

Extremely quick update on `bootstrap_variance_2.pdf`:

In going from (11) to (12), I note that I'm not entirely certain about the conversion of notation. This was for good reason, as I was wrong. Note:

$$\sum_{W_i=1} \widehat{Y_i(0)}^2 \neq \sum_{W_i=0} K_i^2 Y_i^2 \quad (1)$$

This is because simply replacing  $\widehat{Y_i(0)}$  with  $K_i Y_i$  is not valid. Only the statement  $\sum \widehat{Y_i(0)} = \sum K_i Y_i$  is valid. The sum of  $\widehat{Y_i(0)}^2$  is properly represented as  $\sum_{W_i=0} K_i Y_i^2$ . This squares every  $Y_i(0)$ , and zeroes out those which are not matched to anything.

Thus, (13) becomes:

$$E \left[ \frac{1}{N_1^2} \sum_{W_i=1} \left( Y_i(1) - \widehat{Y_i(0)} - \hat{\tau} \right) | X, W \right] = \frac{1}{N_1^2} \sum_{W_i=0} K_i - \frac{1}{N_1^3} \sum_{W_i=0} K_i^2 \quad (2)$$

This is rather worse in the sense it is much farther from what it should be - but (2) can under- *and* over-estimate the proper variance ( $\frac{1}{N_1^2} \sum K_i^2$ ), and can do so by large enough amounts to potentially match my simulation results.