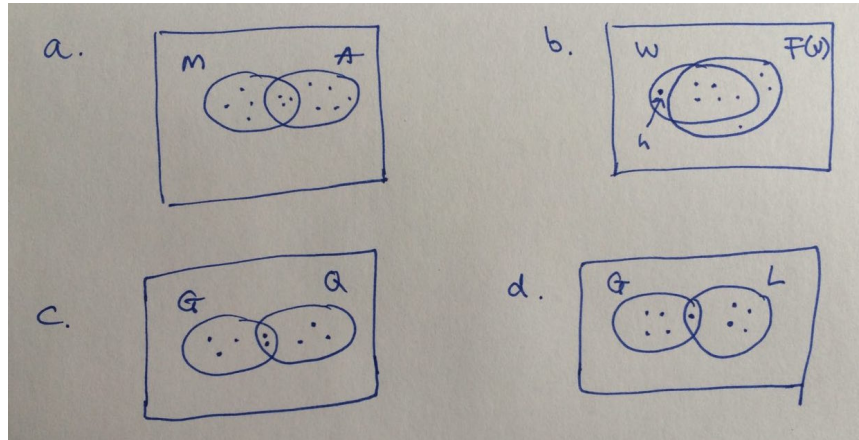


Semantic Theory 2017: Solutions exercise sheet 4

Exercise 1

Give the truth conditions for the following sentences, interpreting the (complex) determiner as a relation between sets. You can interpret each VP as a property (i.e., a set of entities). Illustrate your answer with a graphical representation of a model in which the sentence is true.

- a. $\llbracket \text{Some but not all Muggles are afraid of magic} \rrbracket = 1$ iff
 $\llbracket A \rrbracket \in \{P \subseteq U_M \mid \llbracket M \rrbracket \cap P \neq \llbracket M \rrbracket \text{ and } \llbracket M \rrbracket \cap P \neq \emptyset\}$ iff
 $\llbracket A \rrbracket \in \{P \subseteq U_M \mid \llbracket M \rrbracket \cap P \subset \llbracket M \rrbracket\}$ iff
 $\llbracket M \rrbracket \cap \llbracket A \rrbracket \subset \llbracket M \rrbracket$
- b. $\llbracket \text{Every wizard but Harry fears Voldemort} \rrbracket = 1$ iff
 $\llbracket F(v) \rrbracket \in \{P \subseteq U_M \mid \llbracket W \rrbracket \cap P = \llbracket W \rrbracket \setminus \llbracket h \rrbracket\}$ iff
 $\llbracket W \rrbracket \cap \llbracket F(v) \rrbracket = \llbracket W \rrbracket \setminus \llbracket h \rrbracket$
- c. $\llbracket \text{At most five girls play Quidditch} \rrbracket = 1$ iff
 $\llbracket Q \rrbracket \in \{P \subseteq U_M \mid \text{card}(\llbracket G \rrbracket \cap P) \leq 5\}$ iff
 $\text{card}(\llbracket G \rrbracket \cap \llbracket Q \rrbracket) \leq 5$
- d. $\llbracket \text{Few Gryffindors are lazy} \rrbracket = 1$ iff
 $\llbracket L \rrbracket \in \{P \subseteq U_M \mid \text{card}(\llbracket Griff \rrbracket \cap P) < \text{card}(\llbracket Griff \rrbracket \setminus P)\}$
 (“there are less lazy than non-lazy Gryffindors”) iff
 $\text{card}(\llbracket Griff \rrbracket \cap \llbracket L \rrbracket) < \text{card}(\llbracket Griff \rrbracket \setminus \llbracket L \rrbracket)$



Exercise 2

Determine the monotonicity properties (left and right) of the following determiners. Show how you derived these monotonicity properties.

- a. at least five: $\uparrow mon \uparrow$
- b. at most five: $\downarrow mon \downarrow$
- c. exactly five: $-mon-$
- d. some but not all: $\uparrow mon-$

Derived using the following tests:

$\downarrow mon$		If --- animals walked, then --- dogs walked.
$\uparrow mon$		If --- dogs walked, then --- animals walked.
$mon \downarrow$		If --- dogs walked, then --- dogs walked rapidly.
$mon \uparrow$		If --- dogs walked rapidly, then --- dogs walked.

Exercise 3

Prove that the following statement holds:

- The external negation of an upward monotonic quantifier is a downward monotonic quantifier.

Premise 1: The external negation of Q : $\neg Q = \{P \subseteq U_M \mid P \notin Q\}$

Premise 2: Q is upward monotonic iff for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$.

Premise 3: Q' is downward monotonic iff for all $X, Y \subseteq U$: if $X \in Q'$ and $Y \subseteq X$, then $Y \in Q'$.

Take an arbitrary $A, B \subseteq U_M$ and assume that $A \subseteq B$ and $B \in \neg Q$.

To prove: $A \in \neg Q$.

Proof by Contradiction: If $A \notin \neg Q$, then $A \in Q$ (by Premise 1). Then it follows that $B \in Q$ (by Premise 2). This is in conflict with the assumption that $B \in \neg Q$, which means that $B \notin Q$ (by Premise 1). Therefore, it must hold that $A \in \neg Q$. Given Premise 3, this means that $\neg Q$ is downward monotonic.