Übung 1

Donnerstag, 15. März 2018 12:30

①

$\Omega = \{(K,K,Z), (K,Z,K), (Z,K,K), (K,K,K)\}$

$P[K,K,Z] = \frac{1}{4}$

②

k	p
1	0.25
2	0.15
3	0.6

$E[k] = 1 \cdot 0.25 + 2 \cdot 0.15 + 3 \cdot 0.6 = 2.35$

$V[k] = \sum (k - E[x])^2 \cdot p = 0.728$

③

$f(x) = \frac{1}{T} \cdot e^{-\frac{x}{T}}$

$\Rightarrow$ max likelihood

$\hat{\Theta} = \arg\max \sum_{i=1}^{n} \ln(f_{x_\Theta}(x_i))$

$\hat{\Theta} = \ln\left(\frac{1}{T}\right) \cdot \ln\left(e^{-\frac{x}{T}}\right)$

$= \ln\left(\frac{1}{T}\right) \cdot \left(-\frac{x}{T}\right)$

$= \underbrace{\sum_{i=1}^{n} \ln\left(\frac{1}{T}\right)}_{} - \sum_{i=1}^{n} \frac{x_i}{T}$

macht keinen Sinn $\Rightarrow$ kein x_i

$\hat{\Theta} = n \cdot \ln\left(\frac{1}{T}\right) - \sum_{i=1}^{n} \frac{x_i}{T}$

$\frac{\partial l(L)}{\partial T} = -\frac{n}{T} + \sum_{i=1}^{n} \frac{x_i}{T^2}$

$\emptyset = \frac{\partial l(L)}{\partial T}$

$\emptyset = -\frac{n}{T} + \sum_{i=1}^{n} \frac{x_i}{T^2}$

$\frac{n}{T} = \sum_{i=1}^{n} \frac{x_i}{T^2} \quad | \cdot T^2$

$n \cdot T = \sum_{i=1}^{n} x_i \quad | \cdot \frac{1}{n}$

$T = \sum_{i=1}^{n} \frac{x_i}{n}$ $\Rightarrow$ max. likelihood Schätzwert für T

④ a)

$\hat{m} = 2\bar{x} - 1$

$E[\hat{m}] = \frac{1}{n} \cdot E\Big[\underbrace{\sum_{i=1}^{n} 2\bar{x} - 1}_{\text{Gleichverteilung}}\Big]$

$= \frac{2}{n} \sum_{i=1}^{n} \frac{1}{n} \sum_{i=1}^{n} \bar{x} - 1$ $\Rightarrow$ Gaussche' Summenformel

$= \frac{2}{n^2} \cdot \sum_{i=1}^{n} \frac{n^2+n}{2} - 1$

$$= \frac{2}{n} \left(\frac{n^2+n}{n} \right) - 1$$

$$= \frac{\cancel{2}}{\cancel{n}} \left(\frac{\cancel{n}(n+1)}{\cancel{2}} \right) - 1$$

$$E[\hat{m}] = n$$

c)

$$\hat{m} = 2\bar{x} - 1$$

$$\Rightarrow \frac{34+56+17+22+23+88}{6} - 1 = \frac{240}{6} - 1 = 79$$

=> Sample keine repräsentative Schätzgröße

=> >> m notwendig!