Semantic Theory 2016: Solutions exercise sheet 3

Exercise 1

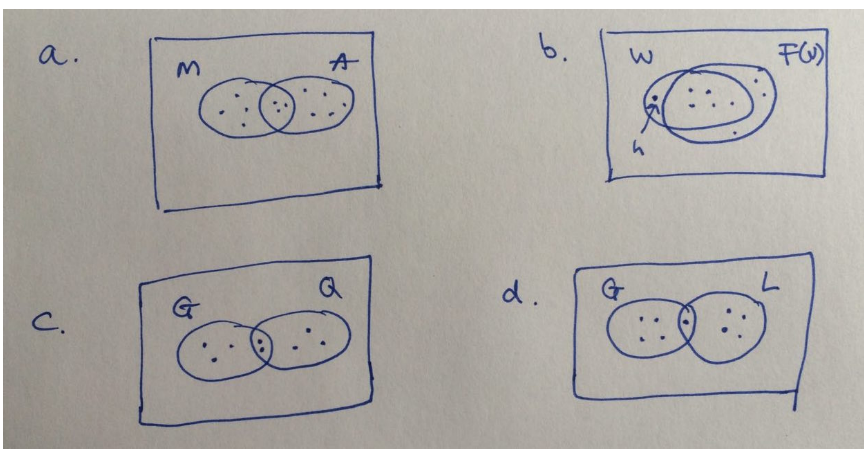
Give the truth conditions for the following sentences, interpreting the (complex) determiner as a relation between sets. You can interpret each VP as a property (i.e., a set of entities). Illustrate your answer with a graphical representation of a model in which the sentence is true.

a. $[\![$ Some but not all Muggles are afraid of magic $]\!] = 1$ iff
$[\![A]\!] \in \{P \subseteq U_M \mid [\![M]\!] \cap P \neq [\![M]\!] \textit{ and } [\![M]\!] \cap P \neq \emptyset\}$ iff
$[\![A]\!] \in \{P \subseteq U_M \mid [\![M]\!] \cap P \subset [\![M]\!]\}$ iff
$[\![M]\!] \cap [\![A]\!] \subset [\![M]\!]$

b. $[\![$ Every wizard but Harry fears Voldemort $]\!] = 1$ iff
$[\![F(v)]\!] \in \{P \subseteq U_M \mid [\![W]\!] \cap P = [\![W]\!] \backslash [\![h]\!]\}$ iff
$[\![W]\!] \cap [\![F(v)]\!] = [\![W]\!] \backslash [\![h]\!]$

c. $[\![$ At most five girls play Quidditch $]\!] = 1$ iff
$[\![Q]\!] \in \{P \subseteq U_M \mid card([\![G]\!] \cap P) \leq 5\}$ iff
$card([\![G]\!] \cap [\![Q]\!]) \leq 5$

d. $[\![$ Few Gryffindors are lazy $]\!] = 1$ iff
$[\![L]\!] \in \{P \subseteq U_M \mid card([\![Grif]\!] \cap P) < card([\![Grif]\!] \backslash P)\}$
("there are less lazy than non-lazy Gryffindors") iff
$card([\![Grif]\!] \cap [\![L]\!]) < card([\![Grif]\!] \backslash [\![L]\!])$



Exercise 2

Determine the monotonicity properties (left and right) of the following determiners. Show how you derived these monotonicity properties.

a. exactly five: $-mon-$

b. at least five: $\uparrow mon \uparrow$

c. at most five: $\downarrow mon \downarrow$

d. some but not all: $\uparrow mon-$

Derived using the following tests:

$\downarrow mon$	If ___ animals walked, then ___ dogs walked.
$\uparrow mon$	If ___ dogs walked, then ___ animals walked.
$mon \downarrow$	If ___ dogs walked, then ___ dogs walked rapidly.
$mon \uparrow$	If ___ dogs walked rapidly, then ___ dogs walked.

Exercise 3

Prove that the following statement holds:

- The external negation of an upward monotonic quantifier is a downward monotonic quantifier.

Premise 1: The external negation of Q: $\neg Q = \{P \subseteq U_M \mid P \notin Q\}$
Premise 2: Q is upward monotonic iff for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$.
Premise 3: Q' is downward monotonic iff for all $X, Y \subseteq U$: if $X \in Q'$ and $Y \subseteq X$, then $Y \in Q'$.

Take an arbitrary $A, B \subseteq U_M$ and assume that $A \subseteq B$ and $B \in \neg Q$.
To prove: $A \in \neg Q$.
Proof by Contradiction: If $A \notin \neg Q$, then $A \in Q$ (by Premise 1). Then it follows that $B \in Q$ (by Premise 2). This is in conflict with the assumption that $B \in \neg Q$, which means that $B \notin Q$ (by Premise 1). Therefore, it must hold that $A \in \neg Q$. Given Premise 3, this means that $\neg Q$ is downward monotonic.