EXPERIMENT

3

Variable and Periodic Stars (VARISTAR)

This experiment was originally written by Prof. Johan van der Walt. It is now under the care of Dr Bruno Letarte. This is the version of: 2021-08-09

3.1 General outcomes

With successful completion of this experiment you should

- Know how to preform a periodical analysis on data and to interpret it;
- Know what differential photometry is;
- Know the different types of variable stars, especially **Delta Scuti** variables (δ Scuti), the type of stars we will work on.
- Understand the basic physics of pulsating stars;
- Know the practical use of pulsating stars in astrophysics;

3.2 Introduction

There are different ways that we can obtain information about the cosmos. All information is currently only transmitted to us by photons. Except for the fact that the spectral information and the absolute flux of photons contain a lot of information about the physical circumstances from which they are emitted, we can also distinguish between time independent and time dependent situations. Even though we can obtain a lot of useful information about astrophysical systems from time independent situations, it is the time dependent cases that teach us something about the dynamics of astrophysical systems.

An example of a time independent case is if we for example study the stars that appear on the main branch of the HR diagram. All these stars are in gravitational equilibrium and burn hydrogen at a constant rate. From the spectra of these stars we can learn many things about their physical conditions and chemical composition. By studying a large number of these stars, we can determine a relation between the luminosity and surface temperature (main branch). There are many other examples in astrophysics where we study processes which are time independent.

On the other side, it became apparent that the universe is not static and that there are many time dependent processes. Except for the fact that it was found that the luminosity of some stars is time dependent, it was also found that the radio emission from some "active galaxies" changes with time. It can be said without a doubt that time dependent effects gives us a new perspective of what is going on in the cosmos. The goal of this experiment is to bring you in contact on a simple level with the practical aspects of data analysis from data of variable stars.

3.2.1 Magnitude scale

The brightness of a star is refereed to as the magnitude, where a lower number is brighter than a higher number, for historical reasons. I will not get into details here, but in order to make things clearer, here are a few definitions.

- The **instrumental magnitude** is what you measure at a telescope. It will depend on your location / altitude, sensitivity of the instruments, sky conditions, etc. It is kind of an apparent magnitude for your telescope.

- The **apparent magnitude** is a universal instrumental magnitude, properly calibrated so that every telescope anywhere on the planet (and in space) will measure the same thing. It usually requires a zero point that will be noted somewhere, usually in the fits file header, to know by how many magnitude you need to shift your measurements so they are compatible with all telescopes. When you look at star databases online (e.g. Simbad), this is the value that will be listed most likely in various filters, U, B, V, R, I, etc. This is useful in an **observational** context, to determine how long you need to expose your detector in order to get sufficient flux/counts for a given star. It is often referred only by a letter from the filter it was taken in, for example, V = 9.761 means an apparent magnitude of 9.761 in the V-band. Sometimes, it's referred to as a small letter m, followed by the filter, like m_V.

- The **apparent magnitude** is the magnitude of that star, if it was located 10 pc away from us. While this is totally useless in terms of planing observations, it's a more **physically** meaningful value when it's time to calculate physical parameters. The Sun is the brightest star in our sky, but if it was pushed to 10 parsec (pc) away from us, and all stars were put at this 10 pc distance, it wouldn't be the brightest star anymore. Many stars are much brighter than the Sun, but because they are very far from us, they appear less bright to us. So when it's time to deal with the physics of a star, the apparent magnitude is a real measure of its brightness, independent of the observer's location. The apparent magnitude is often labeled by a capital M, followed by the filter, like M_V.

- The **bolometric magnitude**. This is the magnitude of a star, integrated for all wavelengths of the EM spectrum. And again here, it can be apparent or absolute, as in m_{bol} and M_{bol} This isn't a value that is directly observed, as we don't have detectors that are able to measure the full EM Spectrum on a telescope.

3.3 Data

The data that you will be using in this experiment is real data obtained from the telescope at Nooitgedacht, looking at **Delta Scuti** variable stars (δ Scuti). You will be given a set of CCD fits images to process in ds9 (or fitsview if you like).

3.4 Method

3.4.1 Week 1 - Constructing the time series of the variable star

Differential photometry requires very little image processing because you compare an image taken at time t_1 to an image taken at time t_2 (where Δt is small) and not much has changed from one image to the next. Sky conditions, position in the sky and on the detector, etc are more or less constant from one image to the next. The following section describes what you need to do for the data processing.

1. Note that you will have to repeat this procedure for both B and V filters.

2. Open a fits file in ds9.

3. Find your star on the "Variable Star Reference Guide", identify the target (variable) star, and your reference stars.

4. We will measure errors in this experiment as well, so take note of your error adding, subtracting, multiplying and dividing formulas. If you have forgotten them, they can easily be found on the internet.

5. To construct a time series, the magnitude must be recorded as function of time. In this experiment, it will be most practical to record the time in decimal day (as most of the observations take place at most in three days). That will be your first column. Then, we need to record the counts of the target star (and rms), the counts of the reference stars (and rms), and the counts of the background (and rms). This will be done as follows:

6. Open a spreadsheet or get a notebook and make a table with the following headings: "Decimal date (days), Ref 1 (counts), Ref 1 σ (counts),..., Ref n (counts), Ref n σ (counts), Target (counts), Target σ (counts), Background 1 (counts), Background 1 σ (counts), ... Background m (counts), Background m RMS(counts)", with n the number of reference stars, m the number of background measurements and σ the RMS of the measured counts.

7. Make a circle in ds9 by click on the edit → region tab. You can then click and drag to make a circle. This will be your "aperture", all the counts within this circle will be counted. Make sure to use the **same size aperture** for **all** measurements. You can click inside the circle, and drag it around. You can also see and edit the radius of the circle by double clicking on it. This opens a window with some information. It will be wise to record the circle radius somewhere, and to set it to that exact value every time you make a new circle.

8. After you have made the circle and opened its window, go to Analysis → Statistics. It will open a new window, with the regions, their sum (counts), some other values and the rms (σ), record these values for the target star, reference stars and at least 2 backgrounds (no stars). Remember to keep the circle size fixed at all times.

9. Repeat this procedure on all other images.

10. The CCD images were recorded over a period of time which means that the atmospheric conditions changed during the observations. When comparing different astronomical measurements it is necessary that we only use data obtained under very good atmospheric conditions. We must thus make sure that the change in observed flux from a star is not from atmospheric change but intrinsic to the star. It is however still possible, when we observe variable stars under not ideal atmospheric conditions, to obtain meaningful data. The CCD images cover a very small region of the sky from which the valid assumption can be made that the atmospheric conditions should affect all the stars on the image equally. If there is a star on the image that is not variable, then it can be used as reference star. If we measure the luminosity of the variable star relative to the reference star on all images, then we effectively removed the effect of the changing atmosphere. This must also be applied to our data.

    The next step is then to find a star or more stars on the image that is not variable (the reference stars). However it is not so simple to determine if a star is variable or not. What we are going to do in this case is to compare the flux of the variable star to the average flux of two or three stars that appear constant. This means that in normal conditions you will have to choose three other star on the image of which the flux should also be recorded. These stars are specified in the reference guide. Write these fluxes to a table and calculate the average flux of the three stars. This average flux of the reference stars then become the reference to which the variable star is measured.

11. You will need to convert all the count values into count rate and then into instrumental magnitudes with the equation below. The exposure time will be used to convert to count rate. You can find it by searching for it in the header. Click file → header in ds9.

    In this case the magnitude is the instrumental magnitude and is defined as

    $$m = -2.5 \log_{10} N, \tag{3.1}$$

    where N is the **count rate** that can be ascribed to the star. Also pay attention that the exposure time of all images may differ and this is also the reason for working with a **count rate** instead of number of counts.

12. Now you have a table with the time of each observation, each reference, target and background, with their errors in instrumental magnitude. Now we will start with the differential photometry.

13. Take the average of the backgrounds (taking into account the errors), and subtract it from all the reference stars and the target.

14. Average the reference stars (taking the errors into account) to get a Ref avg value for each image. As we are assuming this should be constant and equal to the average **apparent magnitudes** of the reference stars. This means that there is some factor $K_i = \bar{M}_{\rm abs}/\bar{M}_{i,\rm inst}$ that we have to multiply with the target and its error with to get the variable stars **apparent magnitude**. $\bar{M}_{\rm abs}$ is the average apparent magnitudes of the reference stars and $\bar{M}_{i,\rm inst}$ is the average instrumental magnitude for a single frame i. Remember, K_i also has an error due to $\bar{M}_{i,\rm inst}$'s error. Then export the decimal days and target apparent magnitude with its error.

15. Plot the apparent magnitude of the target vs decimal days, with its errors.

16. Submit this plot, with your time series to eFundi by the due date.

3.4.2 Week 2 - Determining the distance to the star

The variable star that we encounter here belongs to the class of stars known as Delta Scuti stars. As with other pulsating stars there also exist a relation for this class of stars between the luminosity of the star and the period of variation. This relation is given by M Breger (2000), stated a simple theoretical model for the period luminosity relation for δ Scuti stars:

$$M_{\text{bol}} - M_{\text{bol}\odot} = -3.33\log P + 3.33\log Q - 10\log(T_{\text{eff}}/T_{\text{eff}}) - 1.67\log(M/M_{\odot}) \tag{3.2}$$

with M_{bol} the bolometric magnitude, P the period of the star, Q the pulsation constant, T_{eff} the effective temperature and M the mass of the star. The variables with the $\odot$ symbol are for the sun, and those without are for our stars. The mass term is negligible in this case, $Q = 0.033$d, $M_{\text{bol}\odot} = 4.75$, $T_{\text{eff}\odot} = 5800$K. So that means only the effective temperature, period and bolometric magnitude of the star is unknown. We will use this information to calculate the distance to the star, with the help of some methods described below. We will go about in the following manner:

1. Calculate the period using the Lomb-Scargle method.
2. Estimate the effective temperature.
3. Use the period luminosity relation to calculate the bolometric absolute magnitude.
4. Use the bolometric correction to calculate the absolute V magnitude.
5. Use the distance modulus to calculate the distance.

Determining the period of the star.

The next step is to determine the period. We will use a mathematical method to try to determine the period (if any). A very well known and reliable method is the method of Lomb and Scargle. Basically this method is a Fourier analysis for unevenly spaced data. Suppose that we have a data set with N data points $h_i \equiv h(t_i)$, $i = 1, \ldots, N$. The *normalised periodogram* is then defined as

$$P_N(\omega) \equiv \frac{1}{2\sigma^2}\left\{\frac{\left[\sum_j (h_j - \bar{h})\cos\omega(t_j - \tau)\right]^2}{\sum_j \cos^2\omega(t_j - \tau)} + \frac{\left[\sum_j (h_j - \bar{h})\sin\omega(t_j - \tau)\right]^2}{\sum_j \sin^2\omega(t_j - \tau)}\right\}, \tag{3.3}$$

where

$$\bar{h} \equiv \frac{1}{N}\sum_{i=1}^{N} h_i \tag{3.4}$$

and

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(h_i - \bar{h})^2 \tag{3.5}$$

are the average and variance of the data points respectively, and τ is defined as:

$$\tan(2\omega\tau) = \frac{\sum_{j=1}^{N}\sin 2\omega t_j}{\sum_{j=1}^{N}\cos 2\omega t_j}. \tag{3.6}$$

The figure below is an example of a periodogram. The frequency of pulsation is the frequency where $P_N(\omega)$ has the highest peak. The theory on periodograms is a lot more complicated as what we can explain here. The idea is only that you are introduced to periodical data and to use the periodogram to identify periodicities in data.

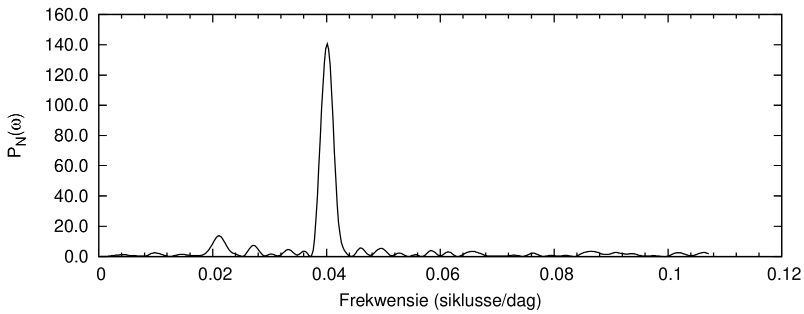
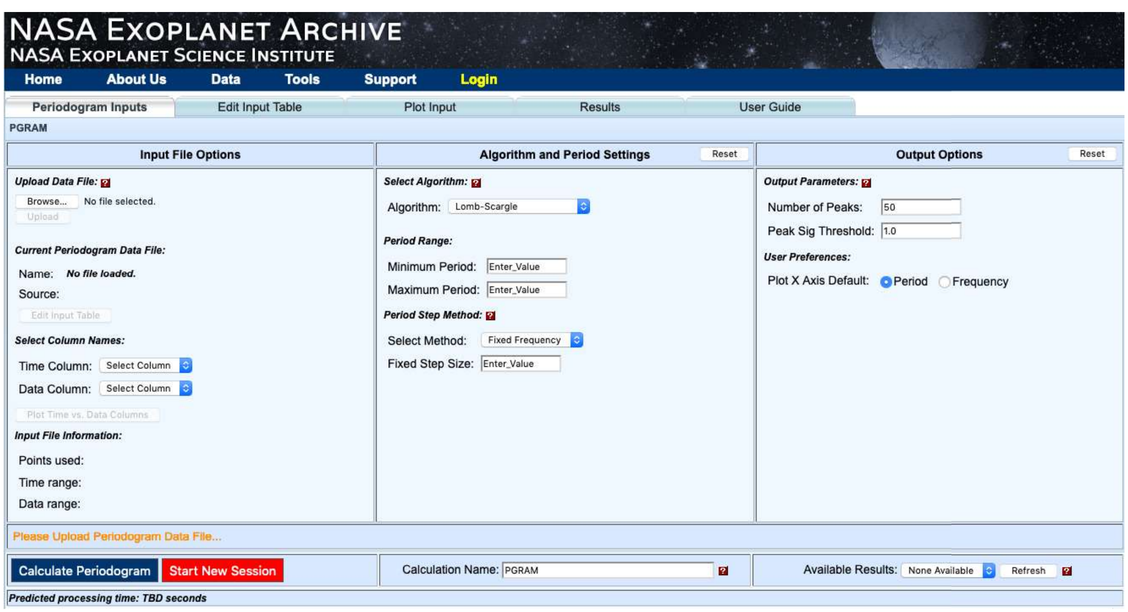


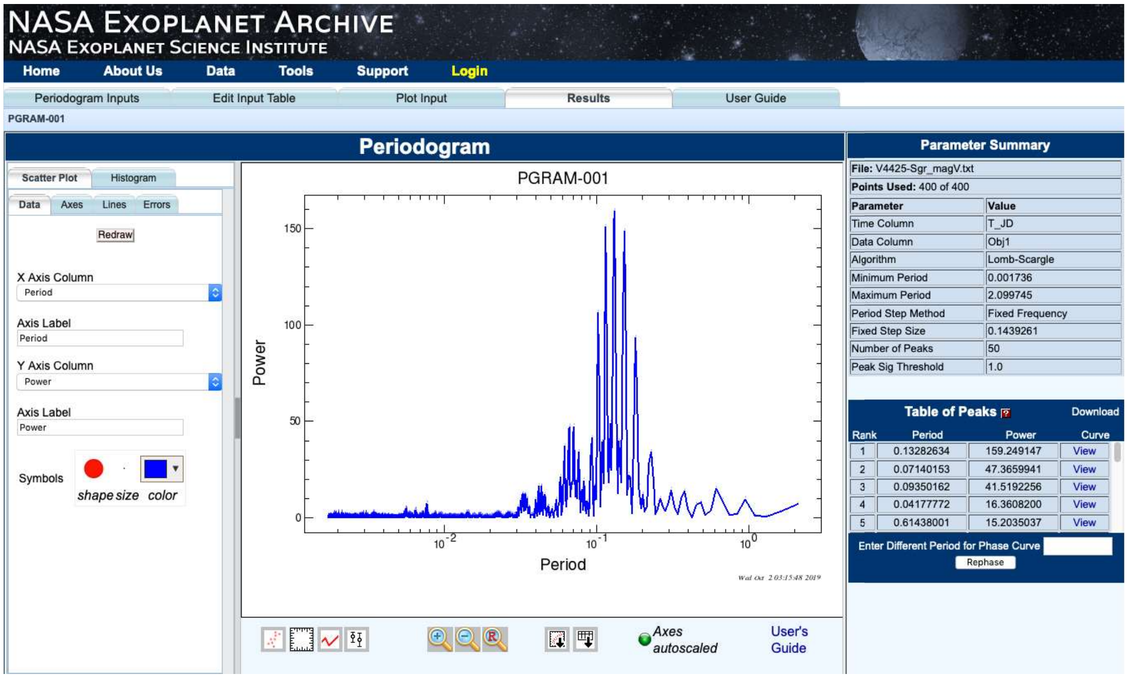
Figure 3.1: Example of a periodogram. Note the peak at around 0.04.

Use this website and use it to do your analysis. You will build a periodigram using the Lomb-Scargle method, a light curve plot (time vs magnitude) and all the rest you might need. Make these plots look nice and save them for your report. A little demo will be given in class.

`https://exoplanetarchive.ipac.caltech.edu/cgi-bin/Pgram/nph-pgram`



In addition to obtaining the light curve and the periodigram (as saved plots, PNG files) for each dataset, note the period given by the interface and present them in a table. We will be interested mostly in the largest period, the first appearing on the table (Rank 1).



Determining the temperature variations of the star

To determine the period using the Lomb-Scargle method is fairly simple. The period alone does not tell us much about the physics of the star yet. An assumption often made is that the radiation spectrum of the star conforms to that of a black body radiator, thus it has a Plank spectrum. The use of filters enable us to determine the flux at different wavelengths. Changes in the temperature of a black body radiator has two consequences: firstly the total energy of the black body increases as T^4, ie. if the temperature increases then the flux should increase significantly. This is the Stefan-Boltzman law. Secondly, the wavelength of the peak in the spectrum shifts according to Wien's law that states that $\lambda_{max} = 0.28978T$, where λ is in centimetres and T in Kelvin. Check your first year physics text book and your modern physics text book again.

Important to note is that we expect the flux at different wavelengths to change with different amounts if the temperature of the star changes. Inversely, if we observe different changes of the flux at different wavelengths, it implies changes in the temperature of the star. The question now arises whether the changes that we see of the flux in the different filters could be linked to the temperature changes in the star. If the flux at different wavelengths varied with the same factor, then the observed variations cannot be linked to variations in the temperature of the star.

What should we do now to find out if the changes in the flux can be related to a temperature variation? The answer is actually simple: We need to investigate the change in $B - V$ as a function of time. B is the B-filter magnitude and V, the magnitude for the V-filter. Remember that magnitudes are logarithmic defined so that :

$$B - V \;\propto\; \log_{10} \frac{S_B}{S_V} \tag{3.7}$$

Thus draw a graph of $\Delta m_B - \Delta m_V$ as function of Julian day, and try to interpret what it means. We now go one step further and try to estimate real temperature variations of this specific star. Stars are not perfect black body radiators and therefore the relation between $B - V$ and the temperature should be calibrated empirically. Here we accept the results from such a calibration and use it on the star that we investigate. Table 3.1 gives the relation between $B - V$ and the temperature for main sequence stars.

$B - V$	T_{eff} (K)	$B - V$	T_{eff} (K)	$B - V$	T_{eff} (K)	$B - V$	T_{eff} (K)
-0.33	42000	-0.02	9700	+0.52	6250	+0.91	4830
-0.31	34000	+0.05	9000	+0.58	5940	+1.15	4410
-0.30	30000	+0.15	8180	+0.63	5790	+1.40	3840
-0.24	20900	+0.30	7300	+0.68	5560	+1.49	3520
-0.17	15200	+0.35	7000	+0.74	5310	+1.64	3170
-0.11	11400	+0.44	6650	+0.81	5150	...	...

Table 3.1: Empirical calibration values of $B - V$ vs T_{eff} for main sequence stars

Plot the calibration values on a graph of $B-V$ as function of T_{eff}. Indicate the region on the y-axis over which $B - V$ vary and estimate the temperature variation of the star from that. Also draw the following relation between T_{eff} and $B - V$ on the same graph:

$$B - V = -3.684 \log T_{\text{eff}} + 14.551 \quad \text{for} \log T_{\text{eff}} < 3.961 \tag{3.8}$$

and

$$B - V = 0.344 \log^2 T_{\text{eff}} - 3.402 \log T_{\text{eff}} + 8.037 \quad \text{for} \log T_{\text{eff}} > 3.961. \tag{3.9}$$

Also include in the results the temperature variation that is deduced from this and compare that to the temperature variation found earlier. These coefficients are from Reed (1998).

Calculating the absolute V magnitude

Equation (3.2) gives the relation between the absolute bolometric magnitude and the period of the star. Use this relation to determine the absolute magnitude M_{bol}. To convert M_{bol} into M_V you will need to apply a bolometric correction. Because stars can be approximated by a blackbody radiator, knowing the flux or magnitude in one part of the spectrum can be informative about the total flux or magnitude (bolometric). This has been empirically calculated by Johnson 1966, and is summarized in this figure.

Basically, what you have to remember is the relation between the bolometric correction, the bolometric magnitude and the magnitude from a specific filter. For example, in the V band, that would be:

$$\text{BC}_V = m_{\text{bol}} - m_V = M_{\text{bol}} - M_V \tag{3.10}$$

To determine the distance to the star, we must also know the real apparent magnitude of the star. Let us call it m_V. If we know both the absolute and apparent magnitudes, then we can calculate the *distance modulus* and thus the distance to the star. To determine the apparent magnitude m_V, just average the apparent magnitude time series you calculated in week 1. Then can then use the distance modulus formula, relating m, M and d (in parsec).

$$m - M = 5 \log_{10} \left(\frac{d}{10} \right) \tag{3.11}$$

Now we have calculated the distance to the variable star! If it was 1970, you would have been able to publish this result! There are some really important period luminosity relations in astrophysics, and you will also have to write paragraph on that for your report. The next section describes the report we want from you.

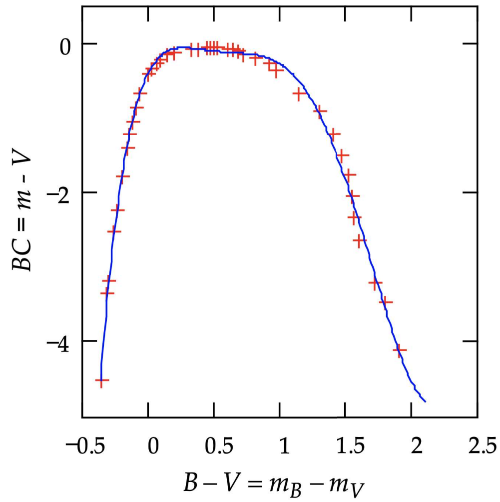


Figure 3.2: Bolometric correction as a function of color index $B - V$, from Johnson 1966 for main sequence stars. Here is data from their table II plotted.

3.5 The Report

Write a report containing the following sections (these are standard in observational astrophysics):

1. Title page with an abstract.
   (a) Write this section last, only after you have finished the whole report.
   (b) Summarize your main results, quoting the numbers you have calculated.
   (c) Do not put any references in this section.
2. Introduction (background information on the topic, references are extremely important in this section.)
   (a) What are variable stars?
   (b) Period luminosity relations, and its importance in astrophysics.
   (c) Delta Scuti stars and their period luminosity relations.
   (d) Differential photometry.
3. Observations and Data reduction (when and how was the star observed, look in the header. And how you got your initial results (week 1's work).)
   (a) All the processing described in week 1, to get the time series.
   (b) Put in your table with the counts, errors and instrumental magnitude. All those things.
4. Results (your results from week 1).
   (a) All the apparent magnitude vs time plots for each star.
   (b) Light curve plots, mag vs time (convert decimal day to hours, starting at zero) per star per filter
   (c) Describe the plots, without interpreting them (please contact your facilitator if this is unclear to you, this is an important point).

5. Discussion (the work from week 2, and your interpretation thereof. Remember to cite all the methods you used. Some of the papers are given in the manual).
   (a) The period luminosity relation for delta scuti stars.
   (b) Calculating the period (you do not need to describe the Lomb Scargle method, merely cite a paper where it was introduced.)
   (c) Periodigram (power vs freq) per star per filter.
   (d) Calculating the effective temperature. Plot of $m_B - m_V$ vs time per star.
   (e) The bolometric absolute magnitude and the bolometric correction. Plot of $B - V$ vs T_{eff} for main sequence stars showing the range of your 3 stars
   (f) Using the distance modulus to get the distance.
   (g) A summary for each star with period, temperature(min) temperature(max) and distance estimate
6. Conclusion (Summarize what you have done and your main results, quote the numbers you have calculated.)
7. References. (This is a lot easier if you use bibtex.)

Mark allocations are as follows:

Section	Mark Allocation %
Abstract	10
Introduction	10
Observations and Data Reduction	15
Results	20
Discussion	30
Conclusion	10
References	5

References

These articles have been made available on eFundi for your convenience. Tip for references: If you copy-paste the information below into Google Scholar, there will be a cite button (shown with a " sign), you can then either copy a written out bibliography item, or copy the bibtex code if you are lazy/smart and use automatic referencing in LaTeX. (If you do not know how this works, please contact your facilitator).

- Flower, 1996, ApJ, 469, 355F
- Johnson, 1966, ARA&A, 4, 193J
- Reed, 1998, JRASC, 92, 36R
- Torres, 2010, AJ, 140, 1158
- Breger, 2000, ASPC, 210, 3B