial \theta} + \frac{1}{F} \frac{\partial C_4}{\partial \theta}\right) - \left(C_1 + \frac{C_2}{F}\right) \frac{1}{F^2} \frac{\partial C_4}{\partial \theta}\right]\right. \\ & \left. - 2 \left(\frac{\partial C_3}{\partial \theta} + \frac{1}{F} \frac{\partial C_4}{\partial \theta}\right) \left[\left(C_3 + \frac{C_4}{F}\right) \frac{C_2}{F^2} - \left(C_1 + \frac{C_2}{F}\right) \frac{C_4}{F^2}\right]\right\} \end{aligned} \tag{A53}$$

$$\frac{\partial^2 Q}{\partial \theta^2} = \frac{-1}{\left(C_3 + \frac{C_4}{F}\right)^3} \left[\left(C_3 + \frac{C_4}{F}\right) \left(C_1 + \frac{C_2}{F}\right)\left(\frac{\partial^2 C_3}{\partial \theta^2} + \frac{1}{F} \frac{\partial^2 C_4}{\partial \theta^2}\right)\right.$$

$$\left. - 2\left(C_1 + \frac{C_2}{F}\right) \left(\frac{\partial C_3}{\partial \theta} + \frac{1}{F} \frac{\partial C_4}{\partial \theta}\right)^2\right] \quad \text{(A54)}$$

where

$$\frac{\partial C_3}{\partial \theta} = \sin (\alpha - \theta) \quad \text{(A55)}$$

$$\frac{\partial C_4}{\partial \theta} = -\cos (\alpha - \theta) \tan \phi \quad \text{(A56)}$$

$$\frac{\partial^2 C_3}{\partial \theta^2} = -\cos (\alpha - \theta) \quad \text{(A57)}$$

$$\frac{\partial^2 C_4}{\partial \theta^2} = -\sin (\alpha - \theta) \tan \phi \quad \text{(A58)}$$

19. Expressions for the various partial derivatives of the variable y_Q in Equations A40 through A44 are as follows:

$$\frac{\partial y_Q}{\partial F} = \frac{-1}{\left(Q_0 \cos \theta_0\right)^2} M_o \frac{\partial Q}{\partial F} \cos \theta_0 \quad \text{(A59)}$$

$$\frac{\partial y_Q}{\partial \theta} = \frac{-1}{\left(Q_0 \cos \theta_0\right)^2} M_o\left(\frac{\partial Q}{\partial \theta} \cos \theta_0 - Q_0 \sin \theta_0\right) \quad \text{(A60)}$$

$$\frac{\partial^2 y_Q}{\partial F^2} = \frac{-1}{Q^2 \cos \theta} M_o \left[\frac{\partial^2 Q}{\partial F^2} - \frac{2}{Q} \left(\frac{\partial Q}{\partial F}\right)^2\right] \quad \text{(A61)}$$

$$\frac{\partial^2 y_Q}{\partial F \partial \theta} = \frac{-1}{Q_0^2 \cos \theta_0} M_o \left(\frac{\partial^2 Q}{\partial F \partial \theta} + \frac{\partial Q}{\partial F} \tan \theta - 2 \frac{\partial Q}{\partial F} \frac{\partial Q}{\partial \theta} \frac{1}{Q_0} \right) \tag{A62}$$

$$\frac{\partial^2 y_Q}{\partial \theta^2} = \frac{-1}{Q_0^2 \cos \theta_0} M_o \left[2 \frac{\partial^2 Q}{\partial \theta^2} \tan \theta_0 - \frac{\partial^2 Q}{\partial \theta^2} + Q_0 + \frac{2}{Q_0} \left(\frac{\partial Q}{\partial \theta} - Q_0 \tan \theta_0 \right)^2 \right] \tag{A63}$$

Solution for remaining unknowns

20. Once the values of the factor of safety and side-force inclination are determined which satisfy the equilibrium Equations A15 and A16, the remaining unknowns are calculated. The remaining unknowns consist of the normal force on the base of the slice N, the side force Z between slices, and the locations of the side forces y_t. The normal forces are calculated from Equation A18 which was derived by summing forces in a direction perpendicular to the base of each slice. The value of the force Q in Equation A18 is calculated from Equation A23. Although the shear force is not actually considered an "unknown" (it is known if N and F are known), the shear force can only be calculated once F is found; the shear force is calculated from Equation A21.

21. Side forces Z are calculated slice-by-slice, beginning with the first, leftmost slice. In general, for the i^{th} slice,

$$Z_{i+1} = Z_i - Q_i \tag{A64}$$

where

Z_i = the side force on the left of the slice

Z_{i+1} = the side force on the right of the slice

Q_i = the resultant of the side forces on each side of the slice

The side forces are shown in Figure A3. The resultant of the side forces Q_i is calculated for each slice from Equation A23 once values for the factor of safety and side-force inclination have been determined. The side forces are then calculated for each slice beginning with the first slice. For the first slice, the side force on the left of the slice Z_1 must be zero and, thus,

$$Z_2 = Q_1 \tag{A65a}$$

where

Z_2 = the force on the right-hand side of the first slice

Q_1 = the value of Q for the first slice

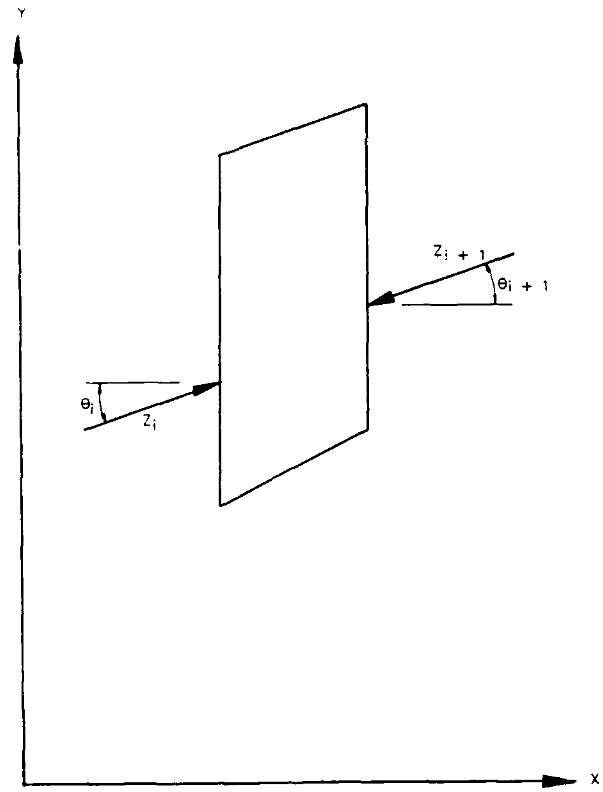

Figure A3. Resultant side forces and inclination acting on the sides of a typical slice

Application of Equation A64 to the next slice then gives

$$Z_3 = Z_2 - Q_2 \tag{A65b}$$

where

Z_3 = the side force on the right of the second slice

Q_2 = the value of Q for the second slice

Equation A64 is applied successively to the remaining slices until all side forces have been calculated.

22. The location of the side forces (line of thrust) is also calculated slice-by-slice beginning with the first slice. By summing moments about the center of the base of the slice, the following equation can be written for any slice:

$$M_o - Z_i \sin\theta \frac{\Delta x}{2} - Z_{i+1} \sin\theta \frac{\Delta x}{2} - Z_i \cos\theta (y_{t,i} - y_b)$$

$$+ Z_{i+1} \cos\theta (y_{t,i+1} - y_b) = 0 \qquad \text{(A66)}$$

which can be rearranged to give the following expression for the location of the side force on the right-hand side of a slice:

$$y_{t,i+1} = y_b - \left[\frac{M_o - Z_i \sin\theta \frac{\Delta x}{2} - Z_{i+1} \sin\theta \frac{\Delta x}{2} - Z_i \cos\theta (y_{t,i} - y_b)}{Z_{i+1} \cos\theta} \right] \qquad \text{(A67)}$$

For the first slice (i = 1), the value of the force Z_i is zero, and the value of the location $y_{t,i}$ is of no significance in Equation A67. Thus, $y_{t,i+1}$, corresponding to the location of the side force on the right-hand side of the first slice, can be calculated once Z_{i+1} has been calculated. This process can be repeated for the next slice once the values of Z_i and Z_{i+1} and the value of $y_{t,i}$ have been calculated. The process is repeated until the locations of all of the side forces have been determined.

Force-Equilibrium Procedures

23. Two force-equilibrium procedures are used in the computer program, UTEXAS2. The first of these procedures uses the Corps of Engineers Modified Swedish side-force assumption, EM 1110-2-1902 (Headquarters, Department of the Army 1970); the second uses the Lowe and Karafiath's (1960) side-force assumption. Both procedures satisfy equilibrium of forces in the vertical and horizontal direction, but do not satisfy moment equilibrium. A statically determinate solution is obtained by assuming the inclination of the resultant side forces between slices. In the case of the Corps of Engineers Modified Swedish side-force assumption, the side forces are assumed to be parallel (all side forces have the same inclination), and the inclination must be selected and input by the user. According to EM 1110-2-1902 (Headquarters, Department of the Army 1970), the side-force inclination would normally be taken to be equal to the "average embankment slope" although in actual practice other inclinations might be assumed. In the case of Lowe and Karafiath's side-force assumption, the side forces are assumed to act at the average of the

inclination of the slope (or ground) surface, directly above, and the shear surface, directly below, each vertical boundary between slices. The side-force inclinations vary from slice to slice.

Derivation of the equilibrium equations

24. In deriving the equations used to compute the factor of safety by the force-equilibrium procedures, it is convenient to express the forces on each side of the slice by the resultant forces Z_i and Z_{i+1} acting on the left and right side of the slice, respectively, and the respective inclinations θ_i and θ_{i+1}. The resultant forces are illustrated in Figure A3. Summation of forces in the vertical direction gives the following equilibrium equation:

$$F_v + Z_i \sin \theta_i - Z_{i+1} \sin \theta_{i+1} + N \cos \alpha + S \sin \alpha = 0 \qquad \text{(A68)}$$

Similarly, the summation of forces in the horizontal direction produces the following equilibrium equation:

$$F_h + Z_i \cos \theta_i - Z_{i+1} \cos \theta_{i+1} - N \sin \alpha + S \cos \alpha = 0 \qquad \text{(A69)}$$

Substituting the expression for the shear force S given by Equation A68 into Equations A68 and A69, respectively, gives the following two equations:

$$F_v + Z_i \sin \theta_i - Z_{i+1} \sin \theta_{i+1} + N\left(\cos \alpha + \frac{\tan \bar{\phi}}{F} \sin \alpha\right) + \left(\bar{c} - u \tan \bar{\phi}\right) \frac{\Delta \ell}{F} \sin \alpha = 0 \qquad \text{(A70)}$$

and

$$F_h + Z_i \cos \theta_i - Z_{i+1} \cos \theta_{i+1} + N\left(\sin \alpha + \frac{\tan \bar{\phi}}{F} \cos \alpha\right) + \left(\bar{c} - u \tan \bar{\phi}\right) \frac{\Delta \ell}{F} \cos \alpha = 0 \qquad \text{(A71)}$$

Equations A70 and A71 can be combined to eliminate the unknown normal force N and solved for the side force Z_{i+1} acting on the right side of the slice to give

$$Z_{i+1} = \frac{1}{\cos(\alpha - \theta_{i+1}) + \frac{\tan\bar{\phi}}{F}\sin(\alpha - \theta_{i+1})}\left\{F_v \sin\alpha + F_h \cos\alpha\right.$$

$$+ Z_i \cos(\alpha - \theta_i) + \frac{\tan\bar{\phi}}{F}\left[-F_v \cos\alpha + F_h \sin\alpha - u\Delta\ell\right.$$

$$\left.\left. + Z_i \sin(\alpha - \theta_i)\right] + \frac{\bar{c}\Delta\ell}{F}\right\} \tag{A72}$$

Solution of equilibrium
equations for the factor of safety

25. The solution for the factor of safety is obtained using an iterative procedure based on Newton's method. A factor of safety is assumed, and the side forces are computed slice-by-slice using Equation A73. Beginning with the first slice and noting that for the first slice Z_i is zero, the force Z_{i+1} on the right of the slice is calculated. The force Z_{i+1} calculated for the right of the first slice becomes the force Z_i on the left of the next slice, and the force on the right of the next slice can then be calculated once again using Equation A72. This procedure is repeated slice-by-slice to the last slice where the force on the right side of the last slice is calculated. If the force on the right of the slice is acceptably small, the factor of safety is considered to be correct, and the remaining unknowns can be calculated as described in the following paragraphs. Otherwise, a new value is assumed for the factor of safety, and the process is repeated until the force on the right of the last slice is acceptably small.

26. In the computer program UTEXAS2, an iterative procedure based on Newton's method is used to compute the factor of safety. A factor of safety F_0 is assumed, and a new estimate for the factor of safety F_1 is obtained from

$$F_1 = F_0 + \Delta F \tag{A73}$$

where

$$\Delta F = \frac{Z_{i+1}}{\frac{\partial Z_{i+1}}{\partial F}} \tag{A74}$$

The values of Z_{i+1} and $\partial Z_{i+1}/\partial F$ in Equation A74 are evaluated for the last, rightmost slice using the assumed value for the factor of safety F_0. In evaluating the partial derivative $\partial Z_{i+1}/\partial F$, it is convenient to write Equation A72 in the form

$$Z_{i+1} = \frac{C_1 + \dfrac{C_2}{F}}{C_3 + \dfrac{C_4}{F}} \tag{A75}$$

where

$$C_1 = F_v \sin \alpha + F_h \cos \alpha + Z_i \cos (\alpha - \theta_i) \tag{A76}$$

$$C_2 = \tan \bar{\phi} \, [-F_v \cos \alpha + F_h \sin \alpha - u\Delta\ell + Z_i \sin (\alpha - \theta_i)] + \bar{c}\Delta\ell \tag{A77}$$

$$C_3 = \cos(\alpha - \theta_{i+1}) \tag{A78}$$

$$C_4 = \tan \bar{\phi} \sin (\alpha - \theta_{i+1}) \tag{A79}$$

The partial derivative $\partial Z_{i+1}/\partial F$ then becomes

$$\frac{\partial Z_{i+1}}{\partial F} = \left\{\left(C_3 + \frac{C_4}{F}\right)\left[\frac{\partial C_1}{\partial F} + \frac{1}{F^2}\left(F\,\frac{\partial C_2}{\partial F} - C_2\right)\right] + \left(C_1 + \frac{C_2}{F}\right)\cdot\frac{C_4}{F^2}\right\}\left(C_3 + \frac{C_4}{F}\right)^{-2} \tag{A80}$$

where

$$\frac{\partial C_1}{\partial F} = \frac{\partial Z_i}{\partial F} \cos(\alpha - \theta_i) \tag{A81}$$

$$\frac{\partial C_2}{\partial F} = \frac{\partial Z_i}{\partial F} \sin(\alpha - \theta_i) \tan \bar{\phi} \tag{A82}$$

Equation A6 is used to compute successive trial values for the factor of safety until the changes on successive trials ΔF and the force imbalance Z_{i+1} on the last slice become acceptably small.

Solution for remaining unknowns

27. Once the value of the factor of safety is determined by satisfying the equilibrium requirement that the side force Z_{i+1} must be essentially zero for the last (right-most) slice, the remaining unknowns are calculated. The remaining unknowns consist of N on the base of the slice and Z between slices. However, the side forces are calculated as part of the iterative procedure used to calculate the factor of safety, and, thus, only N on the base of each slice remains to be calculated. The expression used to calculate the normal forces is obtained by resolving forces in a direction perpendicular to the base of the slice and by solving the resulting equilibrium equation for the normal force to give

$$N = -F_v \cos \alpha + F_h \sin \alpha + Z_i \sin(\alpha - \theta_i) - Z_{i+1} \sin(\alpha - \theta_{i+1}) \qquad \text{(A83)}$$

Although the shear force is not actually considered an unknown (it is known if N and F are known), the shear force can only be calculated once F is found; the expression used to calculate the shear force is derived by summing forces in a direction parallel to the base of the slice and by solving the resulting equilibrium equation for the shear force to give

$$S = -F_v \sin \alpha - F_h \cos \alpha - Z_i \cos(\alpha - \theta_i) + Z_{i+1} \cos(\alpha - \theta_{i+1}) \qquad \text{(A84)}$$

Simplified Bishop Procedure

28. The Simplified Bishop procedure (Bishop 1955) is based on the assumption of a circular shear surface. Side forces are assumed to act in the horizontal direction; i.e., there is assumed to be no shear force between slices. The Simplified Bishop procedure satisfies equilibrium of forces in the vertical direction for each slice and equilibrium of moments about the center of the circular shear surface for the entire free body composed of all slices (overall moment equilibrium). This procedure has been extended by Wright (1986) to account for external loads.

Derivation of the equilibrium equations

29. An expression for the normal force on the base of each slice is obtained first by summing forces in the vertical direction. The resulting equation for equilibrium of forces in the vertical direction is

Solution of equilibrium
equation for the factor of safety

31. Newton's method is used in the computer program UTEXAS2 to solve Equation A91 for the factor of safety. For an assumed factor of safety F_0, Equation A91 is written as

$$M_i = \Sigma M_o + \Sigma F_v(x_b - x_c) - \Sigma F_h(y_b - y_c)$$

$$+ \frac{R}{F_0} \Sigma[\bar{c}\Delta x + (-F_v - u\Delta x) \tan \bar{\Phi}] \, m_\alpha \qquad \text{(A92)}$$

where M_i represents the moment imbalance based on the assumed factor of safety. The new estimate for the factor of safety F_1 is written as

$$F_1 = F_0 + \Delta F \qquad \text{(A93)}$$

where, by Newton's method, ΔF is expressed as

$$\Delta F = \frac{M_i}{\dfrac{\partial M_i}{F}} \qquad \text{(A94)}$$

In computing the partial derivative $\partial M/\partial F$, it is convenient to write Equation A92 as

$$M_i = \Sigma M_o + \Sigma F_v(x_b - x_c) - \Sigma F_h(y_b - y_c) + R\Sigma \frac{C_1}{F_0 C_2 + C_3} \qquad \text{(A95)}$$

where

$$C_1 = \bar{c}\Delta x - (F_v + u\Delta x) \tan \bar{\Phi} \qquad \text{(A96)}$$

$$C_2 = \cos \alpha \qquad \text{(A97)}$$

$$C_3 = \sin \alpha \tan \bar{\Phi} \qquad \text{(A98)}$$

Then

$$\frac{\partial M_i}{\partial F} = -R\Sigma \frac{C_1}{(C_2F_0 + C_3)^2} C_2 \qquad \text{(A99)}$$

Solution for remaining unknowns

32. The only unknown, in addition to the factor of safety, which is calculated in the Simplified Bishop procedure (Bishop 1955) is the normal force on the base of the slice. The expression used to calculate the normal force is obtained by substituting the expression for the shear force (Equation A9), obtained from the Mohr-Coulomb equation, into the expression for the normal force (Equation A86), obtained by resolving forces in the vertical direction, which gives

$$N = \frac{1}{\cos\alpha}\left\{-F - \frac{1}{F}\left[\bar{c}\Delta x \sec\alpha + (N - u\Delta x \sec\alpha)\tan\bar{\phi}\right]\sin\alpha\right\} \qquad \text{(A100)}$$

Rearranging Equation A100 gives the following equation for the normal force on the base of each slice:

$$N = \left[-F_v - \frac{1}{F}(\bar{c} - u\tan\bar{\phi})\Delta x \tan\alpha\, m_\alpha\right] \qquad \text{(A101)}$$

where m_α is defined in Equation A89.

APPENDIX B: NOTATION

c	Total stress equivalent
$\bar{c}$	Shear-strength parameter in terms of effective stresses
c_m	Mobilized strength parameter
d_c	Depth of crack
E	Horizontal side forces between slices
E_i	Horizontal forces on right boundaries of slices
E_{i-1}	Horizontal forces on left boundaries of slices
F	Factor of safety
F_o	Assumed values of factor of safety
F_h	Resultant expressing components of known forces acting in the horizontal direction on a slice
F_v	Resultant expressing components of known forces acting in vertical direction on a slice
F_1	New estimate for the factor of safety
h_t	Location of side forces on the side of the slice
H_B	Water forces on side
H_L	Water forces on left side
H_R	Water forces on right side
KW	Horizontal force representing body force for seismic loading in a pseudo-static analysis
ℓ_N	Location of normal force on base of slice
M_o	Moment produced above center of base of slice by known forces
n	Number of slices
N	Normal forces on base of slice
P	External surface loads
Q	Total resultant force of side forces
$Q_{1,2}$	Value of Q for first and second slices

R	Force acting on base of slice to represent internal reinforcement
s	Available shear strength
S	Shear force on base of slice
T	External shear forces
u	Pore water pressure
U	Water forces on base of slice
W	Weight of the slice
x_b	Coordinate of the center of base of slice
X_i	Shear forces on right boundary of slice
x_p	Coordinate on top of slice
x_r	Coordinate on base of slice
X	Vertical shear forces on slice boundary
X_{i-1}	Vertical shear forces on left boundary of slice
y_b	Coordinate of the center of base of slice
y_k	Cooridnate of the centroid of the slice
y_p	Coordinate on top of slice
y_r	Coordinate on base of slice
y_t	Cooridnate of side force location
y_Q	Coordinate of resultant force
Z	Resultant side force
Z_i	Resultant side force on left of slice
$Z_{1,2,3}$	Resultant side force on right-hand side of the first, second, and third slices
α	Shear-surface inclinations
β	Ground-surface inclination
γ	Unit weight of soil
ΔF	Adjustment to assumed value of F
$\Delta \ell$	Length of slice base

$\Delta\theta$	Adjustment to assumed value of θ
θ	Inclimation of resultant interslice force
θ_0	Assumed value of side-force inclination
σ	Total normal stress
$\sigma - u$	Effective normal stress
ϕ	Total stress equivalent
ϕ_m	Modilized strength parameter
$\bar{\phi}$	Shear-strength parameter in terms of effective stresses
$\bar{\phi}_m$	Mobilized friction angle of cohesionless soil
τ	Shear stress required for just-stable, static equilibrium
ψ	Inclination of reinforcement force

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	Title	Date
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Miscellaneous Paper GL-82-1	Geotechnical Computer Program Survey	Mar 1982
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	Report 2. Interactive Graphics Options	Mar 1981
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Instruction Report K-81-7	User's Guide: Computer Program for Design or Investigation of Orthogonal Culverts (CORTCUL)	Mar 1981
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	Title	Date
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Instruction Report K-83-2	User's Guide: Computer Program for Generation of Engineering Geometry (SKETCH)	Jun 1983
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	Title	Date
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