Appendix

8.1 Analysis of P'_c

To simplify the analysis, we assume that each bit in Bloom filter is set to 0 with probability p_0 and 1 with probability $1 - p_0$. Note that we do not consider the case of *cost exchange*. When all buckets mapped by e_{sk} through all hash functions in H_c are *conflict after adjustment*, we cannot adjust the hash functions of e_{sk}, so we get

$$P'_c = 1 - \prod_{h \in H_c(e_{sk})} (1 - (1 - p_0^{k-1})^{\chi(h(e_{sk}))}), \tag{35}$$

where $\chi(i)$ represents the number of keys in the i^{th} bucket of Γ. Moreover, according to average value inequality, we have

$$\begin{aligned}1 - P'_c \leq & (\frac{|H| - k - \sum_{h \in H_c} (1 - p_0^{k-1})^{\chi(h(e_{sk}))}}{|H| - k})^{|H|-k} \\ \leq & (1 - \frac{1}{|H| - k} \sum_{h \in H_c} (1 - p_0^{k-1})^{\chi(h(e_{sk}))})^{|H|-k} \\ \leq & (1 - \prod_{h \in H_c} (1 - p_0^{k-1})^{\frac{\chi(h(e_{sk}))}{|H|-k}})^{|H|-k}. \end{aligned} \tag{36}$$

It is easy to prove that: $\forall 0 < \alpha < 1, \beta \in \mathbb{N}, (1 - \alpha)^\beta < 1 - \alpha^\beta$, which is similar to Lemma 1, then we have

$$\begin{aligned}1 - P'_c <& 1 - (1 - p_0^{k-1})^{\sum_{h \in H_c} \chi(h(e_{sk}))} \\ P'_c >& (1 - p_0^{k-1})^{\sum_{h \in H_c} \chi(h(e_{sk}))}. \end{aligned} \tag{37}$$

Since function $g''(x) = (1 - p_0^{k-1})^x$ is a convex function, by the Jensen inequality, we get

$$E(P'_c) > (1 - p_0^{k-1})^{E(\sum_{h \in H_c} \chi(h(e_{sk})))}. \tag{38}$$

Let $\psi = \sum_{h \in H_c} \chi(h(e_{sk}))$, and we assume that $\forall h \in H, e_{sk} \in S$, for a certain unit u in V, the probability that u is mapped by e_{sk} through h is only determined by $p(u)$, so we have

$$E(\psi) = E(\sum_{u=1}^{m} \sum_{p \in H_c} \chi(u) p(u)) = E(\sum_{u=1}^{m} \chi(u) \sum_{p \in H_c} p(u)), \tag{39}$$

where $\chi(u) = |O| \sum_{p' \in H_0} p'(u)$, for $\forall p_\alpha \in H_0$, $p_\gamma \in H_c$, p_α and p_γ are independent of each other, we have

$$\begin{aligned}E(\psi) =& \sum_{u=1}^{m} |O| E(\sum_{p \in H_0} p(u)) \cdot E(\sum_{p \in H_c} p(u)) \\ <& \sum_{u=1}^{m} \frac{|O|}{4} (\sum_{p \in H} E(p(u)))^2 = \frac{|O| \cdot |H|^2}{4m}. \end{aligned} \tag{40}$$

Since $0 < (1 - p_0^{k-1}) < 1$, then

$$E(P'_c) > (1 - p_0^{k-1})^{\frac{|O| \cdot |H|^2}{4m}}. \tag{41}$$