考试科目名称 离散数学 （A卷）

考试方式：开卷 考试日期________年 ___月____日 教师______赵建华，姚远______

系（专业）__软件学院（软件工程）__ 年级____________ 班级________________

学号______________________ 姓名____________________ 成绩________________

注意：所有作答请写在答题纸上。

1. (8 points) Let p be the statement “It’s raining”, q be the statement “The field is wet”, r be the statement “The flowers need watering”. Please represent the following statements as logical formulas.
   a) It’s raining, the filed is wet, and the flowers need watering.
   b) It is not raining, the field is wet, and the flowers need watering.
   c) If it is raining and the filed is wet, the flowers need watering.
   d) If the flowers don’t need watering, then it is not raining or the field is not wet.

2. (8 points) Suppose there are 10 persons and each of them flips a coin. We know that the probability of the ‘HEAD’ outcome of the i-th person is 1/(2i+1). What is probability that the number of ‘HEAD’ outcomes is even?

3. (8 points) Let relation R be a reflexive and transitive relation on the set A. Define relation R' as x R' y if and only if x R y and y R x.
   a) Prove that R' is an equivalence relation.
   b) Let R_P be a relation on the quotient set A/R' defined as:
   [x] R_P [y] if and only if x R y
   Prove that R_P is a partial order relation on A/R'.

4. (8 points) Define relation $\mathrm{R}=\{(\mathrm{x},\mathrm{y})|y=x+1\}$ on the set of all integers. Give and prove the transitive closure of R.

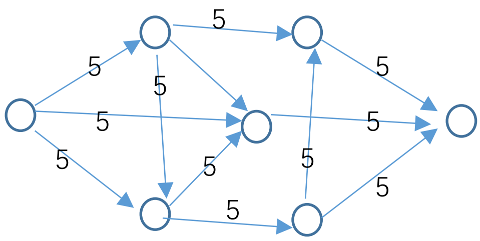
5. (8 points) Prove the following properties by mathematical induction.
   a) For any two elements a, b in a communitive group (S,*), and any positive integer n, $ab^n=b^na$.
   b) Using the above property to prove that $a^mb^n=b^na^m$ holds for any two elements a, b in S, and any two positive integers m, n.

6. (8 points) Given a sequence of m numbers, prove that there must be a continuous subsequence such that the sum of this subsequence can be divided by m.

7. (8 points) Prove that a connected graph G has a unique minimal span tree if the weights of the edges of G are mutually different with each other.

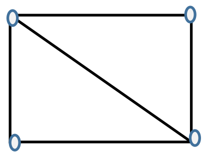
8. (10 points) A subset of set A = {1, 2, 3,... , n} is called *alternating*, if the first number is odd and odd numbers and even numbers alternatingly appear after we sort all its elements in ascending order. For example, {1} and {1,2,3,4} are alternating; {2}, {1,3,4} and {1,4,6} are not alternating. Define that the empty set is alternating. Find the number of alternating subsets of A.

9. (8 points) Given the following network:



a) Calculate the maximal flow of this network.
b) Give the minimal cut of this network.

10. (8 points) Calculate the number of different ways to color the following graph with 5 different colors such that any two adjacent vertexes have different colors. The calculation process is required.



11. (8 points) Prove that: on a 4*n Chinese chessboard, it is impossible for "horse" to traverse each grid exactly once and return to the origin.

12. (10 points) For a set S with n elements, let $A_1, A_2, \dots, A_n$ be n mutually unequal subsets of S. Prove that: there exists an element x in S such that $A_1 \cup \{x\}, A_2 \cup \{x\}, \dots, A_n \cup \{x\}$ are still n mutually unequal subsets.

中文参考

1. (8 分) 假设 p 表示“天正在下雨”，q 表示“地上是湿的”，r 表示“花需要浇水”，请用逻辑公式表示下列命题:
   a) 天正在下雨，地上不是湿的，并且花需要浇水
   b) 天不在下雨，地上是湿的，花需要浇水
   c) 如果天在下雨并且地上是湿的，那么花不需要浇水
   d) 如果花需要浇水，那么天不在下雨或者地上不是湿的

2. (8 分) 假设第 i 个人抛硬币正面向上的概率是 1/(2i+1)。10 个人抛硬币，正面向上的个数是偶数的概率是多少?

3. (8 分) 假设集合 A 上的关系 R 是自反的和传递的。定义 A 上的关系 R'如下:

   $x\ R'\ y$ 当且仅当 $x\ R\ y$ 且 $y\ R\ x$。

   a) 请证明 R'是一个等价关系。
   b) A 的商集 A/R'上的关系 R_P 定义如下:

   $[x]\ R_P\ [y]$ 当且仅当 $x\ R\ y$

   请证明 R_P 是 A/R'上的偏序关系。

4. (8 分) 定义整数集上的关系$\mathrm{R} = \{(\mathrm{x}, \mathrm{y}) | y = x + 1\}$，请给出 R 的传递闭包并证明之。

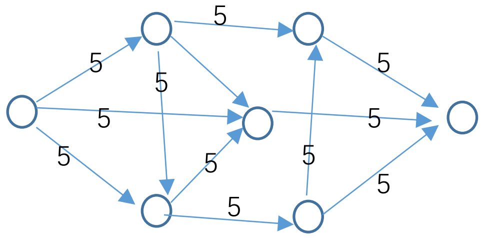
5. (8分) 使用数学归纳法证明下列性质:
   a) 可交换群(S,*)的元素a和b，对于任意的正整数n，都有$ab^n=b^na$。
   b) 利用这个性质证明对于S中的任意元素a,b和任意正整数m,n，$a^mb^n=b^na^m$。

6. (8分) 给定m个数组成的序列，请证明一定能够从该序列中选出一个连续子序列，使得这个子序列的和能够被m整除。

7. (8分) 请证明如果图G中各条边的权重各不相同，那么G的最小生成树是唯一的。

8. (10分) 集合$A = \{1, 2, 3, \ldots, n\}$的某个子集称为是交替的，如果其元素按照升序排列时是奇数、偶数交替出现的，且第一个数是奇数。例如$\{1, 2, 3, 4\}$是交替的，$\{1, 3, 4\}$与$\{1, 4, 6\}$不是交替的。规定空集是交替的。求A的交替子集的个数。

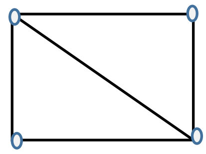
9. (8分) 已知网络如下:



a）请计算出这个图的最大流。

b）给出这个图的最小割。

10. (8分) 要使用5种颜色对下图中顶点进行染色并使得相邻顶点的颜色不同。请问总共有多少种染色方法。请给出演算过程。



11. (8分) 试证明：在4*n的中国象棋棋盘上，“马”不可能不重复的遍历每一个格子并回到原点。

12. (10分) 对于一个含有n个元素的集合S，令$A_1, A_2, \ldots, A_n$为S的n个互不相等的子集。试证明：存在S的元素x，使得$A_1 \cup \{x\}, A_2 \cup \{x\}, \ldots, A_n \cup \{x\}$依然是n个互不相等的子集。