Semantic Theory 2019: Solutions exercise sheet 4

Exercise 1

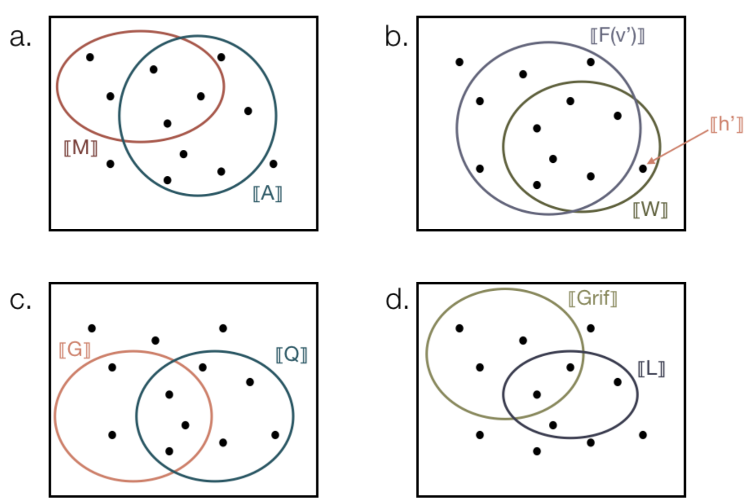
Give the truth conditions for the following sentences, interpreting the (complex) determiner as a relation between sets. You can interpret each VP as a property (i.e., a set of entities). Illustrate your answer with a graphical representation of a model in which the sentence is true.

a. $\llbracket$ Some but not all Muggles are afraid of magic $\rrbracket = 1$ iff
$\llbracket A \rrbracket \in \{P \subseteq U_M \mid \llbracket M \rrbracket \cap P \neq \llbracket M \rrbracket \textit{ and } \llbracket M \rrbracket \cap P \neq \emptyset\}$ iff
$\llbracket A \rrbracket \in \{P \subseteq U_M \mid \llbracket M \rrbracket \cap P \subset \llbracket M \rrbracket\}$ iff
$\llbracket M \rrbracket \cap \llbracket A \rrbracket \subset \llbracket M \rrbracket$

b. $\llbracket$ Every wizard but Harry fears Voldemort $\rrbracket = 1$ iff
$\llbracket F(v) \rrbracket \in \{P \subseteq U_M \mid \llbracket W \rrbracket \cap P = \llbracket W \rrbracket \backslash \llbracket h \rrbracket\}$ iff
$\llbracket W \rrbracket \cap \llbracket F(v) \rrbracket = \llbracket W \rrbracket \backslash \llbracket h \rrbracket$

c. $\llbracket$ At most five girls play Quidditch $\rrbracket = 1$ iff
$\llbracket Q \rrbracket \in \{P \subseteq U_M \mid card(\llbracket G \rrbracket \cap P) \leq 5\}$ iff
$card(\llbracket G \rrbracket \cap \llbracket Q \rrbracket) \leq 5$

d. $\llbracket$ Few Gryffindors are lazy $\rrbracket = 1$ iff
$\llbracket L \rrbracket \in \{P \subseteq U_M \mid card(\llbracket Grif \rrbracket \cap P) < card(\llbracket Grif \rrbracket \backslash P)\}$
("there are less lazy than non-lazy Gryffindors") iff
$card(\llbracket Grif \rrbracket \cap \llbracket L \rrbracket) < card(\llbracket Grif \rrbracket \backslash \llbracket L \rrbracket)$



Exercise 2

Determine the monotonicity properties (left and right) of the following determiners. Show how you derived these monotonicity properties.

a. at least five: $\uparrow mon \uparrow$

b. at most five: $\downarrow mon \downarrow$

c. exactly five: $-mon-$

d. some but not all: $\uparrow mon-$

Derived using the following tests:

$\downarrow mon$	If ___ animals walked, then ___ dogs walked.
$\uparrow mon$	If ___ dogs walked, then ___ animals walked.
$mon \downarrow$	If ___ dogs walked, then ___ dogs walked rapidly.
$mon \uparrow$	If ___ dogs walked rapidly, then ___ dogs walked.

Exercise 3

Prove that the following statement holds:

- The external negation of an upward monotonic quantifier is a downward monotonic quantifier.

Premise 1: The external negation of Q: $\neg Q = \{P \subseteq U_M \mid P \notin Q\}$
Premise 2: Q is upward monotonic iff for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$.
Premise 3: Q' is downward monotonic iff for all $X, Y \subseteq U$: if $X \in Q'$ and $Y \subseteq X$, then $Y \in Q'$.

Take an arbitrary $A, B \subseteq U_M$ and assume that $A \subseteq B$ and $B \in \neg Q$.
To prove: $A \in \neg Q$.
Proof by Contradiction: If $A \notin \neg Q$, then $A \in Q$ (by Premise 1). Then it follows that $B \in Q$ (by Premise 2). This is in conflict with the assumption that $B \in \neg Q$, which means that $B \notin Q$ (by Premise 1). Therefore, it must hold that $A \in \neg Q$. Given Premise 3, this means that $\neg Q$ is downward monotonic.