Semantic Theory 2020: Solutions exercise sheet 2

Exercise 1

Derive the types of the underlined expressions in the following sentences. The subscripts indicate the types of the relevant expressions. Also provide the (simplified) logical form you assume for the sentences.

a. (is$_{\langle e,et\rangle}$ (($\underline{\text{the father of}}$)$_{\langle e,e\rangle}$ Luke$_e$)$_e$)$_{\langle et\rangle}$(Darth Vader)$_e$
   or: (is$_{\langle et,et\rangle}$ (($\underline{\text{father of}}$)$_{\langle e,et\rangle}$ Luke$_e$)$_{\langle et\rangle}$)$_{\langle et\rangle}$(Darth Vader)$_e$

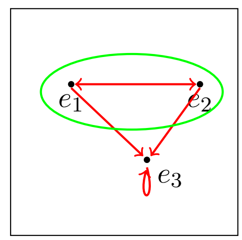
b. ($\underline{\text{Every}}_{\langle et,\langle et,t\rangle\rangle}$ Jedi$_{\langle et\rangle}$)$_{\langle et,t\rangle}$ (has$_{\langle e,et\rangle}$ (a lightsaber)$_e$)$_{\langle et\rangle}$

c. (on$_{\langle e,\langle et,et\rangle\rangle}$(Naboo$_e$)(is (the (($\underline{\text{most}}_{\langle\langle et,et\rangle,\langle et,et\rangle\rangle}$ beautiful$_{\langle et,et\rangle}$)$_{\langle et,et\rangle}$ woman$_{\langle et\rangle}$)$_{\langle et\rangle}$))$_{\langle et\rangle}$)$_{\langle et\rangle}$
   (Padmé Amidala)$_e$ — NB. types for "is" and "the" are left out here; see (a).

Exercise 2

The diagram on the right graphically represents a model structure $M = \langle U_M, V_M\rangle$ with a universe consisting of three entities: $U_M = \{e_1, e_2, e_3\}$. The interpretation function V_M describes the first-order property "Jedi" (indicated by the green circle) and the two-place relation "to help" (indicated by the red arrows).

M:



2.1 Give the type-theoretic denotation of the interpretation function V_M for the following non-logical constants (i.e., using functions, not sets):

a. For instance: $V_M(\text{anakin'}) = e_1$, $V_M(\text{yoda'}) = e_2$, $V_M(\text{padmé'}) = e_3$

b. $V_M(\text{jedi'}) = [e_1 \to 1, e_2 \to 1, e_3 \to 0]$

c. $V_M(\text{help'}) =$
   $[e_1 \to [e_1 \to 0, e_2 \to 1, e_3 \to 0],$
   $e_2 \to [e_1 \to 1, e_2 \to 0, e_3 \to 0],$
   $e_3 \to [e_1 \to 1, e_2 \to 1, e_3 \to 1]]$

2.2 Compute the type-theoretic denotations of the following expressions relative to the given model structure M and some arbitrary variable assignment g. Here, x is a variable of type e, and F is a variable of type $\langle e,t\rangle$.

a. $[\![\textit{help'(padmé')}]\!]^{M,g} = [\![\textit{help'}]\!]^{M,g}([\![\textit{padmé'}]\!]^{M,g}) = V_M(\textit{help'})(V_M(\textit{padmé'})) =$
   $[e_1 \to 1, e_2 \to 1, e_3 \to 1] \in D_{\langle et\rangle}$

b. $[\![\forall x(\textit{help'}(x)(x) \rightarrow \neg \textit{jedi'}(x))]\!]^{M,g} = 1$ *iff*
for all $d \in U_M$ $[\![\textit{help'}(x)(x) \rightarrow \neg \textit{jedi'}(x)]\!]^{M,g[x/d]} = 1$ *iff*
for all $d \in U_M$ $[\![\textit{help'}(x)(x)]\!]^{M,g[x/d]} = 0$ or $[\![\neg \textit{jedi'}(x)]\!]^{M,g[x/d]} = 1$ *iff*
for all $d \in U_M$ $[\![\textit{help'}]\!]^{M,g[x/d]}([\![x]\!]^{M,g[x/d]})([\![x]\!]^{M,g[x/d]}) = 0$ or $[\![\textit{jedi'}(x)]\!]^{M,g[x/d]} = 0$ *iff*
for all $d \in U_M$ $V_M(\textit{help'})(d)(d) = 0$ or $[\![\textit{jedi'}]\!]^{M,g[x/d]}([\![x]\!]^{M,g[x/d]}) = 0$ *iff*
for all $d \in U_M$ $V_M(\textit{help'})(d)(d) = 0$ or $V_M(\textit{jedi'})(d) = 0$.
Given $U_M = \{e_1, e_2, e_3\}$ these truth conditions are satisfied: for e_1 and e_2 this holds because $V_M(\textit{help'})(e_1)(e_1) = 0$ and $V_M(\textit{help'})(e_2)(e_2) = 0$, and for e_3 this holds because $V_M(\textit{jedi'})(e_3) = 0$.

c. $[\![\forall F \exists x(F(x))]\!]^{M,g} = 1$ *iff*
for all $P \in D_{\langle et \rangle}$, $[\![\exists x(F(x))]\!]^{M,g[F/P]} = 1$ *iff*
for all $P \in D_{\langle et \rangle}$, there is a $d \in U_M$ s.t. $[\![F(x)]\!]^{M,g[F/P][x/d]} = 1$ *iff*
for all $P \in D_{\langle et \rangle}$, there is a $d \in U_M$ s.t. $[\![F]\!]^{M,g[F/P][x/d]}([\![x]\!]^{M,g[F/P][x/d]}) = 1$ *iff*
for all $P \in D_{\langle et \rangle}$, there is a $d \in U_M$ s.t. $P(d) = 1$.
For, $V_M(\textit{jedi'}) \in D_{\langle et \rangle}$, this is true: take $d = e_1$. Are there any other functions in $D_{\langle et \rangle}$? Since $V_M(\textit{help'}) \in \langle e, \langle et \rangle\rangle$, which is defined as the set of all functions from D_e to $D_{\langle et \rangle}$, we could argue that $V_M(\textit{help'})(e_1)$, $V_M(\textit{help'})(e_2)$ and $V_M(\textit{help'})(e_3)$ should also be elements of $D_{\langle et \rangle}$ (then the statement still holds, take $d = e_2$, $d = e_1$ and $d = e_1$, respectively). On the other hand, if we strictly interpret the domain $D_{\langle et \rangle}$ as the set of <u>all</u> functions from D_e to D_t, this will by definition also include the function $[e_1 \rightarrow 0, e_2 \rightarrow 0, e_3 \rightarrow 0]$, which means the truth conditions can never be satisfied; i.e., (c) is a contradiction.