南京大学
人工智能学院
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02 系统的时域分析

系统的描述以及卷积算子

线性系统的判定

- 判断以下系统是否是线性系统
- 1. $y(t) = t^2 x(t)$
- 2. $y(t) = 3x(t) + 4$
- 3. $y(t) = 4\frac{\mathrm{d}x(t)}{\mathrm{d}t}$

线性系统的判定

- 判断以下系统是否是线性系统
- 1. $y(t) = t^2 x(t)$
  - 齐次性： $x_1(t) \rightarrow t^2 x_1(t)$，$Kx_1(t) \rightarrow t^2 K x_1(t)$
  - 可加性

$$x_1(t) \rightarrow t^2 x_1(t), \qquad x_2(t) \rightarrow t^2 x_2(t)$$

$$x_1(t) + x_2(t) \rightarrow t^2 [x_1(t) + x_2(t)]$$

是线性系统

线性系统的判定

- 判断以下系统是否是线性系统
- 2. $y(t) = 3x(t) + 4$

$$x_1(t) \to 3x_1(t) + 4$$

$$Kx_1(t) \to 3Kx_1(t) + 4$$

不满足齐次性

线性系统的判定

- 判断以下系统是否是线性系统
- 3. $y(t)=4\frac{\mathrm{d}x(t)}{\mathrm{d}t}$
  - 齐次性：$x_1(t)\to 4\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}$，$Kx_1(t)\to 4\frac{\mathrm{d}Kx_1(t)}{\mathrm{d}t}=4K\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}$
  - 可加性

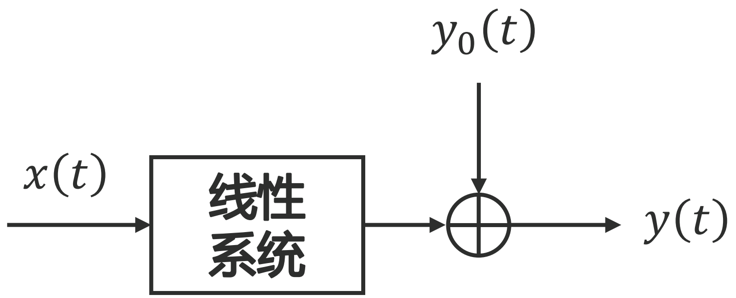
$$x_1(t)\to 4\frac{\mathrm{d}x_1(t)}{\mathrm{d}t},\quad x_2(t)\to 4\frac{\mathrm{d}x_2(t)}{\mathrm{d}t}$$

$$x_1(t)+x_2(t)\to 4\frac{\mathrm{d}[x_1(t)+x_2(t)]}{\mathrm{d}t}=4\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}+4\frac{\mathrm{d}x_2(t)}{\mathrm{d}t}$$

是线性系统

线性系统的判定

- 系统$y(t) = 3x(t) + 4$不是线性系统
  - 齐次性：$x(t) \rightarrow 3x(t) + 4;\ Kx(t) \rightarrow 3Kx(t) + 4$

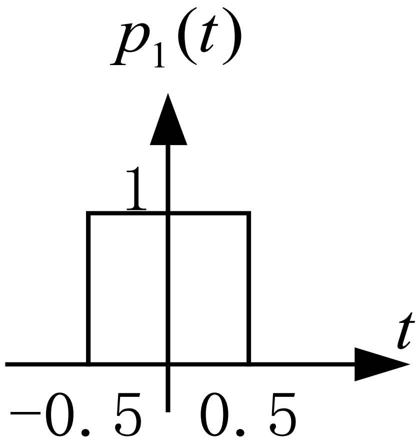


  - **增量**线性系统：系统可表示为一个线性系统的输出与一个等于该系统**零输入响应**（zero-input response）的信号$y_0(t) = 4$之和

卷积和计算

- 已知$R_K[n] = \begin{cases} 1, 0 \leq n \leq K-1 \\ 0, \quad\quad\quad\quad o.w. \end{cases}$, 计算$y[n] = R_N[n] * R_N[n]$

- 计算 $y(t) = p_1(t) * p_1(t)$



卷积和计算

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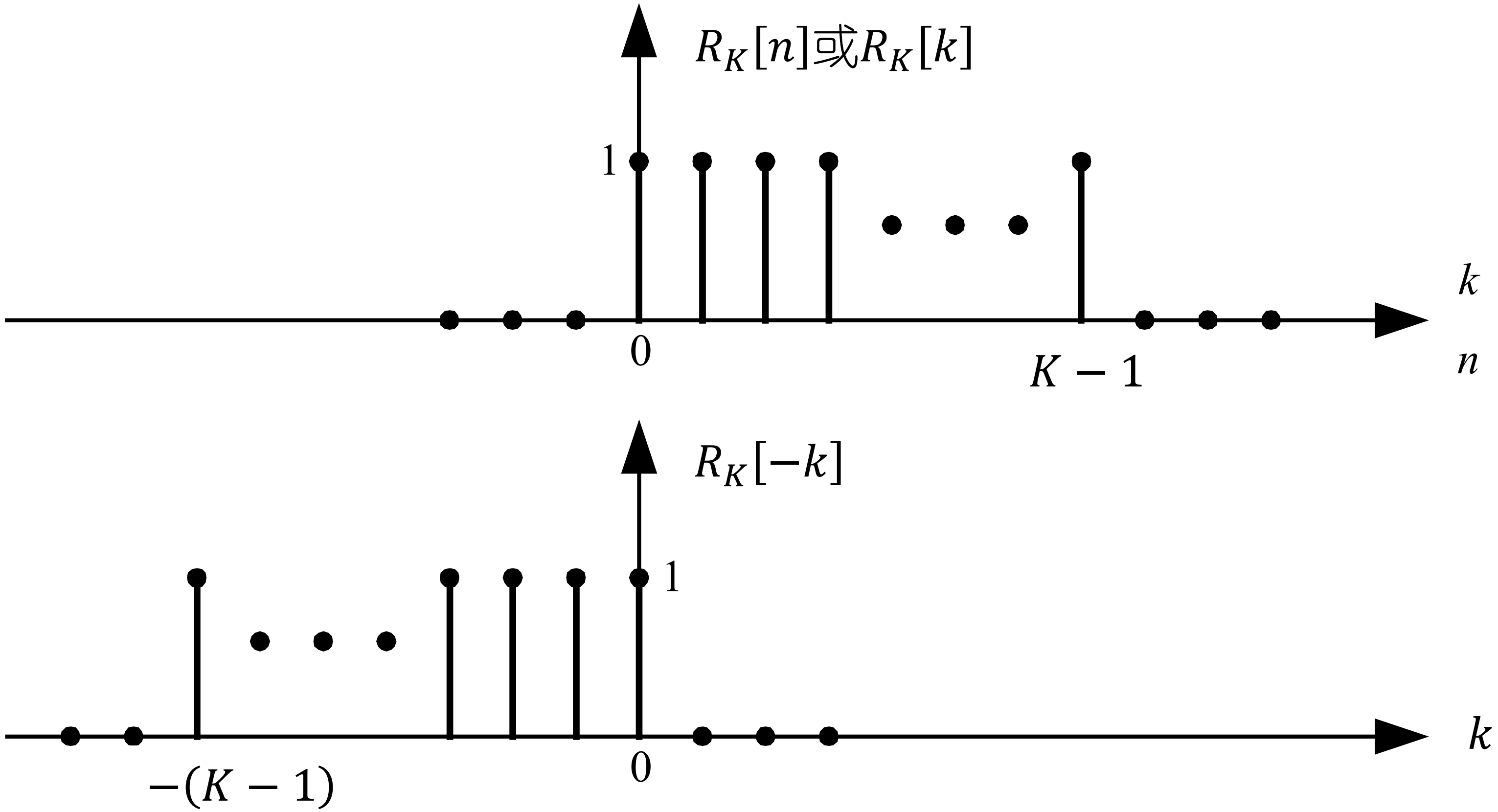
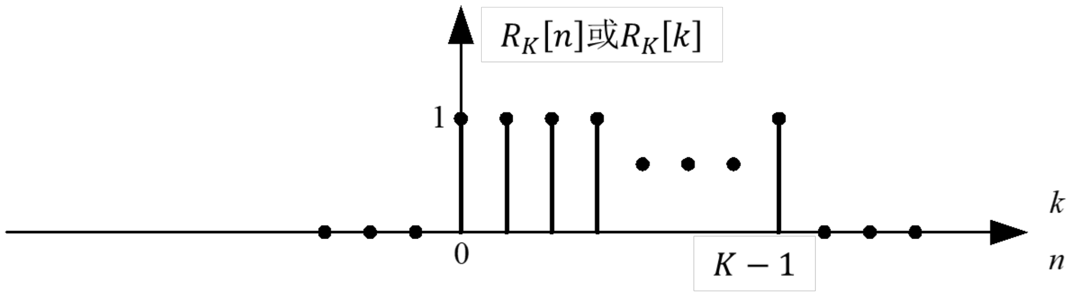
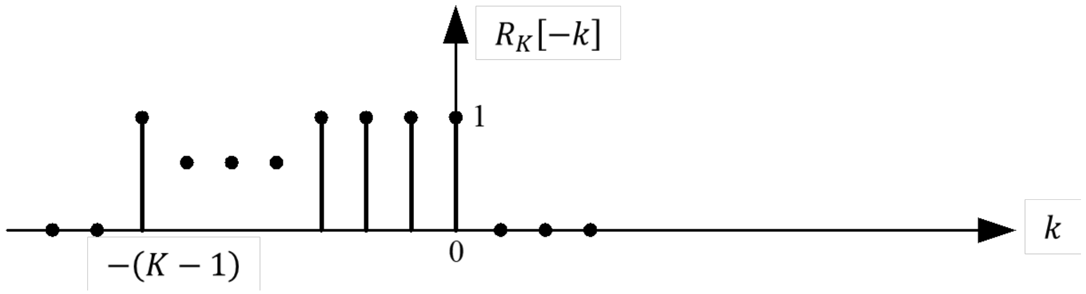


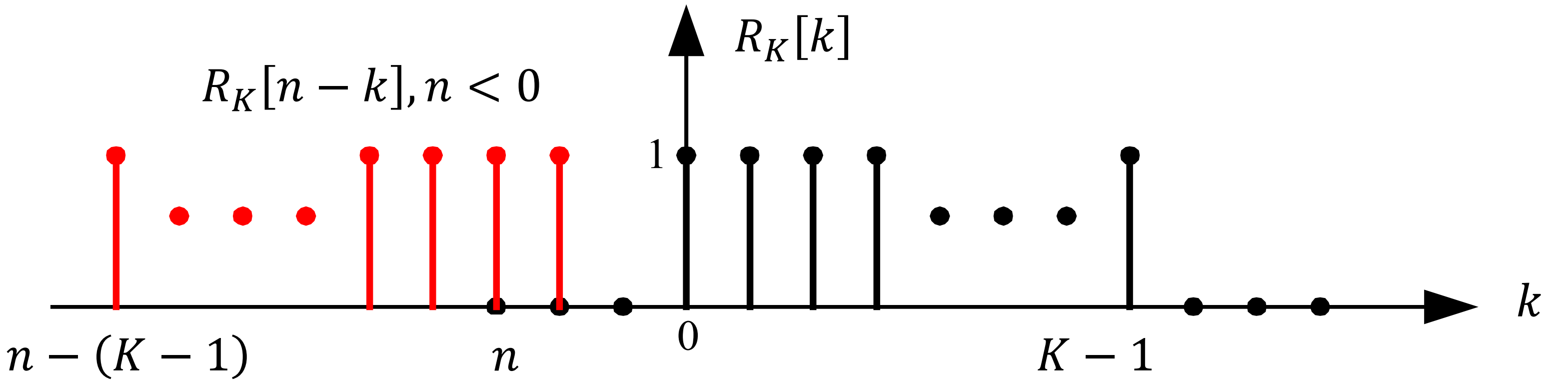
卷积和计算

- 已知$R_K[n]=\begin{cases}1, 0\le n\le K-1\\ 0, \qquad\qquad o.w.\end{cases}$，计算$y[n]=R_N[n]*R_N[n]$



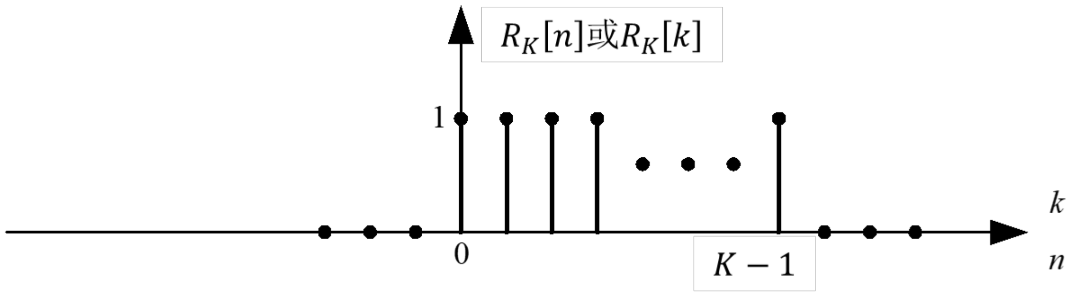


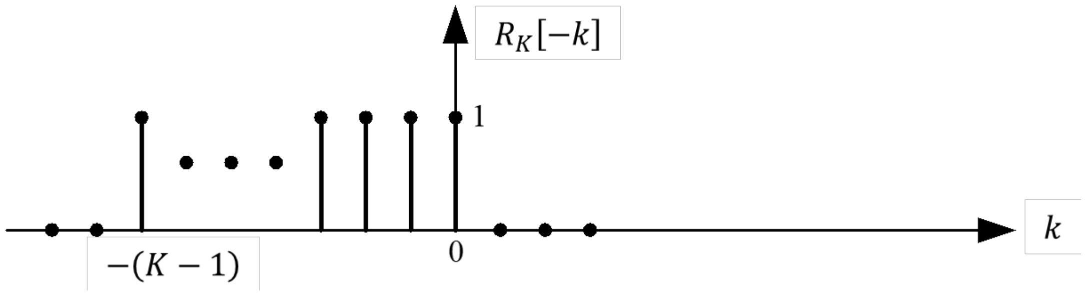
  - $n<0$时，$R_K[k]$与$R_K[n-k]$图形没有相遇，$y[n]=0$



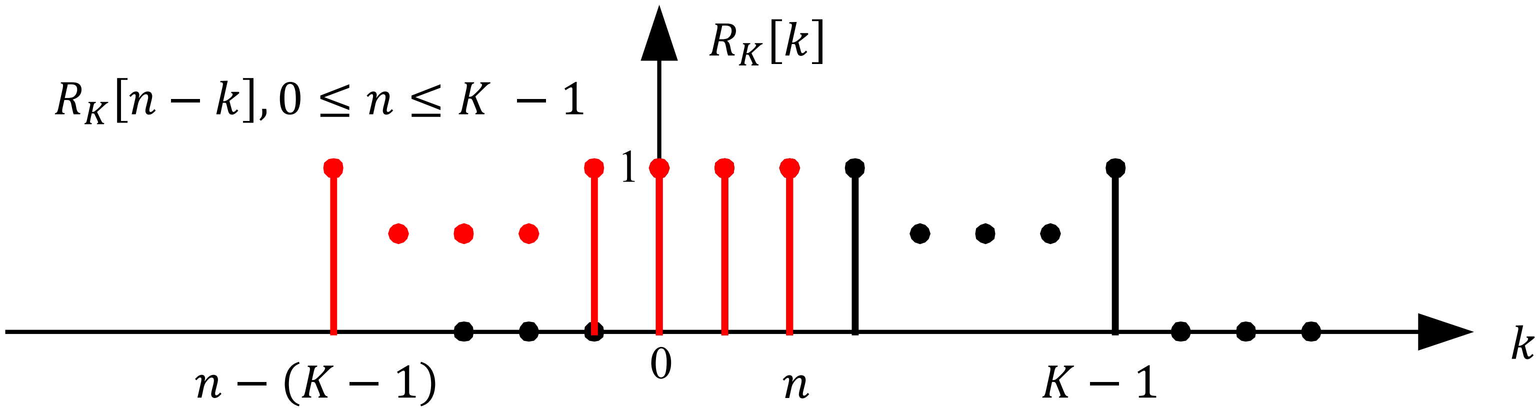
卷积和计算

- 已知$R_K[n]=\begin{cases}1, 0\le n\le K-1\\ 0, \quad\quad\quad o.w.\end{cases}$, 计算$y[n]=R_N[n]*R_N[n]$



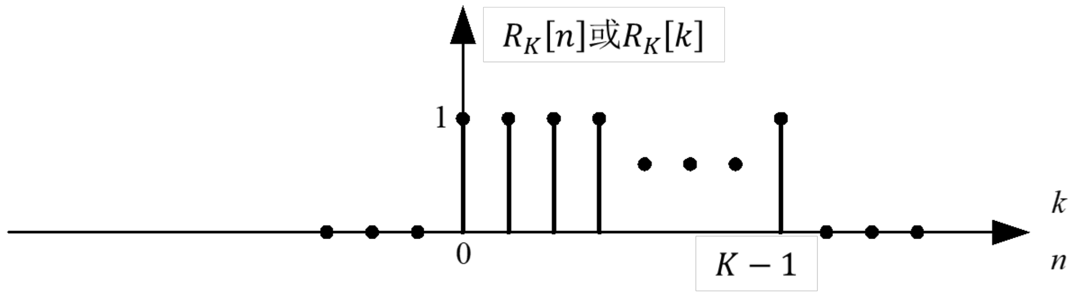


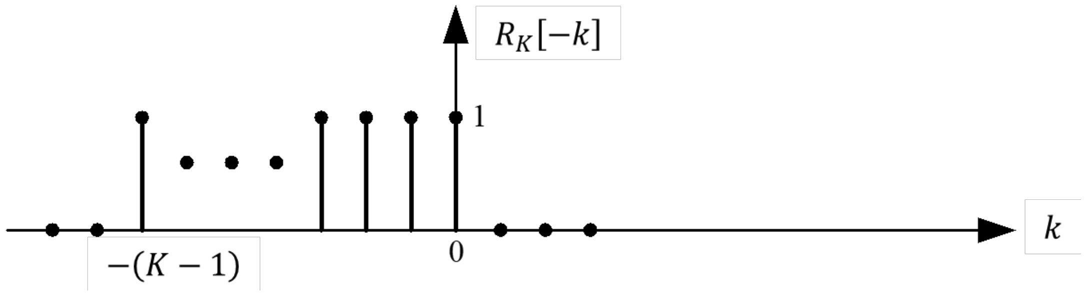
  - $0\le n\le K-1$时, $RK\ [k]$与$R_K\ [n-k]$图形重合区间为$[0,n]$, $y[n]=\sum_{k=0}^{n}1=n+1$



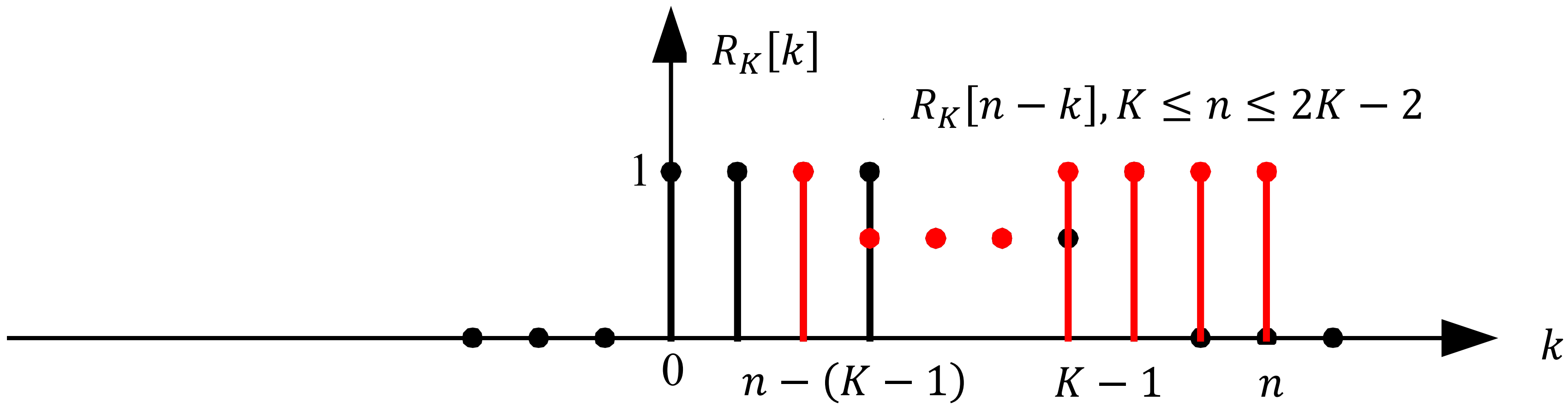
卷积和计算

- 已知$R_K[n]=\begin{cases}1, 0\le n\le K-1\\ 0, \qquad\qquad o.w.\end{cases}$, 计算$y[n]=R_N[n]*R_N[n]$



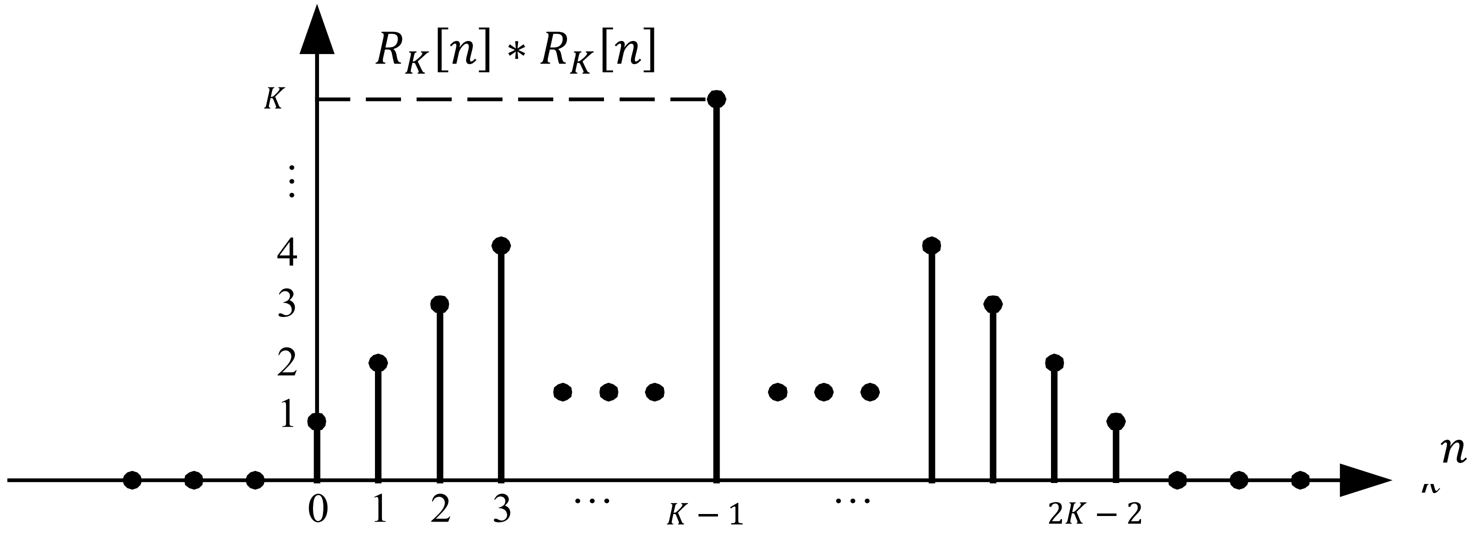


  - $K\le n\le 2K-2$时, $R_K[k]$与$R_K[n-k]$图形重合区间为$[n-(K-1), K-1]$, $y[n]=\sum_{k=n-(K-1)}^{K-1}1=2K-1-n$



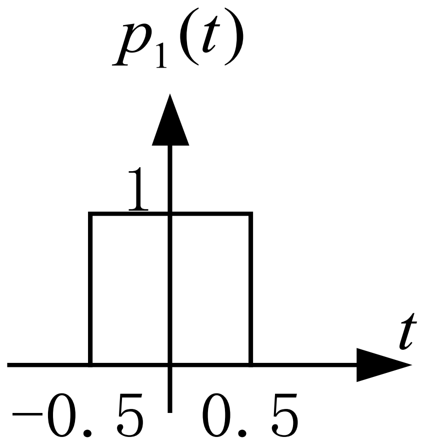
卷积和计算

- 已知$R_K[n]=\begin{cases}1, 0\le n\le K-1\\ 0, \quad\quad\quad\quad o.w.\end{cases}$, 计算$y[n]=R_N[n]*R_N[n]$
  - $n\ <\ 0$时, $R_K\ [k]$与$R_K\ [n-k]$图形没有相遇, $y[n]=0$
  - $0\le n\le K\ -1$时, $R_K\ [k]$与$R_K\ [n-k]$图形重合区间为$[0,n]$, $y[n]=\sum_{k=0}^{n}1=n+1$
  - $K\le n\le 2K-2$时, $R_K\ [k]$与$R_K\ [n-k]$图形重合区间为$[n-(K-1),K-1]$, $y[n]=\sum_{k=n-(K-1)}^{K-1}1=2K-1-n$
  - $n>2K-2$时, $R_K\ [k]$与$R_K\ [n-k]$图形没有相遇, $y[n]=0$

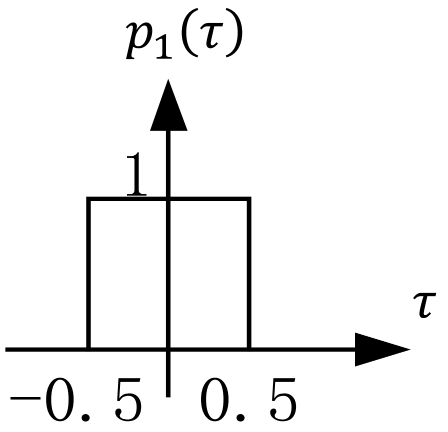


卷积计算

- 计算 $y(t) = p_1(t) * p_1(t)$



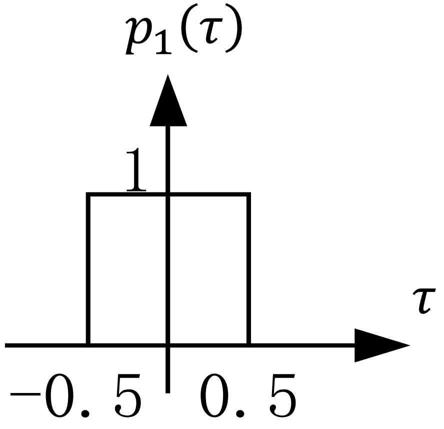
  - 将信号的自变量由t改为τ

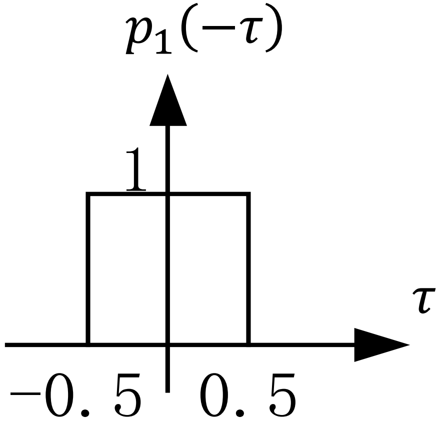


  - 将$h(\tau)$翻转得$h(-\tau) = h(\tau)$

卷积计算

- 计算 $y(t) = p_1(t) * p_1(t)$

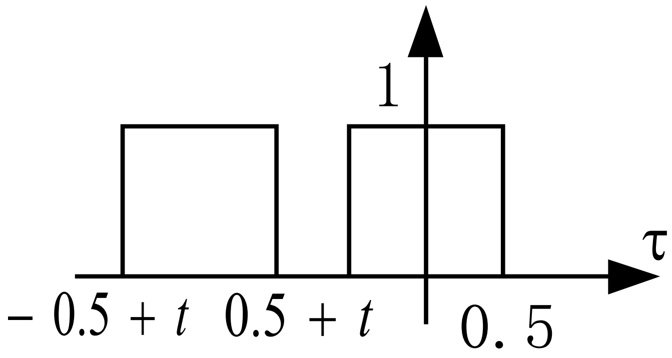




  - $-\infty < t \leq -1, y(t) = 0$

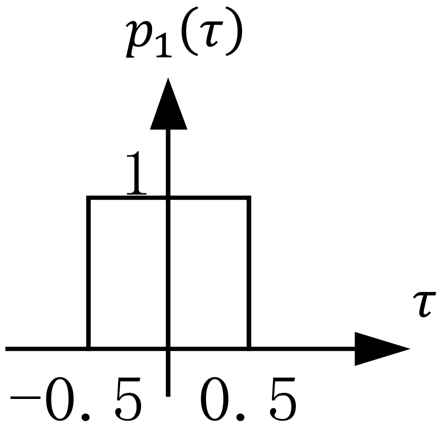
$-\infty < t < -1$

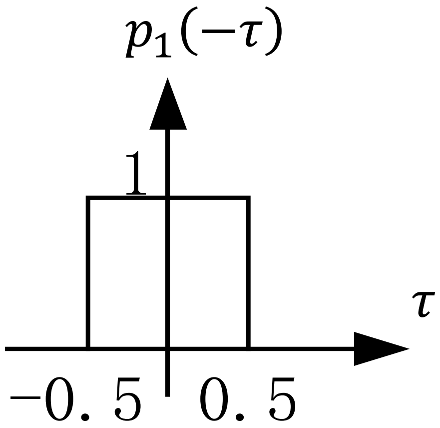
$p_1(\tau)p_1(t-\tau)$



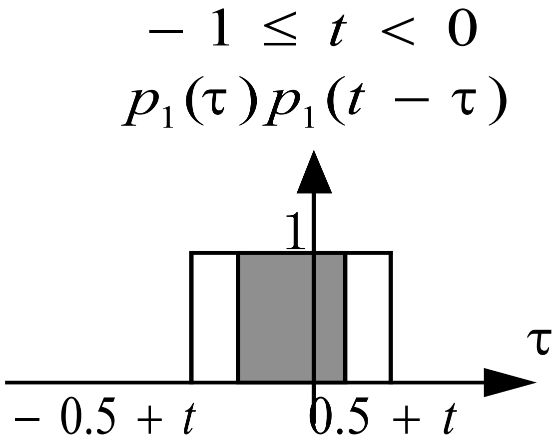
卷积计算

- 计算 $y(t) = p_1(t) * p_1(t)$



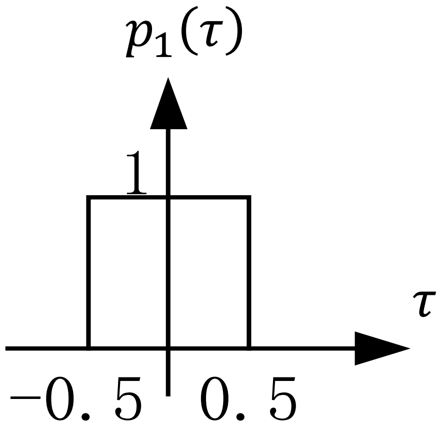


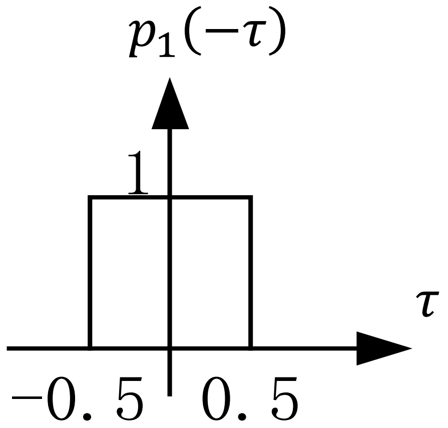
- $-1 < t \leq 0,\ y(t) = \int_{-0.5}^{0.5+t} \mathrm{d}t = 1 + t$



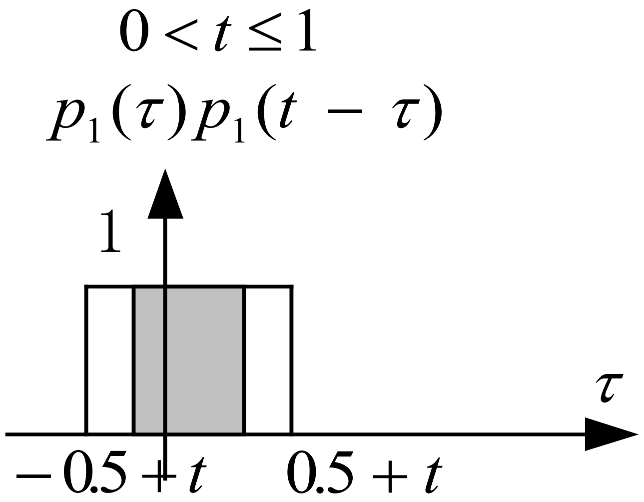
卷积计算

- 计算 $y(t) = p_1(t) * p_1(t)$



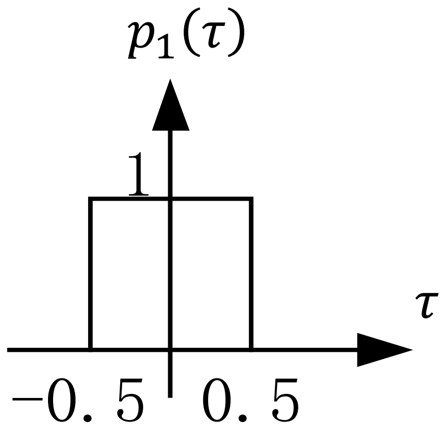


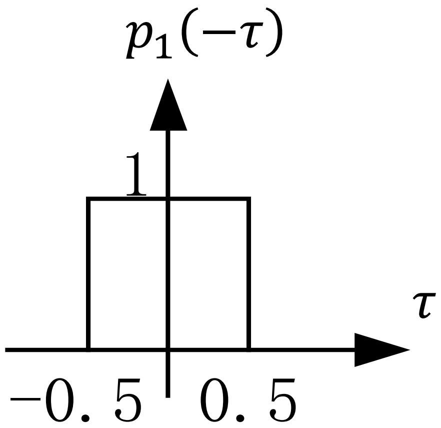
- $0 < t \leq 1, y(t) = \int_{-0.5+t}^{0.5} \mathrm{d}t = 1 - t$



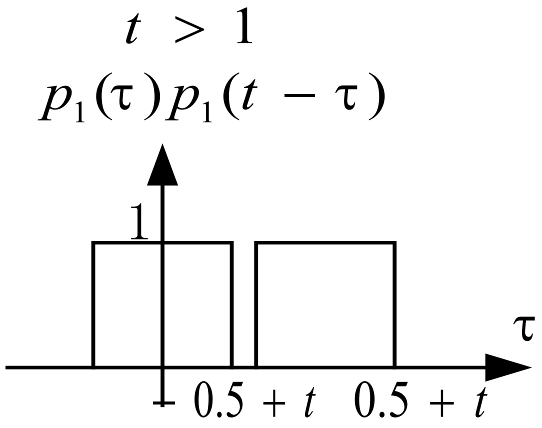
卷积计算

- 计算 $y(t) = p_1(t) * p_1(t)$



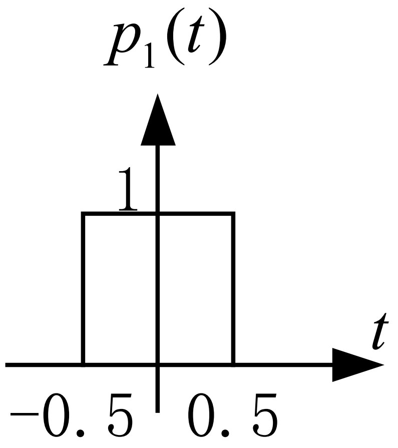


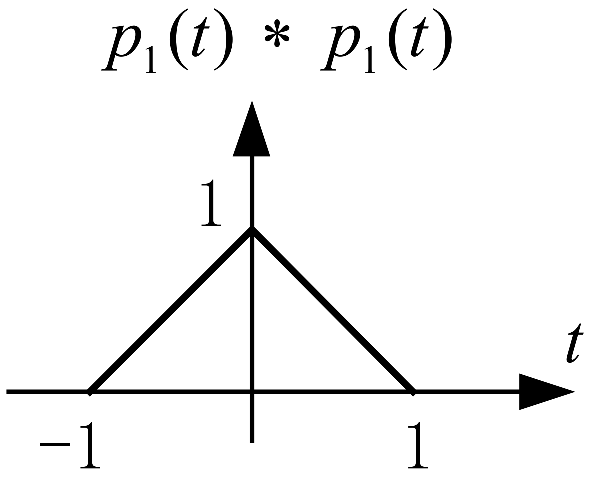
- $t > 1, y(t) = 0$



卷积计算

- 计算 $y(t) = p_1(t) * p_1(t)$
  - $-\infty < t \leq -1, y(t) = 0$
  - $-1 < t \leq 0, y(t) = \int_{-0.5}^{0.5+t} \mathrm{d}t = 1 + t$
  - $0 < t \leq 1, y(t) = \int_{-0.5+t}^{0.5} \mathrm{d}t = 1 - t$
  - $t > 1, y(t) = 0$





利用卷积特性简化卷积运算

- 已知 $y(t) = x_1(t) * x_2(t)$，求$y'(t)$和 $y^{(-1)}(t)$
  - 利用卷积的微分特性，结合律

$$y'(t) = y(t) * \delta'(t) = [x_1(t) * x_2(t)] * \delta'(t)$$
$$= x_1'(t) * x_2(t) = x_1(t) * x_2'(t)$$

  - 利用卷积的积分特性，结合律

$$y^{(-1)}(t) = y(t) * u(t) = [x_1(t) * x_2(t)] * u(t)$$
$$= x_1^{(-1)}(t) * x_2(t) = x_1(t) * x_2^{(-1)}(t)$$

利用卷积特性简化卷积运算

- 计算$2e^{-2t}u(t) * 3e^{-t}u(t)$

- 计算$2e^{-2(t-1)}u(t-1) * 3e^{-(t-2)}u(t-2)$

- 计算$2e^{-2t}u(t-1) * 3e^{-t}u(t-2)$

利用卷积特性简化卷积运算

- 计算$2e^{-2t}u(t) * 3e^{-t}u(t)$

$$2e^{-2t}u(t) * 3e^{-t}u(t)$$

$$= \int_{-\infty}^{\infty} 2e^{-2\tau}u(\tau) * 3e^{-(t-\tau)}u(t-\tau)\,\mathrm{d}\tau$$

$$= \begin{cases} 6\int_0^t e^{-2\tau}e^{-(t-\tau)}\mathrm{d}\tau \\ 0, t < 0 \end{cases} = 6\left(e^{-t} - e^{-2t}\right)u(t)$$

利用卷积特性简化卷积运算

- 计算$2e^{-2(t-1)}u(t-1) * 3e^{-(t-2)}u(t-2)$
  - 利用平移性质

$$2e^{-2(t-1)}u(t-1) * 3e^{-(t-2)}u(t-2)$$
$$= 6\left(e^{-(t-3)} - e^{-2(t-3)}\right)u(t-3)$$

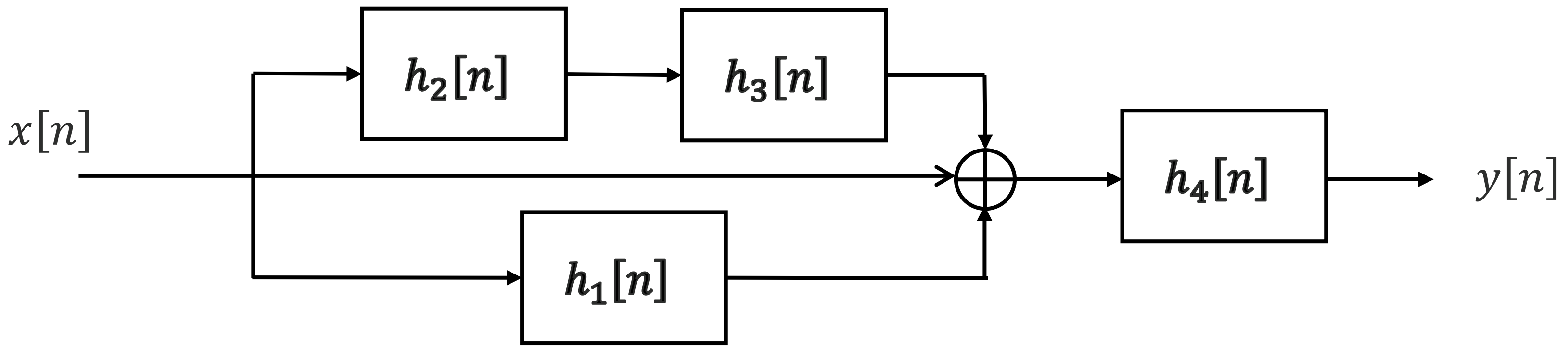
- 计算$2e^{-2t}u(t-1) * 3e^{-t}u(t-2)$

$$2e^{-2t}u(t-1) * 3e^{-t}u(t-2)$$
$$= 2e^{-2}e^{-2(t-1)}u(t-1) * 3e^{-2}e^{-(t-2)}u(t-2)$$
$$= 2e^{-4}e^{-2(t-1)}u(t-1) * 3e^{-(t-2)}u(t-2)$$
$$= 6e^{-4}\left(e^{-(t-3)} - e^{-2(t-3)}\right)u(t-3)$$

利用卷积分析系统

求图示系统的单位脉冲响应，

其中$h_1[n] = 2^n u[n]$，$h_2[n] = \delta[n-1]$，$h_3[n] = 3^n u[n]$， $h_4[n] = u[n]$



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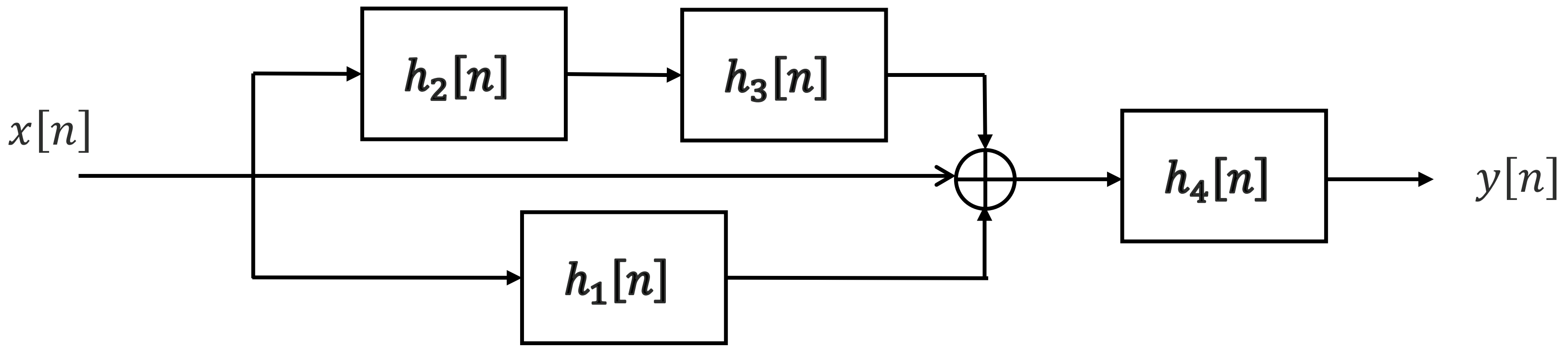

- 子系统$h_2[n]$与$h_3[n]$ 级联，$h_1[n]$支路、全通支路与$h_2[n]$ $h_3[n]$ 级联支路并联，再与$h_4[n]$级联

$$h[n] = \{h_1[n] + \delta[n] + h_2[n] * h_3[n]\} * h_4[n]$$

$$= (2(2)^n - 1)u[n] + u[n] + \left[1.5(3)^{n-1} - 0.5\right]u[n-1]$$