

考试科目名称 离散数学 (B 卷)

考试方式：开卷 考试日期_____年 ____月 ____日 教师 赵建华，姚远

系（专业） 软件学院（软件工程） 年级_____ 班级_____

学号_____ 姓名_____ 成绩_____

注意：所有作答请写在答题纸上。

1. (10 points) Symbolize the following propositions, and provide the logic reasoning steps:

“Natural numbers are all integers, and integers are all rational numbers; some rational numbers are not integers. So, natural numbers are all rational numbers, and there are rational numbers that are neither natural numbers nor integers.”

2. (8 points) A player rolls three dices. Calculate the probability that two of them have same point and the other one has a strictly larger point.

3. (8 points) Let S be the set $\{a, b, c, d, e, f\}$, and A, B be two relations defined on S as follows.

$A = \{(a, c), (a, d), (a, e), (b, a), (b, d), (b, e), (b, f), (c, a), (c, b), (c, e), (c, f), (d, c), (d, d), (e, f), (f, f), (f, a)\}$

$B = \{(a, c), (a, f), (b, b), (b, e), (b, f), (c, b), (c, c), (d, e), (e, c), (e, f)\}$

Please give the relations $A \circ B$ and A^2 .

4. (8 points) Given a set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Give the Hasse diagram of A in terms of the divisibility relation on A , and list all the minimal value(s) and maximal value(s) of A on the divisibility.

5. (8 points) Suppose set A has four elements. Give the total number of equivalence relations on A .

6. (8 points) Assume that n is a positive even number.

(1) How many functions $f: \{0, 1\}^n \rightarrow \{0, 1\}^n$ are there satisfying $\forall x: f(x) \neq x$.

(2) For $x \in \{0, 1\}^n$, let x^r be the reverse string of x . How many strings are there satisfying $x^r = x$.

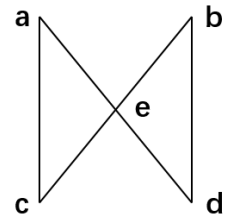
7. (8 points) Let $Z_m = \{0, 1, \dots, m-1\}$, $+_m$ be the addition modulo m , and $*_m$ be the multiplication modulo m .

(1) Prove that $(Z_m, +_m)$ is a group.

(2) Give and prove the sufficient and necessary condition of $(Z_m - \{0\}, *_m)$ being a group.

8. (8 points) Given a simple graph G with a limited number of vertices, and assume that we can delete vertices in G step by step as follows: we can delete only vertices with degrees less than 2. Prove that: all the vertices in G can be deleted if and only if there is no circuit in G .

9. (10 points) Consider a simple Euler graph G ($|G| > 2$). A vertex v in G is called *extendible* if any simple path from v can be extended to an Euler circuit. For example, as shown in the figure below, vertex e is extendible; vertex a is not extendible, as the simple path aec cannot be extended to an Euler circuit. Prove that: vertex v is extendible if and only if $G - v$ is a forest.



10. (8 points) Represent the expression $(a-b) * (c + d) + (e / f) * g$ as a rooted binary tree. Then, give the sequences travelling this tree in preorder and postorder, respectively.

11. (8 points) Let G be an Abelian group. Prove that the function $f: G \rightarrow G$ defined as $f(a) = a^3$ is a homomorphism.

12. (8 points) Let (L, \leq) be a lattice. For any element x, y, z in L , prove the following formulas:

$$(1) x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$$

$$(2) (x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$$

中文参考

1. (10分) 符号化以下命题，并给出推理的证明过程。

自然数都是整数，整数都是有理数，有些有理数不是整数，所以自然数都是有理数，并且存在既不是自然数也不是整数的有理数。

2. (8分) 计算掷出三个骰子后，有两个骰子点数相同，且另外一个的点数大于这个点数的概率。

3. (8分) 已知集合 $S=\{a,b,c,d,e,f\}$ 和 S 上的关系 A, B 如下：

$A=\{(a,c), (a,d), (a,e), (b,a), (b,d), (b,e), (b,f), (c,a), (c,b), (c,e), (c,f), (d,c), (d,d), (e,f), (f,f), (f,a)\}$

$B=\{(a,c), (a,f), (b,b), (b,e), (b,f), (c,b), (c,c), (d,e), (e,c), (e,f)\}$

请给出关系 $A \circ B$ ，和 A^2 。

4. (8分) 已知整数集合 $A=\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ，画出 A 相对于整除关系的 Hasse 图。并指出 A 中相对于整除关系的所有极小值和极大值。

5. (8分) 若集合 A 有四个元素，给出 A 上的所有等价关系的个数。

6. (8分) 假设 n 是一个正偶数，试分别回答以下问题：

(1) 存在多少函数 $f: \{0,1\}^n \rightarrow \{0,1\}^n$ ，满足 $\forall x: f(x) \neq x$ 。

(2) 对于 $x \in \{0,1\}^n$ ，令 x^r 表示 x 的倒序串，存在多少这样的 x 满足 $x^r = x$ 。

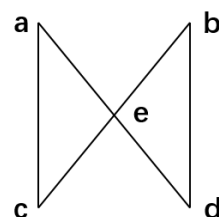
7. (8分) 设 $Z_m=\{0,1,\dots,m-1\}$ ， $+_m$ 是模 m 加法运算， $*_m$ 是模 m 乘法运算。

(1) 证明 $(Z_m, +_m)$ 是群。

(2) 给出 $(Z_m-\{0\}, *_m)$ 是群的充要条件，并证明之。

8. (8分) 给定一个顶点个数有限的简单图 G ，假定我们只可以通过如下方式逐步删除 G 中的顶点：每一步可以删除度数小于2的顶点。试证明：如果 G 中的所有顶点能被删除当且仅当 G 中没有回路。

9. (10分) 简单图 G 是满足 $|G|>2$ 的欧拉图，定义 G 中的节点 v 是可延展的(extendible)指：从节点 v 出发的任意简单通路都可以继续延展成欧拉回路。例如下图所示，只有节点 e 是可延展的；对于节点 a ， aec 这条简单通路无法继续延展成欧拉回路。试证明：节点 v 是可延展的当且仅当 $G-v$ 是一个森林。



10. (8分) 给出表达式 $(a-b) * (c + d) + (e / f) * g$ 的二叉树表示, 然后分别给出按照preorder和postorder遍历这棵树得到的序列。

11. (8分) 假设 G 是一个阿贝尔群, 函数 $f: G \rightarrow G$ 定义为 $f(a) = a^3$, 请证明 f 是一个同态映射。

12. (8分) 设 (L, \leq) 为格, 对于 L 的任意元素 x, y, z , 证明下式成立:

$$(1) x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$$

$$(2) (x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$$