学号

注意：所有作答请写在答题纸上。

1 (10 scores)

For each of the following arguments, if it is valid, give a derivation, and if it is not valid, show why.

(1) If Susan likes fish, then she likes onions. If Susan does not like garlic, then she does not like onions. If she likes garlic, then she likes chicken. She likes fish or she likes pork. She does not like chicken. Therefore, Susan likes pork.

(2) It is not the case that Fred plays both guitar and violin. If Fred does not play guitar and he does not play violin, then he plays both piano and horn. If he plays horn, then he plays piano. Therefore Fred plays piano

2 (10 scores)

Write a derivation for each of the following arguments.

(1) Every cake that is hard is not pleasant to eat. Every cake that is not hard is sweet. Therefore every cake that is pleasant to eat is sweet.

(2) Each high school student in Nanjing who takes an art class is cool. There is a high school student in Nanjing who is smart and not cool. Therefore there is a high school student in Nanjing who is smart and not taking an art class.

3 (10 scores)

Let A, B, and C be sets. Prove that:

$$A \cup B \cup C = (A - B) \cup (B - C) \cup (C - A) \cup (A \cap B \cap C)$$

4 (10 scores)

Prove that the following relations are equivalences, and describe the corresponding partitions. Your description of each partition should have no redundancy, and should not refer to the name of the relation.

(1) Let $\sim$ be the relation on $\mathbb{R}-\{0\}$ defined by $x\sim y$ if and only if $xy > 0$, for all $x,y\in \mathbb{R}-\{0\}$, where R is the set of real numbers.

(2) Let L be the set of all lines in $\mathbb{R}^2$, and let $\sim$ be the relation on L defined by $l_1 \sim l_2$ if and only if l_1 is parallel to l_2 or is equal to l_2, for all $l_1, l_2 \in L$.

5 (10 scores)

Let A and B be sets, and let $S \subseteq A$. Let f: $A \to B$ be a function, $f|S=\{(a,b)|\ (a,b)\in f,\ a\in S\}$ is called a restriction of f on S.

(1) Suppose that f is injective. Is the restriction $f|$ S necessarily injective? Give a proof or a

counterexample.

(2) Suppose that g is surjective. Is the restriction $g \mid S$ necessarily surjective? Give a proof or a counterexample.

6 (10 scores)

(1) Prove that the set of circle of radius 3 on R^2 centered at (1,2) is equipotent as the set of that centered at the origin.

(2) Let A and B be sets, let $X \subseteq A$ be a subset and let f: $A \to B$ be a function. Suppose that f is injective. Prove that $X \sim f(X)$.

7 (10 scores)

A certain network component receives "encoded messages" from other components over very "noisy" data channels where 20% of the messages received contain errors. Assuming that the errors are independent, calculate:

(1) prob(exactly 3 out of 13 messages contain errors).

(2) prob(fewer than 3 out of 13 messages contain errors).

(3) prob(more than 3 out of 13 messages contain errors).

(4) The expected number of error-free messages in 13 transmissions.

(5) Suppose also that the component can detect errors, and when a message is received that contains an error, the component requests that the message be sent again. Calculate prob(a message is sent exactly 6 times before it is correctly received). What is the expected number of times any message must be sent?

8 (10 scores)

Let G be a graph with $n \geq 2$ vertices.

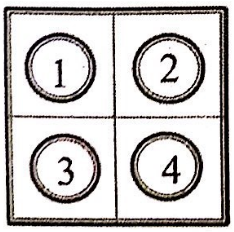
(1) Prove that if G has at least $\binom{n-1}{2}+1$ edges, then G is connected.

(2) Show that the result in (a) is best possible, that is there is a graph with $\binom{n-1}{2}$ edges that is not connected.

9 (10 scores)

Let G be a connected graph that is not Eulerian. Prove that it is possible to add a single vertex to G, together with some edges from this new vertex to some old vertices such that the new graph is Eulerian.

10 (10 scores)



Given a pattern as the left. Three movements are allowed on the pattern: exchanging the column, exchanging the row, and flipping1-4 and 2-3 simultaneously. You can make a sequence of these operations in any order, and for any finite times.

Using a commutative group model to give a sufficient and necessary condition about the sequence of operations to stop at the same pattern as the starting.

附：试题中文翻译（注：若中英版本含义有别，以英文为准。）

1（10 分）

对以下每一个命题，如果你认为正确，给出推导过程，否则说明不成立的理由：

(1) 如果苏珊喜欢吃鱼，她也会喜欢洋葱。如果苏珊不喜欢咖喱，她也不喜欢洋葱。如果她喜欢咖喱，她也会喜欢吃鸡。她喜欢鱼或者喜欢猪肉。她不喜欢吃鸡。因此她喜欢猪肉。

(2) 弗雷德不是既弹吉他又拉小提琴。如果弗雷德不弹吉他也不拉小提琴，他会既弹钢琴有吹喇叭。如果他吹喇叭，他也一定弹钢琴。因此他弹钢琴。

2（10 分）

用逻辑表达式给出以下推导过程。

(1) 硬的饼都不好吃。不硬的饼都是甜的。所以好吃的饼都是甜的。

(2) 上了艺术课的高中生都很酷。有的聪明的高中生并不酷。所以有的聪明的高中生并没上艺术课。

3（10 分）

令 A, B, 和 C 为集合。试证明：

$$A \cup B \cup C = (A - B) \cup (B - C) \cup (C - A) \cup (A \cap B \cap C)$$

4（10 分）

证明以下的关系为等价关系，并描述各自所确定的分划。描述中不应包含冗余的内容，也不要用关系的名称。

(1) 设 ~ 是定义在 R-{0} 上的关系：对任意 $x,y\in R-\{0\}$，$x\sim y$ 当且仅当 $xy>0$, for all $x,y\in R-\{0\}$，其中 R 是实数集。

(2) 设 L 是 R^2 上所有直线的集合。 定义 L 上的关系 ~ 如下：对任意 $l_1, l_2\in L$，$l_1\sim l_2$ 当且仅当 l_1 平行于 l_2 或等于 l_2。

5（10 分）

A 和 B 是集合，$S\subseteq A$. $f: A\to B$ 是函数，$f|S=\{(a,b)\mid (a,b)\in f, a\in S\}$称为 f 在 S 上的限制。

(1) 假设 f 是一对一的，是否 $f|S$ 一定是一对一的？证明或给出反例。

(2) 假设 g 是满射，是否 $g|S$ 一定是满射？证明或给出反例

6（10 分）

(1) 证明实数平面上以（1，2）为圆心半径为 3 的圆构成的集合与以原点为圆心半径为 3 的圆构成的集合等势。

(2) A 和 B 是集合，$X\subseteq A$，$f: A\to B$ 是函数。假设f 是一对一的。证明 $X\sim f(X)$。

(7)（10 分）

网络上某个站点接收另一个站点发来的“编码报文”。由于线路质量问题，接受到的报文中有 20% 含有误码。假设误码的出现相互是独立的，计算：

(1) 13 份报文中恰好有 3 个有错的概率；

(2) 13 份报文中最多有 3 个有错的概率；

(1)

13×2 11
32

(3) 13 分报文中有错的大于 3 个的概率；
(4) 13 个报文中不出错的报文个数的期望值；
(5) 假设接收站点能发现有错，一旦有错就要求重发。恰好发 6 次才正确的概率是多少？任一报文发的次数的期望值是多少？

8（10 分）

R W
3 2
3.
4

假设 G 是顶点数 $n\geq 2$ 的图，

(1) 证明：如果 G 至少有 $\binom{n-1}{2}+1$ 条边，则 G 是连通图。

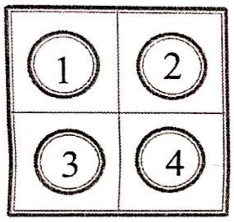
[4]

(2) 证明：上述结果是最佳下限，即存在图，恰好有 $\binom{n-1}{2}$ 条边，但不连通。

9（10 分）

假设 G 是连通图，但不是欧拉图。证明：一定可以通过增加一个顶点以及若干条将此新顶点与部分原顶点相连接的边，将原图改造成欧拉图。

10（10 分）



对于左图所描绘的格局，可以施行如下操作：两列互换，两行互换，1-4 与 2-3 两组对角的元素同时互换。你可以用任意顺序执行上述操作各任意有限多次。

对于任意满足上述条件的操作序列，给出一个结束时与开始时状态恰好完全一样的充分必要条件。建立一个合适的可交换群模型来证明你的结果。