Semantic Theory 2019
Supplementary materials: Practice exam

1 Type Theory: Semantics

Let U is a non-empty set of entities. For every type τ, the domain of possible denotations D_τ is given by:

- $D_e = U$
- $D_t = \{0, 1\}$
- $D_{\langle\sigma,\tau\rangle}$ is the set of functions from D_σ to D_τ.

A model structure is a pair $M = \langle U_M, V_M \rangle$ such that U_M is a non-empty set of individuals, and V_M is a function assigning every non-logical constant of type τ a member of D_τ.

Interpretation:

- $[\![\alpha]\!]^{M,g} = V_M(\alpha)$ if α is a constant
- $[\![\alpha]\!]^{M,g} = g(\alpha)$ if α is a variable
- $[\![\alpha(\beta)]\!]^{M,g} = [\![\alpha]\!]^{M,g}([\![\beta]\!]^{M,g})$
- $[\![\lambda v\alpha]\!]^{M,g}$ = that function $f : D_\sigma \to D_\tau$ such that for all $a \in D_\sigma$, $f(a) = [\![\alpha]\!]^{M,g[v/a]}$ (for v a variable of type σ)
- $[\![\alpha = \beta]\!]^{M,g} = 1$ iff $[\![\alpha]\!]^{M,g} = [\![\beta]\!]^{M,g}$
- $[\![\neg\varphi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 0$
- $[\![\varphi \wedge \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 1$ and $[\![\psi]\!]^{M,g} = 1$
- $[\![\varphi \vee \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 1$ or $[\![\psi]\!]^{M,g} = 1$
- $[\![\varphi \to \psi]\!]^{M,g} = 1$ iff $[\![\varphi]\!]^{M,g} = 0$ or $[\![\psi]\!]^{M,g} = 1$

- $[\![\exists v\varphi]\!]^{M,g} = 1$ iff there is an $a \in D_\tau$ such that $[\![\varphi]\!]^{M,g[v/a]} = 1$ (for v a variable of type τ)
- $[\![\forall v\varphi]\!]^{M,g} = 1$ iff for all $a \in D_\tau$, $[\![\varphi]\!]^{M,g[v/a]} = 1$ (for v a variable of type τ)

2 Generalized Quantifiers

Definition 1 (Persistence: $\uparrow$ *mon*)**.** A determiner D is persistent in M iff: for all X, Y, Z: if $D(X, Z)$ and $X \subseteq_M Y$, then $D(Y, Z)$

Definition 2 (Antipersistence: $\downarrow$ *mon*)**.** A determiner D is antipersistent in M iff: for all X, Y, Z: if $D(X, Z)$ and $Y \subseteq_M X$, then $D(Y, Z)$

Definition 3 (Upward Monotinicity: *mon* $\uparrow$)**.** A quantifier Q is upward monotonic (or: monotone increasing) in $M = \langle U, V\rangle$ iff Q is "closed under supersets", i.e.: for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$

Definition 4 (Downward Monotinicity: *mon* $\downarrow$)**.** A quantifier Q is downward monotonic (or: monotone decreasing) in $M = \langle U, V\rangle$ iff Q is closed under inclusion: for all $X, Y \subseteq U$: if $X \in Q$ and $Y \subseteq X$, then $Y \in Q$

Definition 5 (Conservativity)**.** A quantifier Q is conservative in $M = \langle U, V\rangle$ iff for every $A, B \subseteq U$: $D(A, B) \Leftrightarrow D(A, A \cap B)$

3 DRT: Syntax

A discourse representation structure (DRS) K is a pair $\langle U_K, C_K\rangle$ where U_K is a set of discourse referents, and C_K is a set of conditions.
The set of well-formed conditions is defined as follows:

- $R(u_1, ..., u_n)$, where R is an n-place relation and $u_i \in U_K$
- $u = v$, with $u, v \in U_K$
- $u = a$, with $u \in U_K$ and a is a proper name
- $K_1 \Rightarrow K_2$, where K_1 and K_2 are DRSs
- $K_1 \vee K_2$, where K_1 and K_2 are DRSs
- $\neg K_1$, where K_1 is a DRS

4 DRT: Embedding, verifying embedding

Let U_D be a set of discourse referents, $K = \langle U_K, C_K \rangle$ a DRS with $U_K \subseteq U_D$, and $M = \langle U_M, V_M \rangle$ a model structure of first-order predicate logic that is suitable for K. An embedding of U_D into M is a (partial) function from U_D to U_M that assigns individuals from U_M to discourse referents.
An embedding f verifies the DRS K in M ($f \models_M K$) iff

1. $U_K \subseteq Dom(f)$, and
2. f verifies each condition $\alpha \in C_K$.

f verifies a condition α in M ($f \models_M \alpha$) in the following cases:

- $f \models_M R(u_1, ..., u_n)$ iff $\langle f(u_1), ..., f(u_n) \rangle \in V_M(R)$
- $f \models_M u = v$ iff $f(u) = f(v)$
- $f \models_M u = a$ iff $f(u) = V_M(a)$
- $f \models_M K_1 \Rightarrow K_2$ iff for all $g \supseteq_{U_{K_1}} f$ such that $g \models_M K_1$, there is a $h \supseteq_{U_{K_2}} g$ such that $h \models_M K_2$
- $f \models_M \neg K_1$ iff there is no $g \supseteq_{U_{K_1}} f$ such that $g \models_M K_1$
- $f \models_M K_1 \vee K_2$ iff there is a $g_1 \supseteq_{U_{K_1}} f$ such that $g_1 \models_M K_1$, or there is a $g_2 \supseteq_{U_{K_2}} f$ such that $g_2 \models_M K_2$.