Semantic Theory
week 5 – Generalised Quantifiers

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Back to Noun Phrases

Natural language contains a wide variety of NPs, serving as quantifiers

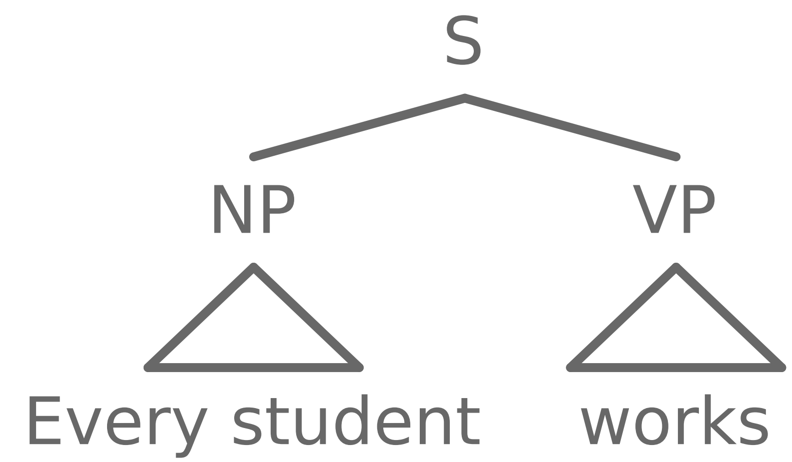
all students, no woman, not every man, everything, nothing, three books, the ten professors, John, John and Mary, only John, firemen, at least five horses, most girls, all but ten marbles, less than half of the audience, John's car, some student's exercise, no student except Mary, more male than female cats, usually, each other.

Aristotle: "Quantifiers are second-order relations between sets"

Frege: "All quantifiers can be defined in terms of ∀ (and ∃)"

NP interpretation

"Every student"



- $\mapsto \lambda P \forall x(\text{student}'(x) \rightarrow P(x))$
- Type: $\langle\langle e, t\rangle, t\rangle$
- Interpretation: "Every student" denotes the *set of properties* that apply to every student (property = set of individuals).
- $[\![\text{Every student}]\!]^M = \{ P \subseteq U_M \mid \text{every student has property } P \}$
  $= \{ P \subseteq U_M \mid [\![\text{student}]\!] \subseteq P\}$
- $[\![\text{Every student works}]\!]^M = 1$ iff $[\![\text{work}]\!]^M \in [\![\text{every student}]\!]^M$

Generalized Quantifiers

Generalized quantifiers are sets of subsets of M (i.e., sets of properties)

every student $\mapsto \lambda P \forall x(\text{student'}(x) \rightarrow P(x))$

- $[\![\text{every student}]\!]^M = \{ P \subseteq U_M \mid [\![\text{student}]\!] \subseteq P\}$

"the set of properties P such that all students are P"

a student $\mapsto \lambda P \exists x(\text{student'}(x) \wedge P(x))$

- $[\![\text{a student}]\!]^M = \{ P \subseteq U_M \mid [\![\text{student}]\!] \cap P \neq \varnothing \}$

"the set of properties P such that at least one student is P"

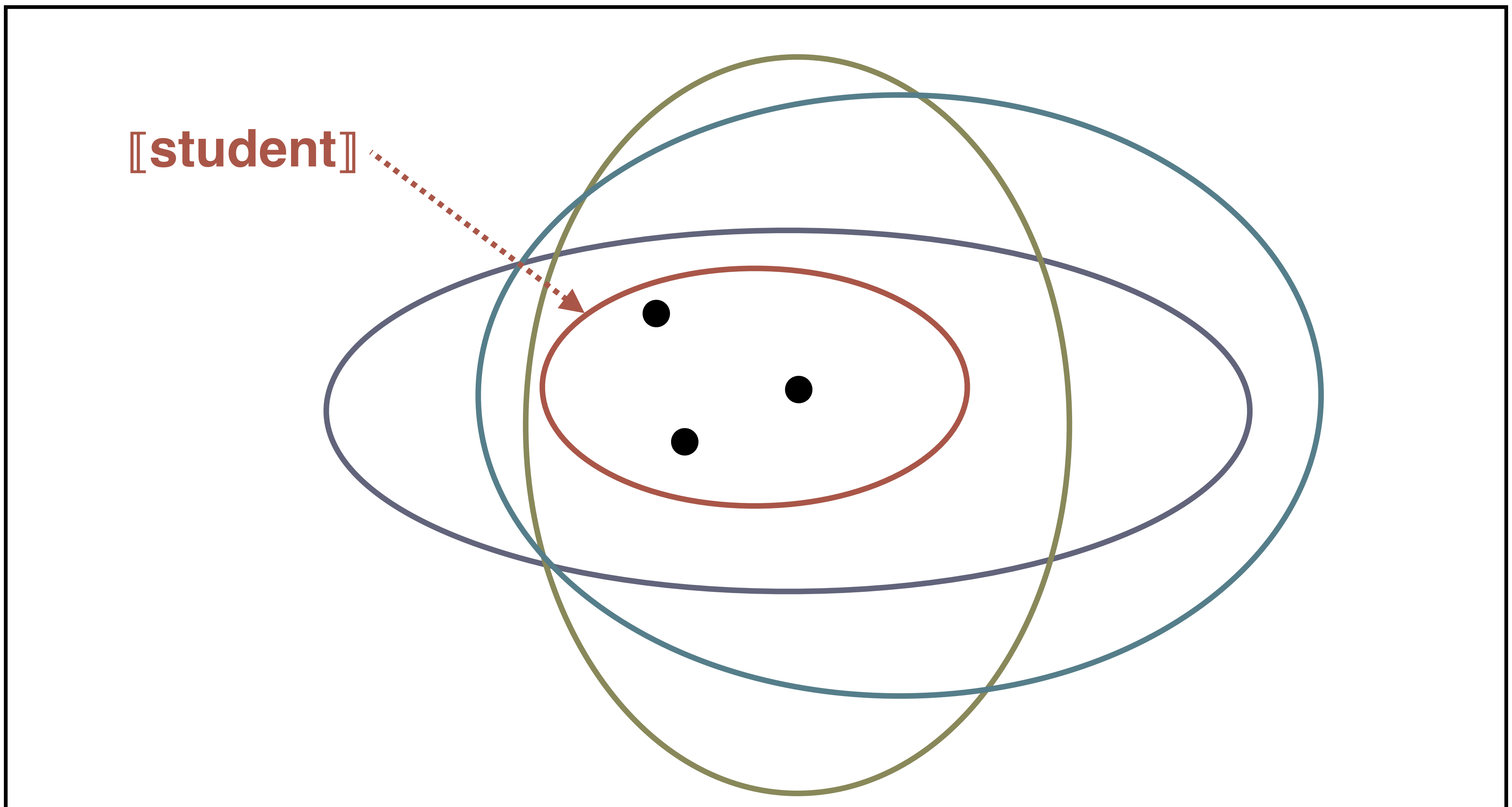
Bill $\mapsto \lambda P.P(b^*)$

- $[\![\text{Bill}]\!]^M = \{ P \subseteq U_M \mid b^* \in P\}$

"the set of properties P, such that Bill is P"

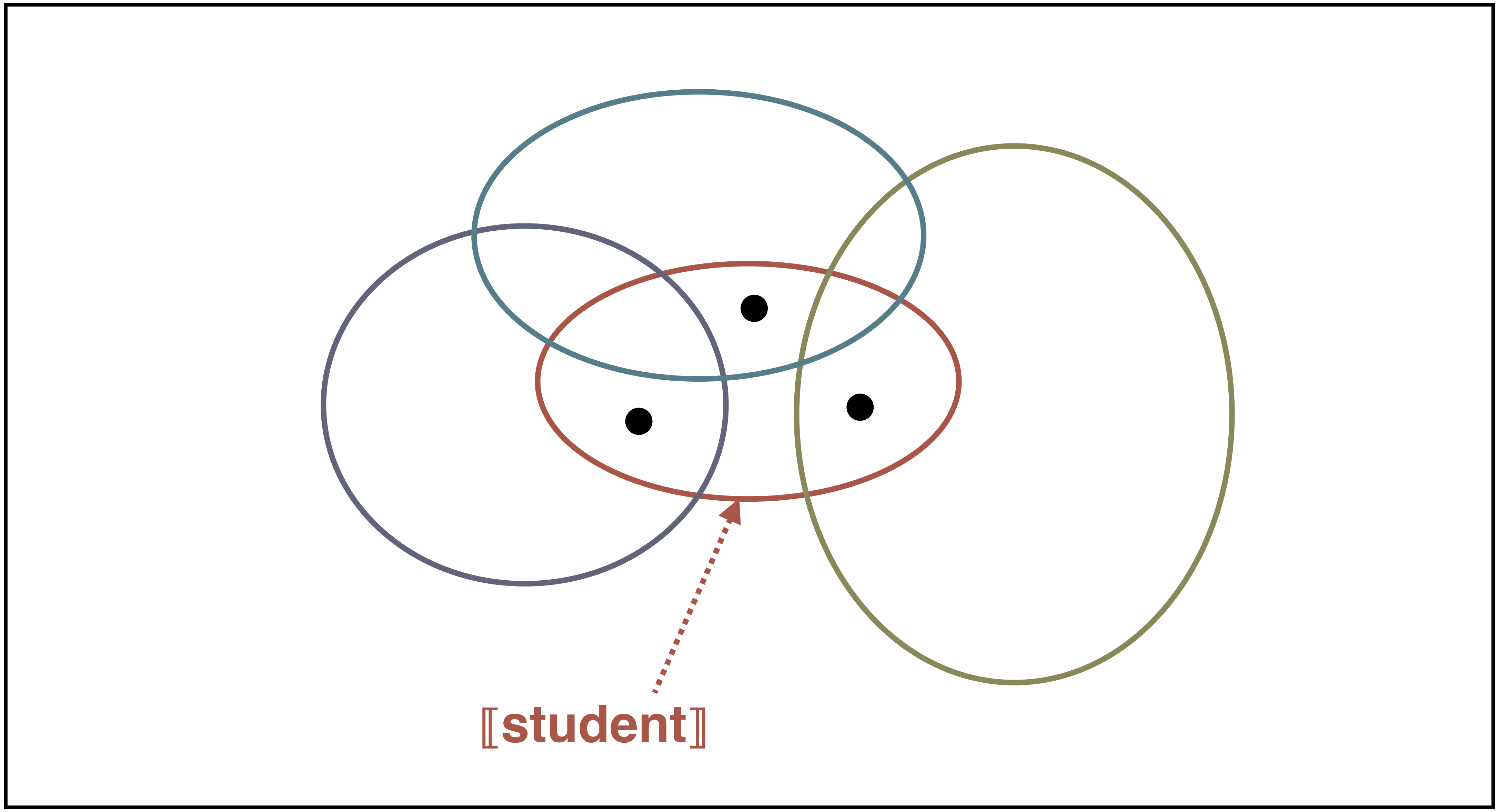
⟦*every student*⟧

- "every student" denotes the set of properties that apply to every student (i.e., all supersets of ⟦student⟧)



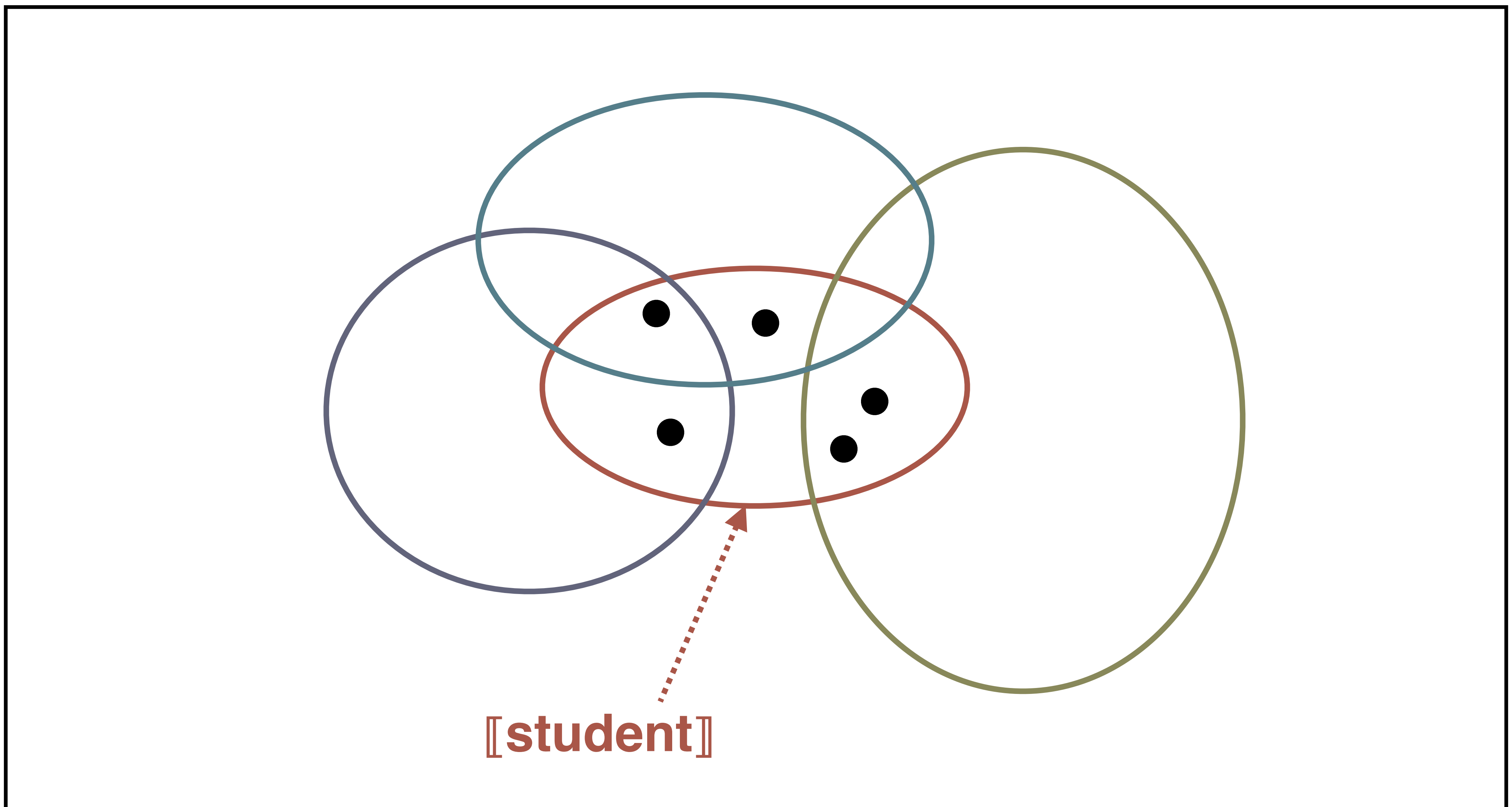
⟦*a student*⟧

- "a student" denotes the set of properties that apply to at least one student.



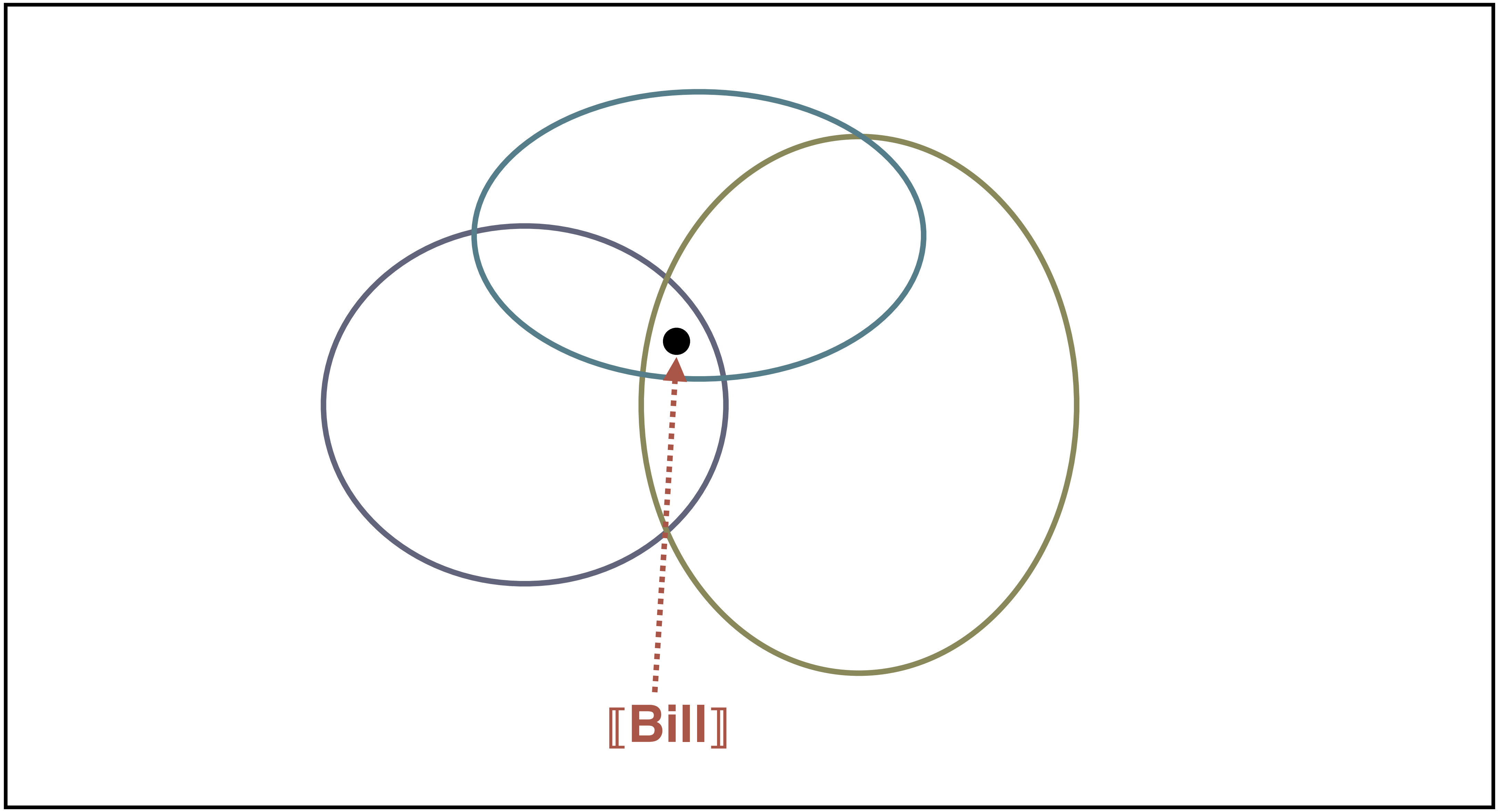
⟦*two students*⟧

- “two students” denotes the set of properties that apply to at least (exactly) two students.



⟦*Bill*⟧

- "Bill" denotes the set of properties that apply to Bill



Noun Phrase Interpretations

$[\![\text{all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = [\![N]\!] \}$

$[\![\text{a(n) N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq \varnothing \}$

$[\![\text{bill}]\!]^M = \{ P \subseteq U_M \mid b^* \in P\}$

$[\![\text{not all N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \}$

$[\![\text{no N}]\!]^M = \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \}$

$[\![\text{exactly n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) = n \}$

$[\![\text{at most n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \leq n \}$

$[\![\text{at least n N}]\!]^M = \{ P \subseteq U_M \mid \text{card}([\![N]\!] \cap P) \geq n \}$

Generalized Quantifier Theory

I. How do generalized quantifiers differ in terms of their formal properties?

II. What universal regularities govern the meaning of terms?

III. Which subclasses actually represent meanings of natural language noun phrases?

Observation 1: Inference Patterns

(1) *All men walked rapidly* ⊨ *All men walked*

(2) *A girl smoked a cigar* ⊨ *A girl smoked*

(3) *No man walked* ⊨ *No man walked rapidly*

(4) *Few girls smoked* ⊨ *Few girls smoked a cigar*

Q: How to explain the different inference patterns for quantifiers?

Observation 2: Negative Polarity Items

NPIs (*need*, *any, ever, …*) can occur only in “negative contexts”

(1) a. *John <u>need</u>n’t go there.*

b. **John <u>need</u> go there.*

(2) a. *Nobody saw <u>anything.</u>*

b. **Somebody saw <u>anything.</u>*

(3) a. *No student has <u>ever</u> been in Saarbrücken.*

b. **Some student has <u>ever</u> been in Saarbrücken.*

Q: What licenses negative polarity items?

Observation 3: Coordination

(1) *No man and few women walked.*

(2) *None of the girls and at most three boys walked.*

(3) **A man and few women walked.*

(4) **John and no woman saw Jane.*

Q: which noun phrases can be coordinated?

Subsets and Supersets

(1) *All men walked rapidly* ⊨ *All men walked*

- Note: ⟦walked rapidly⟧ ⊆ ⟦walked⟧

(2) *A girl smoked a cigar* ⊨ *A girl smoked*

- Note: ⟦smoked a cigar⟧ ⊆ ⟦smoked⟧

Intuitively: For the given quantifiers, sentence [$_S$ NP VP] remains true if the denotation of the VP is made "larger"

Upward Monotonicity

A quantifier Q is upward monotonic (or: *monotone increasing*) in $M = \langle U, V \rangle$ iff Q is "closed under supersets", i.e.:

- for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$

- A noun phrase is upward monotonic if it denotes an upward monotonic quantifier.

Upward Monotonicity Tests

If $\llbracket VP_1 \rrbracket \subseteq \llbracket VP_2 \rrbracket$, then $NP\ VP_1 \vDash NP\ VP_2$

- $\llbracket$walked rapidly$\rrbracket \subseteq \llbracket$walked$\rrbracket$
- *All men walked rapidly* $\vDash$ *All men walked* 🙂
- *No man walked rapidly* $\nvDash$ *No man walked* ☹️

$NP\ VP_1$ and $VP_2 \vDash NP\ VP_1$ and $NP\ VP_2$

- *All men smoked and drank* $\vDash$ *All men smoked and all men drank* 🙂
- *No man smoked and drank* $\nvDash$ *No man smoked and no man drank* ☹️
- Note: $\llbracket VP_1$ and $VP_2 \rrbracket = \llbracket VP_1 \rrbracket \cap \llbracket VP_2 \rrbracket$

Upward Monotonicity and logical operators

Upward monotonic quantifiers are *closed under* conjunction and disjunction:

- *All boys and a girl walked rapidly* ⊨ *All boys and a girl walked*
- *John or a student arrived late* ⊨ *John or a student arrived*
- Note: $[\![\text{NP}_1 \text{ and NP}_2]\!] = [\![\text{NP}_1]\!] \cap [\![\text{NP}_2]\!]$
  $[\![\text{NP}_1 \text{ or NP}_2]\!] = [\![\text{NP}_1]\!] \cup [\![\text{NP}_2]\!]$

The intersection/union of two upward monotonic quantifiers is an upward monotonic quantifier.

Downward Monotonicity

(3) *No man walked* $\vDash$ *No man walked rapidly* $[\![\textit{walked}]\!] \supseteq [\![\textit{walked rapidly}]\!]$

(4) *Few girls smoked* $\vDash$ *Few girls smoked a cigar.* $[\![\textit{smoked}]\!] \supseteq [\![\textit{smoked a cigar}]\!]$

A quantifier Q is downward monotonic (or: *monotone decreasing*) in $M = \langle U, V\rangle$ iff Q is closed under inclusion:

- for all $X, Y \subseteq U$: if $X \in Q$ and $X \supseteq Y$, then $Y \in Q$

- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier.

Downward Monotonicity Tests

If $[\![VP1]\!] \supseteq [\![VP2]\!]$, then NP VP1 $\vDash$ NP VP2

- $[\![\text{walked}]\!] \supseteq [\![\text{walked rapidly}]\!]$
- *No man walked* $\vDash$ *No man walked rapidly* 🙂
- *All men walked* $\nvDash$ *All men walked rapidly* ☹️

NP VP1 or VP2 $\vDash$ NP VP1 and NP VP2

- *Neither girl was drinking or smoking* $\vDash$
  Neither girl was drinking and neither girl was smoking. 🙂
- *All boys sing or dance* $\nvDash$ *All boys sing and all boys dance.* ☹️
- Note: $[\![VP_1 \text{ or } VP_2]\!] = [\![VP_1]\!] \cup [\![VP_2]\!]$ and $[\![VP_1 \text{ and } VP_2]\!] = [\![VP_1]\!] \cap [\![VP_2]\!]$

Looking for Universals I: Monotonicity Constraint

> "The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers."
> (Barwise & Cooper 1981)

Simple noun phrase: Proper names or NPs of the form $[_{NP}$ DET N]

Monotone quantifiers: quantifiers that are either upward or downward monotonic

Back to
Observation 2: Negative Polarity Items

(1) a. *John <u>need</u>n't go there.*

b. **John <u>need</u> go there.*

(2) a. *Nobody saw <u>anything.</u>*

b. **Somebody saw <u>anything.</u>*

(3) a. *No student has <u>ever</u> been in Saarbrücken.*

b. **Some student has <u>ever</u> been in Saarbrücken.*

- NPIs are licensed only in downward monotonic contexts.

Back to Observation 3: Coordination

(1) *No man and few women walked.*

(2) *None of the girls and at most three boys walked.*

(3) **A man and few women walked.*

(4) **John and no woman saw Jane.*

- (Non-comparative) NPs can be coordinated iff they have the same direction of monotonicity.

(3') *A man but few women walked.*

(4') *John but no woman saw Jane.*

- Coordination with the connective “but” requires NPs with a different direction of monotonicity.

Quantifier Negation

External negation

- $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$

$$\begin{aligned} \neg[\![\text{all } N]\!] &= \{ P \subseteq U_M \mid P \notin [\![\text{all } N]\!] \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap P \neq [\![N]\!] \} \\ &= [\![\text{not all } N]\!] \end{aligned}$$

Internal negation

- $Q\neg = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$

$$\begin{aligned} [\![\text{all } N]\!]\neg &= \{ P \subseteq U_M \mid (U_M - P) \in [\![\text{all } N]\!] \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) = [\![N]\!] \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap (U_M - P) \neq \varnothing \} \\ &= \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \} \\ &= [\![\text{no } N]\!] \end{aligned}$$

- If Q is an *upward monotonic* quantifier, then both ¬Q and Q¬ are *downward monotonic*.
- If Q is an *downward monotonic* quantifier, then both ¬Q and Q¬ are *upward monotonic*.

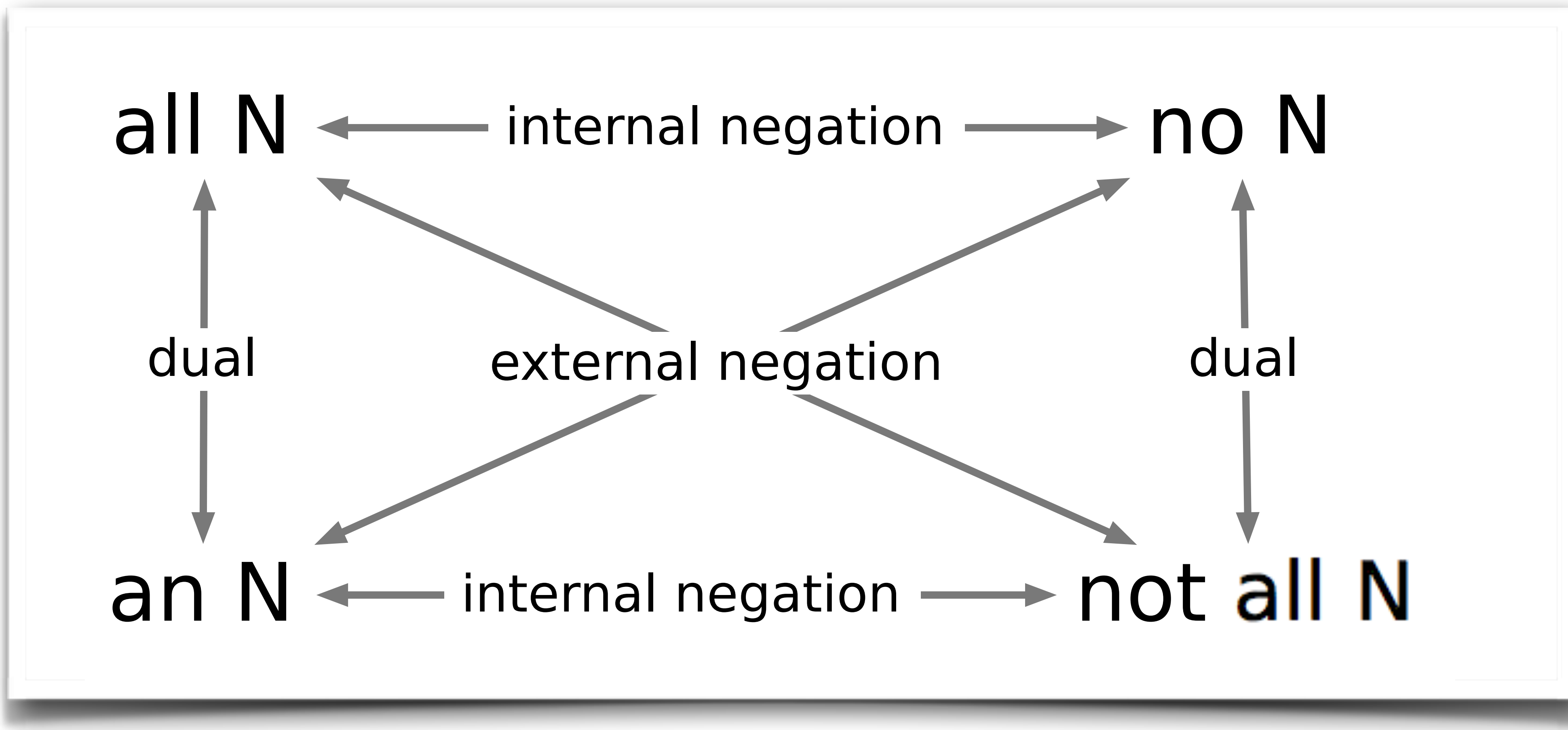
Duals

The dual Q^* of a quantifier Q in M

$$Q^* = \neg Q \neg \quad = \{ P \subseteq U_M \mid (U_M - P) \in \neg Q \}$$
$$= \{ P \subseteq U_M \mid (U_M - P) \notin Q \}.$$

- If Q is *upward monotonic*, then Q^* is *upward monotonic.*
- If Q is *downward monotonic*, then Q^* is *downward monotonic*.

The “Square of Opposition”



From NPs to Determiners

Every man walked $\mapsto \forall x(\text{man'}(x) \rightarrow \text{walk'}(x))$

- *Every* $\Rightarrow \lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))$
- $[\![\text{Every}]\!](A)(B) = 1$ iff $A \subseteq B$

- Syntactically, determiners are expressions that take a noun and a verb phrase to form a sentence.

- Semantically, the interpretation of a determiner can be seen as:

- a *function* from sets of entities to sets of properties: $\langle\langle e, t\rangle,\langle\langle e, t\rangle, t\rangle\rangle$
- a *relation* between two sets A and B, denoted by the NP and VP, respectively

Persistence

A determiner D is *persistent* in M iff: for all X, Y, Z:

- if $D(X, Z)$ and $X \subseteq_M Y$, then $D(Y, Z)$

Persistence test: If $[\![N_1]\!] \subseteq_M [\![N_2]\!]$, then DET N_1 VP $\vDash$ DET N_2 VP

- *Some men walked* $\vDash$ *Some human beings walked*
- *At least four girls were smoking* $\vDash$ *At least four women were smoking.*

Antipersistence

A determiner D is *antipersistent* in M iff: for all X,Y,Z:

- if D(X, Z) and Y ⊆ X, then D(Y, Z)

Antipersistence test: If ⟦N2⟧ ⊆ ⟦N1⟧, then DET N1 VP ⊨ DET N2 VP

- *All children walked* ⊨ *All toddlers walked*
- *No woman was smoking* ⊨ *No girl was smoking*
- *At most three Englishmen agreed* ⊨ *At most three Londoners agreed.*

Persistence and Monotonicity

Persistence (antipersistence)	⇔	upward (downward) monotonicity of the first argument.
		left-monotonicity (↑mon and ↓mon)
Upward (downward) monotonicity of noun phrases	⇔	upward (downward) monotonicity of the second argument of the determiner in the NP.
		right-monotonicity (mon↑ and mon↓)

Left and Right Monotonicity of Determiners

↑mon↑	*some*
↓mon↑	*all*
↓mon↓	*no*
↑mon↓	*not all*

Looking for Universals II: Conservativity

Conservativity:

- for every $A, B \subseteq U$: $D(A, B) \Leftrightarrow D(A, A \cap B)$

- implies that set A (the NP-denotation) is more important than the second set B (the VP-denotation), in other words: *"D lives on A"*

Test: D N VP ⇔ D N are N that VP

- *All students work ⇔ All students are students that work*
- *Some girls are dancing ⇔ Some girls are girls that are dancing*
- *Most teachers are motivated ⇔ Most teachers are teachers that are motivated*

Looking for Universals II: Conservativity (cont.)

The universality of conservativity:

In every natural language, simple determiners together with an N yield an NP which lives on ⟦N⟧. (Barwise & Cooper 1981)

Apparent exception: *only*

Only men smoke cigars ⇎ *Only men are men that smoke cigars*

- "only" not a determiner?

Literature

- L.T.F. Gamut. Logic, Language, and Meaning. Vol 2. Chapter 7.
- Jon Barwise & Robin Cooper. Generalized Quantifiers. Linguistics and Philosophy. 1981.