

Semantic Theory

week 7 – Discussion of exercises

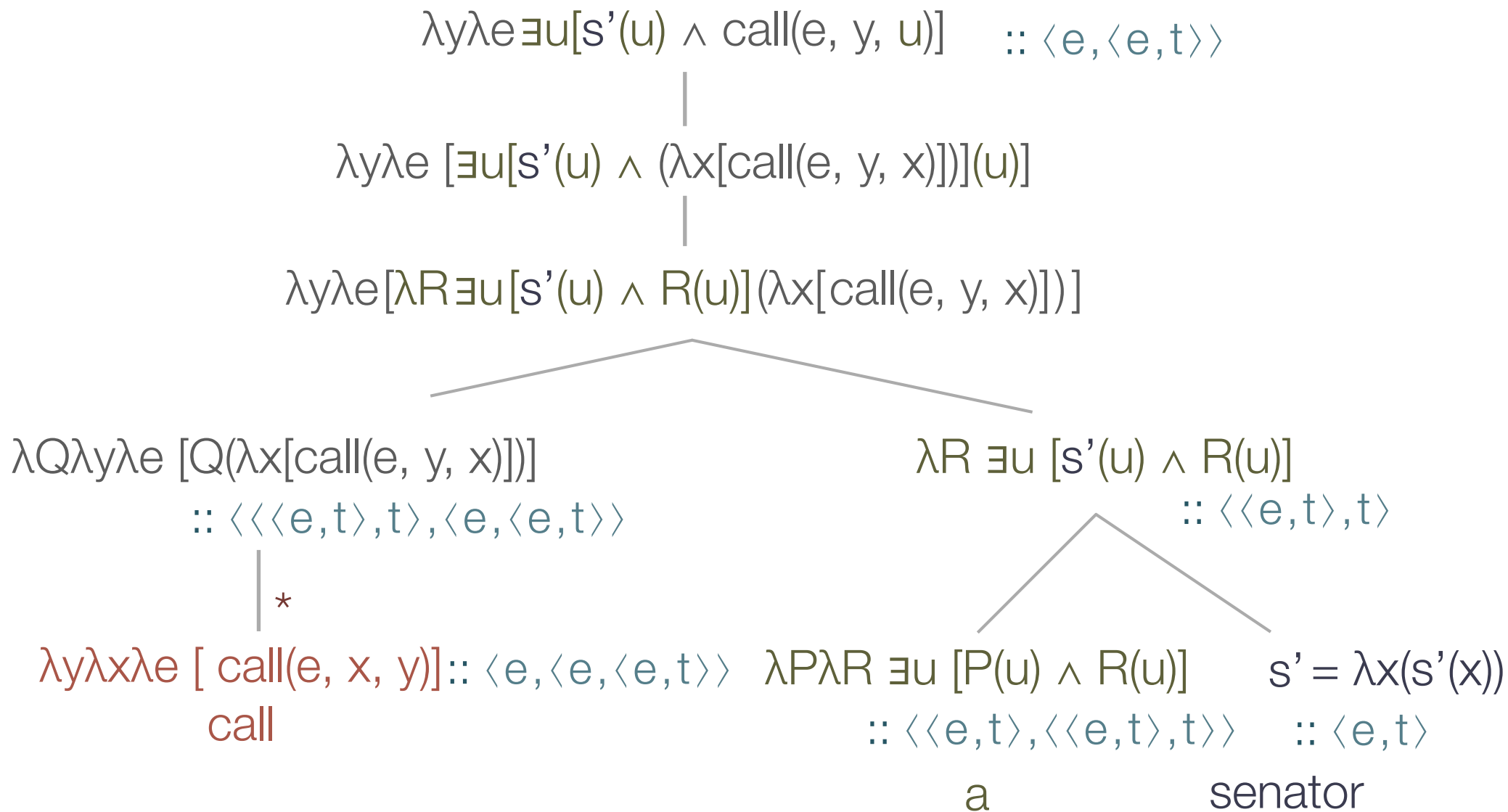
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Exercise sheet 4: Exercise 1b

Beta-reduction and Type-raising



Exercise sheet 3: Exercise 3

A formal proof

The external negation of an upward monotonic quantifier is a downward monotonic quantifier.

Proof: We first collect the three relevant definitions:

- (1) The external negation $\neg Q$ of a quantifier Q is defined as $\{P \subseteq U_M \mid P \notin Q\}$.
- (2) Q is an upward monotonic quantifier iff for all $X, Y \subseteq U_M$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$
- (3) Q' is a downward monotonic quantifier iff for all $X, Y \subseteq U_M$: if $X \in Q'$ and $Y \subseteq X$, then $Y \in Q'$

Take an arbitrary $A, B \subseteq U_M$ and suppose $A \subseteq B$ and $B \in \neg Q$. To prove: $A \in \neg Q$.

Suppose that $A \notin \neg Q$, then it follows from (1) that $A \in Q$. Given that Q is an upward monotonic quantifier, it follows from (2) that $B \in Q$. But this is in conflict with the assumption $B \in \neg Q$, which means that $B \notin Q$ (by (1)). Therefore, it must hold that $A \in \neg Q$. Now it follows from (3) that $\neg Q$ is downward monotonic.