

# Semantic Theory

## Lecture 2 – Predicate Logic

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# Information about this course

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## Recommended literature:

- Gamut: Logic, Language, and Meaning, Vol. 2, University of Chicago Press, 1991
- Kamp and Reyle: From Discourse to Logic, Kluwer, 1993

## Final exam:

- Exam date to be confirmed

# Part I:

## Sentence semantics



# Sentence meaning

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## Truth-conditional semantics:

to know the meaning of a (declarative) sentence is to know what the world would have to be like for the sentence to be true:

Sentence meaning = truth-conditions

## Indirect interpretation:

1. Translate sentences into logical formulas:

Every student works  $\mapsto \forall x(\text{student}'(x) \rightarrow \text{work}'(x))$

2. Interpret these formulas in a logical model:

$\llbracket \forall x(\text{student}'(x) \rightarrow \text{work}'(x)) \rrbracket^{M,g} = 1$  iff  $V_M(\text{student}') \subseteq V_M(\text{work}')$

# Step 1: Translation

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Limits of propositional logic: propositions with internal structure

Every man is mortal.

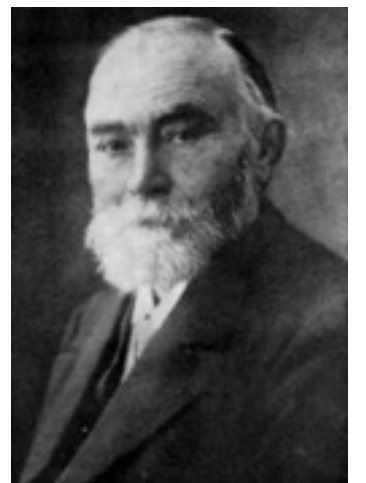
Socrates is a man.

Therefore, Socrates is mortal.

Solution: first-order predicate logic

predicates are expressions  
that contain *arguments*  
(that can be quantified over)

predication & quantification  
over *individuals*



Gottlob Frege

# Predicate Logic: Vocabulary

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## Non-logical expressions:

Individual constants: CON

n-place relation constants:  $\text{PRED}^n$ , for all  $n \geq 0$

Infinite set of individual variables: VAR

Logical connectives:  $\wedge, \vee, \neg, \rightarrow, \leftrightarrow, \forall, \exists$

Brackets: (, )

# Predicate Logic: Syntax

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Terms:  $\text{TERM} = \text{VAR} \cup \text{CON}$

Atomic formulas:

- $R(t_1, \dots, t_n)$  for  $R \in \text{PRED}^n$  and  $t_1, \dots, t_n \in \text{TERM}$
- $t_1 = t_2$  for  $t_1, t_2 \in \text{TERM}$

Well-formed formula (WFF):

1. All atomic formulas are WFFs;
2. If  $\phi$  and  $\psi$  are WFFs, then  $\neg\phi$ ,  $(\phi \wedge \psi)$ ,  $(\phi \vee \psi)$ ,  $(\phi \rightarrow \psi)$ ,  $(\phi \leftrightarrow \psi)$  are WFFs;
3. If  $x \in \text{VAR}$ , and  $\phi$  is a WFF, then  $\forall x\phi$  and  $\exists x\phi$  are WFFs;
4. Nothing else is a WFF.

# Variable binding

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- Given a quantified formula  $\forall x\phi$  (or  $\exists x\phi$ ), we say that  $\phi$  (and every part of  $\phi$ ) is in the **scope** of the quantifier  $\forall x$  (or  $\exists x$ );
- A variable  $x$  is **bound** in formula  $\psi$  if  $x$  occurs in the scope of  $\forall x$  or  $\exists x$  in  $\psi$ ;
- If a variable is not bound in formula  $\psi$ , it occurs **free** in  $\psi$ ;
- A **closed formula** is a formula without free variables.



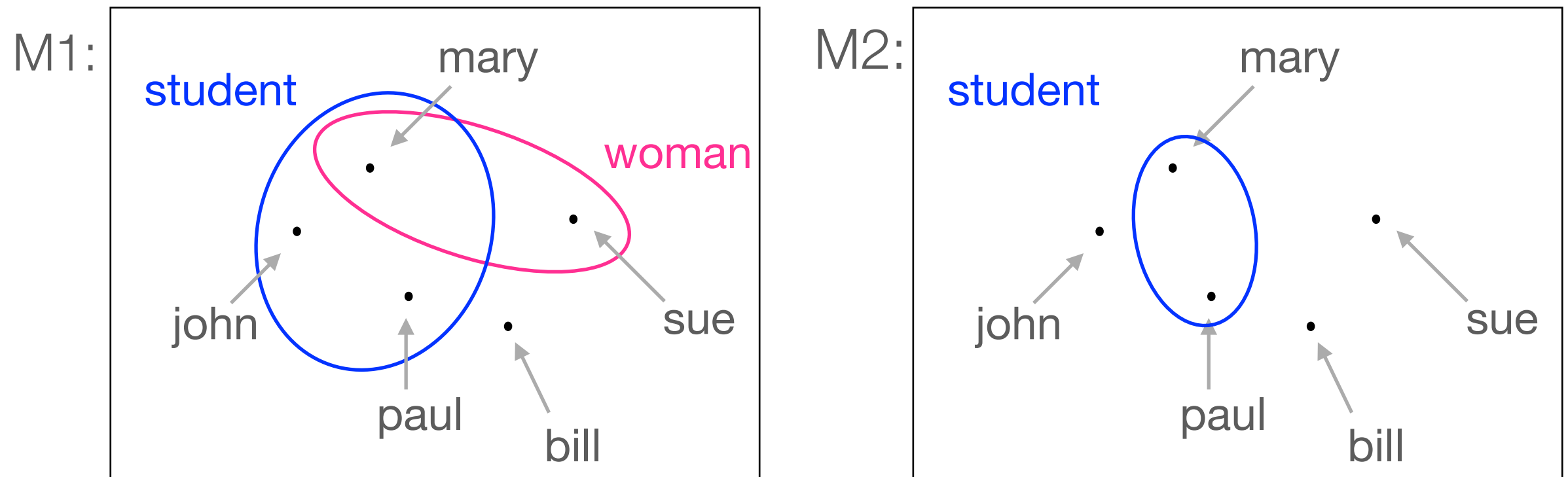
# Formalizing Natural Language

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1. *Bill loves Mary.*
2. *Bill reads an interesting book.*
3. *Every student reads a book.*
4. *Bill passed every exam.*
5. *Not every student answered every question.*
6. *Only Mary answered every question.*
7. *Mary is annoyed when someone is noisy.*
8. *Although nobody makes noise, Mary is annoyed.*

## Step 2: Interpretation

Logical models are simplified representations of the state of affairs in the world



*John is a student* : for any  $M$ ,  $\llbracket \text{student}'(\text{john}) \rrbracket^M = 1$  iff  $V_M(\text{john}) \in V_M(\text{student}')$

$V_{M1}(\text{john}) \in V_{M1}(\text{student}')$  therefore:  $\llbracket \text{student}'(\text{john}) \rrbracket^{M1} = 1$

$V_{M2}(\text{john}) \notin V_{M2}(\text{student}')$  therefore:  $\llbracket \text{student}'(\text{john}) \rrbracket^{M2} = 0$

# A formal description of a model

Model  $M = \langle U_M, V_M \rangle$ , with:

- $U_M$  is the universe of  $M$  and
- $V_M$  is an interpretation function

$U_M = \{e_1, e_2, e_3, e_4, e_5\}$  **universe**

$V_M(\text{john}) = e_1$

...

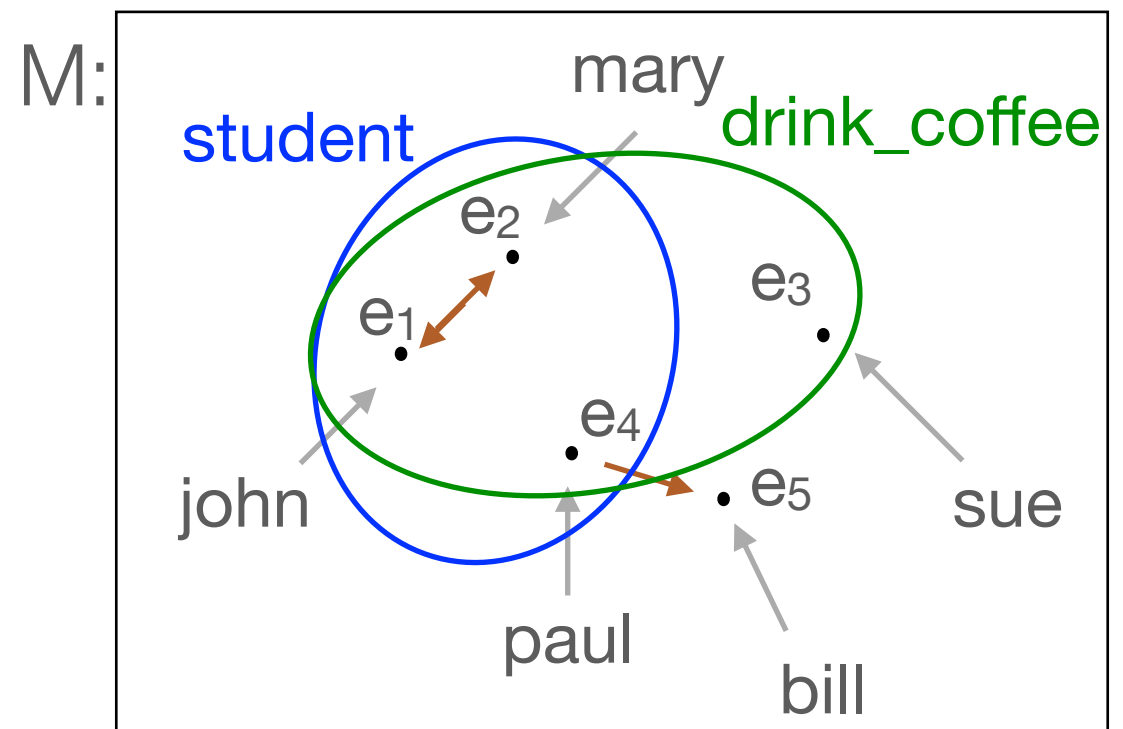
**constants**

$V_M(\text{bill}) = e_5$

$V_M(\text{student}) = \{e_1, e_2, e_4\}$

$V_M(\text{drink\_coffee}) = \{e_1, e_2, e_3, e_4\}$

$V_M(\text{love}) = \{\langle e_1, e_2 \rangle, \langle e_2, e_1 \rangle, \langle e_4, e_5 \rangle\}$



**1-place predicates**

**2-place predicates**

# Interpretation in the model

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$V_M$  is an interpretation function assigning individuals ( $\in U_M$ ) to individual constants and  $n$ -ary relations over  $U_M$  to  $n$ -place predicate symbols:

- $V_M(c) \in U_M$       if  $c$  is an individual constant
- $V_M(P) \subseteq U_M^n$       if  $P$  is an  $n$ -place predicate symbol
- $V_M(P) \in \{0,1\}$       if  $P$  is an 0-place predicate symbol

# Variables and quantifiers

How to interpret the following sentence in our model M:

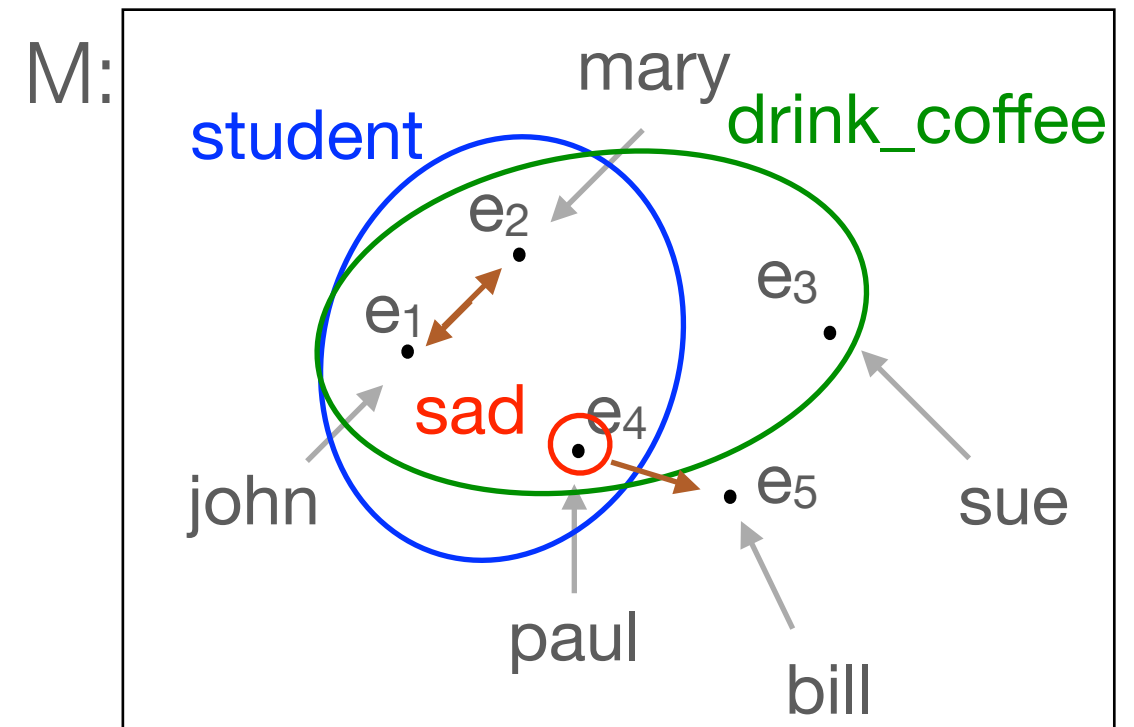
- Someone is sad  $\mapsto \exists x(\text{sad}'(x))$

Intuition:

- find an entity in the universe for which the statement holds:  $V_M(\text{sad}') = e_4$
- replace  $x$  by  $e_4$  in order to make  $\exists x(\text{sad}'(x))$  true

More formally:

- Interpret sentence relative to *assignment function*  $g$ : i.e.,  $\llbracket \exists x(\text{sad}'(x)) \rrbracket^{M,g}$ , such that  $g(x) = e_4$ ; this can be generalised to any  $g'$  as follows:  $g'[x/e_4](x) = e_4$



# Assignment functions

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An assignment function  $g$  assigns values to all variables

- $g :: \text{VAR} \rightarrow U_M$
- We write  $g[x/d]$  for the assignment function  $g'$  that assigns  $d$  to  $x$  and assigns the same values as  $g$  to all other variables.

	$x$	$y$	$z$	$u$	$\dots$
$g$	$e_1$	$e_2$	$e_3$	$e_4$	$\dots$
$g[y/e_1]$	$e_1$	$e_1$	$e_3$	$e_4$	$\dots$
$g[x/e_1]$	$e_1$	$e_2$	$e_3$	$e_4$	$\dots$
$g[y/g(z)]$	$e_1$	$e_3$	$e_3$	$e_4$	$\dots$
$g[y/e_1][u/e_1]$	$e_1$	$e_1$	$e_3$	$e_1$	$\dots$
$g[y/e_1][y/e_2]$	$e_1$	$e_2$	$e_3$	$e_4$	$\dots$

# Interpretation of terms

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Interpretation of terms with respect to a model  $M$  and a variable assignment  $g$ :

$$\begin{aligned} \llbracket \alpha \rrbracket^{M,g} = & \quad V_M(\alpha) \quad \text{if } \alpha \text{ is an individual constant} \\ & g(\alpha) \quad \text{if } \alpha \text{ is a variable} \end{aligned}$$

# Interpretation of formulas

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Interpretation of formulas with respect to a model  $M$  and variable assignment  $g$ :

- $\llbracket R(t_1, \dots, t_n) \rrbracket^{M,g} = 1$       iff       $\langle \llbracket t_1 \rrbracket^{M,g}, \dots, \llbracket t_n \rrbracket^{M,g} \rangle \in V_M(R)$
- $\llbracket t_1 = t_2 \rrbracket^{M,g} = 1$       iff       $\llbracket t_1 \rrbracket^{M,g} = \llbracket t_2 \rrbracket^{M,g}$
- $\llbracket \neg \phi \rrbracket^{M,g} = 1$       iff       $\llbracket \phi \rrbracket^{M,g} = 0$
- $\llbracket \phi \wedge \psi \rrbracket^{M,g} = 1$       iff       $\llbracket \phi \rrbracket^{M,g} = 1$  and  $\llbracket \psi \rrbracket^{M,g} = 1$
- $\llbracket \phi \vee \psi \rrbracket^{M,g} = 1$       iff       $\llbracket \phi \rrbracket^{M,g} = 1$  or  $\llbracket \psi \rrbracket^{M,g} = 1$
- $\llbracket \phi \rightarrow \psi \rrbracket^{M,g} = 1$       iff       $\llbracket \phi \rrbracket^{M,g} = 0$  or  $\llbracket \psi \rrbracket^{M,g} = 1$
- $\llbracket \phi \leftrightarrow \psi \rrbracket^{M,g} = 1$       iff       $\llbracket \phi \rrbracket^{M,g} = \llbracket \psi \rrbracket^{M,g}$
- $\llbracket \exists x \phi \rrbracket^{M,g} = 1$       iff      there is a  $d \in U_M$  such that  $\llbracket \phi \rrbracket^{M,g[x/d]} = 1$
- $\llbracket \forall x \phi \rrbracket^{M,g} = 1$       iff      for all  $d \in U_M$ ,  $\llbracket \phi \rrbracket^{M,g[x/d]} = 1$



# Background reading material

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- Gamut: Logic, Language, and Meaning Vol I/II — Chapter 2
- For a more basic introduction, see:  
<http://www.logicinaction.org> — Chapter 4