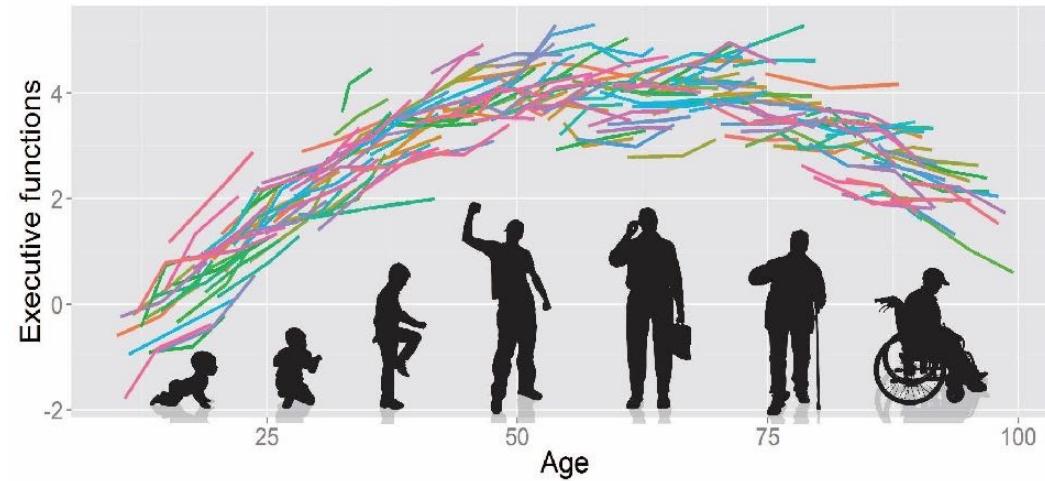


An introduction to Latent Growth Curve Modeling



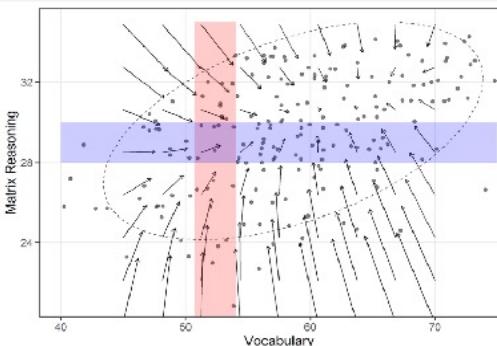
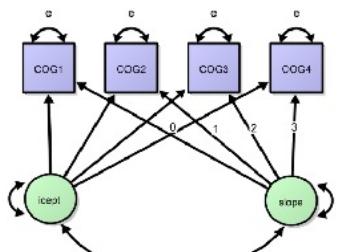
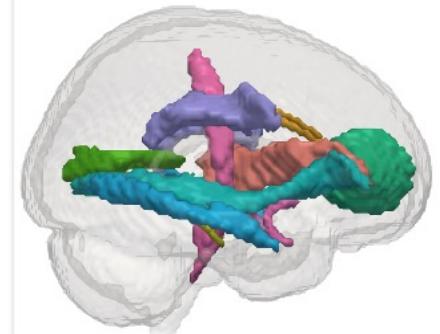
Rogier A. Kievit
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Lifespan Cognitive Dynamics Lab

Brains, Psychology and Psychometrics

LCD LAB PEOPLE RESEARCH INTERESTS PUBLICATIONS OPEN SCIENCE AND RESOURCES JOINING THE LAB



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Jordy van
Langen

Eleni
Zimianiti

Michael
Aristodemou

Lea
Michel

Aran van Hout

Nick Judd

Emma
Meeussen

Feline van
Aagten



DONDERS
INSTITUTE



Outline

Magda Lazari

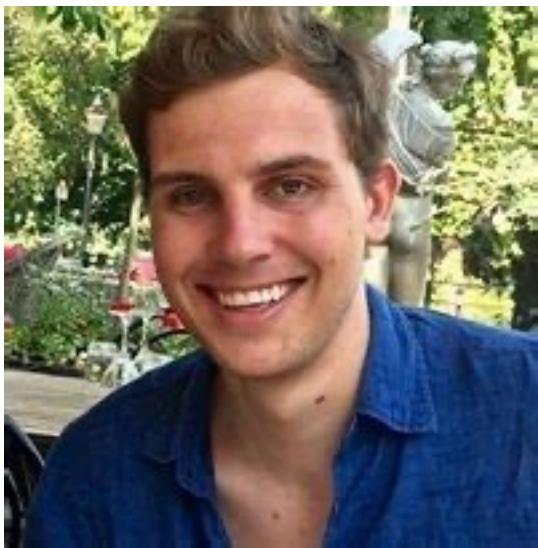


- 1) Why study change?
- 2) Structural Equation Modeling – Getting started
- 3) Latent growth curve modeling
- 4) DIY

Léa Michel

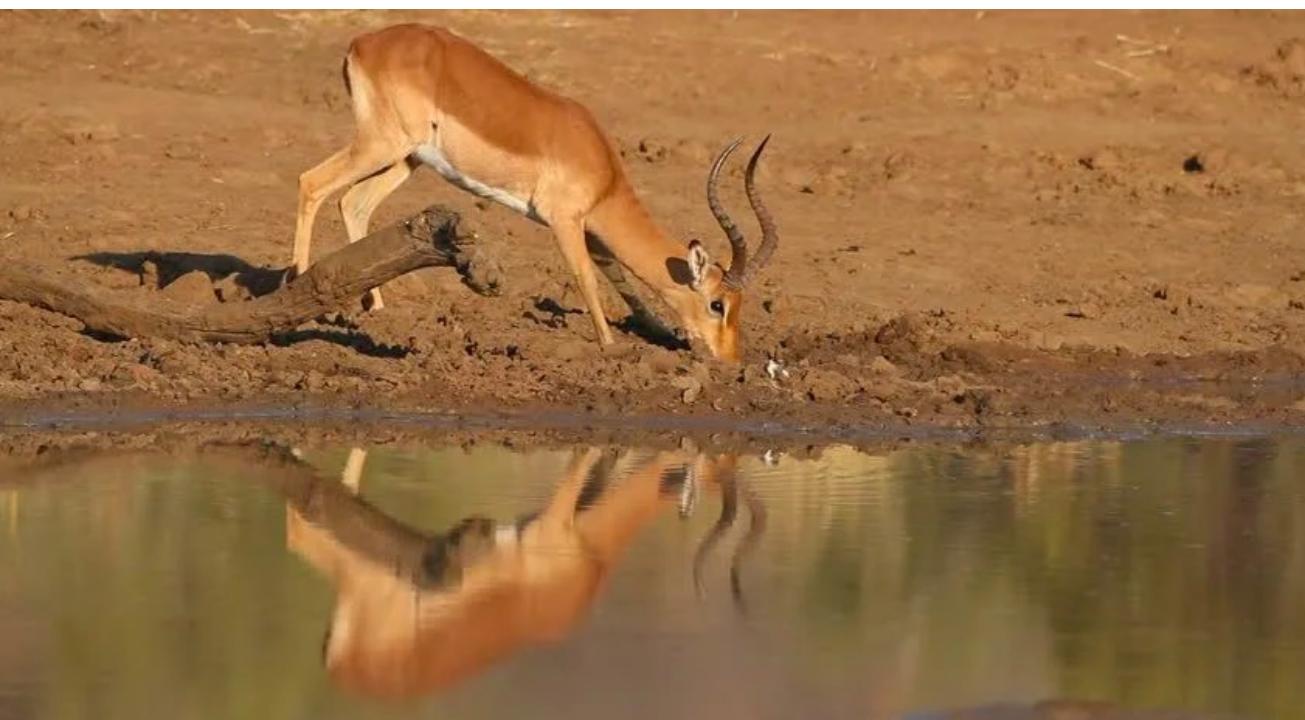


Nick Judd



From snapshots to change

- Classic approach: ‘High resolution’ snapshot of brain structure, behavioural phenotypes
 - 7 Tesla, Connectom; better questionnaires, better tests
- But: Does not capture *change* or *development*
- To study development we should study *change*



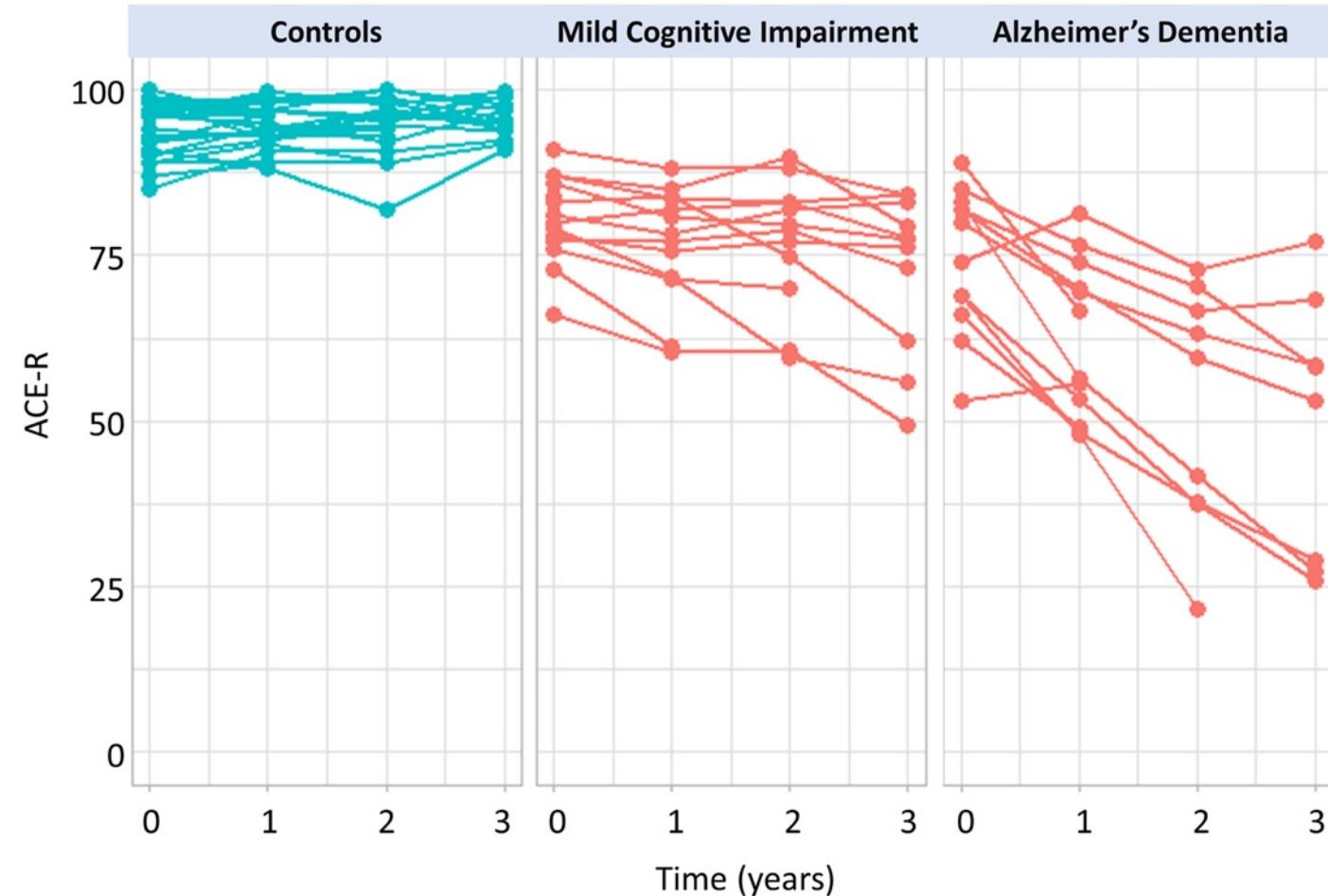
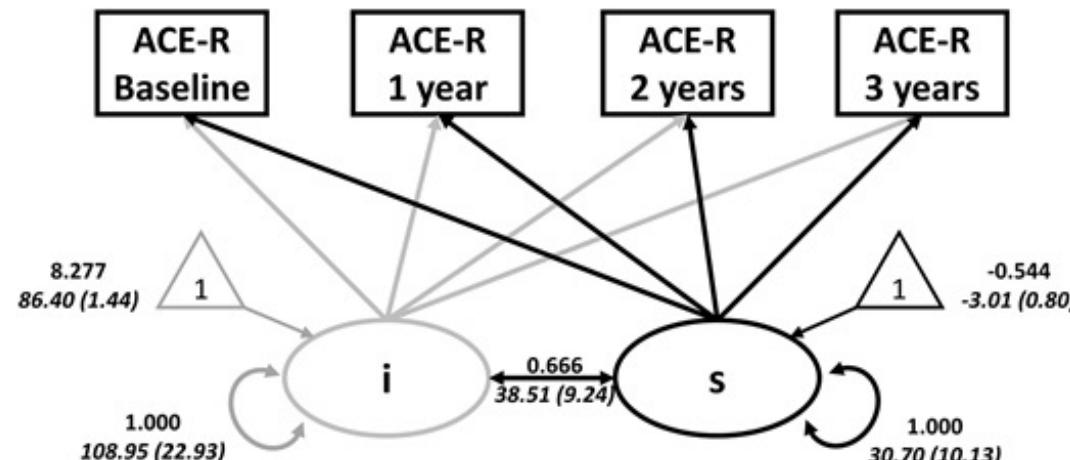
Why study change?

- 1) How do people change?
- 2) How do individuals differ in change?
- 3) How do changes co-occur?
- 4) What predicts – or is predicted by – change?

Baltes, P. B., & Nesselroade, J. R. (Eds.). (1979). *Longitudinal research in the study of behavior and development*. Academic Press.

Example: Modeling decline in dementia

- Group comparison: Do groups differ in their mean linear slope?
- Groups differed significantly in their slopes
- Differences in slopes are predicted by tau burden

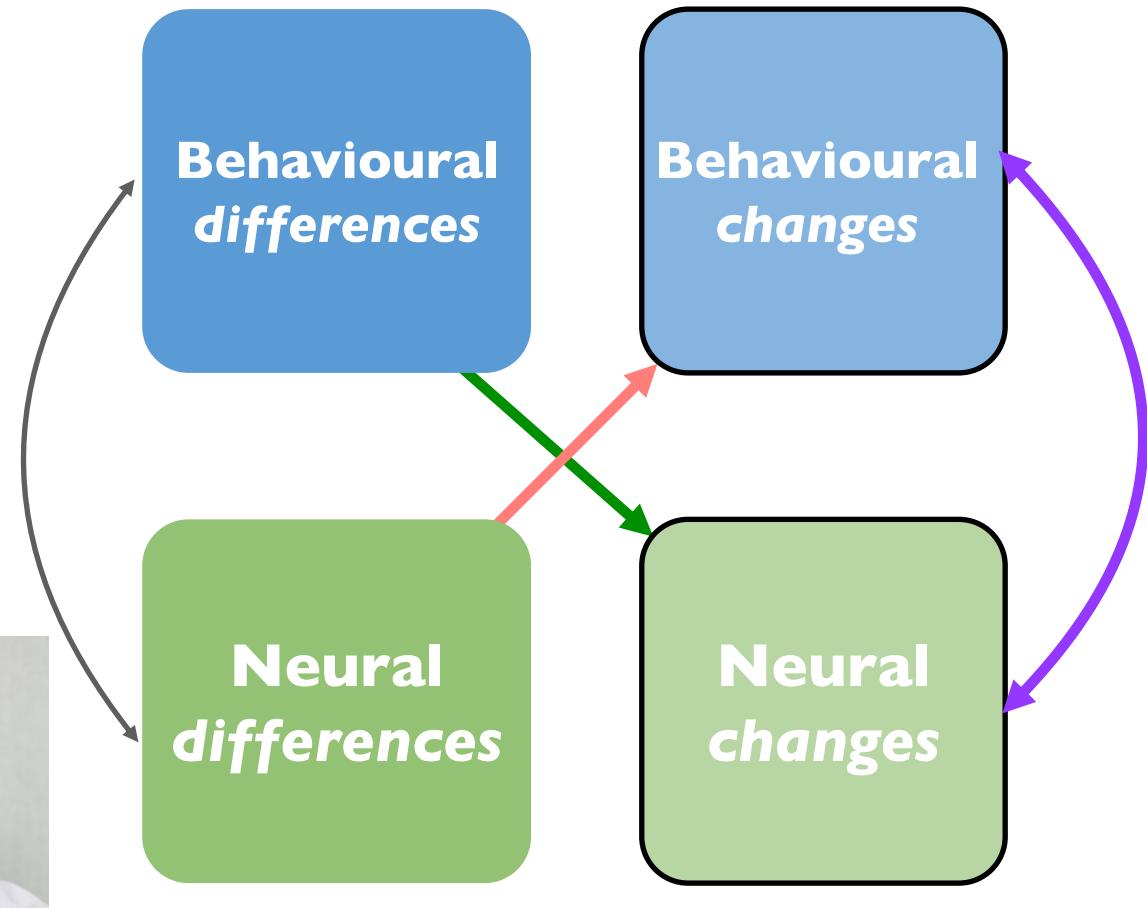
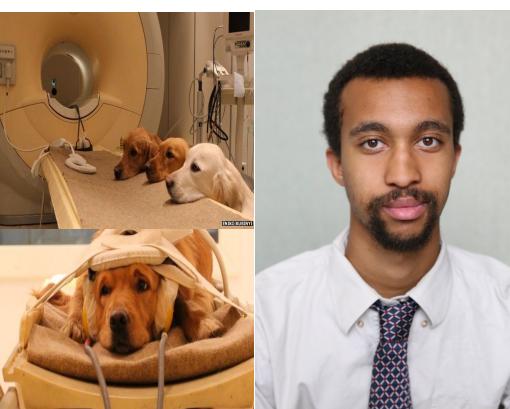


Malpetti, M., Kievit, R. A., Passamonti, L., Jones, P. S., Tsvetanov, K. A., Rittman, T., ... & Rowe, J. B. (2020). Microglial activation and tau burden predict cognitive decline in Alzheimer's disease. *Brain*, 143(5), 1588-1602.

How do behavioural and neural phenotypes co-develop in childhood and early adolescence?

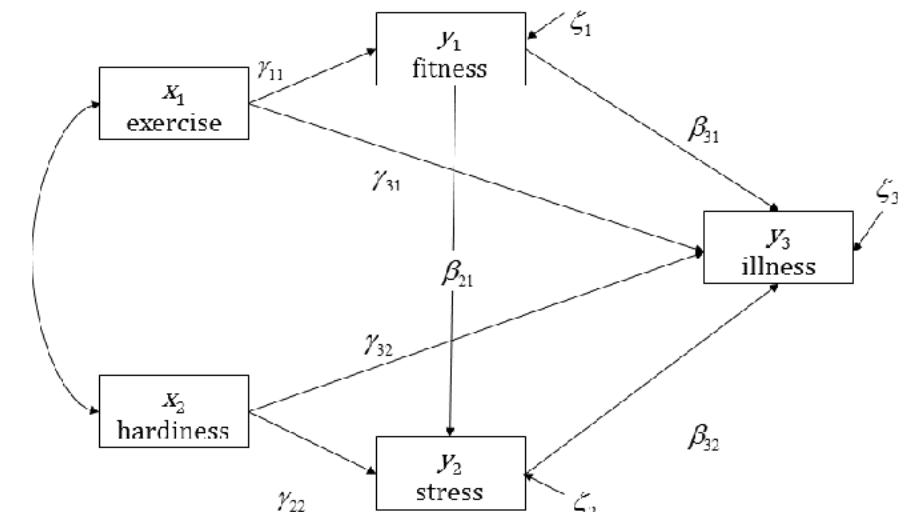
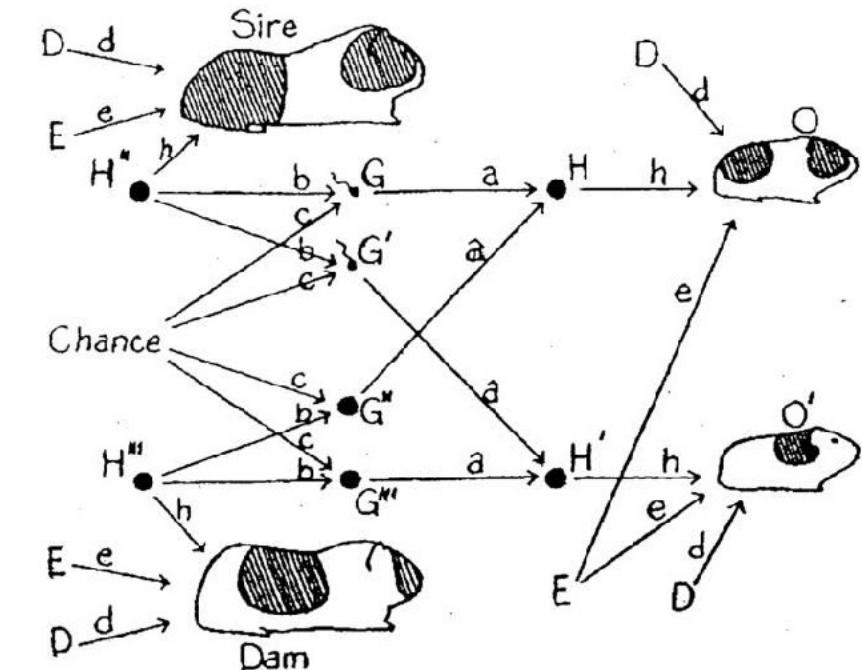
- 99% of Developmental Cognitive Neuroscience: Age heterogeneous differences
- Increase focus on three (empirically) underrepresented questions
- 1) Do changes in brain structure and behavioral characteristics go hand in hand?
- 2) Does brain structure drive the emergence of behavioural characteristics?
- 3) Do behavioural differences drive neural change?

Kievit, R., & Simpson-Kent, I. L. (2019). It's About Time: Towards a Longitudinal Cognitive Neuroscience of Intelligence.
<https://psyarxiv.com/n2yg7/>



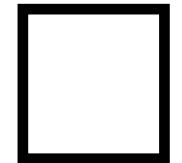
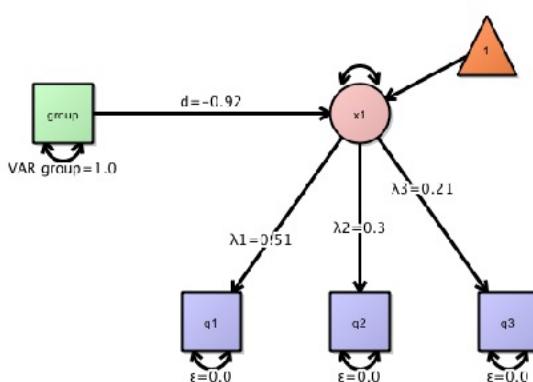
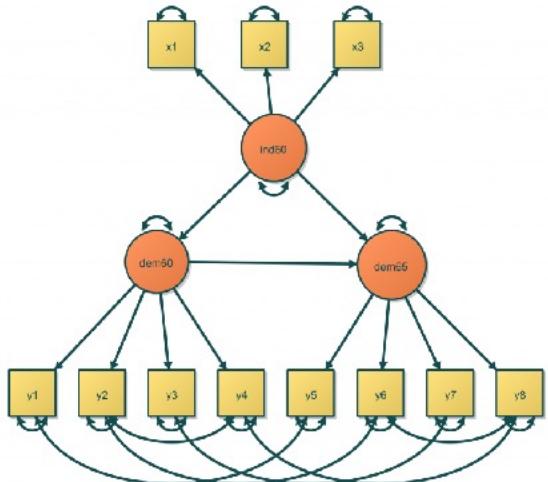
What is structural equation modeling (SEM)?

- A combination of two tools
 - Path analysis
 - Sewall Wright (1921)
 - Unique: path can be the cause and consequence at the same time
 - Estimated simultaneously
 - Latent variable analysis
 - Relating measured variables to hypothesized constructs

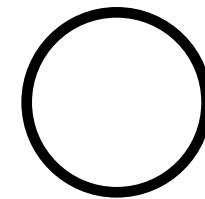


Graphical representation

- One-to-one mapping between graphical representation and matrix algebra
If you can draw your hypothesis, you can (likely) fit it as a model



Observed variable



Latent variable



Directed relationship
(e.g. factor loadings, regressions)



Undirected relationship
(e.g. (co)variance, error)

Model fitting in SEM

- 1. Overall model fit: chi square test
- 2. Model fit: Comparative fit indices
- 3. Model fit: Model misfit
- 4. Comparative model fit
- 5. Parameter estimates

**Evaluating the Fit of Structural Equation Models:
Tests of Significance and
Descriptive Goodness-of-Fit Measures**

Karin Schermelleh-Engel¹ and Helfried Moosbrugger

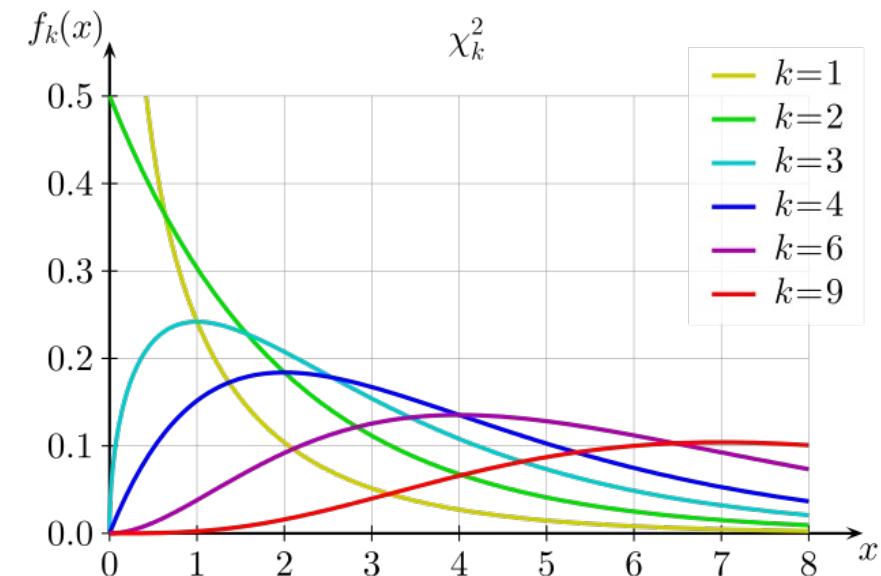
Goethe University, Frankfurt

Hans Müller

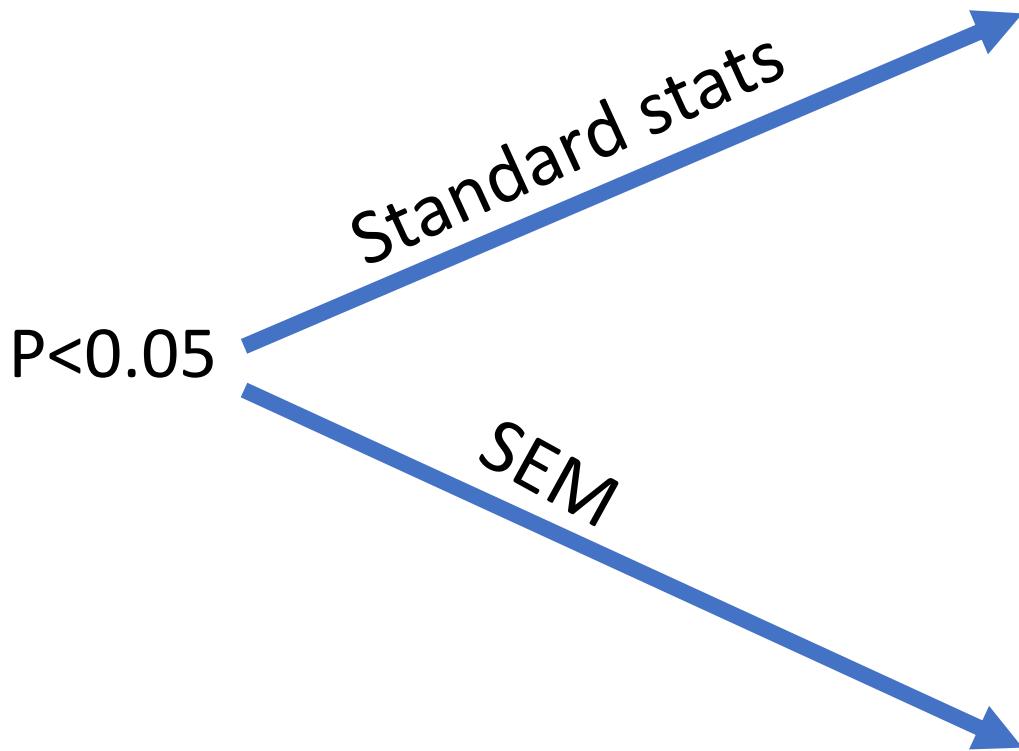
University of Erfurt

1. Chi square test

- Basic principle: Test of perfect model fit
- We have a proposed model
- IF that model generated our data...
- THEN we know the expected misfit of the data to this model
 - Chi square, with k degrees of freedom
- If the data deviates significantly more than expected...
- Then the hypothesis of perfect fit is rejected



Different..



NOVEMBER 5, 2012

New version SPSS will include 'celebratory fireworks' for significant results



An official press release has confirmed that the new version of SPSS will be equipped with 'performance-rewarding features'. The popular data-analysis package will light up with song, whenever a statistical test is significant. "We want to provide a link between the day-to-day experiences of researchers and the excitement of publishing, and the relief that is felt by many when their results appear in the results table."

The level of significance will determine the abundance of fireworks. If the P-value is below 0.05, researchers will automatically hear a short, cheerful tone", according to a company spokesman. "



giantmonster

2. Model fit: Comparative fit indices

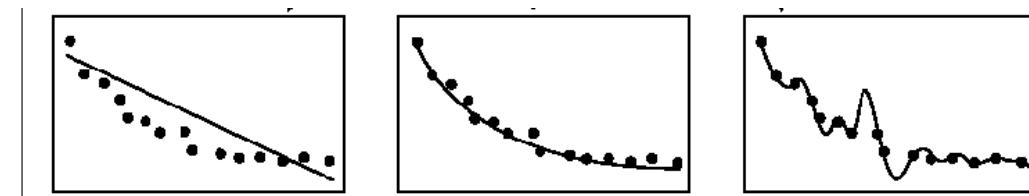
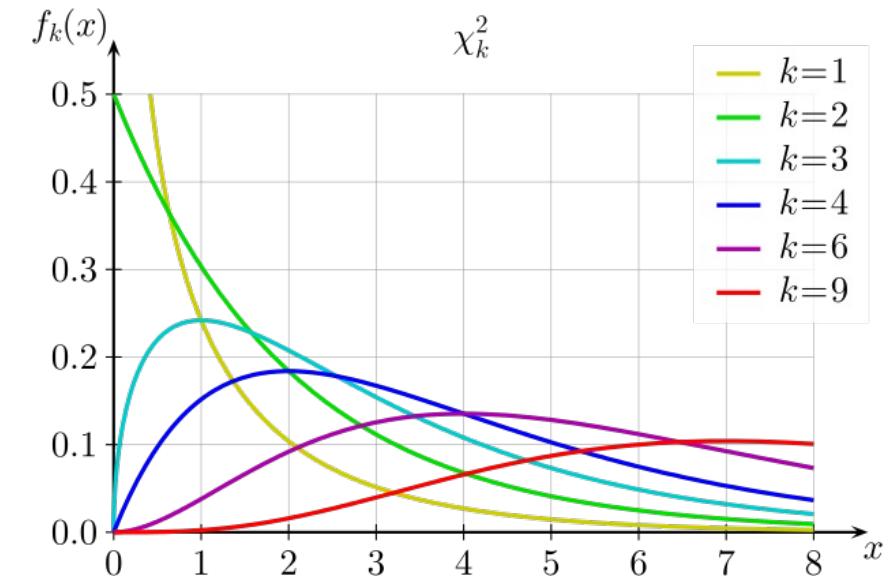
- Basic idea: How much better does my model perform than a baseline, reference (e.g. independence) model?
- Conceptually similar to (adjusted) r-squared: given my model complexity, how well do I explain the data
- Range: 0-1
- Cutoff: >0.95-0.97
- Examples:
 - CFI, TLI, NNFI, GFI

3. Model fit: Model misfit indices

- Basic idea: How *wrong* is my fitted model?
- Conceptually: what is the *average wrongness*
- Examples:
 - RMSEA, SRMR
- Range: ~0-1
- Cutoffs: <0.08-0.05

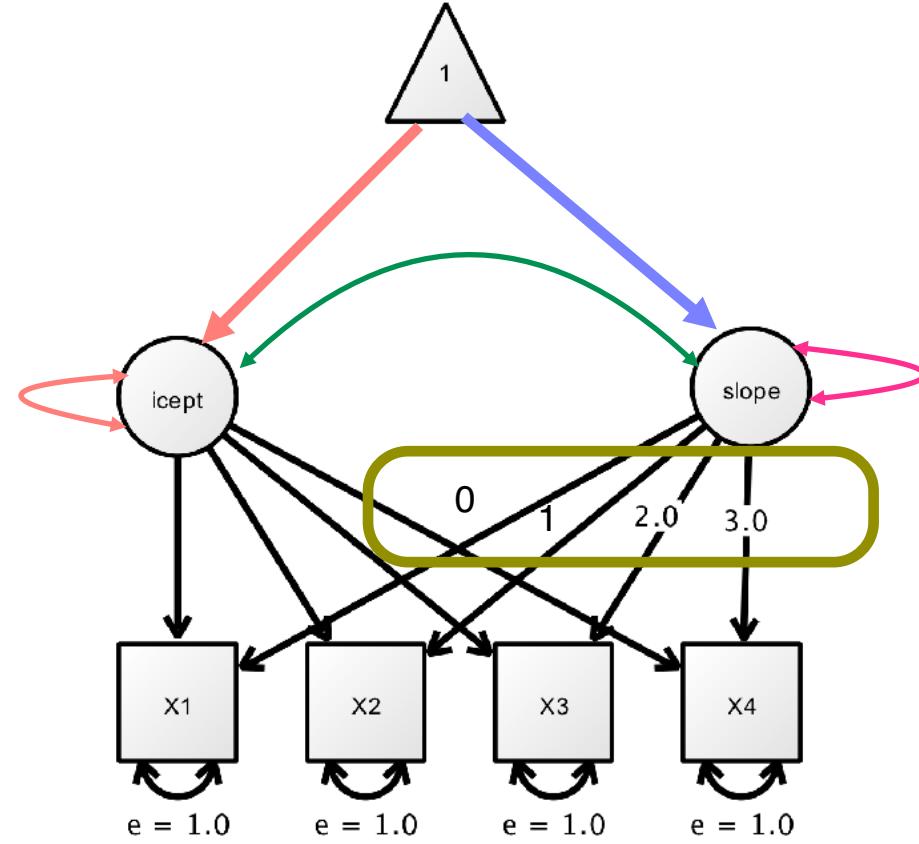
4. Model Comparison A: chi square

- The *difference* in model fit is *also* chi-square distributed
- Only works if models are ***nested***
 - You can turn model 1 into model 2 by simply freeing parameters
- Conceptually:
 - Does the more complicated model fit significantly better than would be expected just by making *any* model more (needlessly) complicated
- Can also use information criteria (AIC/BIC)



A particular type of SEM: *Latent Growth Curve Models*

- Flexible framework to capture:
- Individual differences at baseline
- The shape of change
- Mean change over time (slope)
- Individual differences in change over time (slope variance)
- The association between present and future (slope-intercept covariance)



Kievit et al., (2018). Developmental cognitive neuroscience using latent change score models: A tutorial and applications. *Developmental cognitive neuroscience*, 33, 99-117.

Duncan & Duncan (2004). An introduction to latent curve modelling. *Behaviour Therapy*, 35, 333-363.

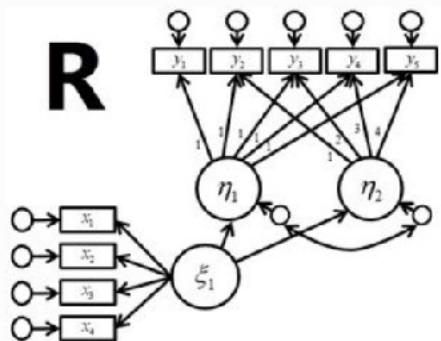
McNeish, D. & Matta, T. (2017). Differentiating Between Mixed Effects and Latent Curve Approaches to Growth Modeling.

Fitting a latent growth curve in Lavaan

- Build up a 3 wave example model
- As *graphical model*
- As *syntax*

Do you have any materials that demonstrate how to estimate structural equation models using lavaan in R?

April 24, 2019



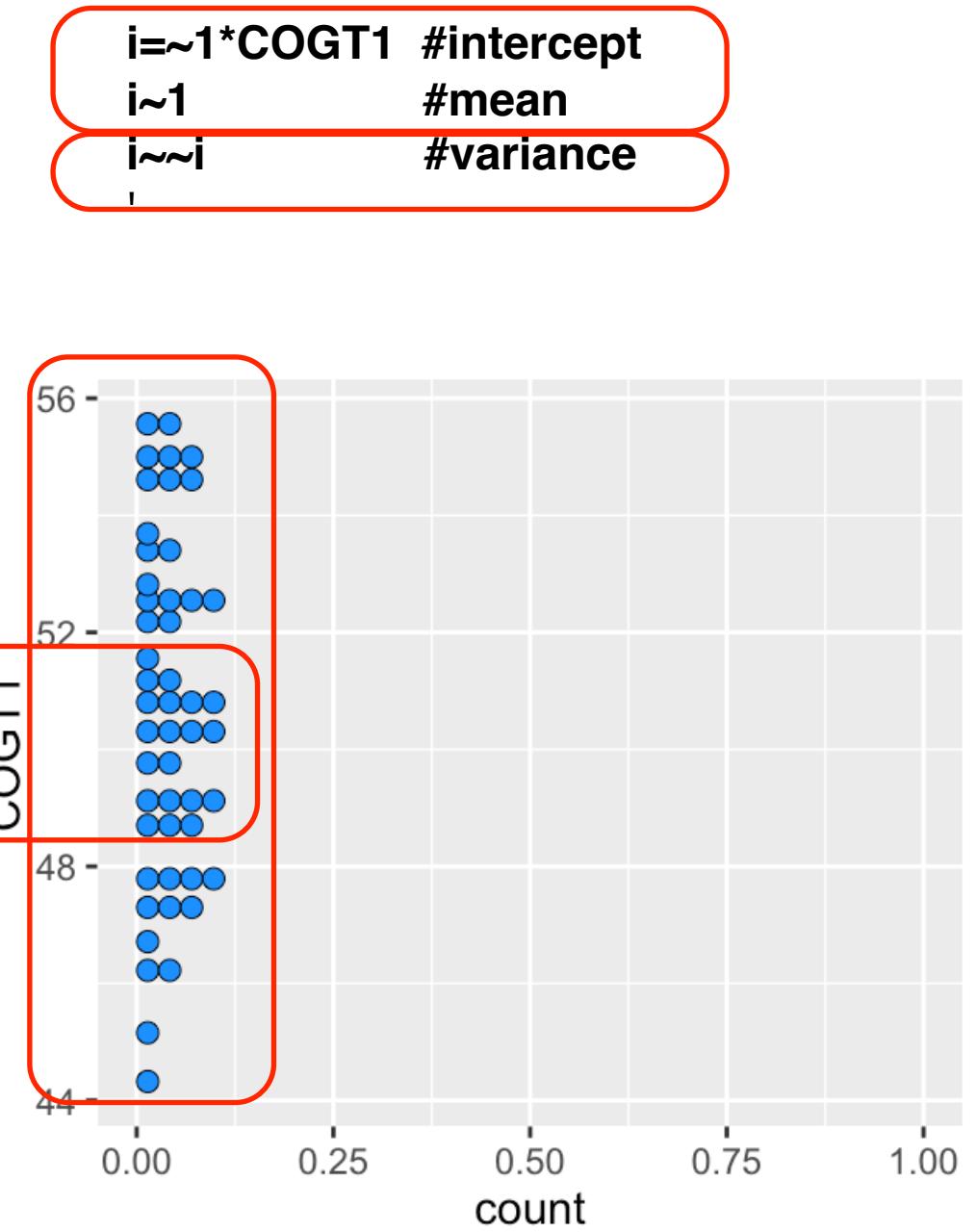
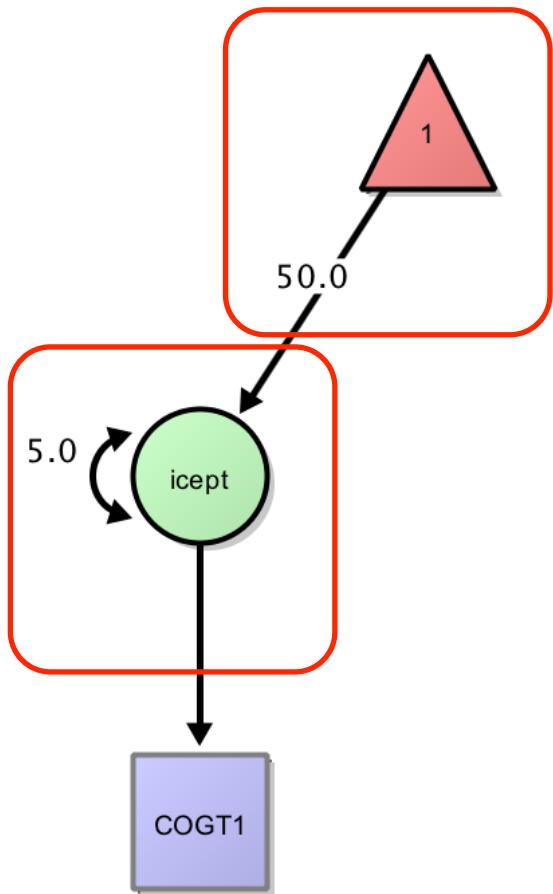
estimate a broad class of SEMs using lavaan and these are now freely available for download.

This is a question we often hear, particularly from students and junior researchers who don't have access to sometimes expensive commercial software for fitting structural equation models. It is possible to estimate a wide array of SEMs, ranging from simple path models to fully latent SEMs to growth curve models and beyond, using the lavaan package within R. For those who may be interested, we have developed detailed demonstrations of how to

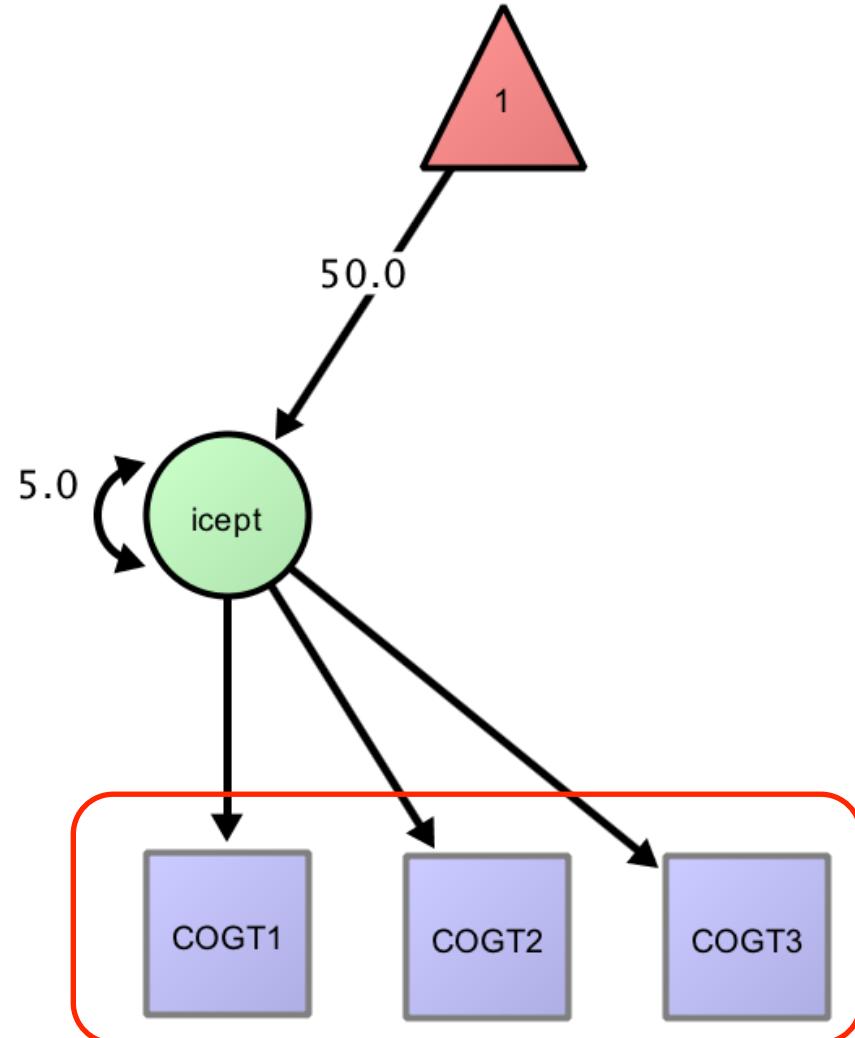
Summary of Formulae Types

| formula type | operator | mnemonic |
|----------------------------|------------|--------------------|
| latent variable definition | $=\sim$ | is measured by |
| regression | \sim | is regressed on |
| (residual) (co)variance | $\sim\sim$ | is correlated with |
| intercept | ~ 1 | intercept |

Time 1



Repeated measures

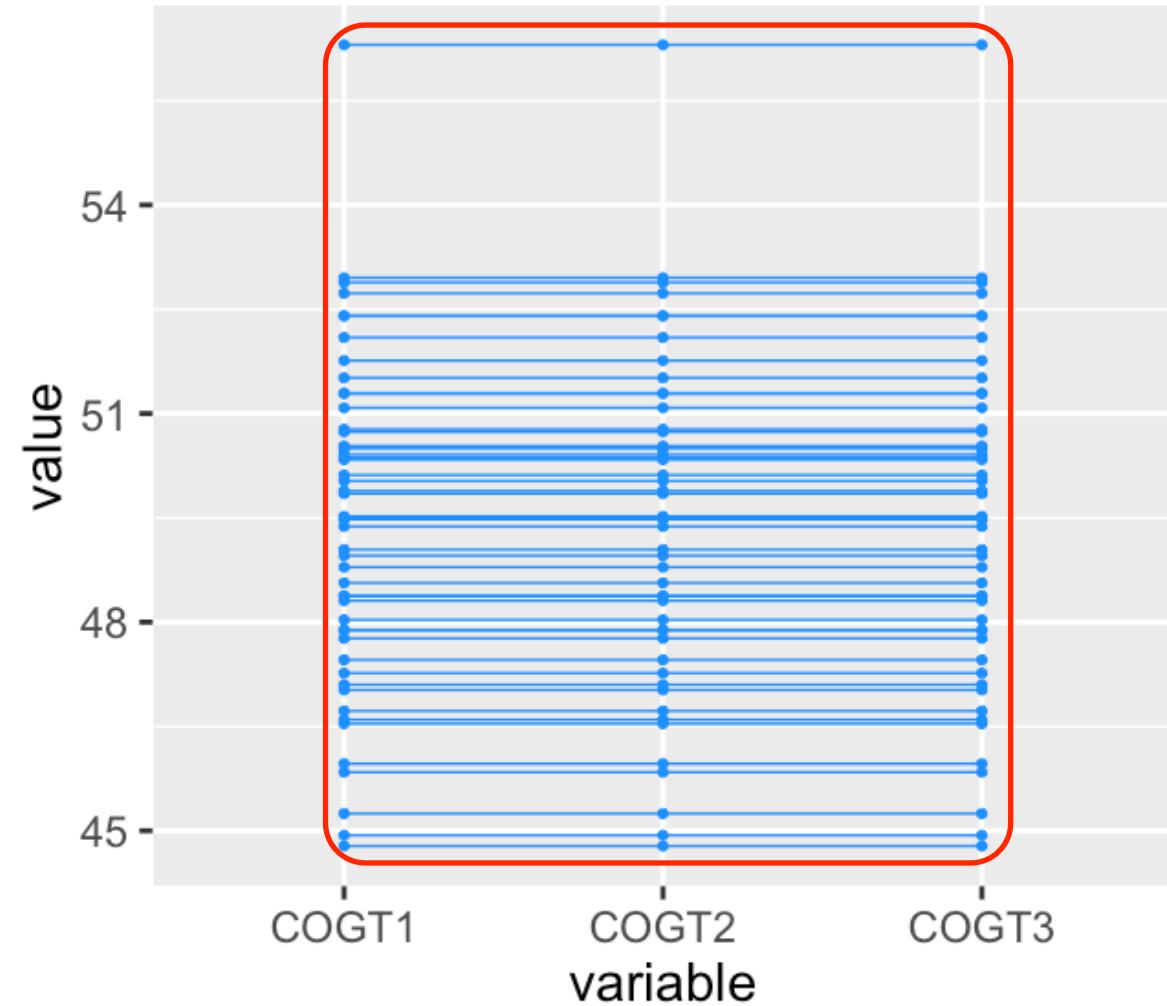


i= $\sim 1^*\text{COGT1} + 1^*\text{COGT2} + 1^*\text{COGT3}$

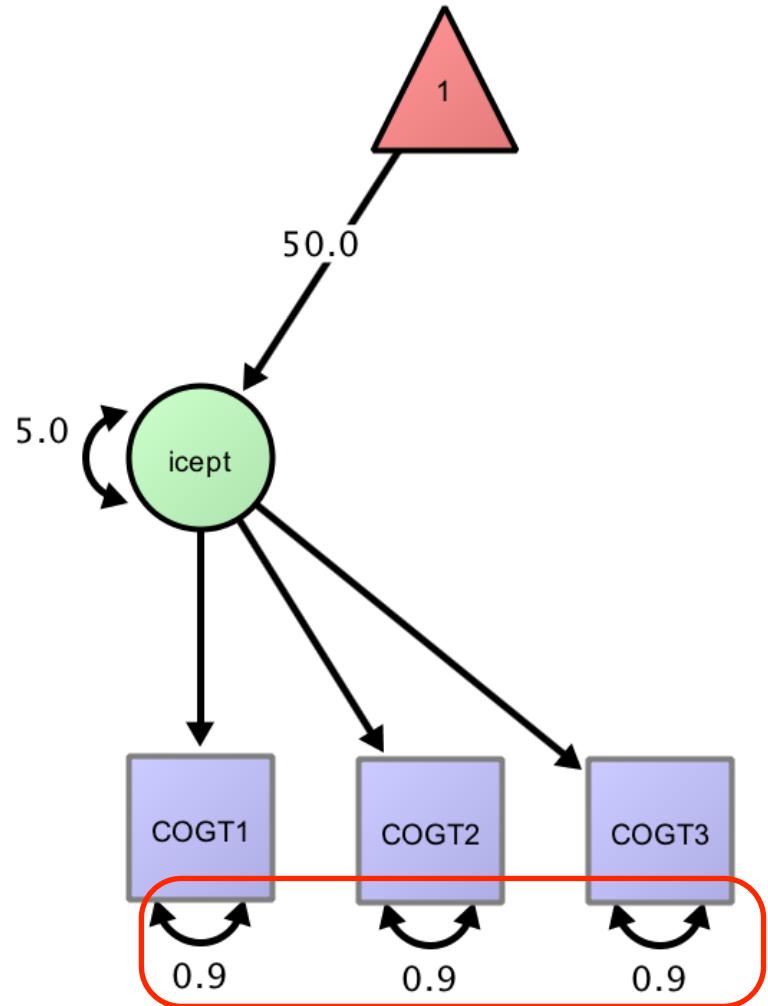
2

i

1

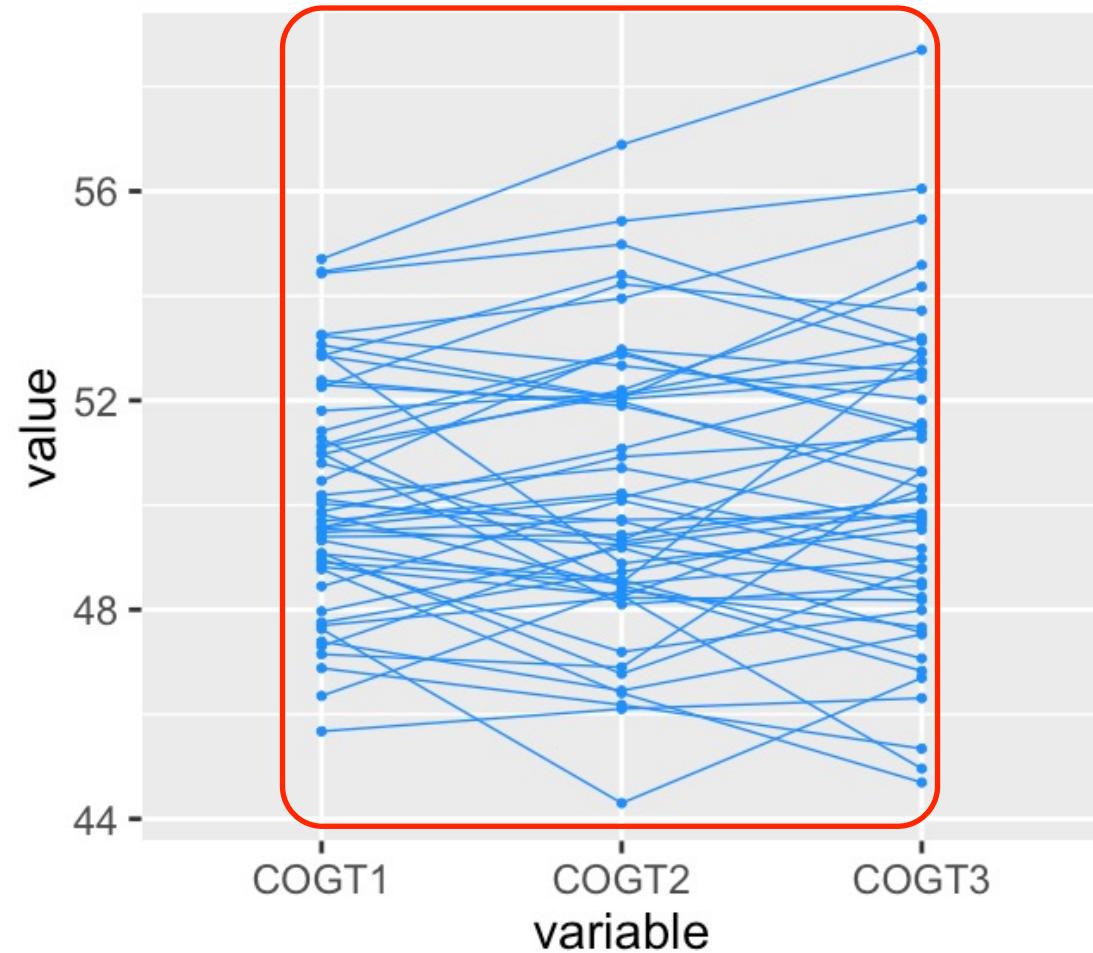


Measurement error

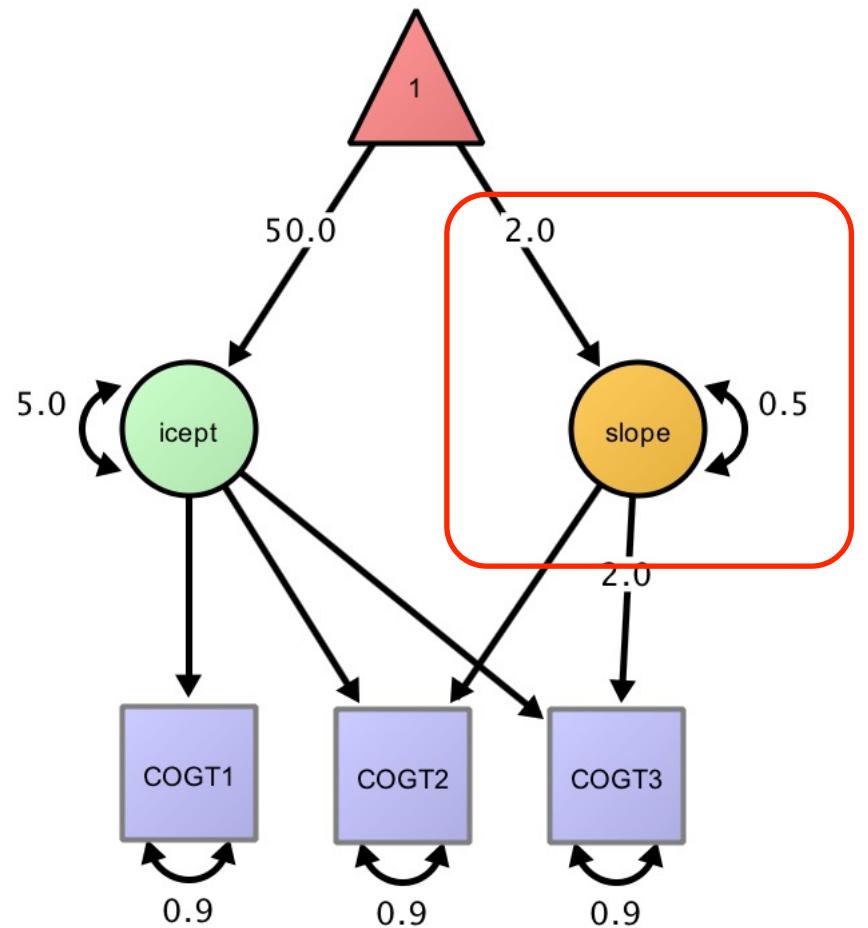


$i = \sim 1 * COGT1 + 1 * COGT2 + 1 * COGT3$
 $i \sim 1$
 $i \sim i$

$COGT1 \sim \text{error} * COGT1$
 $COGT2 \sim \text{error} * COGT2$
 $COGT3 \sim \text{error} * COGT3$



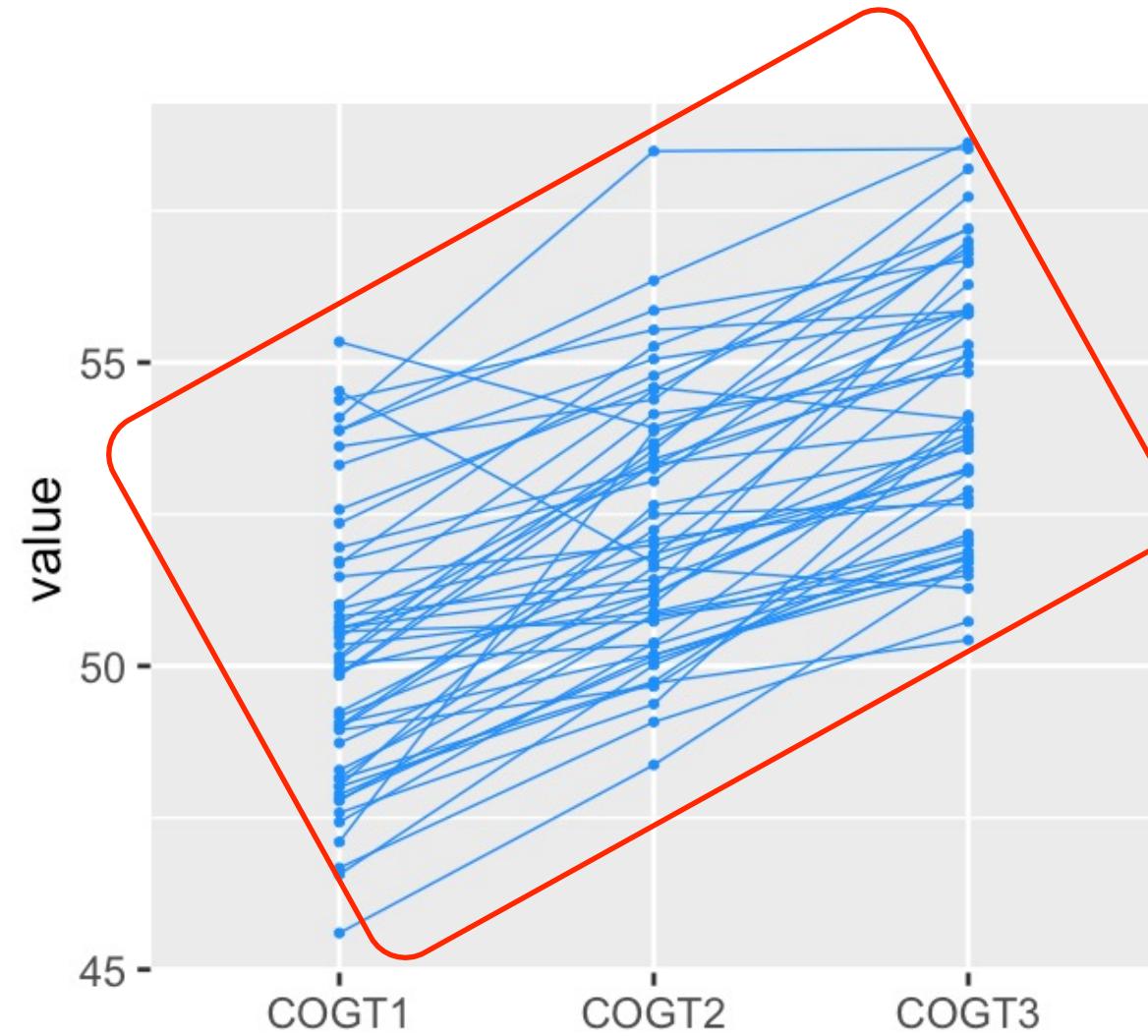
Adding a slope



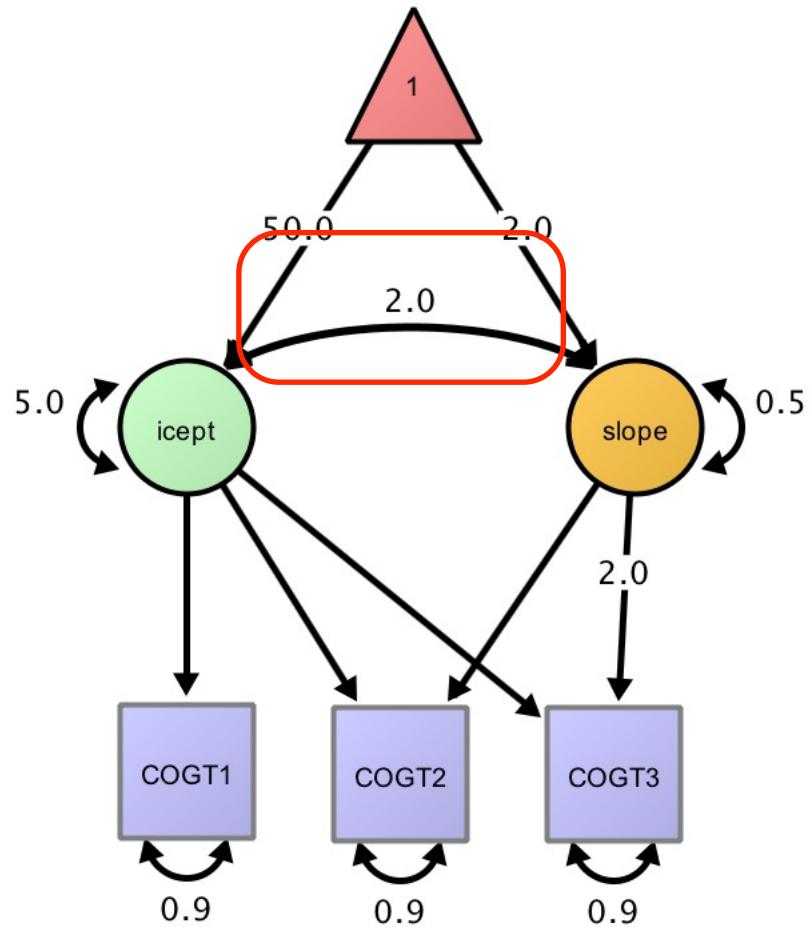
$$i = \sim 1 * \text{COGT1} + 1 * \text{COGT2} + 1 * \text{COGT3}$$

$$s = \sim 0 * \text{COGT1} + 1 * \text{COGT2} + 2 * \text{COGT3}$$

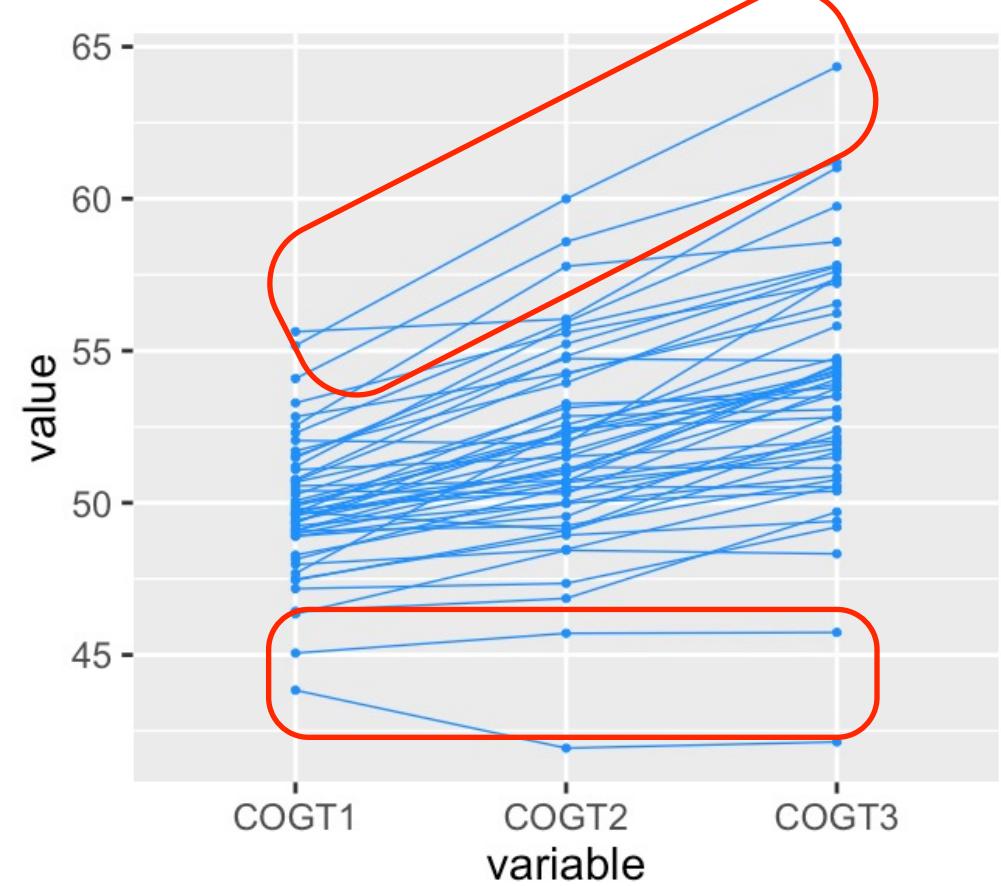
$$s \sim s$$



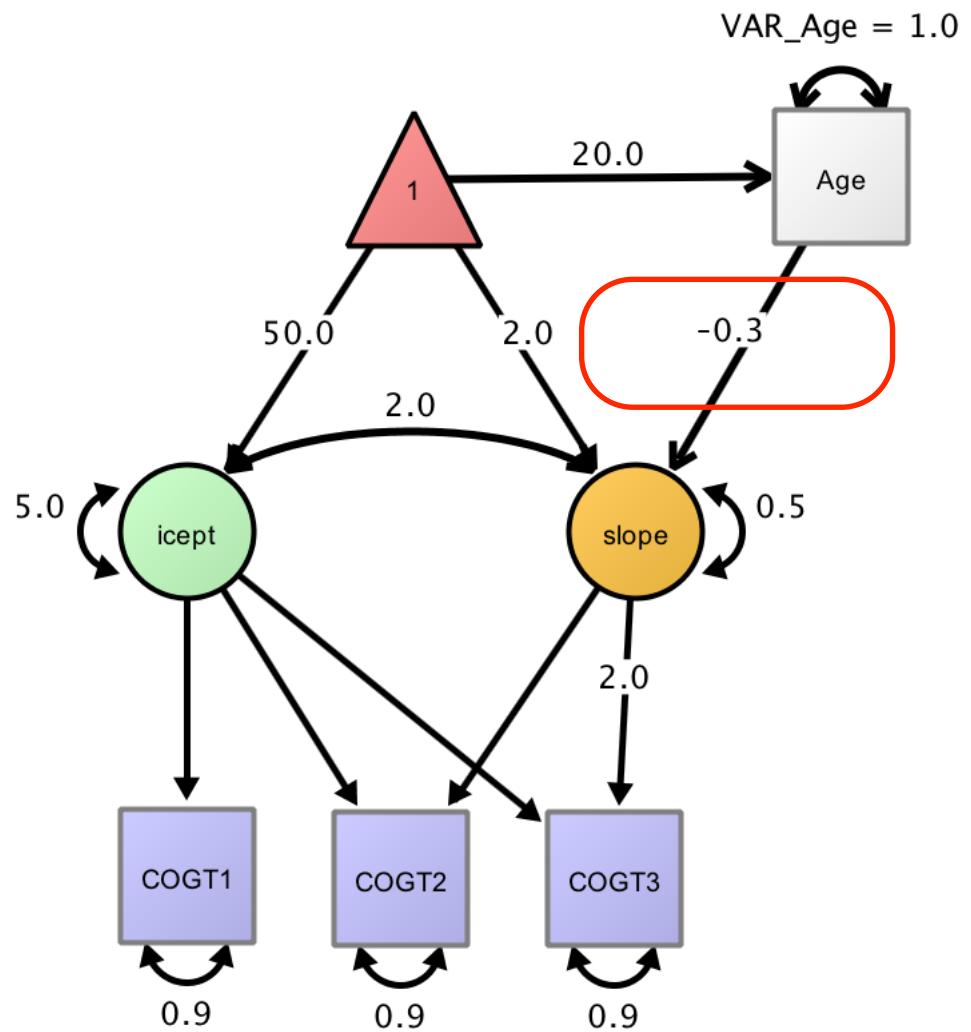
Adding latent covariance



$i = \sim 1 * COGT1 + 1 * COGT2 + 1 * COGT3$
 $s = 0 * COGT1 + 1 * COGT2 + 2 * COGT3$
 $s \sim i$

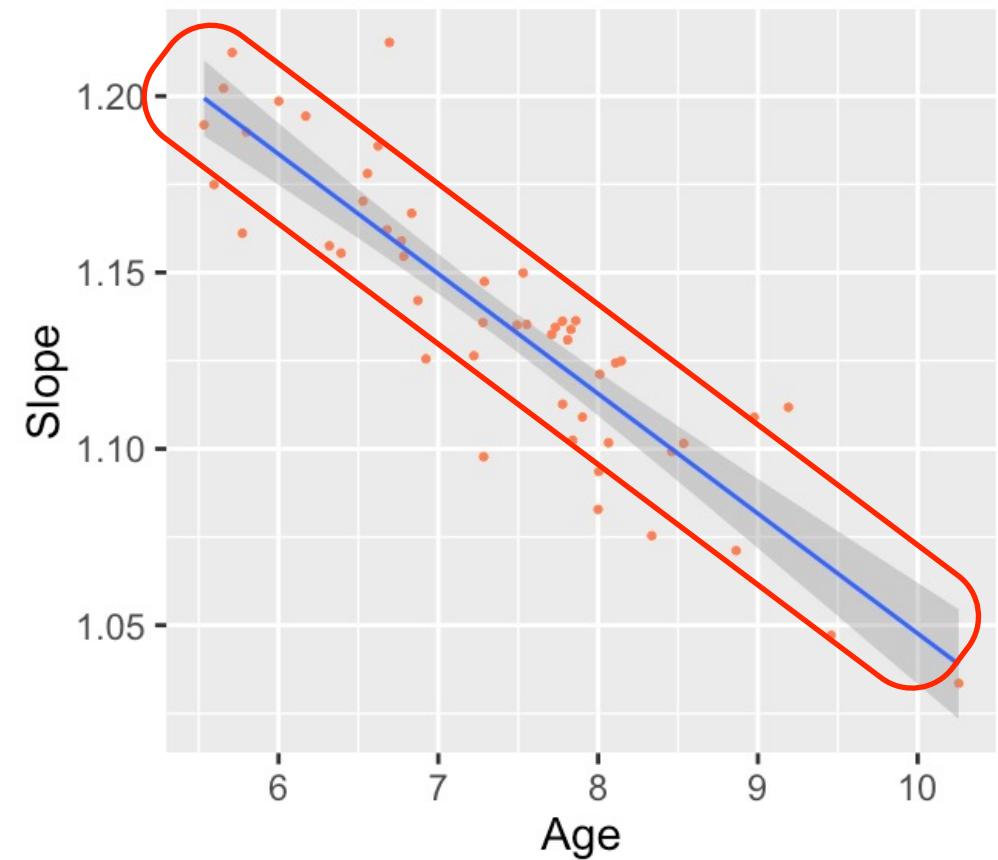


Adding predictors



i= $\sim 1^*\text{COGT1} + 1^*\text{COGT2} + 1^*\text{COGT3}$
s= $\sim 0^*\text{COGT1} + 1^*\text{COGT2} + 2^*\text{COGT3}$
s~i

s~Age

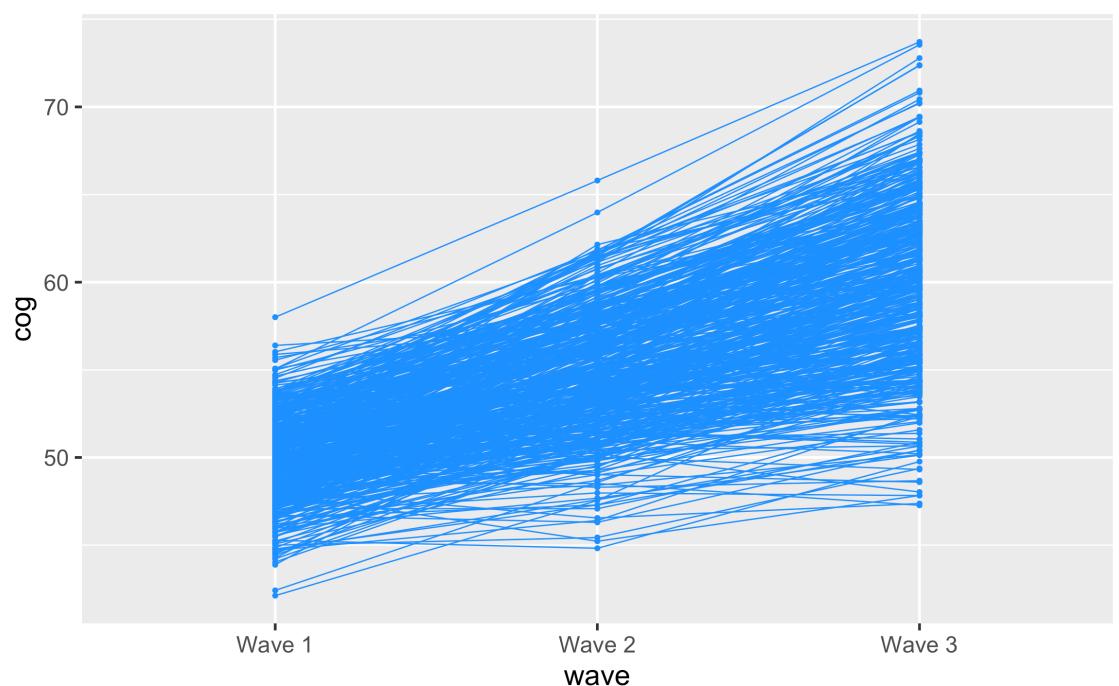


What does this look like in Lavaan?

```
> head(simdat_3waves)
   COGT1    COGT2    COGT3      age
1 53.58734 59.41773 65.45417 14.58179
2 54.64161 58.73914 62.56442 14.13963
3 49.94976 54.86229 63.12696 15.71563
4 53.69024 60.73795 67.02106 17.97347
5 47.70542 55.70050 62.30530 15.96064
6 51.01630 59.79515 65.29828 17.96462
```

```
growth_model_example<-
'
i=~1*COGT1+1*COGT2+1*COGT3
s=~0*COGT1+1*COGT2+2*COGT3
s~~i
s~age
'

#model fitting command
fit_growth_model_example <- growth(growth_model_example, data=simdat_3waves,missing='fiml')
# Output results
summary(fit_growth_model_example, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE, ci=T)
```



- The name of my model is 'growth_model_example'
- The data file is called 'simdat_3waves'

- The model has:
 - an intercept (i)
 - a linear slope (s)
- age predicting the rate of change.
- If data is missing I want to use 'FIML' (i.e. include all data)

- `summary(fit_growth_model_example, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE, ci=T)`

Step 1: Inspect model fit

```
> ## Output results
> summary(fit_growth_model_example, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE, ci=T)
lavaan 0.6-18 ended normally after 123 iterations

Estimator                           ML
Optimization method                 NLMINB
Number of model parameters          9
Number of observations              500
Number of missing patterns          1
Model Test User Model:
Test statistic                      2.561
Degrees of freedom                  3
P-value (Chi-square)                0.464

Model Test Baseline Model:
Test statistic                      1421.790
Degrees of freedom                  6
P-value                            0.000

User Model versus Baseline Model:
Comparative Fit Index (CFI)         1.000
Tucker-Lewis Index (TLI)             1.001

Robust Comparative Fit Index (CFI)   1.000
Robust Tucker-Lewis Index (TLI)      1.001

Loglikelihood and Information Criteria:
Loglikelihood user model (H0)       -3397.408
Loglikelihood unrestricted model (H1) -3396.127

Akaike (AIC)                         6812.816
Bayesian (BIC)                        6850.747
Sample-size adjusted Bayesian (SABIC)  6822.180

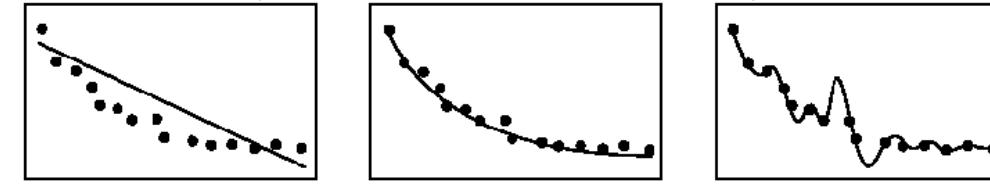
Root Mean Square Error of Approximation:
RMSEA                             0.000
90 Percent confidence interval - lower 0.000
90 Percent confidence interval - upper 0.071
P-value H_0: RMSEA <= 0.050          0.826
P-value H_0: RMSEA >= 0.080          0.025
```



Step 2: interpret parameters

| Latent Variables: | | | | | | | | |
|-------------------|----------|---------|---------|---------|----------|----------|--------|---------|
| | Estimate | Std.Err | z-value | P(> z) | ci.lower | ci.upper | Std.lv | Std.all |
| i ~ | | | | | | | | |
| COGT1 | 1.000 | | | | 1.000 | 1.000 | 2.259 | 0.947 |
| COGT2 | 1.000 | | | | 1.000 | 1.000 | 2.259 | 0.584 |
| COGT3 | 1.000 | | | | 1.000 | 1.000 | 2.259 | 0.396 |
| s ~ | | | | | | | | |
| COGT1 | 0.000 | | | | 0.000 | 0.000 | 0.000 | 0.000 |
| COGT2 | 1.000 | | | | 1.000 | 1.000 | 2.193 | 0.567 |
| COGT3 | 2.000 | | | | 2.000 | 2.000 | 4.386 | 0.769 |
| Regressions: | | | | | | | | |
| | Estimate | Std.Err | z-value | P(> z) | ci.lower | ci.upper | Std.lv | Std.all |
| s ~ age | -0.243 | 0.094 | -2.583 | 0.010 | -0.427 | -0.059 | -0.111 | -0.113 |
| Covariances: | | | | | | | | |
| | Estimate | Std.Err | z-value | P(> z) | ci.lower | ci.upper | Std.lv | Std.all |
| i ~~ .s | 1.901 | 0.288 | 6.612 | 0.000 | 1.338 | 2.465 | 0.386 | 0.386 |
| Intercepts: | | | | | | | | |
| | Estimate | Std.Err | z-value | P(> z) | ci.lower | ci.upper | Std.lv | Std.all |
| i | 50.037 | 0.106 | 471.113 | 0.000 | 49.828 | 50.245 | 22.146 | 22.146 |
| .s | 2.644 | 1.411 | 1.874 | 0.061 | -0.121 | 5.408 | 1.206 | 1.206 |
| Variances: | | | | | | | | |
| | Estimate | Std.Err | z-value | P(> z) | ci.lower | ci.upper | Std.lv | Std.all |
| .COGT1 | 0.592 | 0.267 | 2.219 | 0.026 | 0.069 | 1.115 | 0.592 | 0.104 |
| .COGT2 | 1.216 | 0.222 | 5.519 | 0.000 | 0.911 | 1.591 | 1.216 | 0.283 |
| .COGT3 | 0.609 | 0.633 | 0.962 | 0.336 | -0.632 | 1.849 | 0.609 | 0.019 |
| i | 5.105 | 0.417 | 12.238 | 0.000 | 4.287 | 5.922 | 1.000 | 1.000 |
| .s | 4.747 | 0.381 | 12.464 | 0.000 | 4.000 | 5.493 | 0.987 | 0.987 |
| R-Square: | | | | | | | | |
| | Estimate | | | | | | | |
| COGT1 | 0.896 | | | | | | | |
| COGT2 | 0.917 | | | | | | | |
| COGT3 | 0.981 | | | | | | | |
| s | 0.013 | | | | | | | |

Modeling the shape of change



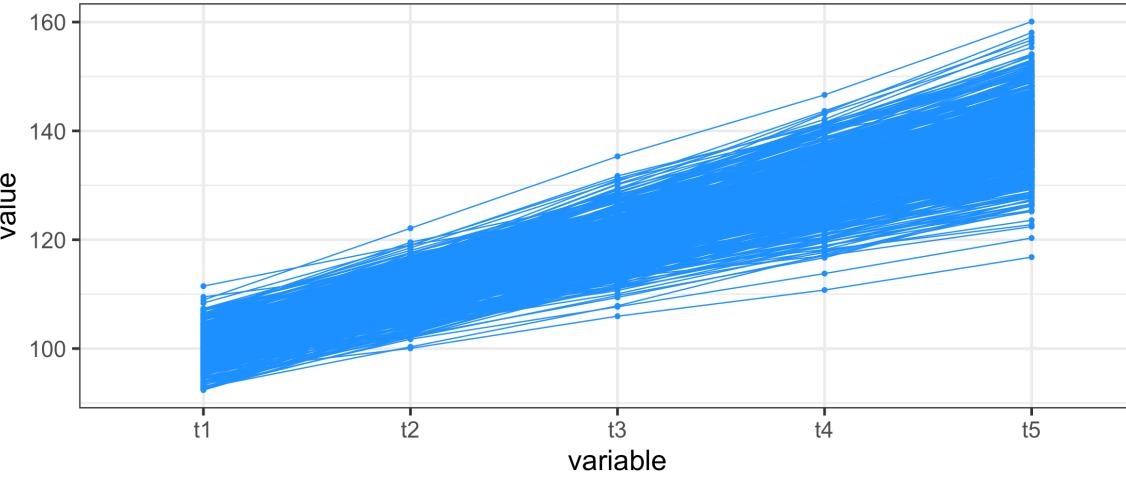
- Think through plausible candidates (given waves)
- Compare model fit using
 - Chi-square: Does the more complex model fit significantly better than expected just by chance
 - AIC/BIC: Does the model fit outweigh the penalty on the number of parameters
- 3 candidates:
 - Linear
 - Quadratic (=linear+quadratic)
 - Free ('basis' model)
- Balance explanatory power (fit to data) and parsimony (# parameters)

$$s_{\text{lin}} \sim 0*t1 + 1*t2 + 2*t3 + 3*t4 + 4*t5$$

$$s_{\text{lin}} \sim 0*t1 + 1*t2 + 2*t3 + 3*t4 + 4*t5$$

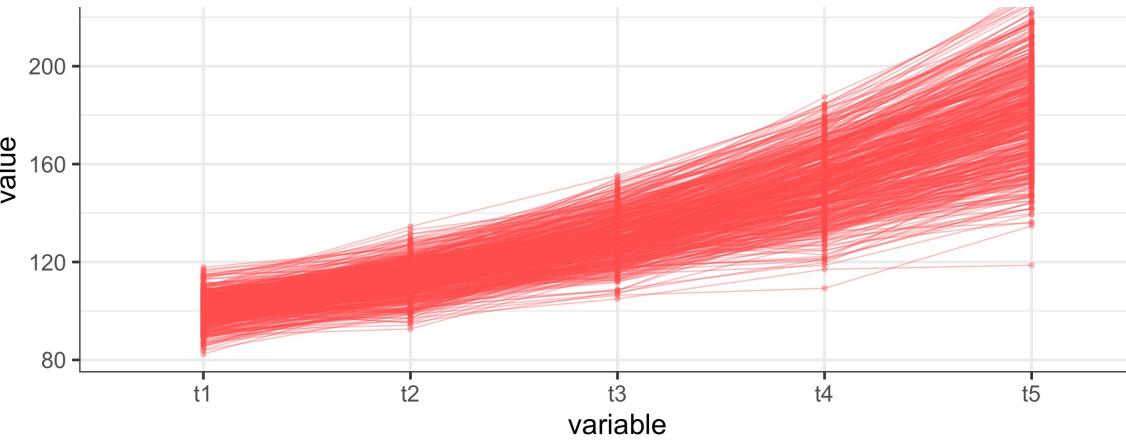
$$s_{\text{quad}} \sim 0*t1 + 1*t2 + 4*t3 + 9*t4 + 16*t5$$

$$s_{\text{basis}} \sim 0*t1 + t2 + t3 + t4 + t5$$



Chi-Squared Difference Test

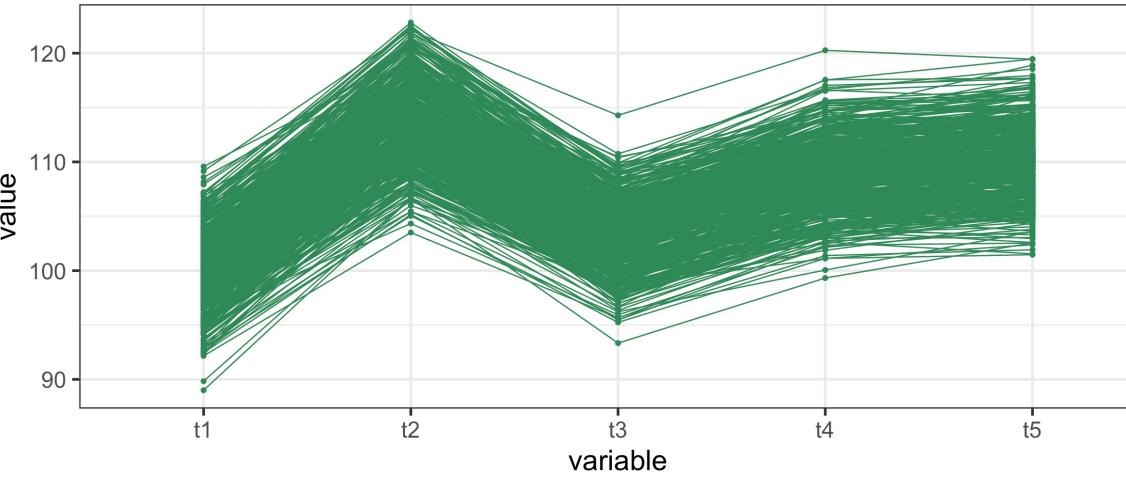
| | Df | AIC | BIC | Chisq | Chisq diff | RMSEA | Df diff | Pr(>Chisq) |
|-----------------|----|-------|-------|--------|------------|-------|---------|------------|
| fit_model_quad | 10 | 10699 | 10741 | 6.4161 | | | | |
| fit_model_basis | 11 | 10697 | 10735 | 6.5470 | 0.13094 | 0 | 1 | 0.7175 |
| fit_model_lin | 14 | 10693 | 10718 | 8.5030 | 1.95596 | 0 | 3 | 0.5816 |
| > | | | | | | | | |



```
> anova(fit_model_lin, fit_model_quad, fit_model_basis)
```

Chi-Squared Difference Test

| | Df | AIC | BIC | Chisq | Chisq diff | RMSEA | Df diff | Pr(>Chisq) |
|-----------------|----|-------|-------|-----------|------------|---------|---------|---------------|
| fit_model_quad | 10 | 16869 | 16911 | 4.4421 | | | | |
| fit_model_basis | 11 | 16894 | 16932 | 31.1694 | 26.73 | 0.22684 | 1 | 2.343e-07 *** |
| fit_model_lin | 14 | 18296 | 18322 | 1439.4909 | 1408.32 | 0.96793 | 3 | < 2.2e-16 *** |
| --- | | | | | | | | |



Chi-Squared Difference Test

| | Df | AIC | BIC | Chisq | Chisq diff | RMSEA | Df diff | Pr(>Chisq) |
|-----------------|----|---------|---------|----------|------------|--------|---------|------------|
| fit_model_quad | 10 | 13331.3 | 13373.4 | 3615.028 | | | | |
| fit_model_basis | 11 | 9741.5 | 9779.4 | 27.225 | -3587.8 | 0.0000 | 1 | 1 |
| fit_model_lin | 14 | 14721.6 | 14746.9 | 5013.336 | 4986.1 | 1.8227 | 3 | <2e-16 *** |



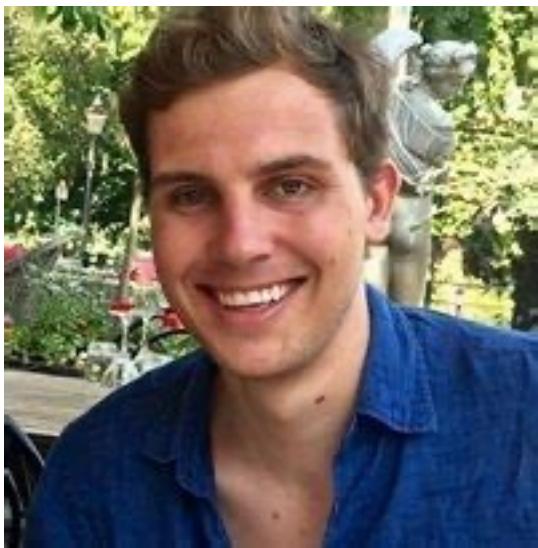
Léa Michel



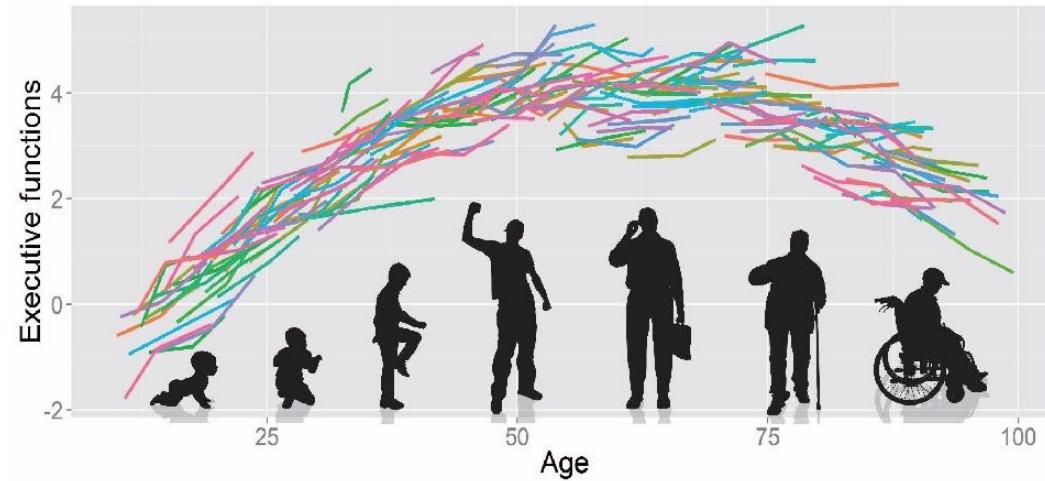
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Nick Judd



An introduction to Latent Growth Curve Modeling



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Simulating data in Lavaan

```
properties cannot be set after the file is saved.

simdat<-'
#basic growth model
i=~1*COGT1+1*COGT2+1*COGT3
s=~0*COGT1+1*COGT2+2*COGT3
#intercepts (mean)
i~50*1
s~2*1
#variances
s~~4*s
i~~5*i

#covariance
i~~2*s

#predictors
s~-0.2*age

#covariate properties
age~15*1
age~~1*age

'

simdat_3waves <- simulateData(simdat, sample.nobs = 500)
colMeans(simdat_3waves)
```