

221900180 田永铭 计算方法作业5

第三章:

22. 解: $\phi = \text{span}\{1, x^2\}$. $\varphi_0 = 1, \varphi_1 = x^2$. 设 $y^* = a_0 + a_1 x^2$.

$(\varphi_0, \varphi_0) = \sum_{i=1}^5 1^2 = 5$, $(\varphi_0, \varphi_1) = (\varphi_1, \varphi_0) = \sum_{i=1}^5 1 \times x_i^2 = 5327$,
 $(\varphi_1, \varphi_1) = \sum_{i=1}^5 x_i^2 \cdot x_i^2 = 7277699$, $(\varphi_0, y) = \sum_{i=1}^5 y_i = 271.4$, $(\varphi_1, y) = \sum_{i=1}^5 x_i^2 y_i = 369321.5$.

$\therefore \begin{bmatrix} 5 & 5327 \\ 5327 & 7277699 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 271.4 \\ 369321.5 \end{bmatrix}$. 解得: $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \approx \begin{bmatrix} 0.9726046 \\ 0.0500351 \end{bmatrix}$.

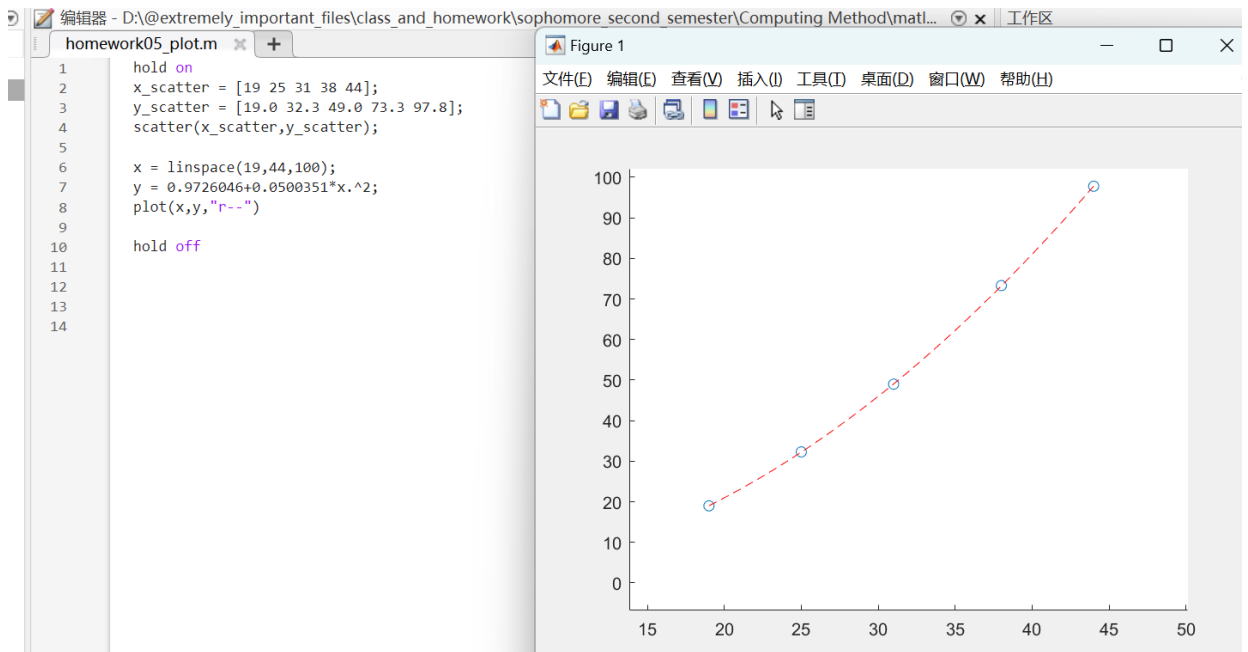
注意: 求解此方程时处理不当会发生溢出 (7277699 \times 5327 超出 int 范围),
 正解: 先解出 a_1 (方程 1×5327 , 方程 2×5 求 a_0), 再代入方程 1 求出 a_0 来.
 得到上述结果.

$\therefore y = 0.9726046 + 0.0500351 x^2$.

$\| \delta^2 \|_2^2 = \| y \|_2^2 - a_0^* (\varphi_0, y) - a_1^* (\varphi_1, y)$
 $= \sum_{i=1}^5 y_i^2 - 0.9726046 \times 271.4 - 0.0500351 \times 369321.5 = 0.01692691$.

$\| \delta^2 \|_2 \approx 0.13$.

为验证拟合效果正确, 参见 Matlab 实验图.



24. 解:

先用Matlab画散点图(见附件), 知近似于指数函数. 由题: 舍去(0,0)数据, 用后11组.

$$\therefore \text{设 } y = ae^{\frac{b}{x}}. \therefore \ln y = \ln a + \frac{b}{x} = \ln a + b \cdot \frac{1}{x}.$$

$$\text{令 } \bar{y} = \ln y, a_0 = \ln a, a_1 = \frac{b}{x}, x = \frac{1}{x}. \therefore \bar{y} = a_0 + a_1 x \text{ 为线性函数.}$$

$$(\bar{y}_0, \bar{y}_0) = \sum_{i=2}^{12} 1^2 = 11, (\bar{y}_0, \bar{y}_1) = \sum_{i=2}^{12} 1 \cdot x_i = \sum_{i=2}^{12} \frac{1}{x_i} = 0.603975469 \approx 0.6039755.$$

$$(\bar{y}_1, \bar{y}_1) = \sum_{i=2}^{12} x_i^2 = \sum_{i=2}^{12} \frac{1}{x_i^2} = 0.06232128776 \approx 0.0623213.$$

$$(\bar{y}_0, \bar{y}) = \sum_{i=2}^{12} \bar{y}_i = \sum_{i=2}^{12} \ln y_i \approx 13.639649.$$

$$(\bar{y}_1, \bar{y}) = \sum_{i=2}^{12} \bar{y}_i \cdot x_i = \sum_{i=2}^{12} \ln y_i \cdot \frac{1}{x_i} \approx 0.53033.$$

$$\therefore \begin{bmatrix} 11 & 0.6039755 \\ 0.6039755 & 0.0623213 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 13.639649 \\ 0.53033 \end{bmatrix} \text{ 解得: } \begin{cases} a_0 \approx 1.651559 \\ a_1 \approx -7.49617 \end{cases}.$$

$$\text{反代出: } a = e^{a_0} \approx 5.21510, b = a_1 \approx -7.49617.$$

$$\therefore \text{拟合公式: } y = 5.21510 e^{\frac{-7.49617}{x}}.$$

为验证拟合效果, 参见Matlab图.

```
hold on
x_scatter = [0 5 10 15 20 25 30 35 40 45 50 55];
y_scatter = [0 1.27 2.16 2.86 3.44 3.87 4.15 4.37 4.51 4.58 4.62 4.64];
scatter(x_scatter, y_scatter);

x = linspace(0, 55, 100);
y = 5.21510 * exp(-7.49617 ./ x);
plot(x, y, 'r--')

hold off
```

