

# Computational methods applied to Particle Physics

## Higgs Boson Machine Learning Challenge

Z. Kassabov

July 2014

## 1 Introduction

- Higgs fermionic decay
- The Higgs Boson Machine Learning Challenge

## 2 Machine Learning

- Types of learning
- An example
- Dimensionality

## 3 Detection of the Higgs boson decays

- Data collection at ATLAS
- Data simulation
- The target process

## 4 Implementation of a classifier

- The classification problem
- Chosen method

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# Discovery of the Higgs boson

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- Mass of 125 GeV.
- Spin 0.
- Decays  $\gamma\gamma$ ,  $ZZ$  and  $W^+W^-$  as predicted by SM.
  - The Brout–Englert–Higgs mechanism is responsible for the mass of the W and Z bosons.

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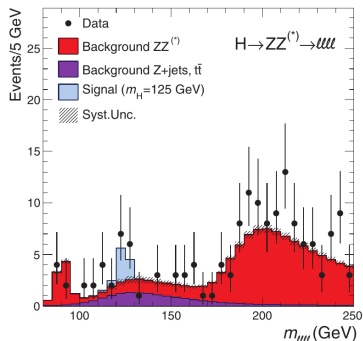
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- Branching ratios predicted to scale with the mass squared of the decay products.
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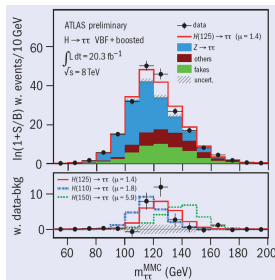
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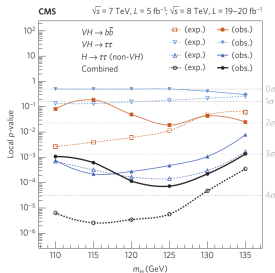
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Higgs challenge **the HiggsML challenge**  
May to September 2014  
When **High Energy Physics** meets **Machine Learning**

info to participate and compete : <https://www.kaggle.com/c/higgs-boson>

ATLAS EXPERIMENT CMS Inria kaggle Google

**Organisation committee**  
Rafael Nisius - ATLAS ML  
Gidon Gelman - CMS ML  
David Rousseau - ATLAS ML  
Glen Cowan - ATLAS ML  
Isabelle Grigor - CMS ML  
Olivier Adam-Souza - ATLAS ML

**Judging committee**  
Theodor Muehlner - ATLAS ML  
Andreas Hocker - ATLAS ML  
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The goal of the challenge is to explore the use of Machine Learning tools to improve the discovery significance of the experiment.

- Simulated samples of data are provided.
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Signal  $\tau\tau$  decay of a Higgs boson.

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*Machine Learning* is a field concerned with developing algorithms that can *learn* from data.

**Supervised learning** Given a set of datapoints where the desired output is *known*, predict the output for unseen datapoints (classification, regression): *Generalize*.

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Given the set of data points  $\mathcal{D} = \{(x_1, y_1), \dots, (x_n, y_n)\}$  with  $x_i, y_i \in \mathbb{R}$ , find a function  $f(x) \rightarrow \mathbb{R}$  that *generalizes* them (ie is able to make a *good* prediction for new points generated by the same underlying model).

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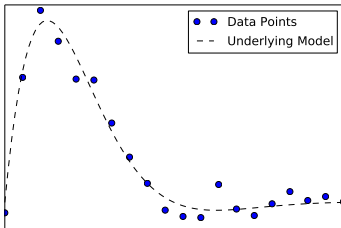
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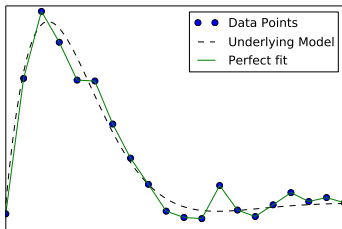


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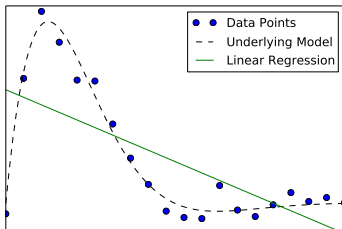
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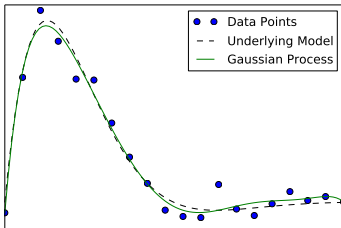
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- Number of points required to sample the space  $\sim 10^d$ .
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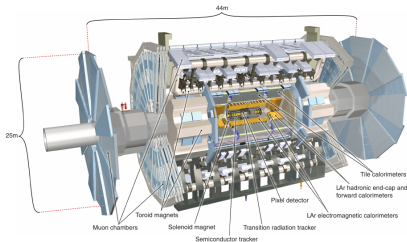
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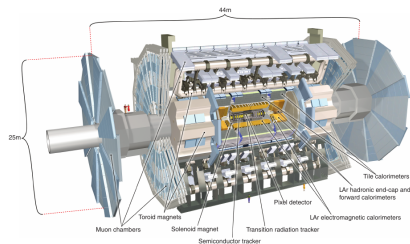


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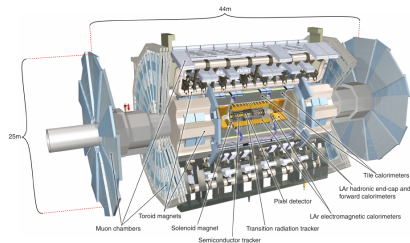
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Momenta of charged particles.
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Energy of photons and electrons.
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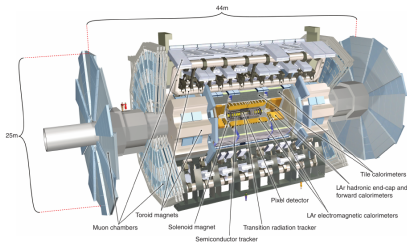
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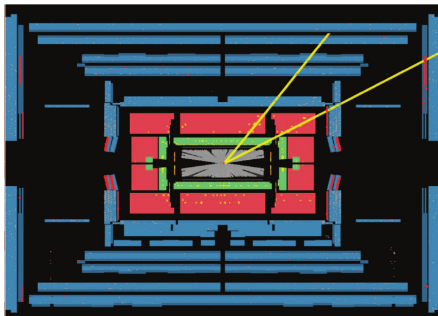
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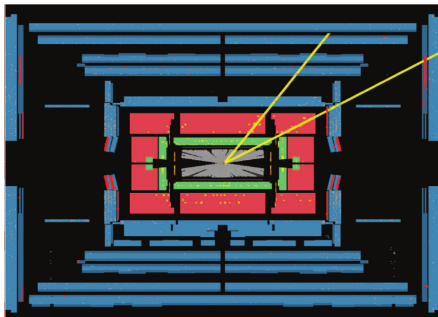
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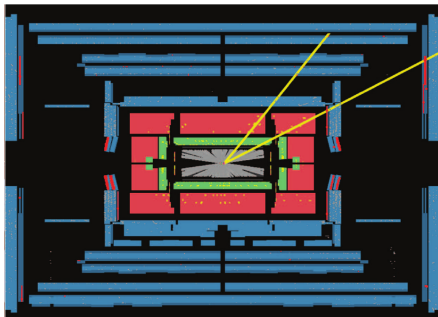
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# Higgs fermionic decay

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$$H \longrightarrow \tau^- \tau^+ \longrightarrow (l + 2\nu) + (\text{hadrons} + \nu)$$

$$l \in \{e^\pm, \mu^\pm\}, \nu \in \{\nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau\}$$

There are *background* events from:

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# The classification problem

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$$\mathcal{D} = \{(x_1, y_1, w_1), \dots, (x_n, y_n, w_n)\}$$

where:

- $x_i \in \mathbb{R}^d$ : Feature vector.
- $y_i \in \{b \equiv \text{"background"}, s \equiv \text{"signal"}\}$ : Label
- $w_i \in \mathbb{R}^+$ : Weight.

Find a classifier  $g : \mathbb{R}^d \rightarrow \{b, s\}$  that maximizes the *Approximate Median Significance*.

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# Gradient Boosting Classifier Formulation

$$g(x) = \sum_{m=1}^M \gamma_m h_m(x)$$

- $h_m$  weak learners (decision trees).
- $h_m$  chosen to minimize some loss function  $L(y_i, x_i)$  at each iteration:

$$g_m(x) = g_{m-1}(x) + \arg \min_h \sum_{i=1}^n L(y_i, g_{m-1}(x_i) - h(x))$$

- The minimization is performed by the steepest descent method:

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The first submission obtained an AMS score of 3.376.

	Score
Random Submission	0.58
Simple Window	1.54
Naive Bayes	2.06
Simple Boosted Trees	3.25
AdaBoost	3.34
My submission	3.38
Best submission	3.81
ATLAS (real significance)	4.1



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Cécile Germain Isabelle Guyon Balázs Kégl David Rousseau Claire Adam-Bourdarios, Glen Cowan.

Learning to discover: the Higgs boson machine learning challenge, 2014.

URL <http://higgsml.lal.in2p3.fr/documentation/>.



ATLAS collaboration et al.

Evidence for Higgs boson decays to the  $\tau^+ \tau^-$  final state with the ATLAS detector.

ATLAS-CONF-2013-108, 2013.



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A particle consistent with the Higgs boson observed with the ATLAS detector at the Large Hadron Collider.

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The CMS Collaboration.

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*Nat Phys*, advance online publication, Jun 2014.

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P K Sinervo.

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P F Harrison.

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