

# Semantic Theory

## Week 9 – Discourse Representation Theory

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# Recap: DRS Syntax

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A discourse representation structure (DRS)  $K$  is a pair  $\langle U_K, C_K \rangle$ , where:

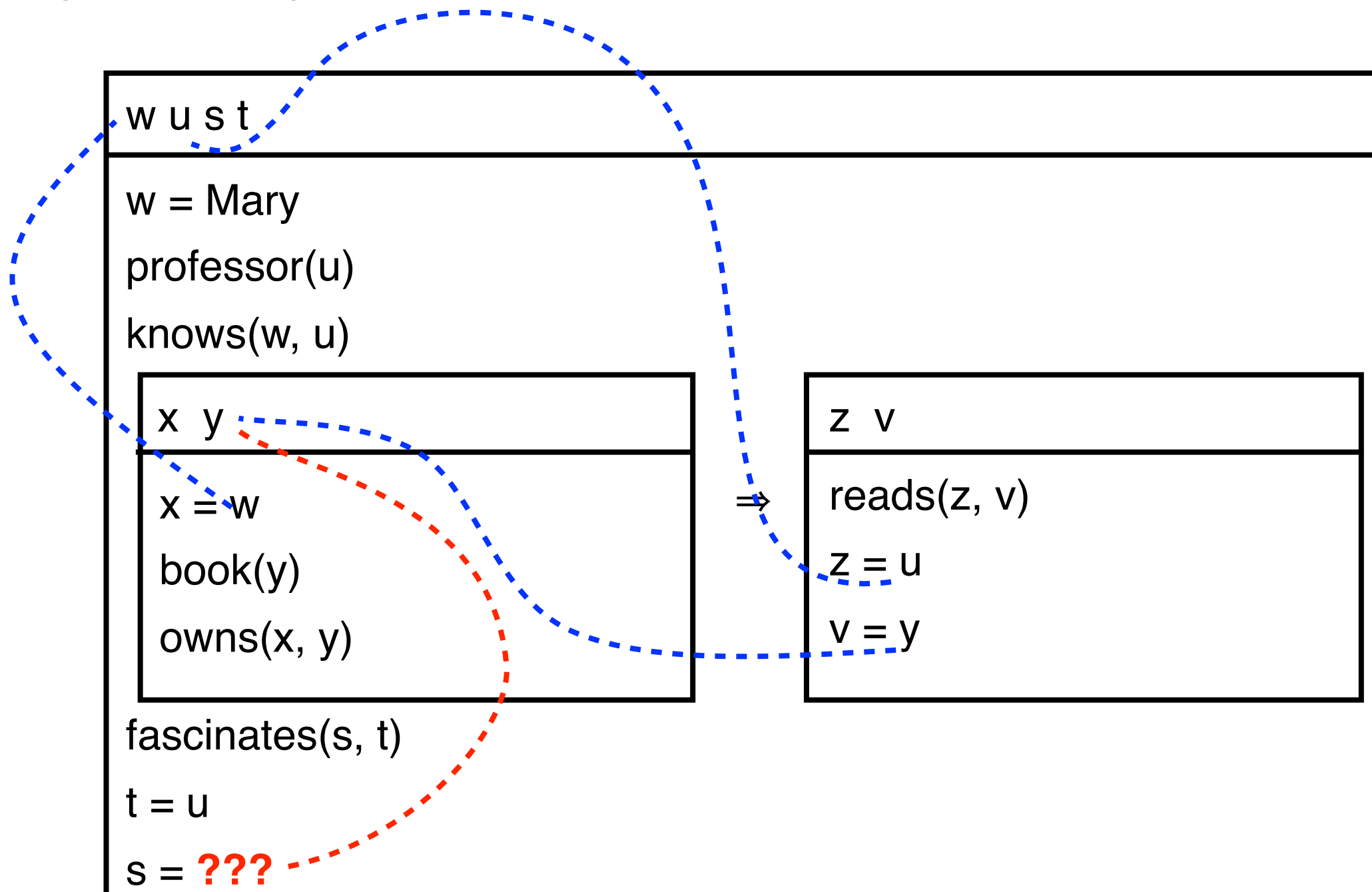
- $U_K \subseteq U_D$  and  $U_D$  is a set of discourse referents, and
- $C_K$  is a set of well-formed DRS conditions

## Well-formed DRS conditions:

- |                         |  |
|-------------------------|--|
| • $R(u_1, \dots, u_n)$  | <i>where:</i> $R$ is an $n$ -place relation, $u_i \in U_D$ |
| • $u = v$               | $u, v \in U_D$   |
| • $u = a$               | $u \in U_D$ , $a$ is a constant                            |
| • $\neg K_1$            | $K_1$ is a DRS   |
| • $K_1 \Rightarrow K_2$ | $K_1$ and $K_2$ are DRSs                                   |
| • $K_1 \vee K_2$        | $K_1$ and $K_2$ are DRSs                                   |

# Anaphora and accessibility

*Mary knows a professor. If she owns a book, he reads it. ?It fascinates him.*



# Non-accessible discourse referents

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## Cases of non-accessibility:

- (1) *If a professor owns a book, he reads it. It has 300 pages.*
- (2) *It is not the case that a professor owns a book. He reads it.*
- (3) *Every professor owns a book. He reads it.*
- (4) *If every professor owns a book, he reads it.*
- (5) *Peter owns a book, or Mary reads it.*
- (6) *Peter reads a book, or Mary reads a newspaper article. It is interesting.*

# Accessible discourse referents

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The following discourse referents are accessible for a condition:

- DRs in the same local DRS
- DRs in a superordinate DRS
- DRs in the universe of an antecedent DRS, if the condition occurs in the consequent DRS.

We need a formal notion of DRS subordination

# Subordination

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A DRS  $K_1$  is an immediate sub-DRS of a DRS  $K = \langle U_K, C_K \rangle$  iff  $C_K$  contains a condition of the form

- $\neg K_1, K_1 \Rightarrow K_2, K_2 \Rightarrow K_1, K_1 \vee K_2$  or  $K_2 \vee K_1$ .

$K_1$  is a sub-DRS of  $K$  (notation:  $K_1 \leq K$ ) iff

- $K_1 = K$ , or
- $K_1$  is an immediate sub-DRS of  $K$ , or
- there is a DRS  $K_2$  such that  $K_1 \leq K_2$  and  $K_2$  is an immediate sub-DRS of  $K$  (i.e. reflexive, transitive closure)

$K_1$  is a proper sub-DRS of  $K$  iff  $K_1 \leq K$  and  $K_1 \neq K$ .

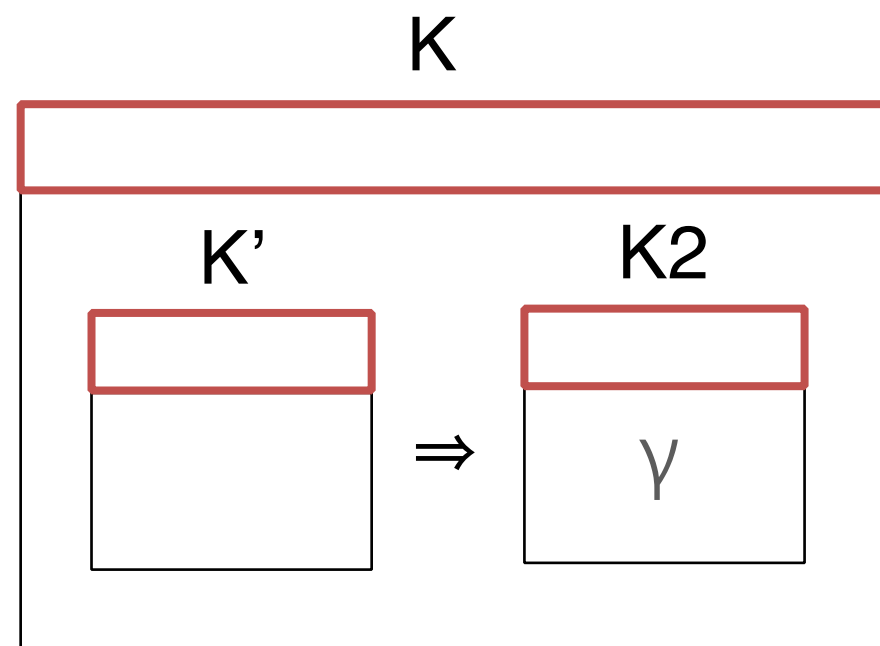
# Accessibility

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Let  $K, K_1, K_2$  be DRSs such that  $K_1, K_2 \leq K, x \in U_{K_1}, \gamma \in C_{K_2}$

$x$  is accessible from  $\gamma$  in  $K$  iff

- $K_2 \leq K_1$  or
- there are  $K_3, K_4 \leq K$  such that  $K_1 \Rightarrow K_3 \in C_{K_4}$  and  $K_2 \leq K_3$



# Free and bound variables in DRT

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A DRS variable  $x$ , introduced in DRS  $K_1$ , is bound in global DRS  $K$  iff there exists a DRS  $K_j \leq K$ , such that:

- (i)  $K_i \leq K_j$ ;
- (ii)  $x \in U(K_j)$ .

**Properness:** A DRS is *proper* iff it does not contain any free variables

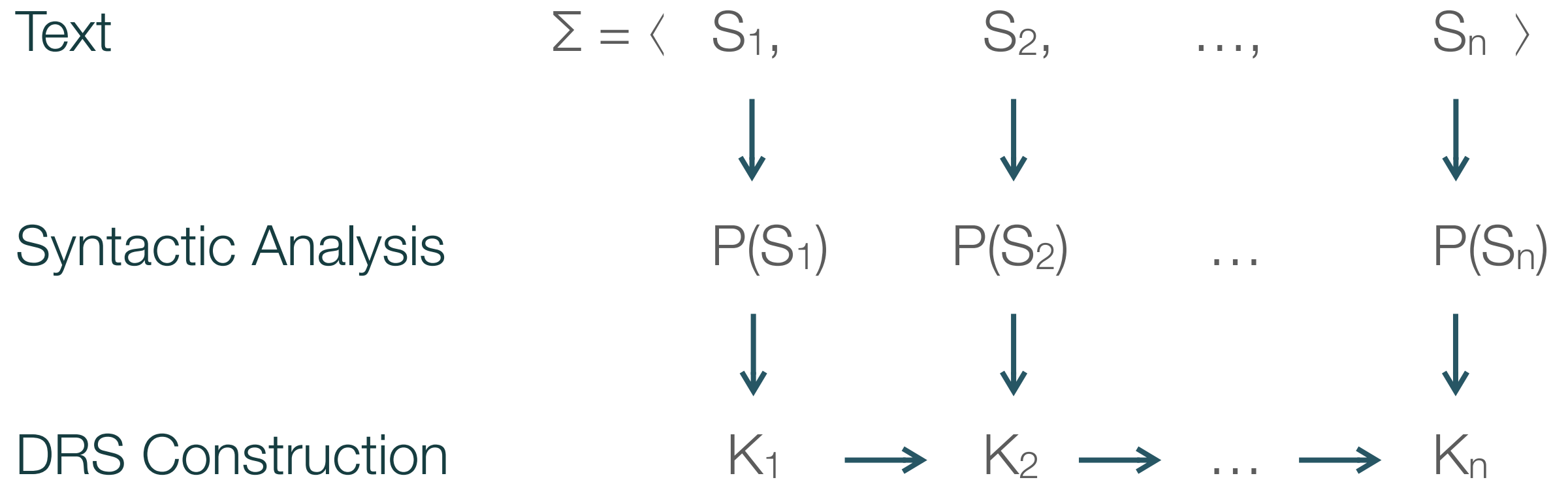
**Purity:** A DRS is *pure* iff it does not contain any *otiose declarations* of variables

  
 $x \in U(K_1)$  and  $x \in U(K_2)$  and  $K_1 \leq K_2$



# From text to DRS

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# DRS Construction Algorithm

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Let the following be a well-formed, *reducible* DRS condition:

- Conditions of form  $\alpha$  or  $\alpha(x_1, \dots, x_n)$ , where  $\alpha$  is a context-free parse tree.

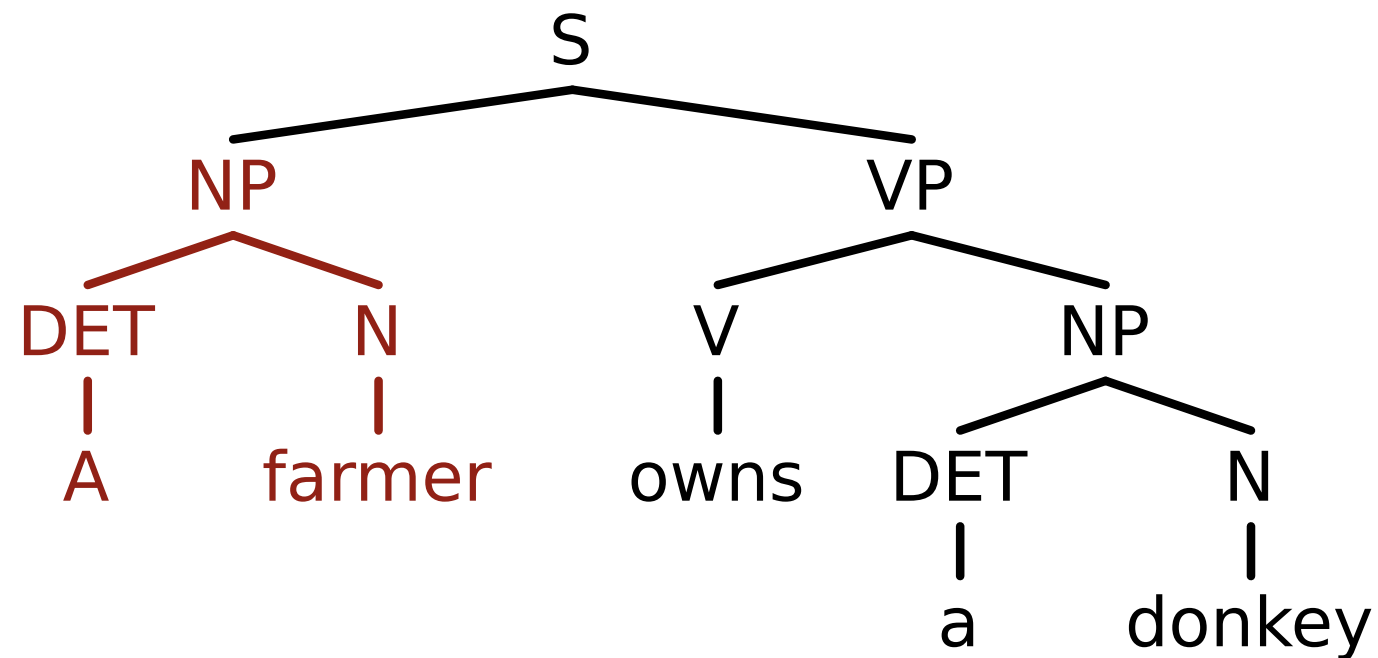
## DRS construction algorithm:

- Given a text  $\Sigma = \langle S_1, \dots, S_n \rangle$ , and a DRS  $K_0 (= \langle \emptyset, \emptyset \rangle$ , by default)
- Repeat for  $i = 1, \dots, n$ :
  - Add parse tree  $P(S_i)$  to the conditions of  $K_{i-1}$ .
  - Apply DRS construction rules to reducible conditions of  $K_{i-1}$ , until no reduction steps are possible any more.
  - The resulting DRS  $K_i$  is the discourse representation of text  $\langle S_1, \dots, S_i \rangle$ .

# DRS Construction Example

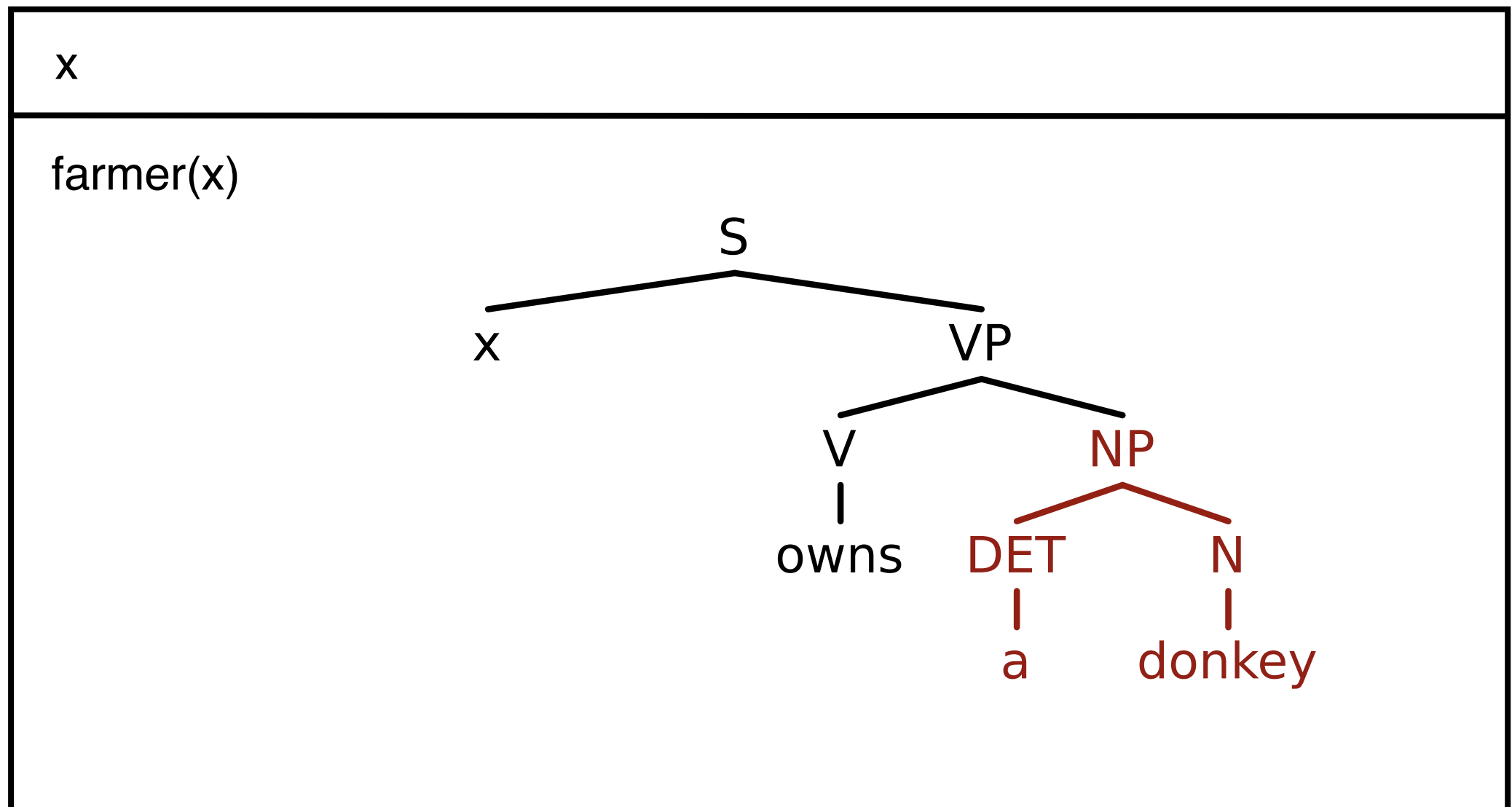
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- A farmer owns a donkey. He beats it.



# DRS Construction Example

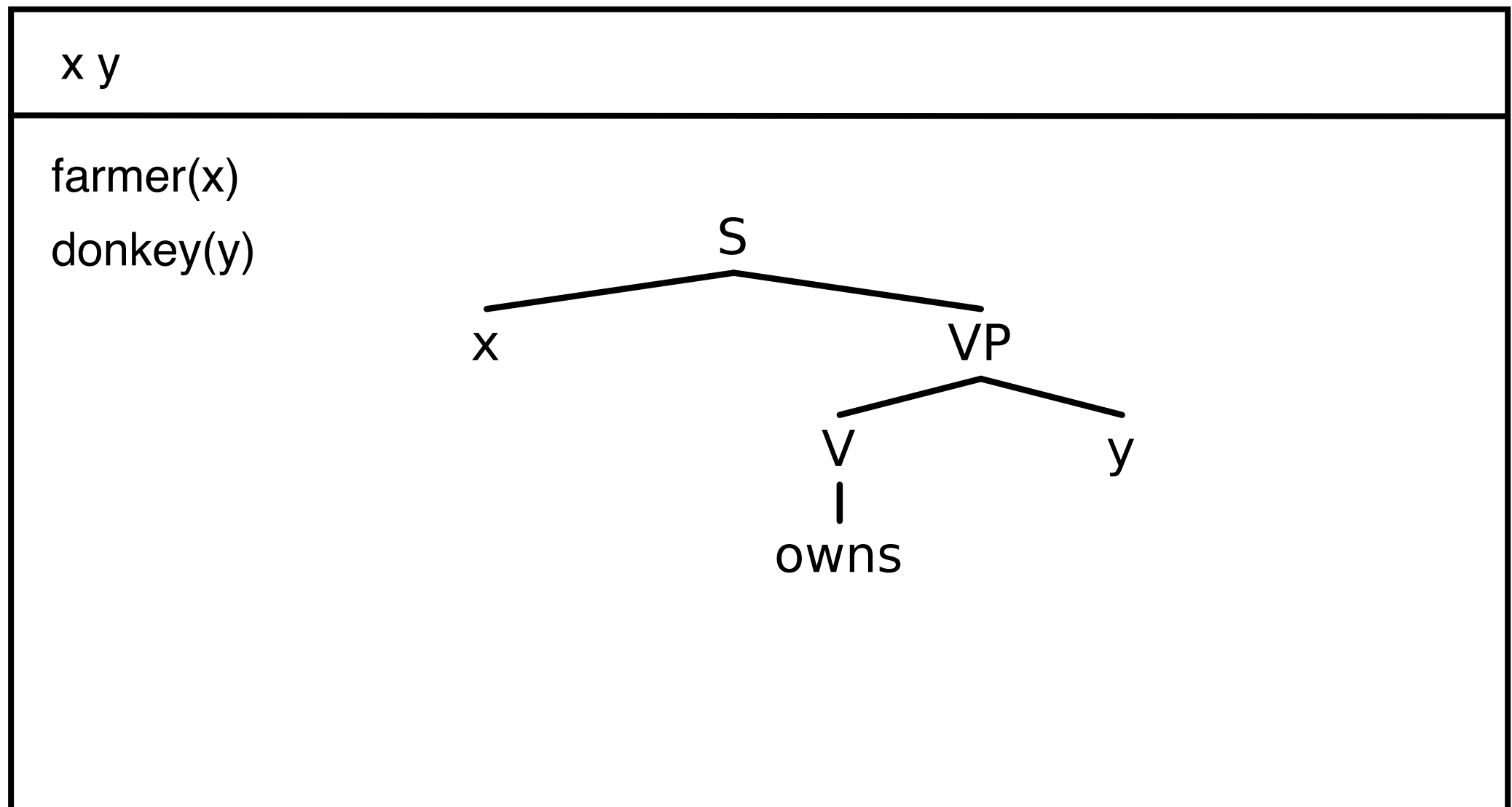
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# DRS Construction Example

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# DRS Construction Example

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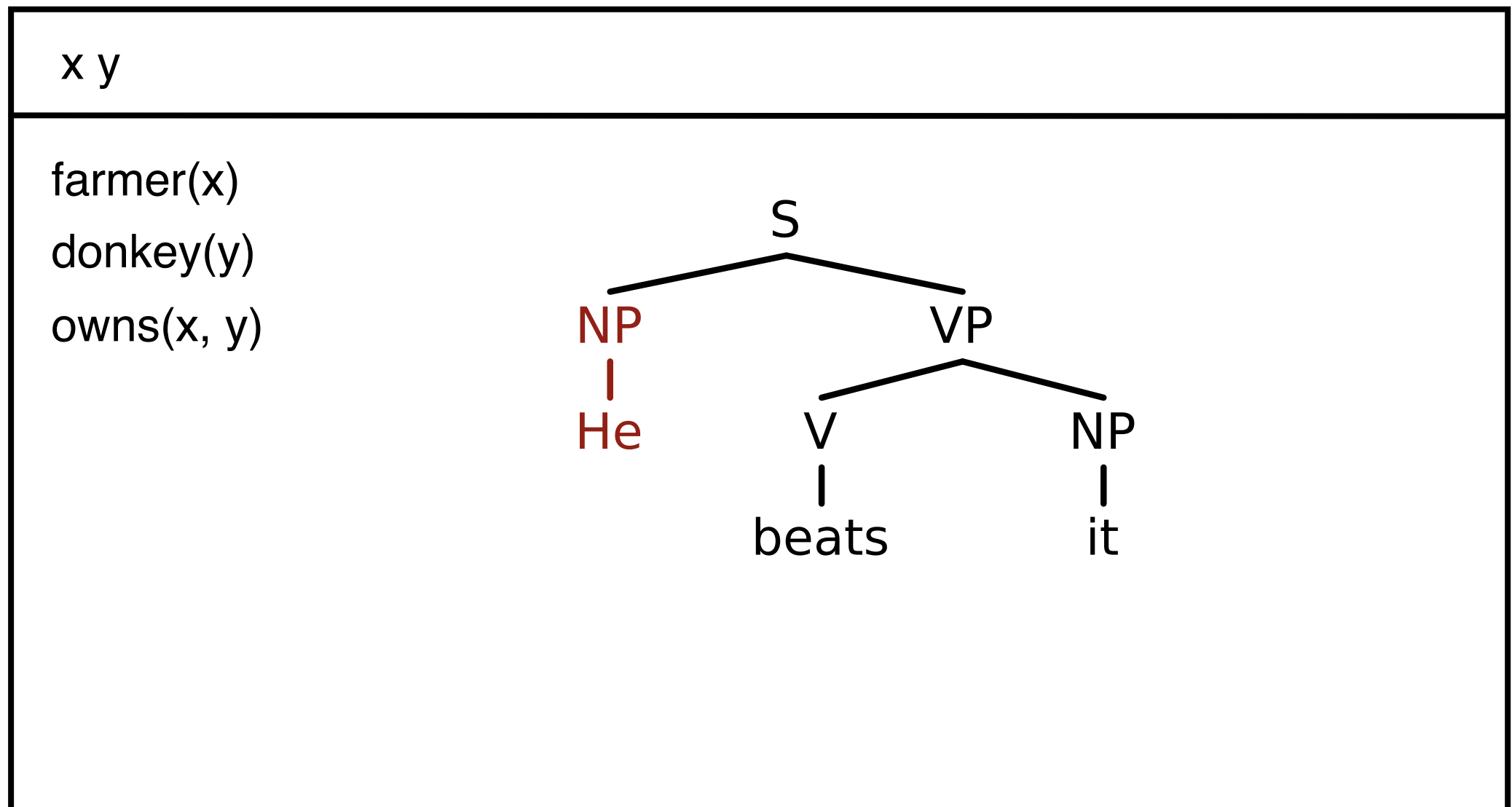
- A farmer owns a donkey. He beats it.

| x y                                  |
|--------------------------------------|
| farmer(x)<br>donkey(y)<br>owns(x, y) |

# DRS Construction Example

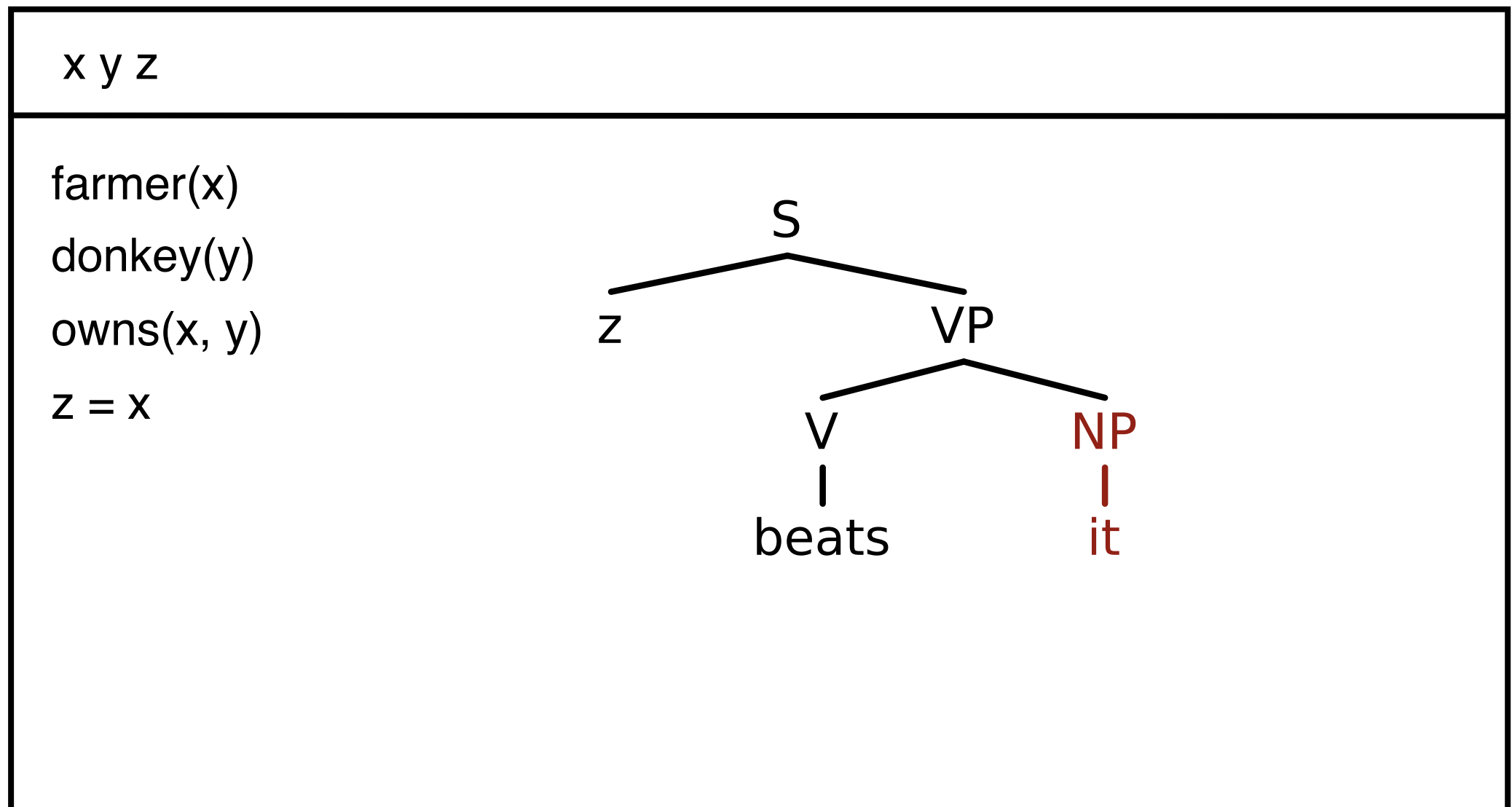
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# DRS Construction Example

- A farmer owns a donkey. He beats it.

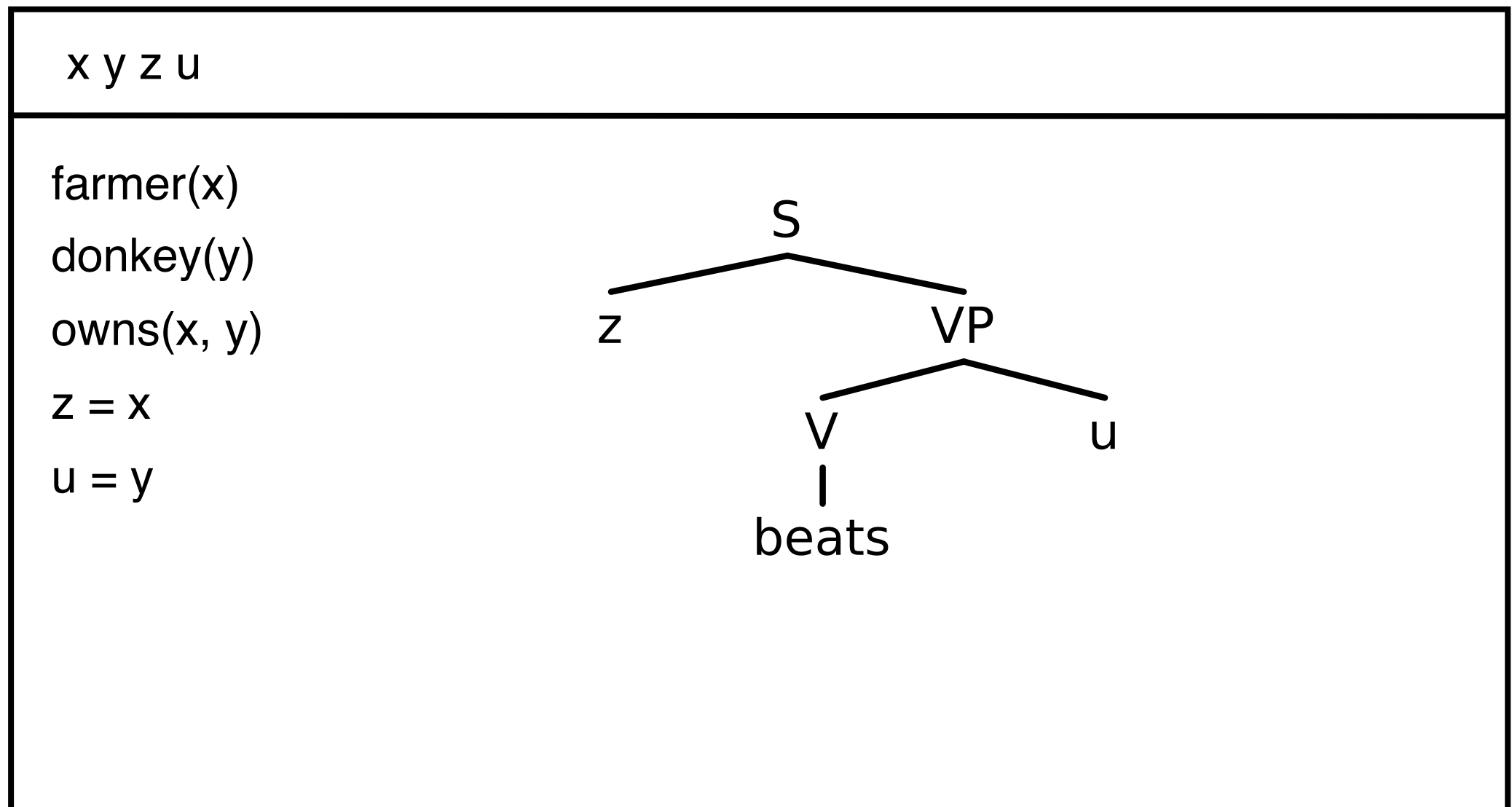




# DRS Construction Example

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- A farmer owns a donkey. He beats it.



# DRS Construction Example

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- A farmer owns a donkey. He beats it.

| x y z u  |
|--|
| farmer(x)<br>donkey(y)<br>owns(x, y)<br>z = x<br>u = y<br>beat(z, u) |

# Construction Rules: Examples

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## Indefinite NPs

- **Trigger:** a reducible condition  $\alpha$  in DRS  $K$  that has a substructure  $[NP \ \beta]$ , such that  $\beta$  is  $\varepsilon\delta$ , where  $\varepsilon$  is an indefinite article
- **Action:** Add new DR  $x$  to  $U_K$ ; Replace  $\beta$  in  $\alpha$  by  $x$ ; Add  $\delta(x)$  to  $C_K$

## Personal Pronouns

- **Trigger:** a global DRS  $K^*$ , and some  $K \leq K^*$ , with a reducible condition  $\alpha$  in  $K$  that has substructure  $[NP \ \beta]$ , such that  $\beta$  is a personal pronoun
- **Action:** Add a new DR  $x$  to  $U_K$ ; Replace  $\beta$  in  $\alpha$  by  $x$ ; Select an appropriate DR  $y$  that is accessible from  $\alpha$  in  $K^*$ ; Add  $x = y$  to  $C_K$

# A constraint on DRS construction

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**Problem:** The basic DRS construction algorithm can derive DRSs for both of the following sentences, with the indicated anaphoric binding:

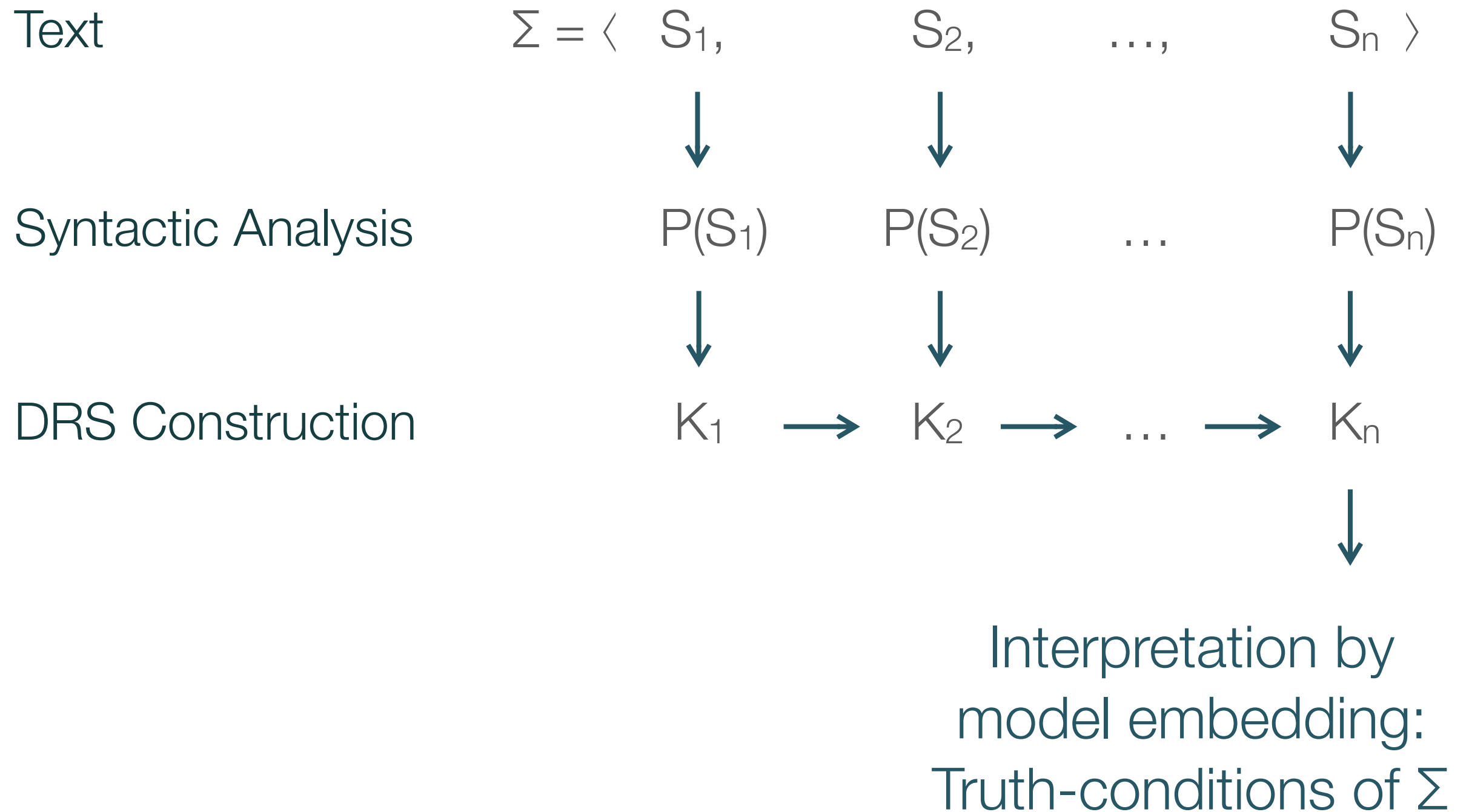
- (1) [A professor]<sub>i</sub> recommends a book that she<sub>i</sub> likes
- (2) She<sub>i</sub> recommends a book that [a professor]<sub>i</sub> likes

**Solution:** If two different triggering configurations occur in a reducible condition, then first apply the construction rule to the highest triggering configuration.

- *The highest triggering configuration* is the one whose top node dominates the top nodes of all other triggering configurations.

# From text to DRS

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# DRS Interpretation

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Given a DRS  $K = \langle U_K, C_K \rangle$ , with  $U_K \subseteq U_D$

Let  $M = \langle U_M, V_M \rangle$  be a FOL model structure appropriate for  $K$ , i.e. a model structure that provides interpretations for all predicates and relations occurring in  $K$

DRS  $K$  is *true* in model  $M$  *iff*

- there is an **embedding function** for  $K$  in  $M$  which verifies all conditions in  $K$

... where: an embedding of  $K$  into  $M$  is a (partial) function **f** from  $U_D$  to  $U_M$  such that  $U_K \subseteq \text{Dom}(\mathbf{f})$ .

# Verifying embedding

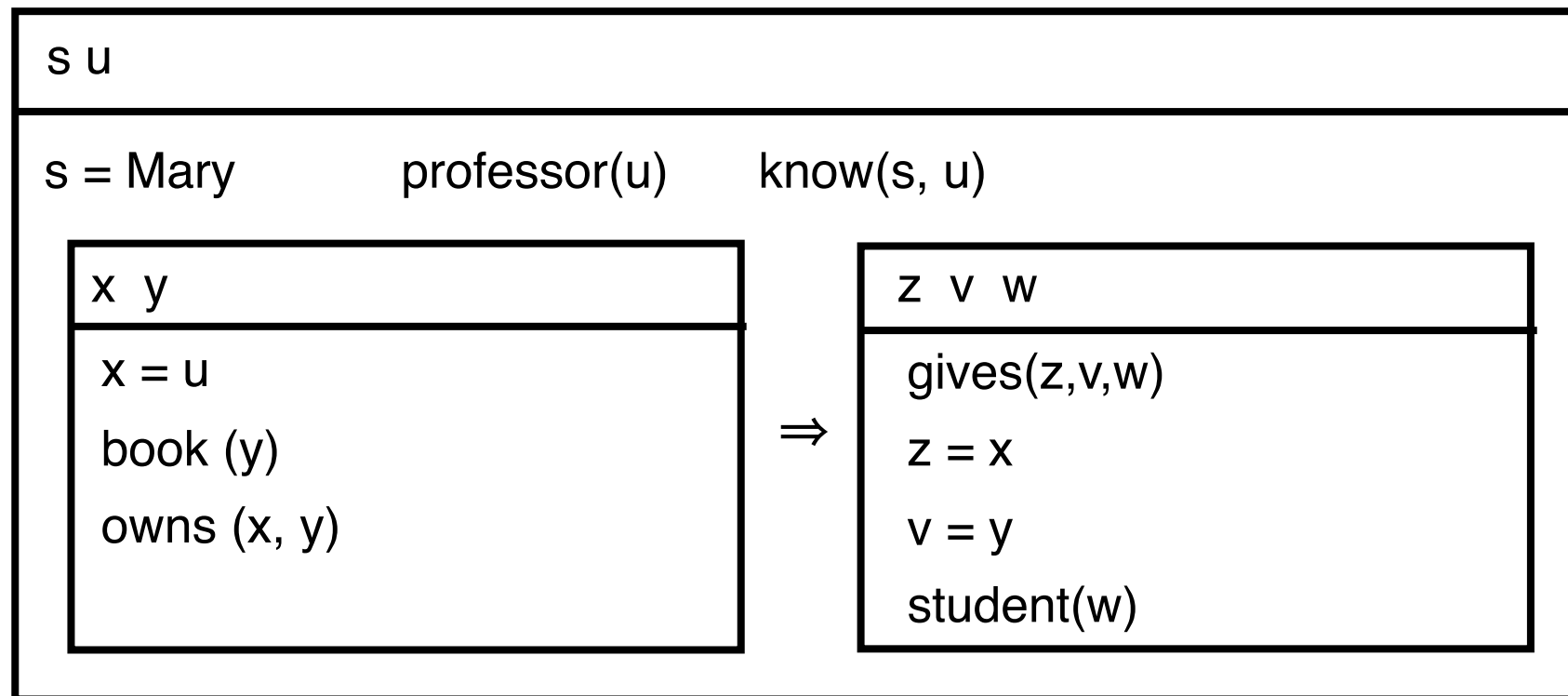
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An embedding  $\mathbf{f}$  of  $K$  in  $M$  **verifies  $K$  in  $M$**  ( $\mathbf{f} \models_M K$ ) iff  $\mathbf{f}$  verifies every condition  $a \in C_K$

- $\mathbf{f} \models_M R(x_1, \dots, x_n)$  iff  $\langle \mathbf{f}(x_1), \dots, \mathbf{f}(x_n) \rangle \in V_M(R)$
- $\mathbf{f} \models_M x = y$  iff  $\mathbf{f}(x) = \mathbf{f}(y)$
- $\mathbf{f} \models_M x = a$  iff  $\mathbf{f}(x) = V_M(a)$
- $\mathbf{f} \models_M \neg K_1$  iff there is no  $\mathbf{g} \supseteq_{U_{K_1}} \mathbf{f}$  such that  $\mathbf{g} \models_M K_1$
- $\mathbf{f} \models_M K_1 \Rightarrow K_2$  iff for all  $\mathbf{g} \supseteq_{U_{K_1}} \mathbf{f}$  such that  $\mathbf{g} \models_M K_1$   
there is a  $\mathbf{h} \supseteq_{U_{K_2}} \mathbf{g}$  such that  $\mathbf{h} \models_M K_2$
- $\mathbf{f} \models_M K_1 \vee K_2$  iff there is a  $\mathbf{g}_1 \supseteq_{U_{K_1}} \mathbf{f}$  such that  $\mathbf{g}_1 \models_M K_1$   
or there is a  $\mathbf{g}_2 \supseteq_{U_{K_2}} \mathbf{f}$  such that  $\mathbf{g}_2 \models_M K_2$

# Verifying embedding: example

*Mary knows a professor. If he owns a book, he gives it to a student.*



...is **true** in  $M = \langle U_M, V_M \rangle$  iff there is an  $\mathbf{f} :: U_D \rightarrow U_M$ , (with  $\{s, u\} \subseteq \text{Dom}(\mathbf{f})$ ) such that:  
 $\mathbf{f}(s) = V_M(\text{Mary})$  &  $\mathbf{f}(u) \in V_M(\text{prof})$  &  $\langle \mathbf{f}(s), \mathbf{f}(u) \rangle \in V_M(\text{know})$ ,  
and for all  $\mathbf{g} \supseteq_{\{x, y\}} \mathbf{f}$  s.t.  $\mathbf{g}(x) = \mathbf{g}(u)$  ( $=\mathbf{f}(u)$ ) &  $\mathbf{g}(y) \in V_M(\text{book})$  &  $\langle \mathbf{g}(x), \mathbf{g}(y) \rangle \in V_M(\text{own})$ ,  
there is a  $\mathbf{h} \supseteq_{\{z, v, w\}} \mathbf{g}$  s.t.  $\langle \mathbf{h}(z), \mathbf{h}(v), \mathbf{h}(w) \rangle \in V_M(\text{give})$  &  $\mathbf{h}(z) = \mathbf{h}(x)$  ( $=\mathbf{g}(x)$ ) & ... etc.



# Translation of DRSs to FOL

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Consider DRS  $K = \langle \{x_1, \dots, x_n\}, \{c_1, \dots, c_k\} \rangle$

|                            |
|----------------------------|
| $x_1 \dots x_n$            |
| $c_1$<br>$\vdots$<br>$c_n$ |

$K$  is truth-conditionally equivalent to the following FOL formula:

$$\exists x_1 \dots \exists x_n [c_1 \wedge \dots \wedge c_k]$$

# DRT and compositionality

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- DRT is non-compositional on truth conditions: The difference in discourse-semantic status of the text pairs is not predictable through the (identical) truth conditions of its component sentences.
- Since structural information which cannot be reduced to truth conditions is required to compute the semantic value of texts, DRT is called a *representational* theory of meaning.

However...

# Wait a minute ...

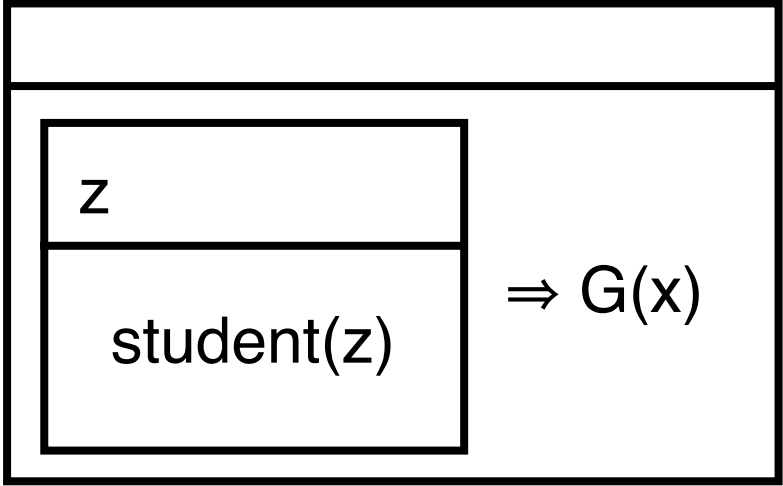
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- Why can't we just combine type theoretic semantics and DRT?
- Use  $\lambda$ -abstraction and reduction as we did before, but:
- Assume that the target representations which we want to arrive at are not First-Order Logic formulas, but DRSs.
- The result is called  $\lambda$ -DRT.

# $\lambda$ -DRSs

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An expression in  $\lambda$ -DRT consists of a lambda prefix and a partially instantiated DRS.

- $every\ student :: \langle \langle e, t \rangle, t \rangle \mapsto \lambda G.$  

Alternative notation:  $\lambda G [ \emptyset \mid [ z \mid student(z) ] \Rightarrow G(z) ]$

- $works :: \langle e, t \rangle \mapsto \lambda x [ \emptyset \mid work(x) ]$

# $\lambda$ -DRT: $\beta$ -reduction

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*Every student works*

$$\mapsto \lambda G [ \emptyset \mid [ z \mid \text{student}(z) ] \Rightarrow G(z) ] (\lambda x [ \emptyset \mid \text{work}(x) ])$$

$$\Rightarrow^\beta [ \emptyset \mid [ z \mid \text{student}(z) ] \Rightarrow (\lambda x [ \emptyset \mid \text{work}(x) ])(z) ]$$

$$\Rightarrow^\beta [ \emptyset \mid [ z \mid \text{student}(z) ] \Rightarrow [ \emptyset \mid \text{work}(z) ] ]$$

How do we define conjunction on DRSs?

# (Naïve) Merge

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The “merge” operation on DRSs combines two DRSs (conditions and universes).

- Let  $K_1 = [ U_1 \mid C_1 ]$  and  $K_2 = [ U_2 \mid C_2 ]$ .

**Merge:**  $K_1 + K_2 = [ U_1 \cup U_2 \mid C_1 \cup C_2 ]$

# Merge: An example

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- *a student*  $\mapsto \lambda G ([z \mid \text{student}(z)] + G(z))$
- *works*  $\mapsto \lambda x [\emptyset \mid \text{work}(x)]$

*A student works*  $\mapsto \lambda G ([z \mid \text{student}(z)] + G(z)) (\lambda x [\emptyset \mid \text{work}(x)])$

$\Rightarrow^\beta [z \mid \text{student}(z)] + \lambda x [\emptyset \mid \text{work}(x)](z)$

$\Rightarrow^\beta [z \mid \text{student}(z)] + [\emptyset \mid \text{work}(z)]$

$\Rightarrow^\beta [z \mid \text{student}(z), \text{work}(z)]$

# Compositional analysis

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- *Mary*  $\mapsto \lambda G ([z \mid z = \text{Mary}] + G(z))$
- *she*  $\mapsto \lambda G. G(z)$

*Mary works. She is successful.*

$$\mapsto \lambda K \lambda K' (K + K') ([z \mid z = \text{Mary}, \text{work}(z)]) ([\mid \text{successful}(z)])$$

$$\Rightarrow^\beta \lambda K' ([z \mid z = \text{Mary}, \text{work}(z)] + K') ([\mid \text{successful}(z)])$$

$$\Rightarrow^\beta [z \mid z = \text{Mary}, \text{work}(z)] + ([\mid \text{successful}(z)])$$

$$\Rightarrow^\beta [z \mid z = \text{Mary}, \text{work}(z), \text{successful}(z)]$$



# Merge again

---

The “merge” operation on DRSs combines two DRSs (conditions and universes).

- Let  $K_1 = [ U_1 \mid C_1 ]$  and  $K_2 = [ U_2 \mid C_2 ]$ .

**Merge:**  $K_1 + K_2 \Rightarrow [ U_1 \cup U_2 \mid C_1 \cup C_2 ]$

*under the assumption that no discourse referent  $u \in U_2$  occurs free in a condition  $\gamma \in C_1$ .*

# Variable capturing

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In  $\lambda$ -DRT, discourse referents are captured via the interaction of  $\beta$ -reduction and DRS-binding:

- $\lambda K'([z \mid \text{student}(z), \text{work}(z)] + K')([ \mid \text{successful}(z)])$

$$\Rightarrow^\beta [z \mid \text{student}(z), \text{work}(z)] + [ \mid \text{successful}(z)]$$

$$\Rightarrow^\beta [z \mid \text{student}(z), \text{work}(z), \text{successful}(z)]$$

But the  $\beta$ -reduced DRS must still be *equivalent* to the original DRS!

So, the potential for capturing discourse referents must be captured into the interpretation of a  $\lambda$ -DRS. Possible, but tricky.

# Literature

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## Reading:

- Hans Kamp and Uwe Reyle: From Discourse to Logic, Kluwer: Dordrecht 1993.

## Link:

- <https://plato.stanford.edu/entries/discourse-representation-theory/>